BLIND MULTIUSER DETECTION FOR CDMA WITH MULTIPATH:
A DIRECT METHOD

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ABSTRACT

In this paper we consider the blind multiuser detection problem for asynchronous DS-CDMA systems operating in a multipath environment. Using only the spreading code of the desired user, we first estimate the column vector subspace of the channel matrix by multiple linear prediction. Then zero-forcing detectors and MMSE detectors with arbitrary delay can be obtained without explicit channel estimation. This avoids any channel estimation error and the resulting methods are therefore more robust and more accurate. The new algorithms are robust against timing estimation errors, and are extremely near-far resistant. Simulations show the effectiveness of these methods.

1. INTRODUCTION

Blind multiuser detection for CDMA systems with multipath channels has aroused much attention and interest recently. Blind joint multiuser detection and channel equalization is a good candidate to reduce both multiple access interference (MAI) and inter-symbol (or chip)-interference (ISI) without any training sequences which will reduce throughput.

Subspace methods (SS) [1] and minimum output energy methods (MOE) [2]-[3] were developed to reduce both ISI and MAI. However, SS suffers from large amount of computation and sensitive rank determination problems while a major drawback of MOE is that there is a saturation effect in the steady state. Another kind of approach is linear prediction method [5]. This approach is promising because linear prediction is computationally efficient and robust. However, all of the above methods depend on the result of channel estimation, which may suffer from system noise and computation errors and deteriorate symbol detection. There are also some direct methods for detector estimation, such as [4]. However, [4] is similar to the SS methods and therefore suffers similar problems as SS.

In this paper, we will present direct CDMA detector estimation methods for joint blind equalization and blind multiuser detection based on channel matrix/vector space estimation by linear predictions. We do not require explicit channel estimation. The resulting algorithms are therefore more robust and more accurate.

![Figure 1: CDMA receiver block-diagram for the jth user and the fth antenna](image)

2. PROBLEM FORMULATION

Consider either a multiple antenna system or a single antenna system with oversampling beyond the chip rate. Channel spreading may extend over several symbol intervals. There are altogether J users and L antennas. bj(n) is the jth user’s symbol sequence. cj(k) is the jth user’s spreading code with length Lc, gj(k) is the jth multipath channel impulse response for the fth antenna (or the fth subchannel in case of oversampling). And v(n) is the additive noise. Note that the received signal x(n) has the same form for all users, which includes MAI (with ICI) due to other users and ICI due to the jth channel gj(k) for the jth user and the fth antenna.

For the jth user, the transmitted signal is

\[ s_j(n) = \sum_k b_j(k)c_j(n - kL_c), \]

the asynchronous channel output with random delay d_j is

\[ x'(n) = \sum_{j=1}^{J} \sum_k b_j(k)h_j^F(n - kL_c - d_j) \]

\[ h_j^F(n) = \sum_{i=0}^{L_c-1} c_j(i)g_j^F(n-i) \]

Define h_j^F,i(n) = h_j^F(nL_c + L_c - i - 1), x_j^F(n) = x'(nL_c + L_c - i - 1) for i = 0, 1, ..., L_c - 1 and h_j^F,i+L_c d_j = [h_j^F,i+L_c d_j(k),

\[ \cdots, h_j^F,i+L_c d_j+L_c-1(k)\]^T, x'(n) = [x_0^F(n), \cdots, x_{L_c-1}^F(n)]^T, then

\[ x'(n) = \sum_{j=1}^{J} \sum_{k=0}^{L_c-1} b_j(n-k)h_j^F,i+L_c d_j(k) \]
where
\[ L_h = \max_{l=1}^{L_c} \left[ \frac{L_c + L_g - 1 + d_j^l}{L_c} \right] \]
(5)

In fact, \( L_h \) can be over estimated in our algorithm.

Now let \( x(n) = [x^0(n) \cdots , x^{L-1}(n)]^T \) and \( \mathcal{X}(n) = [x^T(n), \cdots , x^T(n-N+1)]^T \), we have
\[ \mathcal{X}(n) = \sum_{j=1}^{J} \mathcal{H}_j \mathbf{b}_j(n) + v(n), \]
(6)
where \( \mathcal{H}_j \) is the following matrix
\[
\begin{bmatrix}
\mathbf{h}_j(0) & \cdots & \mathbf{h}_j(L_h - 1) & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & \mathbf{h}_j(L_h - 1)
\end{bmatrix}
\]
(7)
with \( \mathbf{h}_j(i) = [(\mathbf{h}^0_{j,i}, \mathbf{h}^1_{j,i}, \cdots , \mathbf{h}^{L-1}_{j,i})]^T \). Ignoring the noise \( v(n) \) for the time being, the received signal vector is \( \mathcal{X}(n) = \mathcal{H} \mathbf{b}(n) \) where \( \mathcal{H} = [\mathcal{H}_1 , \cdots , \mathcal{H}_J] \) is of dimension \( \text{NLL}_c \times J(L_h + N - 1) \). Similar to other existing multiuser detection approaches, we assume \( \mathcal{H} \) full column rank. A necessary condition is choosing \( N \) such that \( \text{NLL}_c \geq J(L_h + N - 1) \).

Now we need to express matrix \( \mathcal{H}_j \) in terms of the spreading code \( c_j(k) \) and the channel impulse response \( g_j(k) \). From (3),
\[
\begin{bmatrix}
\mathbf{h}_j^T(L_c + L_g - 2) \\
\vdots \\
\mathbf{h}_j^T(0)
\end{bmatrix} = \mathbf{C}_j^T \mathbf{g}_j
\]
where
\[
\begin{bmatrix}
c_j(L_c - 1) \\
\vdots \\
c_j(0)
\end{bmatrix} = \begin{bmatrix}
g_j^T(L_g - 1) \\
\vdots \\
g_j^T(0)
\end{bmatrix}
\]
(8)
Therefore we have
\[
\mathbf{h}_j(k) = \mathbf{C}_j(k) \mathbf{g}_j
\]
(9)
where \( \mathbf{C}_j(k) \) is a block diagonal matrix with the diagonal entries \( \mathbf{C}_j \) and \( \mathbf{g}_j = [(\mathbf{g}^0_j)^T, \cdots , (\mathbf{g}^{L-1}_j)^T]^T \).

3. CHANNEL VECTOR SUBSPACE ESTIMATION BY LINEAR PREDICTION

3.1. Linear transformation
Consider the asynchronous CDMA system (6) - (9). We assume at first the timing of the desired user \( j = 1 \) is \( d_1^1 = 0 \), \( \forall \ell \). The equalizer delay \( d_j \) is between 0 and \( L_h + N - 1 \). For simplicity, we consider only \( d_j \leq N \) case. The \( d_j > N \) case can be similarly obtained. The \( (d_j + 1) \)th column of \( \mathcal{H}_1 \) contains \( \mathbf{h}_1(d_j), \cdots , \mathbf{h}_1(0) \). We construct data vector \( \mathcal{Y}_1(n) = [x(n-d_j + L_h - 1), \cdots , x(n-d_j)] \).

**Proposition 1:** There exists a full row rank \( LLL \times LLL \) matrix \( \mathbf{T} \) such that \( \mathbf{T} \mathcal{Y}_1(n) \) does not contain \( \mathbf{b}_1(n-d_j) \).

**Proof:** See [7].

We note that \( \mathbf{T} \) can be pre-computed and does not completely cancel other symbol components.

3.2. Linear prediction
In order to extract the \( b_1(n-d_j) \) part of \( \mathcal{X}(n) \) using linear prediction, we need to find a data vector which does not contain \( b_1(n-d_j) \) yet the corresponding channel matrix should be full column rank (without considering an all-zero column). Such a data vector is constructed as follows
\[
\begin{bmatrix}
x(n-d_j + L_h - 1 + M_1) \\
\vdots \\
x(n-d_j - 1) \\
x(n-d_j - M_2)
\end{bmatrix}
\]
where \( M_1, M_2 \) have to be sufficiently large. Let \( M = M_1 + M_2 + 1 \). Then we solve the following linear prediction problem
\[
\mathbf{e}_1(n) = \mathcal{X}(n) - \mathbf{P} \mathcal{Y}_2(n)
\]
(10)
where \( \mathbf{P} \) has dimension \( \text{NLL}_c \times \text{NLL}_c \). Assume that the symbols \( b_j(n) \) are uncorrelated in time and that \( b_1(n), \cdots , b_j(n) \) are mutually uncorrelated with variances (powers) \( A_1, \cdots , A_j \). We define
\[
\mathcal{X}(n) \triangleq \mathbf{H}_1 \mathbf{b}_1(n-d_j) + \mathbf{H}_1 \mathbf{b}_1(n)
\]
(12)

**Proposition 2:** The optimal linear prediction matrix \( \mathbf{P} \) gives
\[
\mathbf{e}_1(n) = \mathbf{H}_1 \mathbf{b}_1(n-d_j).
\]
(13)

**Proof:** See [7].

Let \( \mathbf{e}_2(n) = \mathcal{X}(n) - \mathbf{e}_1(n) \), then from (12) and (13) we have \( \mathbf{e}_2(n) = \mathbf{H}_1 \mathbf{b}_1(n) \). Hence the channel matrix vector space is separated into two subspaces by linear prediction.

It can be shown that \( M_1 \) and \( M_2 \) must satisfy
\[
(M_1 + 1) \text{NLL}_c \geq (M_1 + L_h - 1)J \]
\[
(M_2 + 1) \text{NLL}_c \geq (M_2 + L_h - 1)J.
\]
(14)

4. BIND ZERO-FORCING MULTIUSER DETECTION
From equation (12), a zero-forcing detector \( \mathbf{f} \) with dimension \( \text{NLL}_c \times 1 \) and with delay \( d_j \) satisfies
\[
\mathbf{f}^H \begin{bmatrix}
\mathbf{H}_1 & \mathbf{H}_1
\end{bmatrix} = \begin{bmatrix}
1 & 0^H
\end{bmatrix}
\]
(15)

Let \( \mathbf{u} \) be an estimate of the equivalent vector subspace of \( \mathbf{H}_1 \) calculated from
\[
\mathbf{R}_1 = \mathbf{E}\{\mathbf{e}_1(n)\mathbf{e}_1(n)^H\} = \mathbf{A}_1 \mathbf{H}_1 \mathbf{H}_1^H.
\]
(16)
4.1. Batch algorithm

For the linear prediction in (11), Let

\[
R = E\left[ \begin{bmatrix} \mathcal{X}(n) \\ \mathcal{Y}_2(n) \end{bmatrix} \begin{bmatrix} \mathcal{X}^H(n) \\ \mathcal{Y}_2^H(n) \end{bmatrix} \right] = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}.
\]

(17)

**Proposition 3:** The optimal asymptotic solution for the linear prediction problem (11) is

\[
P = R_{11} R_{22}^+.
\]

(18)

where \((\cdot)^+\) denotes pseudoinverse.

*Proof:* See [7].

Similarly to [6], we have

**Proposition 4:** \(f^H \hat{H}_1 \neq 0 \) iff \( f^H (R_{11} - R_{12} R_{22}^+ R_{21}) \neq 0 \).

Therefore we can find the zero-forcing detector by optimizing

\[
\min_{f^H \neq 0} \| f^H R_{12} R_{22}^+ R_{21} \|^2
\]

(20)

Although \( u \) is estimated just as in [5], we only require \( f^H u \neq 0 \) in this case. Therefore the influence of the estimation error of \( u \) is not as significant as in [5].

4.2. Adaptive algorithm

Based on (15) and \( \varepsilon_2(n) = \hat{H}_1 \hat{b}_1 \), we can also solve the following minimization problem to find \( f \) that satisfies (15),

\[
\min_{f} J(n) = \| f^H \varepsilon_2(n) \|^2 \quad \text{subject to} \quad f^H \varepsilon_1(n) \neq 0.
\]

(21)

Since \( u \) can be estimated from (16), (21) becomes

\[
\min_{f} \left\{ J(n) = \| f^H \varepsilon_2(n) \|^2 + \alpha( f^H u - 1)^2 \right\}
\]

(22)

The adaptation of \( f \) by a stochastic gradient algorithm is

\[
f(n+1) = f(n) - \mu [\varepsilon_2(n) \varepsilon_2^H(n) f(n) + \alpha (f^H(n) u(n) - 1) u(n)].
\]

(23)

where \( u(n) \) can also be recursively estimated.

5. BLIND MMSE MULTIUSER DETECTION

Suppose \( \mathcal{X}(n) \) contains noise. The MMSE detector \( \mathbf{m} \) with delay \( d_f \) and dimension \( NLL_c \times 1 \) satisfies

\[
\min_{\mathbf{m}} \| \mathbf{b}_1(n - d_f) - \mathbf{m} \mathcal{X}(n) \|^2.
\]

(24)

After some deduction, one obtains

\[
\mathbf{m} = R_{11}^{-1} H_1
\]

(25)

5.1. Batch blind MMSE detection/equalization

Define the following optimization problem

\[
\min_{\mathbf{m}} J(n) = \| \mathbf{m}^H \mathcal{X}(n) \|^2 \quad \text{subject to} \quad \mathbf{m}^H u = 1.
\]

(26)

Using Lagrange optimization method, one can readily obtain [8]

\[
\mathbf{m} = (u^H R_{11}^{-1} u)^{-1} R_{11}^{-1} u
\]

(27)

Comparing (27) with the MMSE equalizer (25) we find that the only difference is a scalar factor. Thus the optimization of (26) yields an MMSE detector. The advantage is that, similar to the zero-forcing method in Section 4, the role of \( u \) is to make \( \mathbf{m} \) non-zero thus the method may be robust to the estimation errors of \( u \).

5.2. Adaptive blind MMSE detection/equalization

In order to adaptively evaluate (26), we use the instantaneous value of \( R_{11} \) to obtain

\[
\min_{\mathbf{m}} J(n) = \| \mathbf{m}^H \mathcal{X}(n) \|^2 + \alpha(\mathbf{m}^H u - 1)^2.
\]

(29)

Compared with the zero-forcing detector estimation introduced in Section 4.2, the only difference is that we use \( \mathcal{X}(n) \) here instead of \( \varepsilon_2(n) \). Therefore the adaptive method discussed in Section 4.2 applies here. To summarize, we optimize

\[
\min_{\mathbf{m}} J(n) = \| \mathbf{m}^H \mathcal{X}(n) \|^2 + \alpha(\mathbf{m}^H u - 1)^2.
\]

(29)

The adaptation of \( \mathbf{m} \) by a stochastic gradient algorithm is

\[
\mathbf{m}(n+1) = \mathbf{m}(n) + \mu [ \mathcal{X}(n) \mathcal{X}^H(n) \mathbf{m}(n) + \alpha (\mathbf{m}^H u(n) - 1) u(n)]
\]

(30)

where \( u(n) \) can be obtained by linear prediction procedures of Section 3.

6. SIMULATIONS

In all simulations, the channel response of each user is randomly generated [1] [5] by \( g(t) = \sum_{l=1}^{L_d} \alpha q(t - \tau_0) \) where \( L_d \) is the total number of multipaths, \( \tau_0 \) is the associated delay of the qth path, \( \alpha_q \) is the attenuation of the qth path, and \( q(t) \) is raised-cosine pulse function. \( g(t) \) is then sampled and truncated to the length \( L_g = 30 \). The user delay \( d_i \), the multipath delay \( \tau_0 \), and the number of multipath components \( L_d \) are uniformly distributed within [1 \( L_d \)], [0 \( L_d - 1)] \] respectively. We use Gold sequence of length \( L_g = 31 \). All input symbols are drawn from a BPSK constellation.

In Figs. 2-4 we compare our batch algorithms with the subspace algorithm of [1] (SS) and linear prediction based algorithm of [5] (LP). Note that the MOE algorithms of [2]-[3] will not work in this simulation due to the long channel length. The zero-forcing and MMSE adaptive algorithms are then compared with the LMS type linear prediction (LP) adaptive algorithm of [5] in Fig. 5. From the simulations, we see that our algorithms have better performance. Our MMSE algorithms are slightly better than our zero-forcing algorithms.

The initial timing estimation is similar to [5]. When channel order is overestimated, our algorithms is also robust to timing estimation errors just as [5].
Figure 2: 1000 symbols. $L_g = 30$, $J = 10$, Near-far ratio 10dB.

Figure 3: 1000 symbols, $L_g = 30$, SNR=5dB, near-far ratio 10dB.

Figure 4: 1000 symbols, $L_g = 30$, $J = 10$, SNR=5dB.

Figure 5: Performance of the adaptive algorithms. $L_g = 30$, $J = 10$, Near-far ratio 10dB, SNR=10dB.

7. CONCLUSION

In this paper we consider blind joint multiuser detection and channel equalization for CDMA system with multipath channels. We use linear prediction to estimate channel matrix vector subspaces. After that, both zero-forcing detector and MMSE detector can be obtained without explicit channel estimation. Both batch algorithms and adaptive algorithms are obtained. These new methods have better performance when channel estimation error is inevitable.

8. REFERENCES