12.3 Linear Filtering and Decimation & Expansion
Need for Filtering – Decimation

Recall: M-Fold Decimation has no aliasing if original signal is an “Mth Band signal”

→ Usually need to ensure this before decimating
→ Pre-filter w/ M-Band D-T LPF (i.e. a D-T anti-alias filter)
Need for Filtering – Expansion

Recall: Expansion leaves high frequency images that shouldn’t be there

➔ Need to remove these after expanding
➔ Post-filter w/ L-Band D-T LPF (an “anti-image” filter)

\[ x[n] \xrightarrow{\text{Expand \& Filter}} \]

\[ x^f(\theta) \]

\[ L=3 \]

Looks Just Like We Sampled at Higher Rate
So… the ideal post-filter converts the inserted zeros into interpolated samples:

⇒ Expansion-Filtering is called “Interpolation”
Time-Domain Description – Filter/Decimate

To get a time-domain description of filter/decimate:

\[ x[n] \xrightarrow{h[n]} \hat{x}[n] \xrightarrow{\downarrow M} \hat{x}_{(\downarrow M)}[n] \]

\[ \hat{x}[n] = \sum_i x[i]h[n - i] \]

\[ \hat{x}_{(\downarrow M)}[n] = \hat{x}[nM] = \sum_i x[i]h[nM - i] \]
To get a time-domain description of expand/filter:

\[ \hat{x}_{(\uparrow L)}[n] = \sum_{i} x[i]h[n - Li] \]

We’ll see later where this comes from

Compare to filter/decimate:

\[ \hat{x}_{(\downarrow M)}[n] = \sum_{i} x[i]h[nM - i] \]

Staggered Input Index
Staggered Output Index