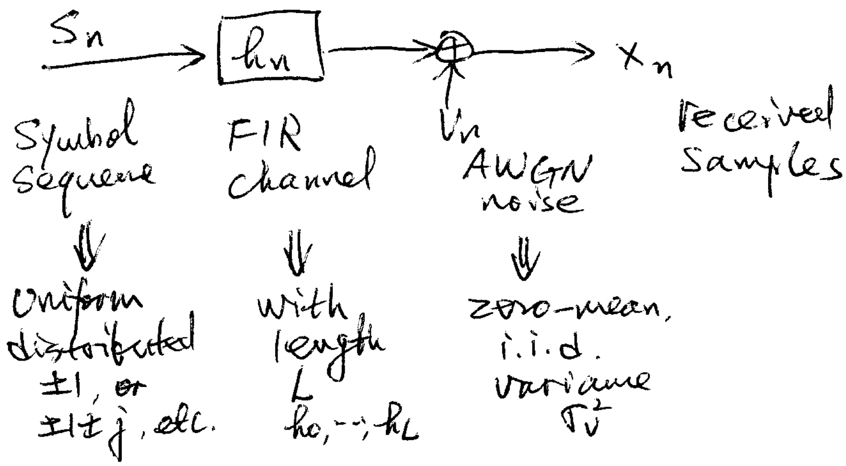
Principle and Simulation
Blind Channel Estimation and Equalization

Outline

1. Training-based methods
    - Channel estimation
    - Equalization
    - Adaptive equalization
2. Blind methods
    - CMA
    - Subspace method
3. CDMA
    - Rake receiver
4. Some MAC-Layer protocol

1. Training-based Methods

1.1 Signal model



- $x_n = \sum_{l=0}^{L} h_l s_{n-l} + v_n$ $\Longrightarrow$ convolution model

- $x_n = [h_0 \cdots h_L] \begin{bmatrix} s_n \\ \vdots \\ s_{n-L} \end{bmatrix} + v_n$ $\Longrightarrow$ vector model

  $x_n = \underline{h}^T \underline{s}_n + v_n$

- $$\begin{bmatrix} x_n \\ \vdots \\ x_{n-N} \end{bmatrix} = \begin{bmatrix} h_0 & \cdots & h_L & & \\ & \ddots & & \ddots & \\ & & h_0 & \cdots & h_L \end{bmatrix} \begin{bmatrix} s_n \\ \vdots \\ s_{n-N-L} \end{bmatrix} + \begin{bmatrix} v_n \\ \vdots \\ v_{n-N} \end{bmatrix}$$

  $(N+1)\times 1$ $\quad$ $(N+1)\times(N+L+1)$ $\quad$ $(N+L+1)\times 1$ $\quad$ $(N+1)\times 1$

  or: $\underline{X}(n) = \mathcal{H} \cdot \underline{S}(n) + \underline{V}(n)$ $\Longrightarrow$ Matrix model

1.2. Training Channel Estimation.

* Let $s_0, \cdots, s_M$ be known

  Let $N = L$,

  Then:

$$\boxed{E[\underline{x}(n) s^*_{n-L}] = \begin{bmatrix} h_L \\ \vdots \\ h_0 \end{bmatrix}} \quad \cdots\cdots (1)$$

  in practice, $E[\underline{x}(n) s^*_{n-L}] = \left(\sum_{n=L}^{M} \underline{x}(n) s^*_{n-L} \right) \frac{1}{M-L+1}$

* Calculate Mean-square error of channel estimator.

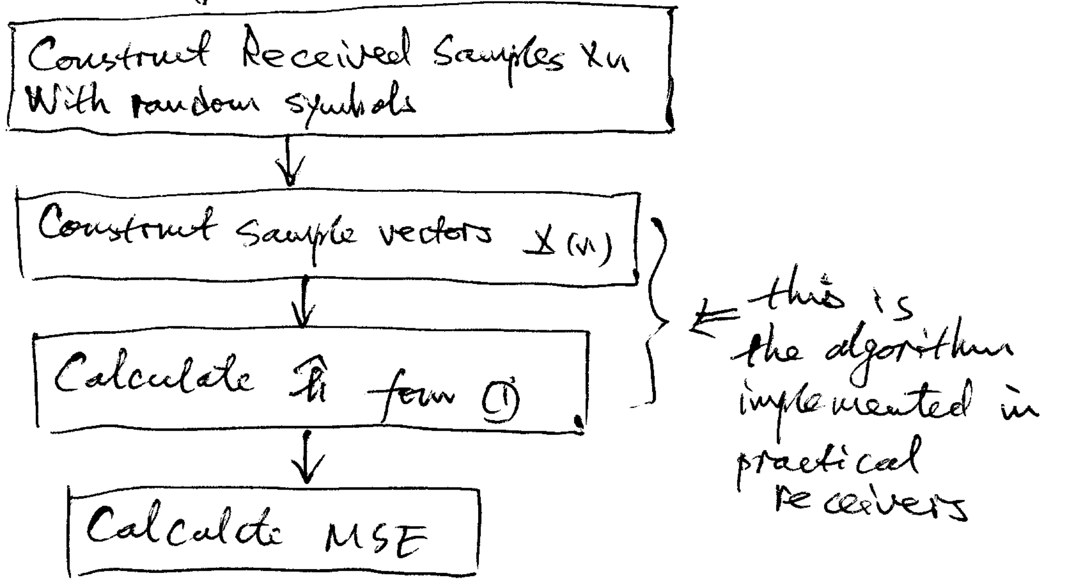
  From (1), we have estimation:

$$\hat{\underline{h}} = \begin{bmatrix} h_0 \\ \vdots \\ h_L \end{bmatrix}$$

  Normalized both $\underline{h}$ and $\hat{\underline{h}}$: $\underline{h}/\|\underline{h}\|$, $\hat{\underline{h}} = \hat{\underline{h}}/\|\hat{\underline{h}}\|$

$$MSE = \|\underline{h} - \hat{\underline{h}}\|$$

1.3. Similation Algorithms:

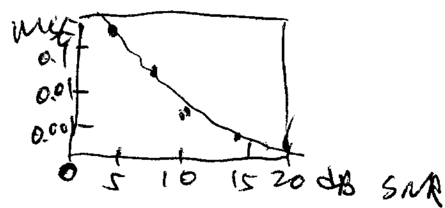


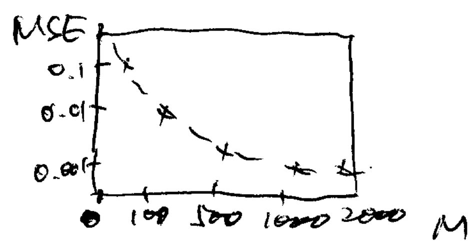
- Sample MATLAB files

- Try to obtain performance plots.

  MSE ~ SNR

  MSE ~ M (training length)





1.4. Traing-based equalization

1.4.1. With estimated channel,

using Viterbi Algorithm, or zero-forcing equalization.

$$\boxed{\hat{\underline{S}}(n) = \mathcal{H}^{+} \underline{X}(n)}$$

$$= \mathcal{H}^{+}\mathcal{H}\, \underline{S}(n) + \mathcal{H}^{+}\underline{V}(n)$$

$$\simeq I\, \underline{S}(n) + \mathcal{H}^{+} \underline{V}(n)$$

1.4.2. MMSE equalization.

- Estimate FIR equalizer $\underline{f} = \begin{bmatrix} f_0 \\ \vdots \\ f_N \end{bmatrix}$ such that
  $\underline{f}^H \underline{x}(n) = s_{n-d}$.
- Estimate $\underline{f}$ by $\min E\left[\|s_{n-d} - \underline{f}^H \underline{x}(n)\|^2\right]$
- Result:

$$\boxed{\underline{f} = \underline{R}_x^{-1} \hat{\underline{h}}_d}$$

where $\underline{R}_x = E[\underline{x}(n)\underline{x}^H(n)]$ ⟸ all available samples

$\hat{\underline{h}}_d = E[\underline{x}(n)\, s_{n-d}^{*}]$ ⟵ training sequence

- In practice:

$$\underline{R}_x = \frac{1}{K}\sum_{n=1}^{K} \underline{x}(n)\underline{x}^H(n)$$

$$\hat{\underline{h}}_d = \frac{1}{M}\sum_{n=1}^{M} \underline{x}(n)\, s_{n-d}^{*}$$

1.4.3. Simulation of Batch Algorithm

① Transmit K symbols, the first M is assumed known

② Obtain received samples x_n

③ Construct sample vectors $\underline{x}(n)$

④ Calculate R_x and $\hat{h}_d$

⑤ Calculate $\underline{f} = R_x^{-1} \hat{h}_d$

⑥ Calculate symbol-error-rate (SER):

i) use $\underline{f}^H \underline{x}(n)$ to estimate symbol $\hat{s}_{n-d}$.

ii) Compare $\hat{s}_{n-d}$ with s_{n-d}.