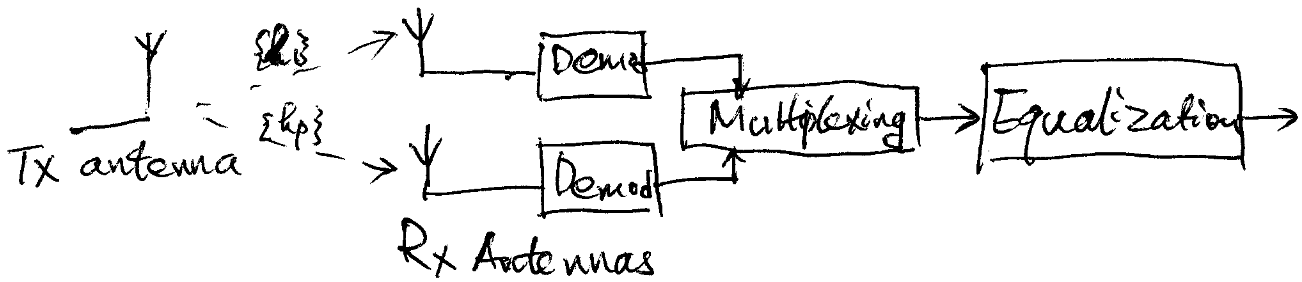
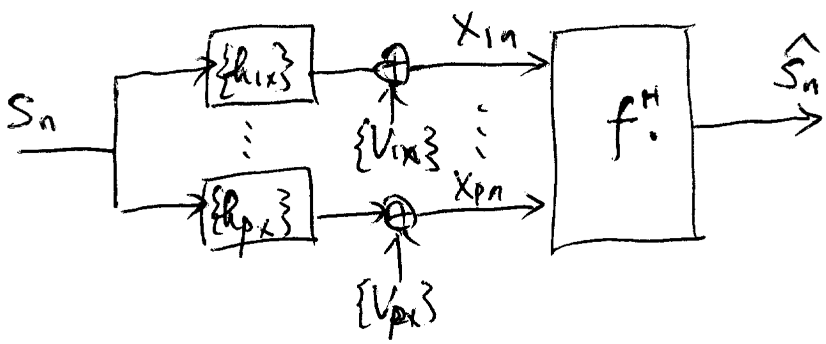
2.2. Blind Equalization with Second-order Statistics

- Perfect equalization
- Fractional space ~ T-space
- Implemented by ~~a~~ receive antenna array or over sampling





- Signal Models:

i)
$$\begin{bmatrix} x_{1n} \\ \vdots \\ x_{pn} \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1L} \\ \vdots & & \vdots \\ h_{p1} & \cdots & h_{pL} \end{bmatrix} \begin{bmatrix} s_n \\ \vdots \\ s_{n-L} \end{bmatrix} + \begin{bmatrix} v_{1n} \\ \vdots \\ v_{pn} \end{bmatrix}$$

ii)
$$\begin{bmatrix} x_{1n} \\ \vdots \\ x_{pn} \\ \vdots \\ x_{1,n-N} \\ \vdots \\ x_{p,n-N} \end{bmatrix} = \begin{bmatrix} \boxed{\begin{matrix} h_{11} & \cdots & h_{1L} \\ \vdots & & \vdots \\ h_{p1} & \cdots & h_{pL} \end{matrix}} & & \\ & \ddots & \\ & & \boxed{\begin{matrix} h_{1,1} & \cdots & h_{1,L} \\ \vdots & & \vdots \\ h_{p,1} & \cdots & h_{p,L} \end{matrix}} \end{bmatrix} \begin{bmatrix} s_n \\ \vdots \\ s_{n-N-L} \end{bmatrix} + \begin{bmatrix} v_{1n} \\ \vdots \\ v_{pn} \\ \vdots \\ v_{1,n-N} \\ \vdots \\ v_{p,n-N} \end{bmatrix}$$

$$\Longrightarrow \quad \underbrace{\underline{X}(n)}_{[P(N+1)]\times 1} = \underbrace{\mathcal{H}}_{[P(N+1)]\times(N+L+1)} \underbrace{\underline{S}(n)}_{(N+L+1)\times 1} + \underbrace{\underline{V}(n)}_{[P(N+1)]\times 1}$$

2.2.1 Fractional-Spaced CMA.

- Used to directly estimate equalizer $\underline{f}$
- Similar to previous CMA
- In fractional space, ~~we~~ it has global Convergence

- $\min J = E[(|\underline{f}^H \underline{x}(n)|^2 - R_2)^2]$

- Update Rule:

$$\underline{f}_{n+1} = \underline{f}_n - \mu \cdot 2[|\underline{f}^H \underline{x}(n)|^2 - R_2]\, \underline{X}(n)\, \underline{X}^H(n)\, \underline{f}_n$$

- Algorithm:

  i) Construct the received samples from P antennas.

  ii) Construct sample vectors $X(n)$

  iii) for $n = 1, 2, \ldots$

  Update $\underline{f_n}$,

  calculate instant error ε_n

  iv) Check SER

2.2.2. Subspace Algorithm for Channel Estimation

- $R_x = E[\underline{x}(n)\,\underline{x}^H(n)] = \mathcal{H}\,\mathcal{H}^H \sigma_s^2 + \sigma_v^2 I$

- $R_y = R_x - \sigma^2 I$ has rank $(N+L+1)$

- Singular-value decomposition (SVD)

$$\underbrace{[U_s}_{[P(N+1)]\times[N+L+1]} \quad \underbrace{U_0]}_{[P(N+1)]\times[P(N+1)-N-L-1]} \begin{bmatrix} \Sigma_s & \\ & 0 \end{bmatrix} \begin{bmatrix} V_s^H \\ V_0^H \end{bmatrix} = R_y$$

- Since $U_0^H R_y = 0$, we have

$$U_0^H \mathcal{H} = 0$$

- Channel estimation by solving equation.

$$A \begin{bmatrix} \boxed{\begin{matrix} h_{10} \\ \vdots \\ h_{1L} \end{matrix}} \\ \vdots \\ \boxed{\begin{matrix} h_{P0} \\ \vdots \\ h_{PL} \end{matrix}} \end{bmatrix} = 0$$

- Algorithm:

  i) Construct samples $\underline{x}_{in}$ from P antennas

  ii) Construct sample vectors $\underline{X}(n)$

  iii) Calculate $R_X = E[\underline{X}(n)\underline{X}^H(n)]$

  iv) Calculate SVD: $[U_s \;\; U_0]\begin{bmatrix}\Sigma_s & \\ & \Sigma_v\end{bmatrix}\begin{bmatrix}U_s^H \\ V_0^H\end{bmatrix} = R_X$.
  Determine rank $N+L+1$.

  v) Restructure U_0 to A

  vi) Solve $A\underline{h} = 0$, by SVD & estimate channel $\underline{h}$

  vii) Check MSE.

- Sample MATLAB algorithm

- Plot: MSE $\sim$ SNR
  MSE $\sim$ T (sample amount)