Example of LT Analysis

Recall from Discussion #3 we found the Diff. Eq. for the speed of a DC motor with an applied voltage $V(t)$:

$$\frac{d^2 V(t)}{dt^2} + \left(\frac{R}{L} + \frac{1}{L} + \frac{1}{C}\right) \frac{dV(t)}{dt} + \left(\frac{K_e K_t + R_s}{L I_o}\right) V(t) = \frac{K_t}{L I_o} X(t)$$

This can be written as:

$$\frac{d^2 V(t)}{dt^2} + a \left(\frac{dV(t)}{dt}\right) + b_0 V(t) = b_0 X(t)$$

These depend on motor parameters.

Now... Assume the motor is not turning (zero IC's) & take LT:
\[ H(s) = \frac{b_0}{s^2 + a_1 s + a_0} \]

Assume that the motor parameters evaluate to

\[ H(s) = \frac{18}{s^2 + 0.65 s + 9} \]

Consider:

\[ x(t) = 1 \]

\[ t \]

**Intent:** Linearly accelerate the motor up to speed then hold steady speed.

\[ x(t) = t u(t) - (t-1) u(t-1) \]
From LT Tables:

\[ X(s) = \frac{1}{s^2} - \frac{1}{s^2} e^{-5} \]

\[ Y(s) = H(s)X(s) \]

\[ = \frac{H(s)}{s^2} - \frac{H(s)}{s^2} e^{-5} \]

\[ \text{Delay of this} \]

Find \( \mathcal{L}^{-1}\left\{ \frac{H(s)}{s^2} \right\} \)

\[ \frac{H(s)}{s^2} = \frac{18}{s^2 (s^2 + 0.65s + 9)} \]

\[ = \frac{18}{s^4 + 0.65s^3 + 9s^2} \]

\[ s = 0.1 \]

"Damping"
No Partial Fractions via MATLAB:

\[
\mathcal{H}(s) = \frac{-0.13}{s} + \frac{2}{s^2} + \frac{0.0667 + j0.33}{s + 0.3 - j2.99} + \frac{0.0667 - j0.33}{s + 0.3 + j2.99}
\]

\[
\mathcal{L}^{-1}\left(\mathcal{H}(s)\right)
\]

\[
y_1(t) = -0.13(u(t) + 2t u(t)) + 0.34 e^{-0.3t} (e^{j1.37t} - e^{-j1.37t})
\]

\[
= -0.13 u(t) + 2t u(t) + 0.34 \ e^{-0.3t} \ \cos(2.99t + 1.37)
\]

The other part is just a delay:

\[
y_2(t) = y_1(t-1)
\]

\[
y(t) = y_2(t) - y_1(t-1)
\]

The response looks like this:
Now, the overshoot occurs because the \( g \) is too low (\( g = 0.1 \)).

The parameter \( g \) provides more damping:

\[
\frac{d^2 \omega(t)}{dt^2} + \left( \frac{R}{L} + \frac{b}{L} \right) \frac{d \omega(t)}{dt} + \left( \frac{K_e K_b + R_b}{L I} \right) \omega(t) = \frac{K_b}{L I} \chi(t)
\]
Suppose we increased $b$ to give

$$H(s) = \frac{18}{s^2 + 7s + 9}$$

was 0.6, now is 7

The a similar analysis leads to:

\[ \sqrt{\text{LAG}} \] or "sluggishness"