Choice of Wavelet for Image Compression

- Wavelet reconstruction formula w/o quantization:

\[ x(t) = \sum_{k=-\infty}^{\infty} \sum_{f=-\infty}^{\infty} d_{j,k} 2^{j/2} \psi(2^j t - k) \]

\[ = \sum_{f=j_0}^{\infty} \sum_{k=-\infty}^{\infty} d_{j,k} 2^{j/2} \psi(2^j t - k) + \sum_{k=-\infty}^{\infty} c_{j_0,k} 2^{j_0/2} \phi(2^{j_0} t - k) \]

- Now consider with quantization:

\[ \hat{x}(t) = \sum_{f=j_0}^{\infty} \sum_{k=-\infty}^{\infty} [d_{j,k} + e_{j,k}^d] 2^{j/2} \psi(2^j t - k) + \sum_{k=-\infty}^{\infty} [c_{j_0,k} + e_{j_0,k}^c] 2^{j_0/2} \phi(2^{j_0} t - k) \]

\[ = x(t) + \sum_{f=j_0}^{\infty} \sum_{k=-\infty}^{\infty} e_{j,k}^d 2^{j/2} \psi(2^j t - k) + \sum_{k=-\infty}^{\infty} e_{j_0,k}^c 2^{j_0/2} \phi(2^{j_0} t - k) \]

error term is sum of wavelets and scaling function

- Thus, if the wavelet and scaling functions are rough, then the error is rough. So we want to make them smooth

  - There are various results that show how to design wavelet systems with specific degrees of smoothness: see the wavelet literature for details

  - One such means is: \[ \int t^k \psi(t) dt = 0 \text{ for } 0 \leq k \leq N \]

  - This is called imposing N vanishing moments and imposes that the wavelet will be N-times continuously differentiable

- Another aspect of vanishing moments:

  - If a wavelet system has N vanishing moments, then polynomials of degree less than N can be represented as a linear combination of translates of the scaling function

  - Thus, any locally-polynomial component of an image having degree less than N gets zeroed out by the high-pass filter because it can be completely handled by the low-pass filter

  - This results in lots of zero values for the wavelet coefficients, which leads to efficient coding (via zerotrees, as we will see).