## Laplace Transform Table

<table>
<thead>
<tr>
<th>Time Signal</th>
<th>Laplace Transform</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u(t)$</td>
<td>$1/s$</td>
</tr>
<tr>
<td>$u(t) - u(t - c), \ c &gt; 0$</td>
<td>$(1 - e^{-ct})/s, \ c &gt; 0$</td>
</tr>
<tr>
<td>$t^n u(t), \ N = 1, 2, 3, \ldots$</td>
<td>$N! / s^{N+1}, \ N = 1, 2, 3, \ldots$</td>
</tr>
</tbody>
</table>

| $\delta(t)$ | $1$ |
| $\delta(t - c), \ c \ \text{real}$ | $e^{-cs}, \ c \ \text{real}$ |
| $e^{-bt} u(t), \ b \ \text{real or complex}$ | $\frac{1}{s + b}, \ b \ \text{real or complex}$ |
| $t^n e^{-bt} u(t), \ N = 1, 2, 3, \ldots$ | $\frac{N!}{(s + b)^{N+1}}, \ N = 1, 2, 3, \ldots$ |

| $\cos(\omega_o t) u(t)$ | $\frac{s}{s^2 + \omega_o^2}$ |
| $\sin(\omega_o t) u(t)$ | $\frac{\omega_o}{s^2 + \omega_o^2}$ |
| $\cos^2(\omega_o t) u(t)$ | $\frac{s^2 + 2\omega_o^2}{s(s^2 + 4\omega_o^2)}$ |
| $\sin^2(\omega_o t) u(t)$ | $\frac{2\omega_o^2}{s(s^2 + 4\omega_o^2)}$ |
| $e^{-bt} \cos(\omega_o t) u(t)$ | $\frac{s + b}{(s + b)^2 + \omega_o^2}$ |
| $e^{-bt} \sin(\omega_o t) u(t)$ | $\frac{\omega_o}{(s + b)^2 + \omega_o^2}$ |
| $t \cos(\omega_o t) u(t)$ | $\frac{s^2 - \omega_o^2}{(s^2 + \omega_o^2)^2}$ |
| $t \sin(\omega_o t) u(t)$ | $\frac{2\omega_o s}{(s^2 + \omega_o^2)^2}$ |

| $A e^{-\varsigma \omega_o t} \sin\left[\omega_o \sqrt{1 - \zeta^2} t\right] u(t)$ | $A = \frac{\alpha}{\omega_n \sqrt{1 - \zeta^2}}$ |

where: $A = \frac{\alpha}{\omega_n \sqrt{1 - \zeta^2}}$

| $A e^{-\varsigma \omega_o t} \sin\left[\omega_n \sqrt{1 - \zeta^2} t + \phi\right] u(t)$ | $\frac{s + \alpha}{s^2 + 2\zeta \omega_n s + \omega_n^2}$ |

$A = \beta \frac{\sqrt{(\alpha - \zeta \omega_o)^2}}{\omega_n^2 (1 - \zeta^2)} + 1 \ \phi = \tan^{-1}\left(\frac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta \omega_o}\right)$

| $te^{-bt} \cos(\omega_o t) u(t)$ | $\frac{(s + b)^2 - \omega_o^2}{((s + b)^2 + \omega_o^2)^2}$ |
| $te^{-bt} \sin(\omega_o t) u(t)$ | $\frac{2\omega_o (s + b)}{((s + b)^2 + \omega_o^2)^2}$ |
# Laplace Transform Properties

<table>
<thead>
<tr>
<th>Property Name</th>
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</tr>
</thead>
<tbody>
<tr>
<td><strong>Linearity</strong></td>
<td>( ax(t) + bv(t) )</td>
</tr>
<tr>
<td><strong>Right Time Shift</strong> (Causal Signal)</td>
<td>( x(t - c), \quad c &gt; 0 )</td>
</tr>
<tr>
<td><strong>Time Scaling</strong></td>
<td>( x(at), \quad a &gt; 0 )</td>
</tr>
<tr>
<td><strong>Multiply by ( t^n )</strong></td>
<td>( t^n x(t), \quad n = 1, 2, 3, \ldots )</td>
</tr>
<tr>
<td><strong>Multiply by Exponential</strong></td>
<td>( e^{a} x(t), \quad a \text{ real or complex} )</td>
</tr>
<tr>
<td><strong>Multiply by Sine</strong></td>
<td>( \sin(\omega_o t) x(t) )</td>
</tr>
<tr>
<td><strong>Multiply by Cosine</strong></td>
<td>( \cos(\omega_o t) x(t) )</td>
</tr>
<tr>
<td><strong>Time Differentiation</strong> 2nd Derivative</td>
<td>( \dot{x}(t) )</td>
</tr>
<tr>
<td><strong>Time Differentiation</strong> n-th Derivative</td>
<td>( \ddot{x}(t) )</td>
</tr>
<tr>
<td><strong>Time Differentiation</strong> n-th Derivative</td>
<td>( x^{(N)}(t) )</td>
</tr>
<tr>
<td><strong>Time Integration</strong></td>
<td>( \int_{-\infty}^{t} x(\lambda) d\lambda )</td>
</tr>
<tr>
<td><strong>Convolution in Time</strong></td>
<td>( x(t) * h(t) )</td>
</tr>
<tr>
<td><strong>Initial-Value Theorem</strong></td>
<td>( x(0) = \lim_{s \to \infty} [sX(s)] )</td>
</tr>
<tr>
<td><strong>Initial-Value Theorem</strong></td>
<td>( \dot{x}(0) = \lim_{s \to \infty} [s^2 X(s) - sx(0)] )</td>
</tr>
<tr>
<td><strong>Initial-Value Theorem</strong></td>
<td>( x^{(N)}(0) = \lim_{s \to \infty} [s^{N+1} X(s) - s^N x(0) - s^{N-1} \dot{x}(0) - \ldots - s x^{(N-2)}(0)] )</td>
</tr>
<tr>
<td><strong>Final-Value Theorem</strong></td>
<td>If ( \lim_{t \to \infty} x(t) ) exists, then ( \lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s) )</td>
</tr>
</tbody>
</table>