Entropy generation due to laminar forced convection in the entrance region of a concentric annulus

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Abstract

This study is focused on the entropy production due to laminar forced convection in the entrance region of a concentric cylindrical annulus. The present hydrodynamic and temperature fields are obtained numerically. Local entropy generation distributions are obtained based on the resulting velocity and temperature fields by solving the entropy generation equation. The effect of different flow parameters on thermal, viscous, and total entropy generation is studied for different thermal boundary conditions. Moreover, the effect of radius ratio on the entropy generation is investigated. Entropy generation was found to be inversely proportional to both Reynolds number and the dimensionless entrance temperature. The results also show that increasing Eckert number and/or the radius ratio will increase the entropy generation. Finally, it is found that thermal entropy generation is relatively dominant over viscous entropy generation.

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1. Introduction

Heat transfer processes and devices are inherently irreversible. In other words, heat transfer phenomena affect the one-way destruction of useful energy (available work, or exergy) [1]. Therefore, conserving useful energy rests heavily on the ability to produce thermodynamically...
# Nomenclature

- \( C_p \): specific heat at constant pressure
- \( Ec \): Eckert number
  - \[ Ec = \frac{u_0^2}{C_P(T_w - T_o)} \] for cases (I) and (O)
  - \[ Ec = \frac{u_0^2}{C_P(T_{wo} - T_{wi})} \] for cases (IO) and (IOE)
- \( Ec_{m} \): modified Eckert number
- \( k \): thermal conductivity
- \( m \): number of axial increments in the numerical mesh network
- \( n \): number of radial increments in the numerical mesh network
- \( N \): annulus radius ratio, \( \frac{r_2}{r_1} \)
- \( p \): local pressure
- \( P \): dimensionless pressure, \( \frac{P}{P_o} \)
- \( P_o \): pressure at annulus entrance
- \( Pr \): Prandtl number, \( \frac{v}{\nu} \)
- \( r \): radial coordinate
- \( r_1 \): inner radius of the annulus
- \( r_2 \): outer radius of the annulus
- \( R \): dimensionless radial coordinate, \( \frac{r}{r_2} \)
- \( Re \): Reynolds number, \( \frac{2\rho u_0 (1-N)}{\mu} \)
- \( s_{gen} \): rate of entropy generation per unit volume
- \( S_{gen} \): dimensionless rate of entropy generation per unit volume
- \( s_{tot} \): total rate of entropy generation
- \( S_{tot} \): dimensionless total entropy generation (viscous and thermal), also called entropy generation number
- \( T \): temperature
- \( T_o \): ambient temperature
- \( T_w \): heated wall temperature
- \( T_{wi} \): inner wall temperature
- \( T_{wo} \): outer wall temperature
- \( u \): axial velocity
- \( u_o \): axial velocity at annulus entrance
- \( U \): dimensionless axial velocity, \( \frac{u}{u_o} \)
- \( v \): radial velocity
- \( V \): dimensionless radial velocity, \( \frac{r v}{u_o} \)
- \( z \): axial coordinate
- \( Z \): dimensionless axial coordinate, \( \frac{2(1-N)z}{r^2 Re} \)
- \( \theta \): dimensionless temperature
  - \[ \theta = \frac{T - T_o}{T_w - T_o} \] for cases (I, O, IE, and OE)
  - \[ \theta = \frac{T - T_{wi}}{T_{wo} - T_{wi}} \] for cases (IO and IOE)
efficient heat transfer processes. This can be done by proper identification of the factors that contribute in entropy generation.

Bejan [2] and San et al. [3] have proposed different analytical solutions for the entropy generation equation in several simple flow situations. Bejan presented an analytical solution to the partial differential equation of the entropy generation encountered in convective heat transfer for different configurations. On the other hand, San et al. investigated the irreversible entropy generation for combined heat and mass transfer in two-dimensional channel where the heat flux is assumed to be constant on both channel walls. Due to the extreme importance of entropy generation in the design of heat exchangers, many studies were carried out in this field, e.g. Liang and Kuhn [4], El-Sayed [5], and Bejan [6,7].

The complexity of the partial differential entropy equation when applied to a convecting fluid in more complex flow fields has been the work of other researchers. Drost and White [8] developed a numerical solution procedure for predicting local entropy generation rates, and they applied that procedure to convective heat transfer associated with a fluid impinging on a heated wall. Baljai et al. [9], Abu-Hijleh [10], and Cheng and Hung [11] also presented numerical solutions for the entropy generation in different configurations.

Heat transfer in an annulus is encountered in many engineering applications, especially in heat exchangers, in some types of nuclear reactors and in water-cooled combustors. In particular, the flow in compact heat exchangers is dominated by developing flow caused by exchanger compactness. In this case, the flow is modeled as steady developing flow since exchangers run for long periods of time. Despite the engineering importance of this flow problem, many researchers have investigated the hydrodynamics and heat transfer of the flow in a concentric annulus, while none have addressed the irreversibility or entropy generation incorporated with the developing flow.

The main objective of the present work is to calculate the entropy production due to laminar forced convection in the entrance region of a concentric annulus. As long as the velocity and temperature fields are known, the entropy generation within the flow domain may be calculated. In a previous work [12], the transient boundary layer equations were solved using finite difference technique to obtain the temperature field in the entrance region of a concentric annulus. In this study, the steady boundary layer equations are solved numerically using a finite difference technique to obtain the velocity and temperature fields in the entrance region. Then, the results

| \( \tau \) | dimensionless entrance temperature | = \( \frac{T_0}{T_w-T_i} \) for cases (I and O) |
| \( \tau \) | | = \( \frac{T_{in}}{T_{wi}-T_{wi}} \) for case (IO) |
| \( \tau \) | | = \( \frac{T_{xo}}{T_{wo}-T_{wi}} \) for case (IOE) |

\( \alpha \) thermal diffusivity, \( \frac{k}{\rho C_p} \)
\( \Delta \) difference
\( \mu \) dynamic viscosity
\( \nu \) kinematic viscosity
\( \rho \) density
\( \Phi \) viscous dissipation
are used to determine the local and total entropy generation in the entrance region subject to different thermal boundary conditions. A parametric study was carried out to see how the controlling parameters of the problem contribute to total entropy being generated.

2. Analysis

Fig. 1a shows a schematic diagram of the flow problem under consideration. The steady state boundary layer equations for laminar forced convection flow in the entrance region of a concentric annulus can be written in the following form (El-Shaarawi and Alkam [12]).

Continuity equation:

$$\frac{\partial u}{\partial z} + \frac{1}{r} \frac{\partial (rv)}{\partial r} = 0$$  (1)

Axial Momentum equation:

$$- \frac{u}{r} \frac{\partial u}{\partial r} = -\frac{1}{\rho} \frac{dp}{dz} + v \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \right]$$  (2)

Fig. 1. (a) Schematic diagram of the annulus. (b) Finite difference network in $R-Z$ plane.
Integral form of continuity equation:
\[ \int_{r_1}^{r_2} 2\pi r u dr = \pi (r_2^2 - r_1^2) u_o \] (3)

Energy equation:
\[ u \frac{\partial T}{\partial z} + v \frac{\partial T}{\partial r} = a \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right] \] (4)

where Eq. (3), the integral form of the continuity, is added to Eqs. (1) and (2) in order to obtain the solution of the hydrodynamic field as suggested by Bodoia and Osterle [14]. Moreover, Eq. (4) is added to obtain the solution of the thermal part of the problem. The effect of viscous dissipation on temperature profiles can be neglected specially when Eckert number is small, however, this effect should not be neglected in the entropy production equation in order to study the relative contribution of both viscous and thermal entropy production to total entropy production [1].

In this study, a fluid with constant physical properties enters the annulus passage with uniform velocity \( u_o \). The problem is investigated under several thermal boundary conditions, as follows.

2.1. Case (I)

The inner wall is kept isothermal at \( T_w \), while the outer wall is kept adiabatic. This corresponds to the following boundary and inlet conditions:

At \( z = 0 \) and \( r_1 < r < r_2 \): \( u = u_o, \; v = 0, \) and \( T = T_o \)
For \( z > 0 \) and \( r = r_1 \): \( u = 0, \; v = 0, \) and \( T = T_w \)
For \( z > 0 \) and \( r = r_2 \): \( u = 0, \; v = 0, \) and \( \frac{\partial T}{\partial r} = 0. \)

2.2. Case (1E)

This is a special case of the previous case (I) in which all conditions remain the same except for the thermal boundary condition at the entrance where \( T_0 \) is changed to \( T_w \). That is:

At \( z = 0 \) and \( r_1 < r < r_2 \): \( u = u_o, \; v = 0, \) and \( T = T_w. \)

2.3. Case (O)

The outer wall is kept isothermal at \( T_w \), while the inner wall is kept adiabatic. This corresponds to the following boundary and inlet conditions:

At \( z = 0 \) and \( r_1 < r < r_2 \): \( u = u_o, \; v = 0, \) and \( T = T_o \)
For \( z > 0 \) and \( r = r_1 \): \( u = 0, \; v = 0, \) and \( \frac{\partial T}{\partial r} = 0. \)
For $z > 0$ and $r = r_2$: $u = 0$, $v = 0$, and $T = T_w$.

2.4. Case (OE)

This is a special case of the previous case (O) in which all conditions remain the same, except for the thermal boundary condition at the entrance where $T_0$ is changed to $T_w$. That is:

At $z = 0$ and $r_1 < r < r_2$: $u = u_o$, $v = 0$, and $T = T_w$.

2.5. Case (IO)

Both inner and outer walls are kept isothermal at $(T_{wi})$ and $(T_{wo})$, respectively, while the fluid enters the annulus at a temperature equivalent to that of the inner wall. This corresponds to the following boundary and inlet conditions:

At $z = 0$ and $r_1 < r < r_2$: $u = u_o$, $v = 0$, and $T = T_{wi}$
For $z > 0$ and $r = r_1$: $u = 0$, $v = 0$, and $T = T_{wi}$
For $z > 0$ and $r = r_2$: $u = 0$, $v = 0$, and $T = T_{wo}$.

2.6. Case (IOE)

Both inner and outer walls are kept isothermal at $(T_{wi})$ and $(T_{wo})$ respectively, while the fluid enters the annulus at a temperature equivalent to that of the outer wall. This corresponds to the following boundary and inlet conditions:

At $z = 0$ and $r_1 < r < r_2$: $u = u_o$, $v = 0$, and $T = T_{wo}$
For $z > 0$ and $r = r_1$: $u = 0$, $v = 0$, and $T = T_{wi}$
For $z > 0$ and $r = r_2$: $u = 0$, $v = 0$, and $T = T_{wo}$.

The local entropy generation can be calculated using the entropy production equation [1]:

$$s''_{gen}(r, z) = \frac{k}{T^2} (\nabla T)^2 + \frac{\mu}{T} \Phi$$

$$= \text{Thermal component} + \text{viscous component}$$

$$= \frac{k}{T^2} \left[ \left( \frac{\partial T}{\partial r} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right] + \frac{\mu}{T} \left\{ 2 \left[ \left( \frac{\partial v}{\partial r} \right)^2 + \frac{v^2}{r^2} + \left( \frac{\partial u}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} \right)^2 \right\}$$

(5)

One important feature of Eq. (5) is that the contributions of different sources of entropy generation are clearly shown. The first bracketed term on the right-hand side of Eq. (5) is the entropy generation due to heat transfer across a finite temperature difference, whereas the second bracketed term is the local entropy generation due to viscous effect.
After solving for the velocity and temperature distribution, Eq. (5) can be evaluated to yield the local entropy generation. Total entropy generation can then be determined by integrating this equation over the region of interest. In our case, the total rate of entropy generation can be expressed in a dimensional form as follows:

$$s_{tot} = 2\pi \int_{r_1}^{r_2} r s_{gen}^m(r, z) \, dr \, dz$$  \hspace{1cm} (6)

Introducing the following non-dimensional groups:

$$U = \frac{u}{u_0}, \quad V = \frac{\rho u_0}{\mu} r^2, \quad Z = \frac{2(1 - N)z}{r_2 Re}, \quad R = \frac{r}{r_2}, \quad P = \frac{P}{\rho u_0^2}, \quad Re = \frac{2(1 - N)}{v} r_2 u_0, \quad N = \frac{r_1}{r_2}$$

$$S_{gen}^m = \frac{s_{gen}^m r_2}{k}, \quad S_{tot} = \frac{(1 - N)s_{tot}}{\pi r_2 k Re}$$

for cases (I, O, IE, and OE):

$$\theta = \frac{T - T_0}{T_w - T_0}, \quad \tau = \frac{T_0}{T_w - T_0}, \quad EcPr = \frac{\mu u_0^2}{k(T_w - T_0)}$$

for cases (IO and IOE):

$$\theta = \frac{T - T_{wi}}{T_{wo} - T_{wi}}, \quad \tau = \frac{T_{wi}}{T_{wo} - T_{wi}} \text{ for case (IO) or } \tau = \frac{T_{wo}}{T_{wo} - T_{wi}} \text{ for case (IOE)},$$

$$EcPr = \frac{\mu u_0^2}{k(T_{wo} - T_{wi})}$$

The above definitions of the non-dimensional variables lead to the following non-dimensional boundary layer equations and boundary conditions in all cases:

Continuity equation:

$$\frac{\partial U}{\partial Z} + \frac{1}{R} \frac{\partial (RV)}{\partial R} = 0$$  \hspace{1cm} (7)

Axial momentum equation:

$$U \frac{\partial U}{\partial Z} + V \frac{\partial U}{\partial R} = -\frac{dP}{dZ} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right)$$  \hspace{1cm} (8)

Integral form of continuity equation:

$$\int_{N}^{1} RU dR = \frac{(1 - N^2)}{2}$$  \hspace{1cm} (9)
Energy equation:

\[
U \frac{\partial \theta}{\partial Z} + V \frac{\partial \theta}{\partial R} = \frac{1}{Pr} \left[ \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \theta}{\partial R} \right) \right]
\]  

(10)

Case (I):

At \( Z = 0 \) and \( N < R < 1 \): \( U = 1, V = 0, \) and \( \theta = 0 \)
For \( Z > 0 \) and \( R = N \): \( U = 0, V = 0, \) and \( \theta = 1 \)
For \( Z > 0 \) and \( R = 1 \): \( U = 0, V = 0, \) and \( \frac{\partial \theta}{\partial R} = 0 \)

Case (IE):

At \( Z = 0 \) and \( N < R < 1 \): \( U = 1, V = 0, \) and \( \theta = 1 \)
For \( Z > 0 \) and \( R = N \): \( U = 0, V = 0, \) and \( \theta = 1 \)
For \( Z > 0 \) and \( R = 1 \): \( U = 0, V = 0, \) and \( \frac{\partial \theta}{\partial R} = 0 \)

Case (O):

At \( Z = 0 \) and \( N < R < 1 \): \( U = 1, V = 0, \) and \( \theta = 0 \)
For \( Z > 0 \) and \( R = N \): \( U = 0, V = 0, \) and \( \frac{\partial \theta}{\partial R} = 0 \)
For \( Z > 0 \) and \( R = 1 \): \( U = 0, V = 0, \) and \( \theta = 1 \)

Case (OE):

At \( Z = 0 \) and \( N < R < 1 \): \( U = 1, V = 0, \) and \( \theta = 1 \)
For \( Z > 0 \) and \( R = N \): \( U = 0, V = 0, \) and \( \frac{\partial \theta}{\partial R} = 0 \)
For \( Z > 0 \) and \( R = 1 \): \( U = 0, V = 0, \) and \( \theta = 1 \)

Case (IO):

At \( Z = 0 \) and \( N < R < 1 \): \( U = 1, V = 0, \) and \( \theta = 0 \)
For \( Z > 0 \) and \( R = N \): \( U = 0, V = 0, \) and \( \theta = 0 \)
For \( Z > 0 \) and \( R = 1 \): \( U = 0, V = 0, \) and \( \theta = 1 \)

Case (IOE):

At \( Z = 0 \) and \( N < R < 1 \): \( U = 1, V = 0, \) and \( \theta = 1 \)
For \( Z > 0 \) and \( R = N \): \( U = 0, V = 0, \) and \( \theta = 0 \)
For \( Z > 0 \) and \( R = 1 \): \( U = 0, V = 0, \) and \( \theta = 1 \)
The non-dimensional entropy equation can then be written as follows:

\[
S''_{gen}(R, Z) = \frac{1}{(\theta + \tau)^2} \left[ \left( \frac{\partial \theta}{\partial R} \right)^2 + \frac{4(1 - N)^2}{Re^2} \left( \frac{\partial \theta}{\partial Z} \right)^2 \right] + \frac{EcPr}{Re^2} \frac{1}{\theta + \tau} \left\{ 8(1 - N)^2 \left[ \left( \frac{\partial V}{\partial R} \right)^2 + \frac{V^2}{R^2} \right] + 8 \left( \frac{\partial U}{\partial Z} \right)^2 + \left[ Re \frac{\partial U}{\partial R} + \frac{4(1 - N)^2}{Re} \frac{\partial V}{\partial Z} \right]^2 \right\} \tag{11}
\]

and the total rate of entropy generation can be expressed in non-dimensional form as follows:

\[
S_{tot} = \int_{0}^{Z} \int_{0.5}^{1} R S''_{gen}(R, Z) \, dR \, dZ \tag{12}
\]

It is important to notice that for cases (IE) and (OE), the entropy generation is due to the viscous effect only, since the local temperature in these cases is constant everywhere and is equivalent to unity. These cases have no significance in real heat exchangers applications, however, they have been studied here to cover all possible combinations of isothermal and adiabatic boundary conditions of the problem at hand. On the other hand, the dimensionless entrance temperature \((\tau)\) cannot be defined for cases (IE, OE), since the difference between the entrance and the heated wall temperature is zero. Therefore, for these case, the entropy equation can be rewritten in terms of the viscous effect only, and the factor that is multiplied by the second bracketed term in Eq. (11) is modified. Thus, the final form of the entropy equation for cases (IE) and (OE) becomes:

\[
S''_{gen} = \frac{Ec_{m} Pr}{Re^2} \frac{1}{\theta} \left\{ 8(1 - N)^2 \left[ \left( \frac{\partial V}{\partial R} \right)^2 + \frac{V^2}{R^2} \right] + 8 \left( \frac{\partial U}{\partial Z} \right)^2 + \left[ Re \frac{\partial U}{\partial R} + \frac{4(1 - N)^2}{Re} \frac{\partial V}{\partial Z} \right]^2 \right\} \tag{13}
\]

where \((Ec, m)\) is the modified Eckert number, which is defined as \(Ec, m = \frac{v^2}{CpT_e}\).

3. Numerical solution

Since the physical properties of the fluid are assumed constant, the equations of continuity and momentum can be solved to determine the axial and radial velocity profiles \((U\) and \(V\), independent of the temperature field. Then the energy equation can be solved using the previously obtained velocities.

In the present work, the independent variables are \(R\) and \(Z\). Therefore, a two-dimensional rectangular grid in the \(R-Z\) plane is imposed on half of the annular flow field due to the symmetry about the \(Z\)-axis. The mesh network for finite difference representation is shown in Fig. 1b. Mesh points are numbered consecutively from an arbitrary origin with the \(i\) progressing in the radial direction, with \(i = 1\) (at the inner wall), 2, 3, \ldots, and \(n + 1\) (at the outer wall). The \(j\) progressing in the axial flow direction, with \(j = 1\) (at the inlet cross-section), 2, 3, \ldots, and \(m + 1\) (at the outlet cross section). Thus, the domain boundaries are located at \(R_1 = N\) (inner wall),
\[ R_{i+1} = 1 \text{ (outer wall), } Z_1 = 0 \text{ (channel entrance) and } Z_{m+1} = 1 \text{ (the chosen channel length).} \]

Therefore, the independent variables are designated as point functions as follows:
\[ R_i = N + (i - 1)\Delta R, \quad Z_j = (j - 1)\Delta Z. \]

The dependent variables are designated as point functions of \((i, j)\).

The finite difference scheme used in this study is identical with that used in [12,13]. It can be shown that the resulting finite difference equations are consistent representation of the boundary layer Eqs. (1), (2) and (4) and stable as long as the downstream axial velocity \((U)\) is non-negative, i.e. there is no flow reversal within the domain of solution. More details on this can be found in reference [12,13].

The present method solves Eqs. (8) and (9) for \(U\) and \(P\) at each cross-section. After that, Eq. (7) is solved for \(V\) at the same level. Now, having obtained values of \(U\) and \(V\) at a cross-section, Eq. (10) can be used to obtain the temperature values at this particular cross-section. The resulting coefficients matrix is tridiagonal and hence the Thomas algorithm is used to get the solution. The same procedure is repeated for other values of \(j\) (other cross-sections) to obtain the temperature field all over the entire annulus entrance length.

Having obtained the velocity and temperature distributions, the local entropy distribution can be calculated by inserting the velocity and temperature values in the entropy equation after being discretized. The entropy equation is discretized using the same finite difference approximations used for the continuity, axial momentum and energy equations (keeping in mind the definition of \((Ec)\) for each case and eliminating the thermal part for cases (IE and OE)).

In this study, the total entropy generation is of considerable importance. This is because there may exist an optimal thermodynamic design that minimizes the amount of entropy generation. To obtain the total entropy generation, it is necessary to solve the continuity, axial momentum, integral form of the continuity, and energy equations first. Having solved these equations for the local velocity and temperature, the local entropy production can be calculated and the results are used to obtain the total entropy production using numerical integration.

4. Results and discussion

This study is focused on the change in total entropy generation as function of the main parameters that define the flow. Results are obtained for different thermal boundary conditions, and then are compared together to optimize the flow parameters. The relative contribution of the viscous and thermal components of entropy generation are obtained and discussed.

In this study, the effect of the different flow parameters is investigated for a fixed value of the annulus radius ratio, namely \(N = \frac{r_1}{r_2} = 0.5\). These parameters involve: the Reynolds number \((Re)\), Prandtl number \((Pr)\), and Eckert number \((Ec)\) for cases (IE) and (OE). In addition, the effect of the dimensionless entrance temperature \((\tau)\) is also investigated for cases (I), (O), (IO), and (IOE). Moreover, the effect of the radius ratio \((N)\) on entropy generation is also illustrated for case (I). That was done in an attempt to optimize the annulus geometry through the radius ratio \((N)\).

The numerical grid used in this study is identical with that used in a previous study in which the transient forced convection heat transfer of the same flow problem was investigated [12]. Extensive numerical test calculations have been carried out to ensure grid insensitive results.
More details on grid sensitivity tests can be found in [12] and will not be repeated here for brevity. Once the grid size is decided, the length of the entrance region of the flow has to be determined. This is done by monitoring the development of the axial velocity profile against axial location. Based on the fact that the velocity profile becomes invariant with axial coordinate in the fully developed region, the entrance length extends from the axial location $z = 0$ to the first axial location beyond which the velocity profile becomes invariant.

To check the adequacy of the present work, the steady state results of the temperature field of the present study were compared with the corresponding numerical results of El-Shaarawi and Alkam [12]. As their work was transient, the comparison is made with their results at fairly large time (i.e. when steady state conditions are prevailing). The obtained steady state temperature profiles for case (I) are presented and compared with those of El-Shaarawi and Alkam [12] in Fig. 2a. It is clear that our results agree very well with the previous work. The results for cases (IO) and (IOE) are also compared with the numerical results of Siegel and Sparrow [15]. A computer run was made with a value of $N$ very close to unity (to approach the parallel plate channel: $N = 0.99$) and with a step temperature change at both walls. The obtained temperature profile is compared with those of Siegel and Sparrow [15] for a single value of $Z$ at which Siegel and Sparrow [15] presented their results, and the result of this comparison is shown in Fig. 2b. Using a uniform velocity profile at the entrance in the present work, Fig. 2b shows that the obtained temperature profile is generally higher than those of Siegel and Sparrow [15]. This is expected since they assumed a fully developed flow, while in the present work the flow velocity is developing and hence enhancing the convection of heat transfer. Overall, we can notice that
Fig. 3. Case (I): (a) Effect of Eckert number on entropy generation: $Re = 1000$, $\tau = 2$. (b) Effect of Eckert number on entropy generation: $Re = 1000$, $Pr = 0.71$ (air). (c) Effect of Reynolds number on entropy generation: $Pr = 0.71$ (Air), $\tau = 2$. (d) Effect of Reynolds number on entropy generation: $Pr = 0.71$ (air), $Ec = 0.1$. (e) Effect of Prandtl number on entropy generation: $Ec = 0.01$, $\tau = 2$. (f) Effect of Prandtl number on entropy generation: $Ec = 0.01$, $Re = 1000$. 
the present results agree very well with those available in the literature. This validates the numerical code of the present study.

4.1. Case (I)

Fig. 3a–f show the contribution of viscous and thermal entropy to the total entropy generation as a function of \( (Re, Pr, Ec) \), and the dimensionless entrance temperature \( (\tau) \) for case (I). Fig. 3a shows the contribution of viscous and thermal entropy as function of Eckert number for different fluids at fixed \( (Re) \) and \( (\tau) \). In this figure we can see that as \( (Ec) \) increases, the total entropy generation increases. This is true, since for a certain value of \( (\tau) \), higher \( (Ec) \) means higher \( (u_o^2) \), and this in turn will result in higher axial velocity gradients. As a result more entropy will be generated. Also, for the same \( (Ec) \), as \( (Pr) \) increases the total entropy generation initially decreases up to a certain critical \( (Pr) \), after which the total entropy generation starts increasing. Higher \( (Pr) \) means higher viscosity, and hence larger viscous entropy contribution. In general, it can be noticed that thermal entropy production is dominant and viscous entropy production is small especially for small values of \( (Pr) \) and \( (Ec) \). This fact is related to Eq. (11) in that \( (Pr) \) and \( (Ec) \) appear in the numerator of the second part of the right-hand side of Eq. (11). At large values of \( (Pr) \) and \( (Ec) \) the viscous entropy will begin to contribute significantly in the total entropy generation due to the increase in both viscosity and velocity gradients. We can also notice that thermal entropy is insensitive to \( (Ec) \). It is worth saying here that for lower values of \( Ec \) \( (Ec \leq 1E - 03) \) viscous generation is negligible, whereas for higher values of \( (Ec) \), viscous generation starts to contribute to the total entropy production.

Fig. 3b shows the effect of \( (Ec) \) on total entropy generation for different values of \( (\tau) \) and fixed \( (Pr) \) and \( (Re) \). It can be observed that increasing \( (Ec) \) will increase the total entropy generation due to the increase in the axial velocity gradient for higher \( (Ec) \). This increase is insignificant at lower values of \( (Ec) \) (e.g. \( Ec < 0.01 \)). Also, for the same inlet conditions, higher \( (\tau) \) means lower wall temperature, and hence lower temperature gradient at the wall, which will result in less entropy generation. Again, it is clear that thermal entropy is dominant and decreases as \( (\tau) \) increases. On the other hand, thermal entropy is not affected by changing \( (Ec) \).

Fig. 3c shows the behavior of entropy generation with \( (Re) \) at fixed values of \( (Pr) \) and \( (\tau) \). This figure shows that for fixed value of \( u_o \) and \( v \) (i.e. \( u_o = \text{constant} \) and \( v = \text{constant} \)), as \( (Re) \) increases, the total entropy generation decreases. Recall that, for fixed value of \( u_o \) and \( v \), \( (Re) \) is related to the flow geometry (i.e. \( Re \) is based on \( r_2 \)), therefore, higher \( (Re) \) means larger \( (r_2) \). Larger \( (r_2) \) leads to lower velocity and temperature gradients in the radial direction, and this will result in lower thermal and viscous entropy generation. Also, for the same \( (Re) \), increasing \( (Ec) \) will increase the rate of entropy generation. The thermal entropy generation is dominant except for high \( (Ec) \) and \( (Re < 160) \), and it is not affected by \( (Ec) \). We can notice that viscous entropy can be neglected for low \( (Ec) \) and \( (Re \geq 500) \).

In Fig. 3d, the effect of \( (Re) \) at different values of \( (\tau) \) is illustrated. For the same inlet conditions, higher \( (\tau) \) means lower wall temperature, and hence smaller magnitude of entropy generation. Increasing \( (Re) \) will decrease the total entropy generation as it was seen in the previous results. For \( (Re > 1000) \) and fixed value of \( (\tau) \), the entropy generation starts to take a constant value, since for these conditions the viscous entropy becomes very small and can be neglected.
Thermal entropy is generally dominant except when \((\tau)\) is low and \((Re < 160)\), and it increases as \((\tau)\) decreases.

Fig 3e illustrates the effect of the fluid type (manifested in the fluid’s Prandtl number \((Pr)\)) at different values of \((Re)\) and at fixed values of \((Ec)\) and \((\tau)\). It can be seen that the total entropy generation initially decreases as \((Pr)\) increases up to a certain value of \((Pr)\) after which it begins to increase. Total entropy generation increases slightly as \((Re)\) decreases. This indicates that total entropy generation is a weak function of \((Re)\) specially at relatively low values of \(Pr\) \((Pr \leq 1)\). The effect of \((Re)\) is more sound at larger values of \(Pr\) \((Pr \geq 1)\). For all values of \((Re)\), thermal entropy is always dominant. Viscous entropy increases with the increase in \((Pr)\) due to higher viscosity, however, for fluids with \(Pr \leq 1\), the viscous entropy generation can be safely neglected.

Finally, looking at Fig. 3f, we can clearly observe that the effect of \((Pr)\) on total entropy generation is similar in trend to that observed in Fig. 3e. On the other hand, entropy generation increases as \((\tau)\) decreases owing to higher values of temperature gradient at the wall associated with cases of lower \((\tau)\). In addition, thermal entropy is dominant while viscous entropy is generally negligible. This result is important from a design point of view and suggests that any effort towards the reduction of total entropy should focus on reducing the thermal contribution.

4.2. Case \((O)\)

The effect of the different parameters of the problem (e.g. \(Re\), \(Pr\), \(Ec\), \(\tau\)) on the viscous, thermal and total entropy production is shown in Fig. 4a–f. It can be observed that qualitatively the trend found in case \((I)\) is also repeated in this case. The difference in the results between the two cases is only quantitative. Therefore, the same discussion presented for case \((I)\) also applies here. However, we can generally state that, the magnitude of the total entropy generation (including the viscous and thermal components) in case \((O)\) is slightly larger than that for case \((I)\). Thus from an optimization point of view, there is no preference between case \((I)\) and case \((O)\).

4.3. Cases \((IE)\) and \((OE)\)

The results of this case are shown in Fig. 5a–c. In this case, the temperature distribution will be constant everywhere and equal to unity. This is because the fluid enters the annulus at the same temperature as that of one of the walls, while the other wall is kept insulated (i.e. no heat transfer is allowed at boundary). Thus there is no reason for the fluid to change its temperature. This was also checked numerically and the result is confirmed. Therefore, the entropy generation now is due to the viscous effect only. The trend of viscous entropy here is found to be the same as that for previous cases. The results for case \((IE)\) and \((OE)\) are identical as seen in Fig. 5a–c. This is because the hydrodynamic field is identical for case \((IE)\) and \((OE)\). It is emphasized here again that these cases have no significance in real heat exchangers applications, however, they have been studied here to cover all possible combinations of isothermal and adiabatic boundary conditions of the problem at hand.
Fig. 4. Case (O): (a) Effect of Eckert number on entropy generation: $Re = 1000$, $\tau = 2$. (b) Effect of Eckert number on entropy generation: $Re = 1000$, $Pr = 0.71$ (air). (c) Effect of Reynolds number on entropy generation: $Pr = 0.71$ (air), $\tau = 2$. (d) Effect of Reynolds number on entropy generation: $Pr = 0.71$ (air), $Ec = 0.1$. (e) Effect of Prandtl number on entropy generation: $Ec = 0.01$, $\tau = 2$. (f) Effect of Prandtl number on entropy generation: $Ec = 0.01$, $Re = 1000$. 
4.4. Cases (IO) and (IOE)

The results are shown in Fig. 6a–f and 7a–f, respectively. The general behavior observed for these cases is identical to that in the previous cases. The only exception is in the effect of \((Pr)\). It can be seen that entropy generation increases as \((Pr)\) increases. This is because larger viscosity is associated with larger \((Pr)\). The effect of \((Pr)\) can be shown in Fig. 6a, e, f and Fig. 7a, e, f. Also, it is observed that the amount of entropy generation in cases (IO) and (IOE) is higher than that for cases (I) and (O). In addition, entropy generation in case (IO) is obviously larger than that in case (IOE).

4.5. Effect of radius ratio

Fig. 8a–d shows the effect of the radius ratio \((N)\) on the total entropy generation for case (I). Recall that the radius ratio defines the geometry of the annulus (i.e. the width of the passage be-
Fig. 6. Case (IO): (a) Effect of Eckert number on entropy generation: $Re = 1000$, $\tau = 2$. (b) Effect of Eckert number on entropy generation: $Re = 1000$, $Pr = 0.71$ (air). (c) Effect of Reynolds number on entropy generation: $Pr = 0.71$ (air), $\tau = 2$. (d) Effect of Reynolds number on entropy generation: $Pr = 0.71$ (air), $Ec = 0.1$. (e) Effect of Prandtl number on entropy generation: $Ec = 0.01$, $\tau = 2$. (f) Effect of Prandtl number on entropy generation: $Ec = 0.01$, $Re = 1000$.  


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between the inner and outer walls for a fixed \( r_1 \) or \( r_2 \). Thus, these figures may be used in attempt to optimize the geometry of the annulus.

Fig. 8a shows the effect of radius ratio for different fluids. It can be clearly seen that for a given fluid (i.e. \( Pr \)) increasing the radius ratio will increase the total entropy produced. Increas-
ing the radius ratio means smaller spacing between inner and outer walls, and hence larger velocity and temperature gradients, which will result in larger entropy production. Also, for a value of \((N)\) approximately less than 0.76, increasing \((Pr)\) number decreases entropy generation up to a certain value after which entropy generation increases. For a value of \((N)\) higher than 0.76, increasing \((Pr)\) will always decrease the entropy production.

Fig. 8b shows the effect of radius ratio on entropy generation for different values of \((Ec)\) for air and water. Generally, the total entropy generation increases with the increase in \((N)\). However, entropy generation is a very weak function of \((Ec)\) in the case of air as the working fluid, whereas the effect of \((Ec)\) is slightly more obvious for water. This can be attributed to the fact that changes in \((Ec)\) will be reflected on the viscous entropy generation, which plays secondary role in total entropy production.

Fig. 8c shows the effect of radius ratio \((N)\) on entropy generation for different values of \((Re)\). It can be observed that entropy generation always increases with the increase in \((N)\). Also, the
results in this figure indicate that entropy generation is slightly affected by changes in \((Re)\) owing to the fact that changes in \((Re)\) will only affect the viscous entropy component, which is very small especially for higher values of \((Re)\).

Fig. 8d shows the effect of radius ratio \((N)\) on entropy generation for different values of \((\tau)\). It can be seen that entropy generation increases with the increase in \((N)\) and the decrease in \((\tau)\). The reasons are the same as those of the previous figure.

Fig. 9 shows the effect of radius ratio on entropy generation for all cases under consideration. It can be obviously seen that with the increase in radius ratio, minimum entropy is generated in cases (IE) and (OE), followed by (I), (O), (IO) and (IOE), respectively. This result should make it very clear as to which boundary condition(s) the best design might be related from a thermo-dynamic (i.e. second law) point of view.

5. Conclusions

In this work, a finite difference analysis was performed to investigate the entropy generation for the developing flow in a concentric annulus subjected to a wide variety of thermal boundary conditions. The obtained results should be used only for the entrance region of a circular concentric annulus and therefore the results of the present study may serve as a guide in future designs of compact heat exchangers. The conclusions from this study can be summarized as follows:

1. The total entropy generation increases with the increase in Eckert number.
2. The total entropy generation decreases with the increase in Reynolds number.
3. In cases (I), (O), (IO) and (IOE): the total entropy generation decreases as the dimensionless entrance temperature increases.
4. For cases (I) and (O): increasing \((Pr)\) will decrease entropy generation up to a certain value of \((Pr)\) after which it starts to increase.
5. For cases (IE), (OE), (IO) and (IOE): increasing \((Pr)\) will increase the entropy generation.
6. The general behavior of the entropy generation is the same for all cases under consideration, whereas the magnitude of entropy differs from one case to another.
7. Thermal entropy generation is generally dominant, and the viscous entropy can be neglected in most cases due to its very small contribution to the total entropy generation.
8. A larger amount of entropy is generated in case (O) than case (I). On the other hand, less entropy is generated in case (O) than that of case (IO) and (IOE). Also, more entropy is generated in case (IO) than case (IE). The viscous entropy produced in cases (IE) and (OE) is greater than that for cases (I), (O), (IO) and (IOE).
9. Increasing the annulus radius ratio increases the entropy generation.
10. Since thermal entropy is dominant over the viscous generation, any effort toward the reduction of total entropy generation should focus on reducing the thermal contribution.

References