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PERFORMANCE OF A BI-STABLE RESONATOR WITH RANDOM INPUT VIBRATIONS

Wei Yang
Shahrzad Towfighian
State University of New York at Binghamton
Binghamton, NY, 13905
Email: wyang24@binghamton.edu
stowfigh@binghamton.edu

ABSTRACT

By converting ambient mechanical energy to electricity, vibration energy harvesting, enable powering of low-power remote sensors. However, realistic ambient vibrations are random and spread over a wide frequency spectrum, which means linear resonators fail to perform effectively because of their narrow frequency bandwidth. Hence, there is a need for thorough investigation of performance of nonlinear resonators with Gaussian random vibration. This article presents a simulation study on the use of magnets to improve a nonlinear oscillator for energy harvesting from broadband low frequency random excitation. The resonator response to Gaussian distribution random input is investigated using root mean square value and power spectral density of voltage. The obtained results show that in a broadband low frequency spectrum the nonlinear system performs better than linear resonance. The optimal performance is found when the distance between two magnets is near the mono-stable to bi-stable transition regime.

INTRODUCTION

Scavenging the prevalent ambient energy (vibration energy harvesting) enables a vast number of sensing applications, including wireless sensor networks (WSN) whose required power meets the levels of harvestable power. In the next years, WSNs will become employed in a wide range of applications such as structural monitoring, industrial sensing, remote healthcare, military equipment, surveillance, logistic tracking and automotive monitoring. An important question that must be addressed by any energy harvesting technology is related to the type of energy available. Among various renewable energy present in the environment such as solar, radio frequency (RF), temperature difference and biochemical, kinetic energy is the most suitable source because of its abundance and power density for applications to micro-energy generation [1].
Vibration energy harvesters are mechanical oscillators that convert kinetic energy via capacitive, inductive, or piezoelectric transducers. To have high energy conversion efficiency, the resonant frequency of the oscillator should match the spectral region where most of the energy is available. However, in the vast majority of cases the ambient vibrations have their energy distributed over a wide spectrum of frequencies, with significant predominance of low frequency components [2]. Several methods have been explored. A resonance frequency tunable energy harvester based on a magnetic force technique and a variable stiffness system has been studied [3]. Researchers have recently exploited the nonlinearities to avoid frequency tuning after the initial set up of the harvester and improve broadband energy harvesting. Several works have been presented in the literature concerning nonlinear macro-oscillators [4-10].

Some approaches to introduce nonlinearity into the system for broadening the bandwidth include mechanical stoppers [11], and magnetic interactions. Stanton et al. [12] investigated the response of a bi-stable resonator with magnets to harmonic base excitation. Considering the actual ambient vibration, Ferrari [13], Daqaq [14] and Ando [15] analyzed the advantage of bi-stable system under random vibration input. In Tang`s work [16], an experimental study was conducted to investigate the use of magnets for improving the functionality of bi-stable energy harvesters under various vibration random vibration scenarios. However, there was not an analytical study that could predict the experimentally observed enhancement. To design high performance energy harvesters, there is a need for an accurate model for optimization.

A bi-stable system outperforms a linear system in some conditions. However, when the excitation is not strong enough, the oscillator under the bi-stable regime does not provide the significant advantages as expected. If there is not enough excitation energy in a bi-stable system, a barrier prevents the resonator from oscillating from one stable point to the other. The height of barrier is determined by the distance between two magnets. Therefore, although the response of bi-stable resonators using magnetic force to random vibrations has been studied, there is a need to investigate parameters of system in order to maximize output.

The contribution of this paper is to demonstrate how the optimal distance between two magnets improves energy harvesting in bi-stable energy harvesters using an analytical approach. Placing two magnets at an optimal distance apart improves energy harvesting by broadening the frequency bandwidth at low frequencies. We use Gaussian random input that best resembles ambient mechanical vibrations. The root mean square method and the power spectral density method are applied to scrutinize how to obtain the largest response and voltage at low frequencies where most of the ambient vibrations spread.

The content of this article is organized as follows. In the next section, a mathematical modeling of a bi-stable resonator is developed. Both mono-stable and bi-stable configurations are considered. Then with Gaussian distribution random input, time response and frequency output are obtained with varied distance between two magnets numerically.

1 MATHEMATICAL MODELLING

The model of a nonlinear oscillator for energy harvesting is shown as Fig. 1. It consists of a piezoelectric cantilever beam with a magnetic tip facing another magnet. The whole system is on the source of vibration. The two magnets face each other with the same pole. The repulsive force acts between two magnets, and it increases in magnitude as the distance between the two magnets decreases.

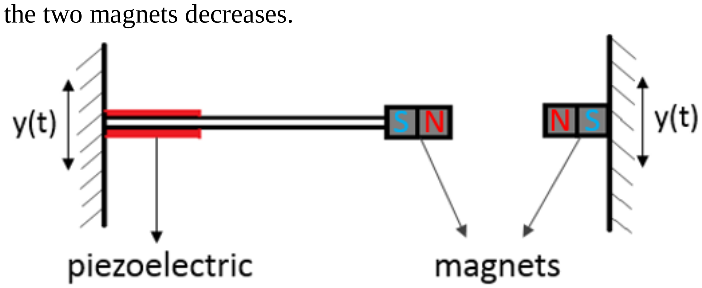

Figure 1. A schematic diagram of the bi-stable resonator.

To allow an approximate analysis, the system can be reduced to a simple spring-damping-mass vibrations model as shown in Fig. 2.

The mass, m, accounts for the effective mass of the 1st mode of the cantilever beam plus the magnet attached in the beam. The effective stiffness of the spring is the elastic reaction of the cantilever. The deflection at the tip of the beam is represented by x (Fig. 2). As the horizontal magnetic force is balanced by longitudinal stiffness of the beam, we only consider the vertical magnetic force to change the beam vertical deflection.

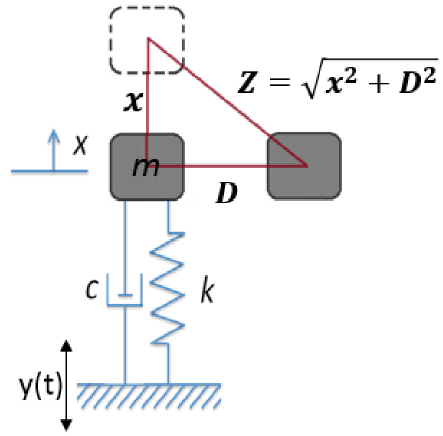

Figure 2. Simplified model of the bi-stable resonator with two magnets.

The magnetic force between two magnets is given [17] as

$$F_{mag} = \frac{3\mu_0 M^2}{2\pi Z^4} \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7} \mathrm{NA}^{-2}$is the permeability constant. M is the effective magnetic moments, and Z is the distance between two magnets. From Fig.2, geometry shows $Z = \sqrt{x^2 + D^2}$ where x is the tip magnet displacement and D is the horizontal distance between two magnets .Integrating the magnetic force, the corresponding potential energy is obtained:

$$U_{mag} = -\int F_{mag}\, dz = \frac{\mu_0 M^2}{2\pi Z^3} \tag{2}$$

Substituting $Z = \sqrt{x^2 + D^2}$ into equation (2), and expanding with a Taylor series up to three terms at $x = 0$ yields

$$U_{mag} = \frac{\mu_0 M^2}{2\pi}\left(\frac{15}{8}\frac{x^4}{D^7} - \frac{3}{2}\frac{x^2}{D^5} + \frac{1}{D^3}\right) \tag{3}$$

Because of the piezoelectric effect, there is electric energy, which can be represented as:

$$U_e = -\theta\lambda(\dot{t}) - \frac{1}{2}c_p\lambda(\dot{t})^2$$

Where θ is the electromechanical coupling term, c_p denotes the properties of the piezoelectric material.$\lambda(t)$ is the flux linkage and $\lambda(\dot{t}) = v(t)$ where v(t) is the voltage generated by the piezoelectric . The total potential energy is the summation of spring energy plus magnetic potential energy and electric energy.

$$U = U_k + U_{mag} - U_e \tag{4}$$

Using equation (3), the total potential energy can be rewritten as:

$$U = \frac{1}{2}kx^2 + \frac{3\mu_0 M^2}{2\pi}\frac{5}{8D^7}x^4 + -\frac{3\mu_0 M^2}{2\pi}\frac{1}{2D^5}x^2 + \frac{\mu_0 M^2}{2\pi}\frac{1}{D^3} - \theta\lambda(\dot{t}) - \frac{1}{2}c_p\lambda(\dot{t})^2 \quad (5)$$

where k is the spring stiffness. Using Lagrange's equation, the governing equation of motion is obtained as:

$$m\ddot{x} + c\dot{x} + k_1 x + k_3 x^3 - \theta\lambda(\dot{t}) = f(t) \quad (6)$$

$$c_p\lambda(\ddot{t}) + \theta\dot{x} + \frac{\lambda(\dot{t})}{R} = 0 \quad (7)$$

Equation (6)is a Duffing-type equation, where $k_1 = k - \frac{FR}{D^5}$, $k_3 = \frac{5FR}{2D^7}$, $FR = \frac{3\mu_0 M^2}{2\pi}$, R is the resistence load.$f(t)$ is the base excitation and is a random vibration input in our case. Effective mass of the system is m explained previously, c is the damping, and$k$ is the effective stiffness of the cantilever. FR is a constant determined by property of magnets,k_1 is the linear stiffness of the system that changes with the disance between two magnets, andk_3 is the cubic nonlinear stiffness introduced by the magnetic froce. That means the stronger the magnetization, the larger the nonlinearites will be. We assigned values for all the parameters of the two magnets, the cantilever beam and the piezoelectric so that the only variable is D, the distance between the magnets. In the following section, numerical analysis will be conducted to study the effect of$D$ on the nature of the response and voltage.

2 NUMERICAL ANALYSIS OF THE BI-STABLE RESONATOR

Potential Energy Function

The distance between the two magnets changes the potential energy function and consequently the time response, which is studied here using numerical simulations. Using a cantilever beam with dimensions as table 1.

Table 1. Geometrical and material properties of the resonator.

Parameters	Polymer	PZR	Magnet
Length(mm)	72	30	8
Width(mm)	10	10	8
Thickness(mm)	1	1	8
Density(kg/m^3)	1220	7800	7500
Modulus(GPa)	2.344	66	-
Magnetic moment(A^2/m)	-	-	0.5

One can calculate the parameters of equations (6) and (7) as $m = 0.0063\text{kg}$, $k = 78.9\text{N/m}$, $M = 0.5\text{Am}^2$, $\theta = -0.011$, $c_p = 1.1 \times 10^{-7}$ and $R = 1000\Omega$ .The damping coefficient shall be found from experiments, here it is assumed$c = 0.2\text{N/(m/s)}$.

The changes in the potential energy function as the distance between two magnets varies (equation (5)) are illustrated in Fig. 3. As it can be seen, the potential energy changes from a mono-stable to a bi-stable state beyond a threshold distance, D_c.

The threshold distance between two magnets, $D_c = 18mm$, is found by setting $k_1 = 0$.This threshold distance between two magnets divides the potential functions to two regimes:

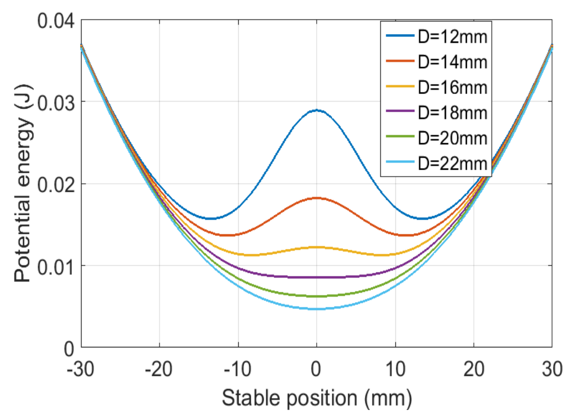

Figure 3. The variation of the potential energy function as the distance between two magnets (D) changes.

(1) Mono-stable when $D \gg D_c$ (k_1 is positive). The system is mono-stable, i.e. it has one stable equilibrium point. The dynamics are characterized by quasilinear oscillations around the single minimum located at zero displacement. (2) Bi-stable when $D \ll D_c$($k_1$is negative). The potential energy is double-well with a barrier between the two wells. The system is bi-stable, i.e. it has two stable and one unstable equilibrium points. The dynamics of the system can contain oscillations in one well or between the two wells.

Time Responses

To find the time response of the resonator, a random input of Gaussian distribution was applied with a root mean square acceleration of$a_{rms} = 0.5g$. Displacements of the beam tip in the mono-stable and bi-stable regimes at corresponding distance between magnets are presented as follows.

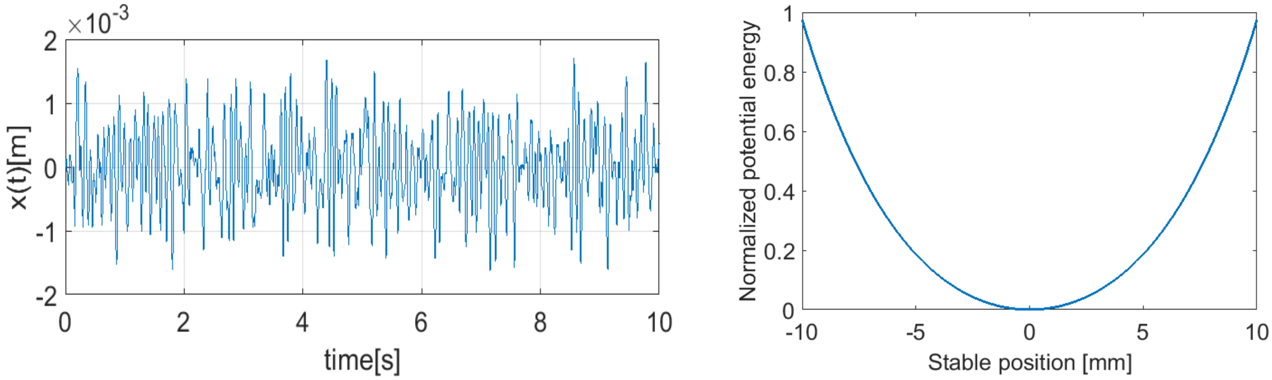

Figure 4. Case I: when D=20mm, Left: displacement time response. Right: Potential energy function.

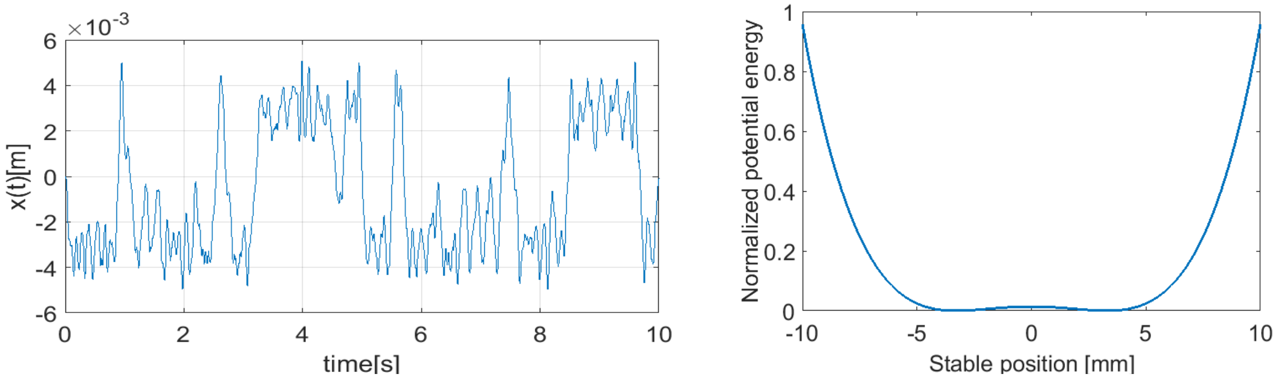

Figure 5. Case II: when D=17.7mm, Left: displacement time response. Right: Potential energy function.

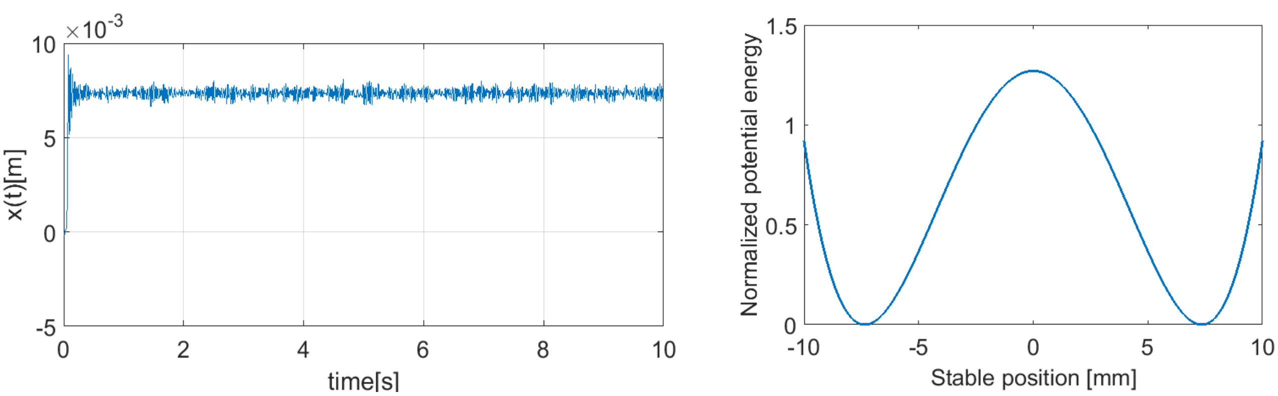

Figure 6. Case III: when D=15mm, Left: displacement time response. Right: Potential energy function.

When the distance is larger than the threshold (Case I: D=20mm), the time response and potential energy are represented in Fig. 4. The time response is quasilinear. There is only one stable equilibrium position, where the beam oscillates about.

When the distance is close to the threshold (Case II: D=17.7 mm), as shown in Fig. 5, the potential energy has two distinct equilibrium points separated by a trivial energy barrier. The displacement response shows an evident bi-stable behavior with the displacement x that switches frequently between the two corresponding potential minima positions. The displacement response contains oscillation around each of the two equilibrium positions and large excursions from one to the other.

With further decreasing of the distance between the two magnets (Case III: D=15 mm), the energy barrier of the double-well potential energy becomes more prominent, which causes the jumps between two wells less probable as it requires larger input. The displacement response gets confined in one well and has lower amplitude (Fig. 6). This behavior is similar to that of a linear oscillator with large distance between magnets as shown in Fig.4.

Probability Density Function

In order to understand the displacement response better, the probability of displacement response is presented in Fig. 7, 8 and 9for three cases of distance between magnets. The probability analysis indicates the number of times a specific displacement (x) occurs.

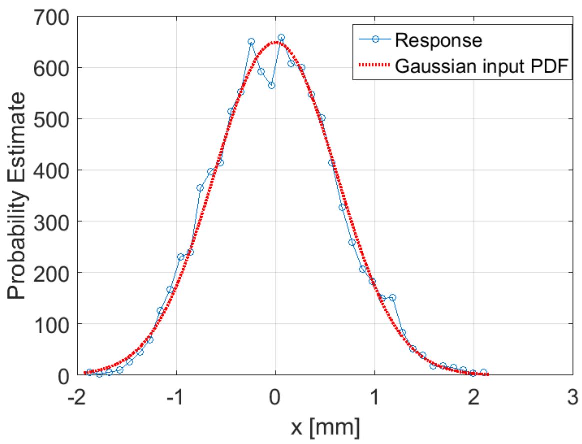

Figure 7. Probability density function of displacement for case I (D=20mm).

The distribution of the probability gives a clear picture of the number of equilibrium points for the resonator. For Case I, the maximum number of times occurs mostly around zero, which is the equilibrium position for the mono-stable system. Moreover, the response is very similar to the Gaussian random input. The little difference comes from the nonlinearities in the governing equation presented by cubic stiffness.

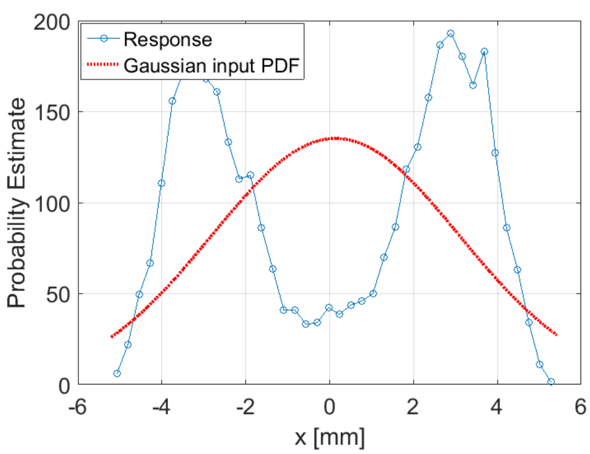

Figure 8. Probability density function of displacement for case II(D=17.7mm).

For Case II, maximum probabilities occur around two stable equilibrium points that can be explained by time data of response (Fig. 5). The cantilever beam oscillates around one stable point and then jumps to another one. Thus there are two peaks. It does not vibrate around the origin anymore. Moreover, the response is not similar as Gaussian distribution probability density because of its bi-stable nature.

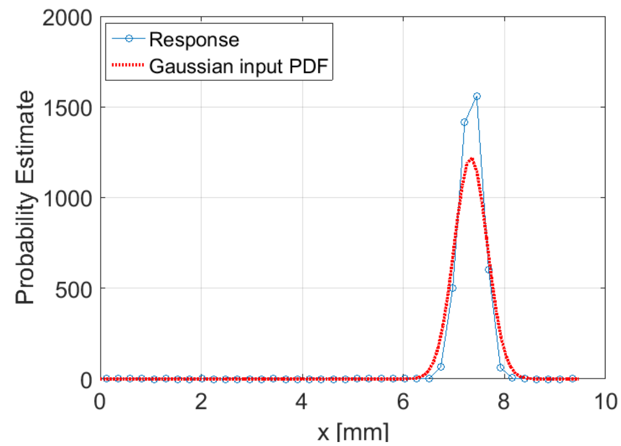

Figure9. Probability density function of displacement for case III (D=15mm).

For Case III, the most probable displacement occurs around one of the stable equilibrium positions of the bi-stable system. Because the two magnets are close to each other, the magnetic force increases the barrier between two valleys. It is hard for the cantilever beam to overcome the barrier at low excitation levels. However, the probability is similar to Gaussian distribution, which means the response is similar to that of a linear oscillator. The oscillator frequency is high in this case, because the route is short.

Power Spectral Density

To scrutinize the response of the resonator for a large variation of the distance between two magnets (D), the average of response amplitude and power spectral density (PSD) of the voltage are presented in Figs. 10 and 11, respectively.

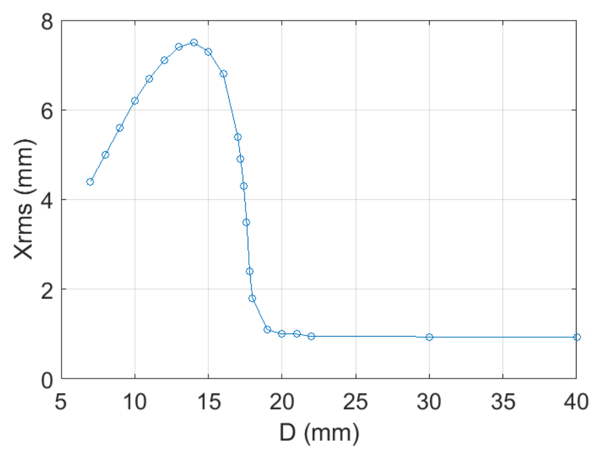

Figure 10. Position x_{rms} as a function of D.

Fig. 10 shows when $D \gg D_c = 18mm$, the average amplitude does not change significantly. The resonator works in the mono-stable regime with a quasilinear behavior. When D decreases to near D_c, the average amplitude starts to increase and it reaches a peak at $D < D_c$. That means the system experiences a large response in the bi-stable regime. With further decrease in D, the response decreases, because the potential energy barrier, produced by the magnetic force, is higher. From the analysis of the root mean squares, it seems that the system performs better in terms of amplitude in the bi-stable region.

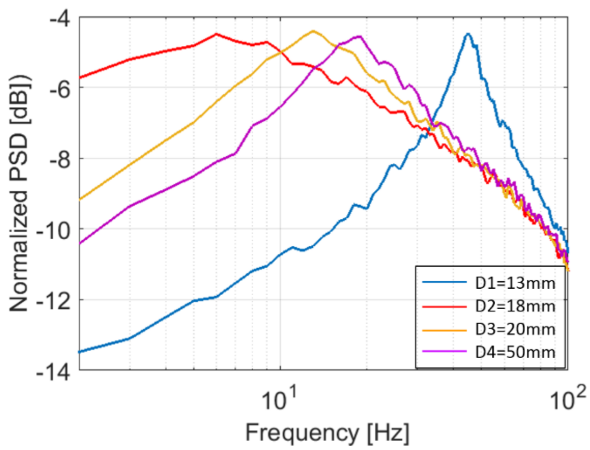

Figure 11. Power spectral density of voltage as the distance between magnets (D) varies when $a_{rms} = 0.5g$.

To characterize the bi-stable resonator response in the frequency domain and study its bandwidth, power spectral densities of voltage are illustrated in Figs. 11 and 12 for two excitation levels. In Figs. 11 and 12, it presents equivalent peak voltage as 1.8V and 2.7V respectively. The behavior at the two excitation levels is qualitatively the same. One observe that the threshold distance D_c=18mm does not have a peak. Decreasing or increasing D from the threshold distance, increases the peak resonant frequency. As D is near D_c, the threshold distance, a larger voltage is obtained at low frequencies compared to other distances (e.g. response of D=20 mm compared to those of D=50 mm and D=13 mm).

For small D, the peak shifts to high frequency. It shows a good performance in relatively high frequency, but in low frequency region, the voltage is even worse than the mono-stable state. In other words, for the bi-stable system, the energy is spread over a larger frequency range, however the amplitude is reduced. In contrast, when D is close to the transition regime from the mono-stable to the bi-stable state, a large voltage is obtained at low frequencies where most of the ambient energy exists. It should be noted that the characteristic of the voltage does not change at different excitation levels as shown in Figs. 11 and 12.

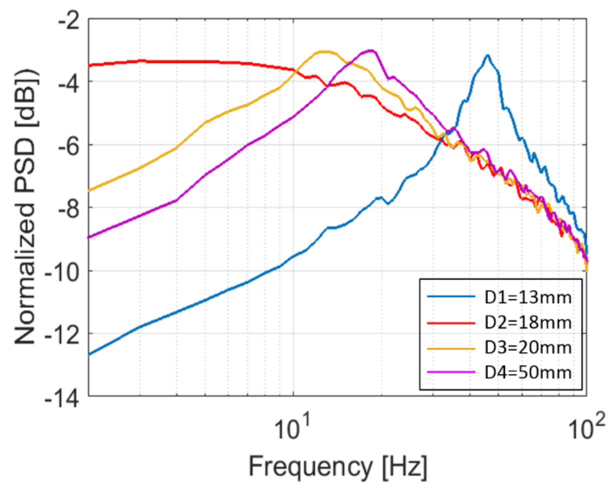

Figure 12. Power spectral density of voltage as the distance between magnets varies when $a_{rms} = 1g$.

CONCLUSION

This article represents an analytical investigation on using magnets to improve the functionality of bi-stable resonator for vibration energy harvesting with random excitations. Simulation results and analysis show that the nonlinear system performs better than a linear system by having a broadband low frequency spectrum. The configuration for the optimal performance of a nonlinear oscillator is found when the distance between two magnets is near the mono-stable to bi-stable transition regime (threshold distance).The presented model provides a fundamental understanding of the behavior of the system in the transition region and the simulations conform to previous experimental results [16].Our results indicate both bi-stable and mono-stable systems can have high performance at low frequencies when the distance between two magnets is close to the threshold distance. This represents the case, where the potential energy function has a trivial energy barrier enabling large amplitude oscillations between two wells regardless of the excitation levels. This concept can be used to increase energy conversion efficiency in vibration energy harvesters.

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