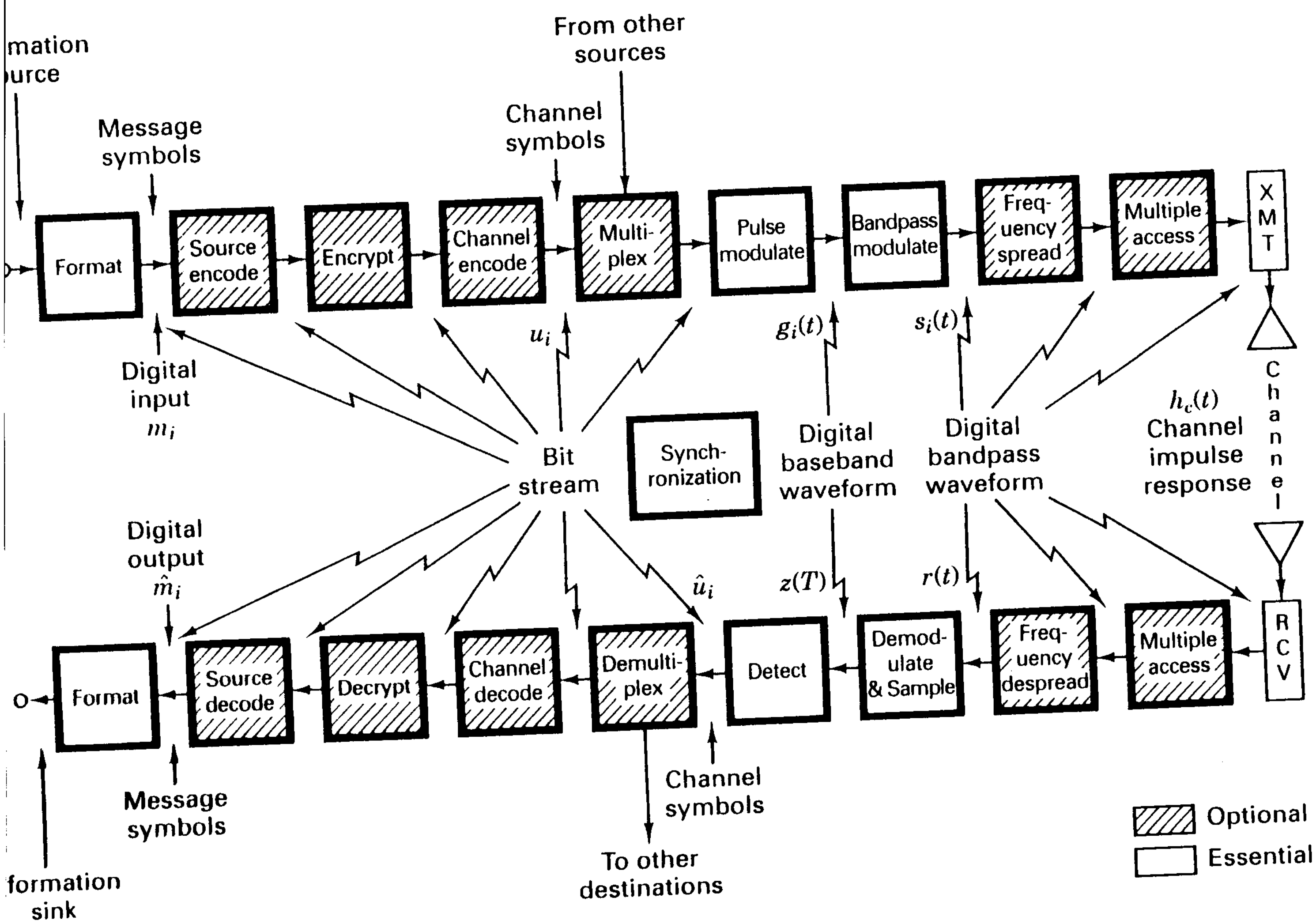
Block Diagram of Digital Comm. Systems



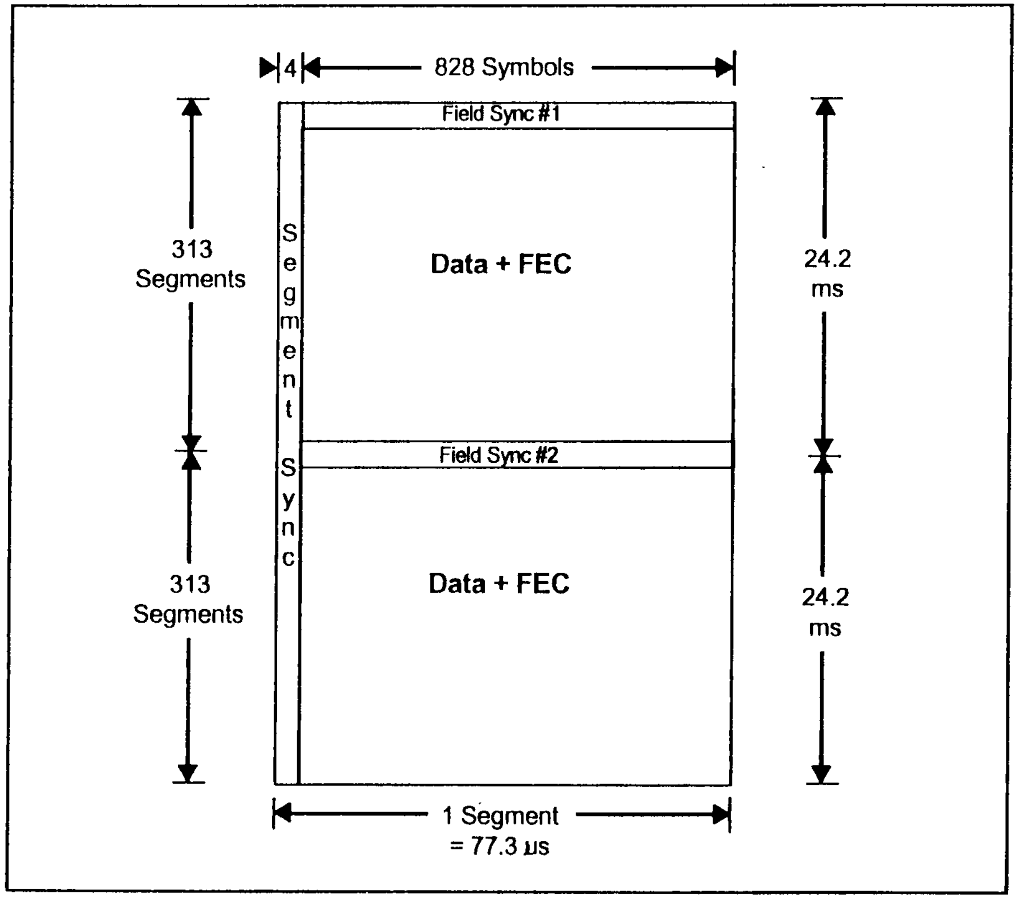


Figure 2. VSB data frame. Synchronization

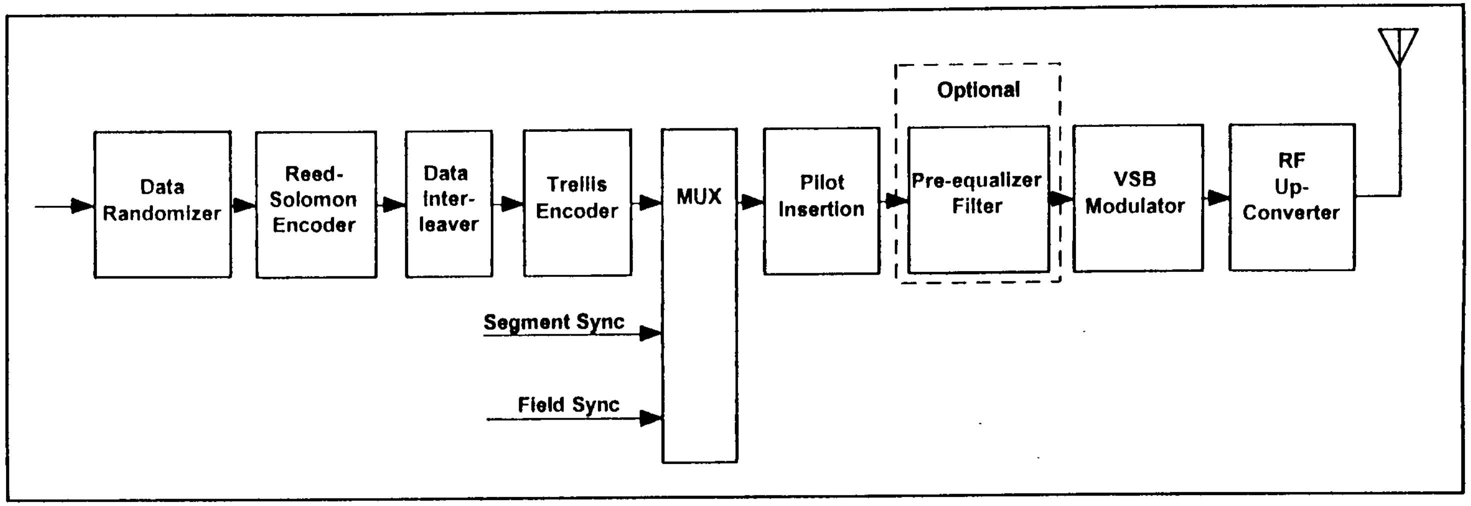


Figure 1. VSB transmitter. HDTV Transmitter

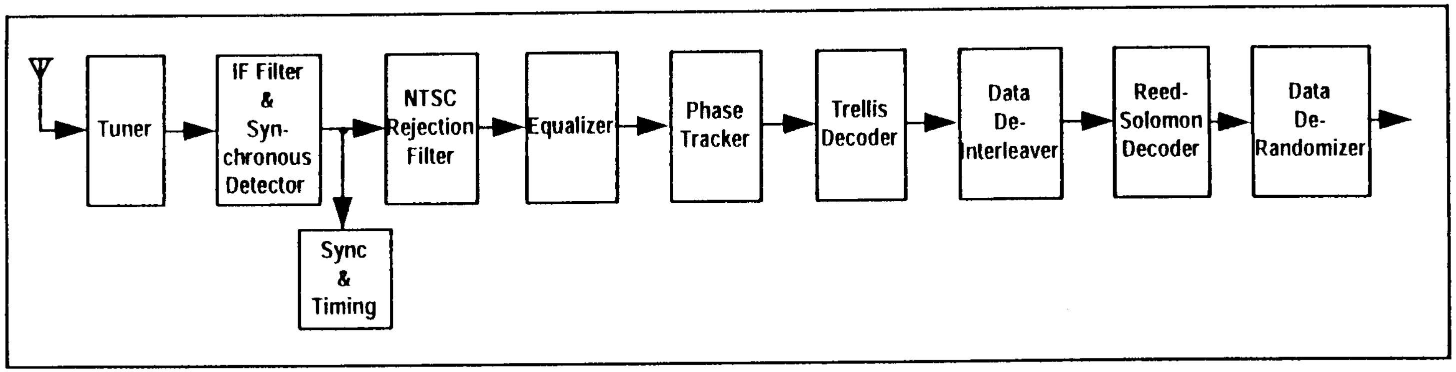


Figure 10.1. VSB receiver. HDTV Receiver

IS-95 CDMA Cellular Phone

1. base station → mobile user

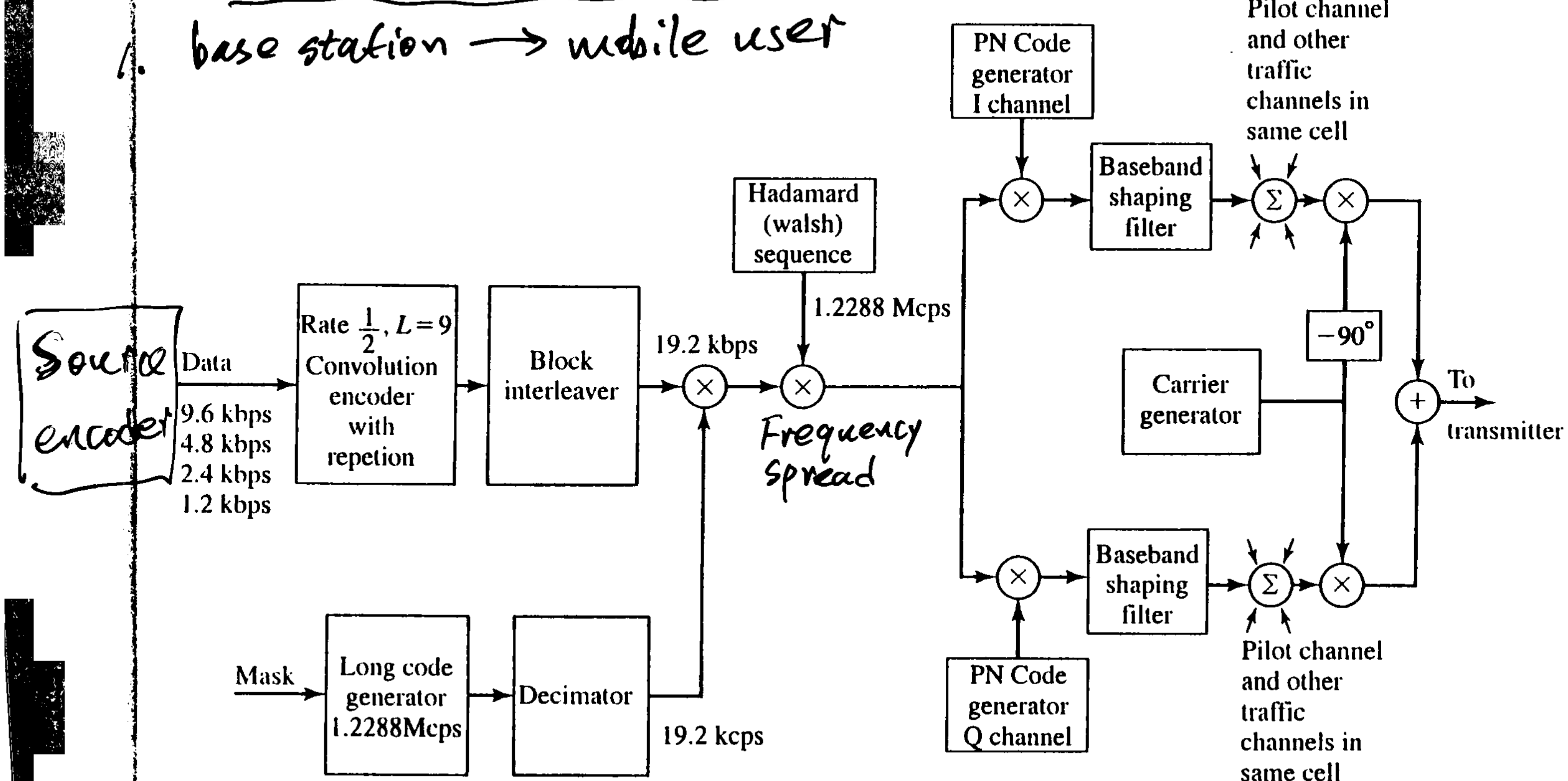


Figure 10.65 Block diagram of IS-95 forward link.

IS-95 CDMA Cellular Phone

2. Mobile user ⟶ base station

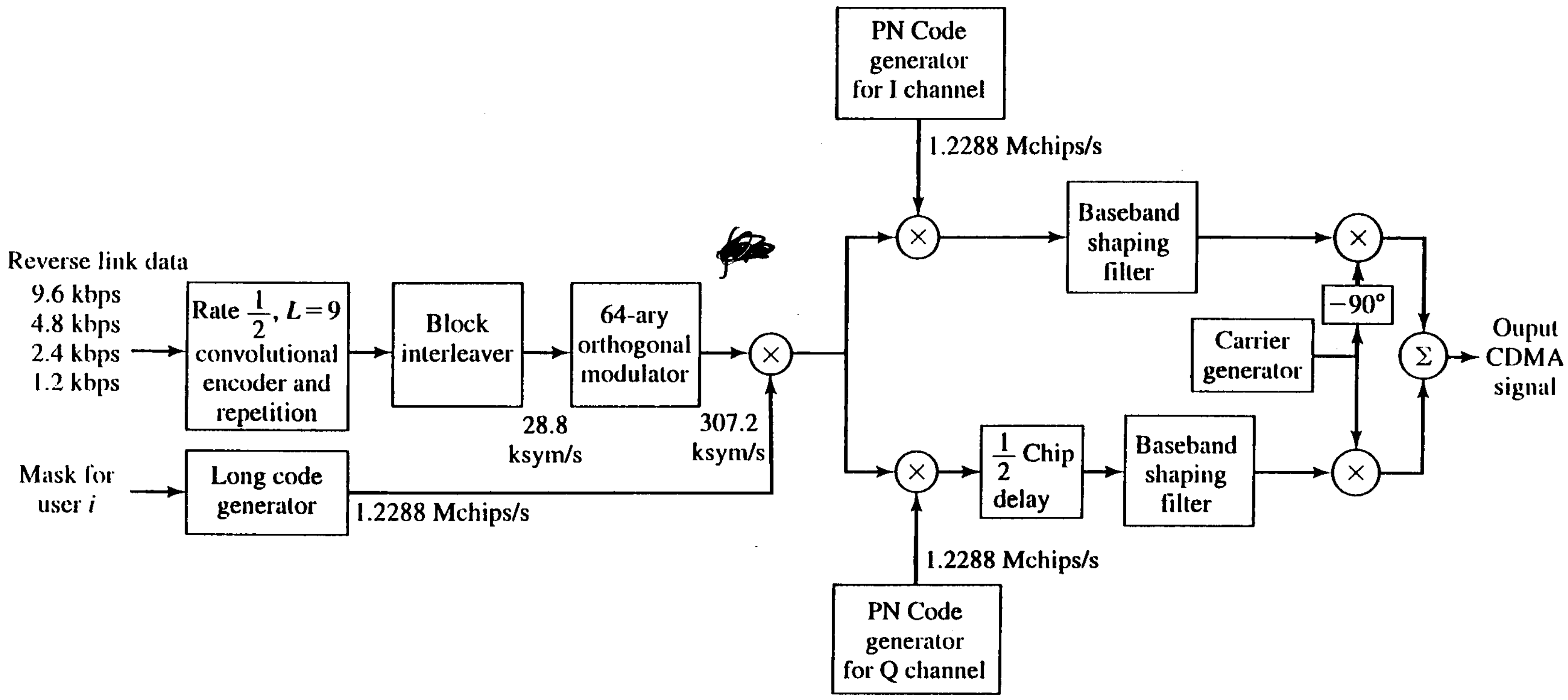


Figure 10.66 Block diagram of IS-95 reverse link.

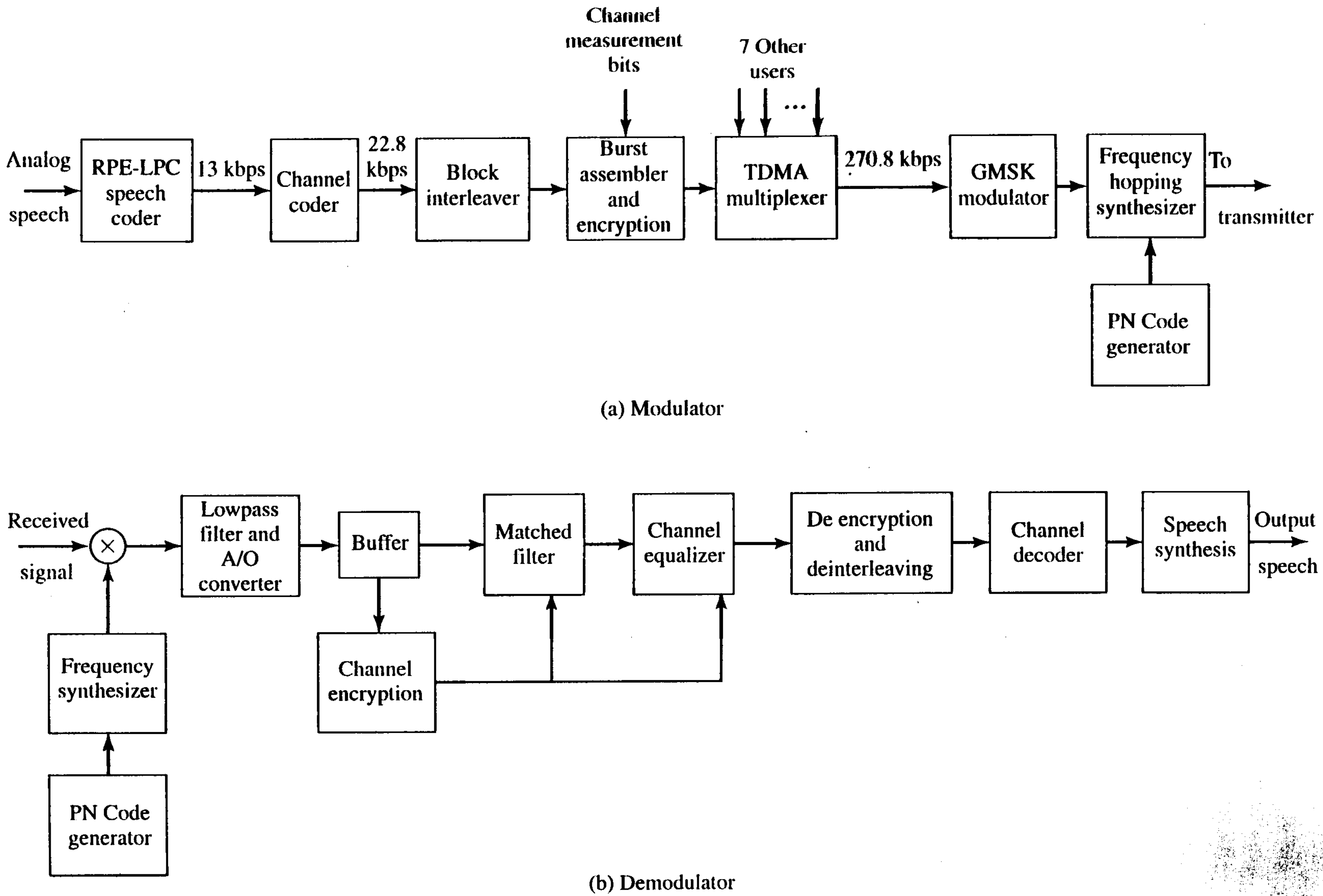


Figure 10.64 Functional block diagram of modulator and demodulator for GSM.

Global System of Mobile Communications

- Baseband ?
  passband ?

- Background knowledge:
  1. Probability and random process
  2. Signal processing: filter, Fourier
  3. System theory
  4. Analog communications

Chapter 1: Signals and Spectra (A Review)

1.1.3 Basic nomenclature

- Information source $\longrightarrow$ binary sequence
- bit $\longrightarrow$ symbol $(M = 2^k)$
- Symbol alphabet, symbol stream
- Data rate: (bits/sec)

  Symbol rate R_s, bit rate R_b

  Symbol interval T_s, bit interval T_b

  what are their relationship?
- Symbol ~~sequ~~ stream $\longrightarrow$ digital waveform
- Probability of error (P_E) v.s.

  Signal to noise ratio (SNR)

Chapter 1.3 ~ 1.4

1) Auto-correlation

Power Signal	Energy Signal
$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,x(t+\tau)\,dt$	$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\,x(t+\tau)\,dt$

• Property: i) $R_x(\tau) \le R_x(0)$: signal power/energy

ii) $R_x(\tau) = R_x(-\tau)$

iii) $R_x(\tau) \underset{\text{IFT}}{\overset{\text{FT}}{\longleftrightarrow}} \begin{cases} G_x(f) : \text{PSD} \\ \Psi_x(f) : \text{ESD} \end{cases}$

• Example: $x(t) = \sin(2\pi f_c t)$, $R_x(\tau) = ?$

2) Spectral density.

Power Signal (Power Spectral density)	Energy Signal (Energy Spectral density)
$G_x(f) = \lim_{T\to\infty} \frac{1}{T} \lvert X_T(f) \rvert^2$	$\psi_x(f) = \lvert X(f) \rvert^2$

- Property: i) real, non-negative, even
  ii) distribution of signal power/energy in the frequency domain
  iii) Averagy power: $P_x = \int_{-\infty}^{\infty} G_x(f)\,df$
  $= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\,dt$
  Averagy energy: $E_x = \int_{-\infty}^{\infty} \psi_x(f)\,df$

- Example: Find the spectral density of
  $x(t) = \sin(2\pi f_c t)$

3. Applications.

- $R_x(\tau)$: correlator receiver
- $G_x(f)$: noise power

1.5 Random signals

1.5.1 Random variable X(A)

- Propobability density function (pdf)

  $p_X(x)$: $\quad P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} p_X(x)\,dx$

- Property: i) $p_X(x) \geq 0$

  ii) $\int_{-\infty}^{\infty} p_X(x)\,dx = 1$

- Averagy parameters

  i) mean: $m_X = E[X] = \int_{-\infty}^{\infty} x\, p_X(x)\,dx$

  ii) Variance: $\sigma_X^2 = E\{[X - m_X]^2\}$

  $= E[X^2] - m_X^2$

- Two important random variables

  i) Gaussian distribution

  ii) Uniform distribution

1.5.2 Random Process:

- $X(A, t)$ — A: event, t: time

  Compare: $X(A, 0)$, $X(0, t)$

- Stationary

- Averagy parameters

  i) mean: $m_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x(t) P_X(x)\, dx$

  ii) Auto-correlation:

  $$R_X(\tau) = R_X(t_1 - t_2) = E[X(t) X(t+\tau)]$$

  $$\left(= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, dt \right)$$

  iii) Power spectral density (PSD)

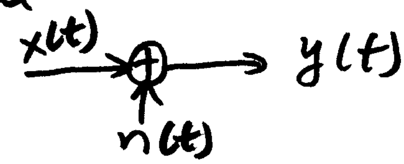
  $G_X(f)$

- Important random processes

  i) AWGN (additive white Gaussian noise)

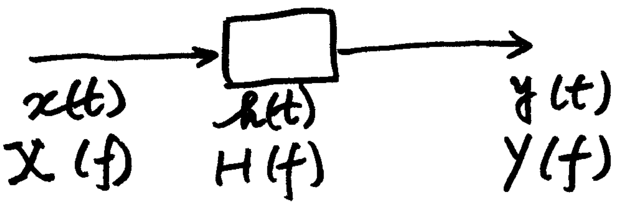
  $G_N(f) = \frac{N_0}{2}$, $P_N(n)$: Gaussian

  ii) Received noisy signal



1.6 Signal transmission through linear systems

- Model: deterministic signals



$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$Y(f) = X(f)\, H(f)$$

- Model: random process signals

$$\begin{cases} G_y(f) = G_x(f)\, |H(f)|^2 \\ y(t) = x(t) * h(t) \end{cases}$$

- Ideal filter: no distortion.
  with constant magnitude response
  and linear phase shift

$$H(f) = |H(f)|\, e^{-j\theta(f)}$$

where $\begin{cases} H(f) = \begin{cases} 1, & \text{passband} \\ 0 & \text{stop band} \end{cases} \\ \theta(f) = 2\pi f t_0 \end{cases}$

$h(t) = ?$

- Example:
  input: white noise with PSD: $G_n(f) = \frac{N_0}{2}$.
  system: ideal lowpass filter
  What are output?

Chapter 2. Baseband Modulation

- Terminology
  - i) Format
  - ii) Transmit Formatting
  - iii) Source coding
  - iv) Line coding
  - v) Baseband
  - vi) Baseband signaling (waveform)

- Compare:

$$\text{PAM} \xrightarrow[\text{Source coding}]{\text{Format}} \text{PCM} \xrightarrow[\text{Line coding}]{\text{Baseband}} \text{baseband waveform}$$

2.5 Sources of corruption

- Error & noise in sampling / Quantizing
- Noise (AWGN)
- Channel effect: ISI
  (inter-symbol interference)
  (due to: $\begin{cases} \text{limited bandwidth} \\ \text{Multipath propagation} \end{cases}$
- Methods to reduce corruption
  i) Companding
  ii) Optimal detector (matched filter)
  iii) Equalization

- M-ary pulse modulation waveform

1) binary PCM bit sequence → $\boxed{\text{K-bit tuple}}$ → symbol sequence → $\boxed{\text{M-ary pulse waveform}}$ → ?

2) Three ways: $\begin{cases} \text{PAM: amplitude} \\ \text{PPM: position} \\ \text{PDM: width} \end{cases}$

3) PAM: multi-level signaling:
(smallest bandwidth requirement)

$$\boxed{\begin{array}{l} M = 2^k : \text{ \# of levels} \\ \text{Symbol rate } R_s = \dfrac{R_b}{k} \end{array}}$$

- <u>Example:</u> Speech signal with 4 KHz bandwidth, 8KHz sampling, represented by 8-bit PCM. If transmit in an $M = 256$ ary PAM waveform, bandwidth efficiency = ?

CHAPTER 3 Baseband Demodulation
Baseband Detection

- **Definitions:**
  - Demodulator
  - Detector

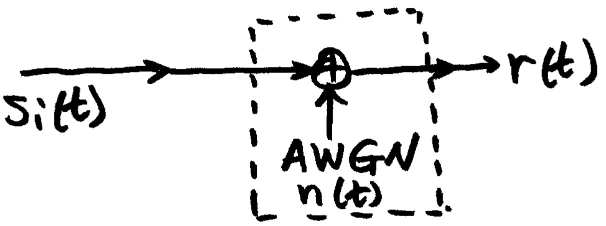
- Objective:
  - obtain best SNR
  - Eliminate ISI

- Equivalence theorem
  - Baseband processing ~ passband processing
  - Baseband channel ~ passband systems

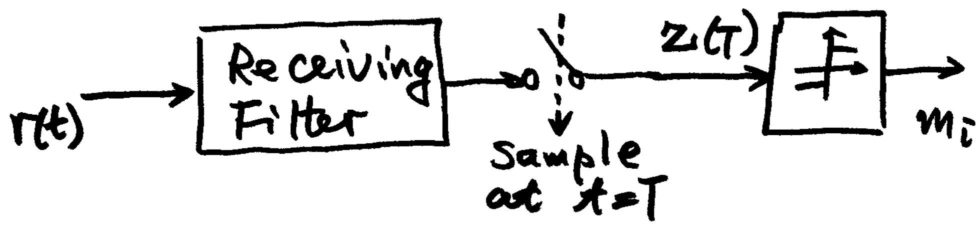
3.1 Systems, signals and noise

- Systems:

  i) Transmitter + channel



  ii) Receiver (demodulator + detector)



- Signals:

  i) Sent: $s_i(t) = \begin{cases} s_1(t), & 0 \le t \le T, \text{ for binary "1"} \\ s_2(t), & 0 \le t \le T, \text{ for binary "0"} \end{cases}$

  ii) Received: $r(t) = s_i(t) + n(t)$, $i = 1, 2$, $0 \le t \le T$

  iii) Noise: $n(t)$: AWGN

  with PSD: $G_N(f) = \frac{N_0}{2}$

  pdf: $f(n_0) = \frac{1}{\sigma_n \sqrt{2\pi}} e^{-\frac{n_0^2}{2\sigma_n^2}}$

  where $\sigma_n^2 = N_0/2$.

- Signal processing procedure

i) Receiving filter $h(t)$

$$y(t) = h(t) * r(t)$$

ii) Sampling at time $t=T$.

$$z(T) = y(T) + n_0(T)$$

which is simplified to

$$\boxed{z = y + n_0}$$

where $y = \pm A$, (corresponding to "1", "0")

n_0: Gaussian, relative to $n(t)$, $h(t)$.

iii) Detection: use threshold γ:

$$\boxed{z \underset{H_2}{\overset{H_1}{\gtrless}} \gamma}$$

where hypothesis: $\begin{cases} H_1: \text{ in favor of binary "1"} \\ H_2: \text{ in favor of binary "0".} \end{cases}$

- Key points:

i) Demodulated by a linear filter $h(t)$

ii) Detected by the value at $t=T$ only, not by all the values in $y(t)$.

- <u>Properties of matched filter</u>

  i) In noiseless case, MF & sampler's output is the energy of the received signal

  $$\boxed{y(T) = \int_0^T s^2(\tau)\,d\tau}$$

  ii) In AWGN case, MF maximize the output (SNR)

  - If $\begin{cases} r(t) = s(t) + n(t), \\ h(t) = s(T-t) \end{cases}$

    then $(SNR)_o \le \frac{2}{N_0} \int_0^T s^2(\tau)\,d\tau = \frac{E_s}{N_0/2}$

    $$\boxed{\text{and } (SNR)_o = \frac{E_s}{N_0/2} \text{ when } h(t) = k\,s(T-t).}$$

  iii) Frequency response:

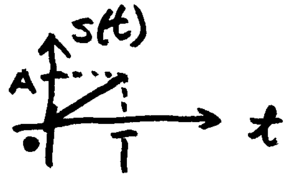
  MF: $H(f) = S^*(f)\,e^{-j2\pi fT}$.

  output: $Y(f) = |S(f)|^2 e^{-j2\pi fT}$.

- Example: If $s(t) = \frac{A}{T}t$.

  what's $h(t)$ MF?

  output $y(T)$?



- Signal space (vector) representation of MF

When dimension $N \geq 2$, vector MF is required.

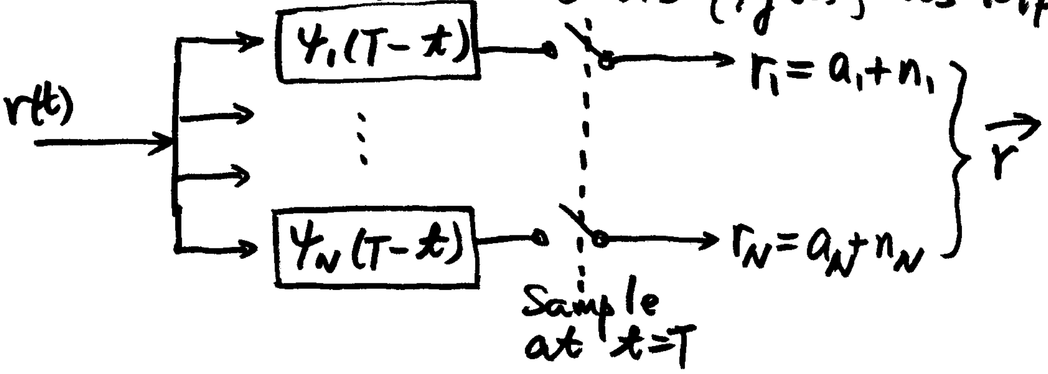
(i) Time-domain Signal | Vector Representation

$$\begin{cases} s(t) = \sum_{j=1}^{N} a_j \psi_j(t) \\ r(t) = s(t) + n(t) \end{cases}$$

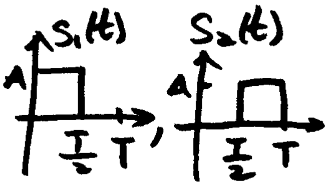
$\vec{s} = [a_1, \cdots, a_N]$

$\vec{r} = [a_1, \cdots, a_N] + [n_1, \cdots, n_N] = \vec{s} + \vec{n}$

(ii) Use the orthonormal basis $\{\psi_j(t)\}$ as MF



Keypoint: the output is directly the vector representation of the received signal.

- Example: For two signal waveforms, 

Find MF and output.

Three Common Signal-Space Representation

1. Binary PAM.

Signal Waveform	Basis	Vector	Constellation
$s_1(t) = A$, $s_2(t) = -A$, $0 \le t \le T$	$\psi(t) = \sqrt{\frac{1}{T}}$, $0 \le t \le T$	$\vec{s_1} = \sqrt{E_b}$, $\vec{s_2} = -\sqrt{E_b}$, $(E_b = A^2T)$	$s_2(t)$: $-\sqrt{E_b}$ $(-A\sqrt{T})$ $s_1(t)$: $\sqrt{E_b}$ $(A\sqrt{T})$ $\psi_1(t)$

2. M-ary PAM. $(M=4)$ $(E_b = A^2T)$

Signal Waveform	Basis	Vector	Constellation
$s_1(t) = 3A$, $s_2(t) = A$, $s_3(t) = -A$, $s_4(t) = -3A$, $0 \le t \le T$	$\psi(t) = \sqrt{\frac{1}{T}}$ $0 \le t \le T$	$\vec{s_1} = 3\sqrt{E_b}$ $\vec{s_2} = \sqrt{E_b}$ $\vec{s_3} = -\sqrt{E_b}$ $\vec{s_4} = -3\sqrt{E_b}$	$\vec{s_4}$ $\vec{s_3}$ $\vec{s_2}$ $\vec{s_1}$ $\psi_1(t)$ $-3\sqrt{E_b}$ $-\sqrt{E_b}$ $\sqrt{E_b}$ $3\sqrt{E_b}$

3. PSK (~~Partial~~) $(E_s = A^2T/2)$

Signal Waveform	Basis	Vector	Constellation
$s_1(t) = A\cos(2\pi f_0 t)$ $s_2(t) = -A\cos(2\pi f_0 t)$ $s_3(t) = A\sin(2\pi f_0 t)$ $s_4(t) = -A\sin(2\pi f_0 t)$ $0 \le t \le T$	$\psi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_0 t)$ $\psi_2(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_0 t)$ $0 \le t \le T$	$\vec{s_1} = [\sqrt{E_s}, 0]$ $\vec{s_2} = [-\sqrt{E_s}, 0]$ $\vec{s_3} = [0, \sqrt{E_s}]$ $\vec{s_4} = [0, -\sqrt{E_s}]$	$\psi_2(t)$ s_3 s_2 s_1 ψ_1 s_4

3.3 Baseband System design (Transmitting Filter Design)

3.3.1 Intersymbol interference (ISI)

- happens when channel impulse response longer than 1 symbol interval, due to:

  e.g. limited bandwidth of channel.
  transmitting / receiving filters
  multipath propagation

- Result: symbol detection errors

- System design objective.

  zero-ISI with limited bandwidth (of transmitted signal)

- Method:

  i) Ideally: Nyquist filter

  ii) Practical: raised-cosine filter

  iii) If channel unknown: equalizer.

3.3.2 Ideal Nyquist filter

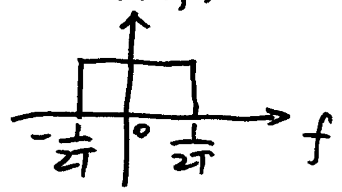
- System:

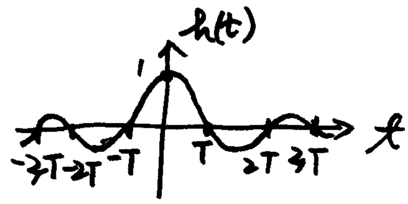
$$\longrightarrow \boxed{\text{Transmitting filter } H_t(f)} \longrightarrow \boxed{\text{channel } H_c(f)} \longrightarrow \boxed{\text{receiving filter } H_r(f)} \longrightarrow$$

$$\begin{cases} H(f) = H_t(f)\, H_c(f)\, H_r(f) \\ h(t) = h_t(t) * h_c(f) * h_r(t) \end{cases}$$

- Design $H_t(f)$ & $H_r(f)$ such that $H(f)$ is Nyquist filter:

$$\begin{cases} H(f) = \begin{cases} T, & -\frac{1}{2T} < f \le \frac{1}{2T} \\ 0, & \text{else} \end{cases} \\ \\ h(t) = \operatorname{sinc}\left(\frac{t}{T}\right) \end{cases}$$





i) Keypoint: $h(t) = 0$ at all $t = kT$.

ii) Bandwidth of transmitted signal:

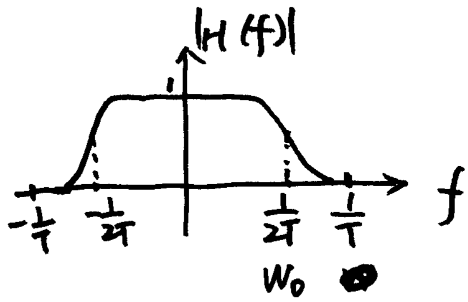
$$\boxed{W = \frac{1}{2T} = \frac{R_s}{2}}$$

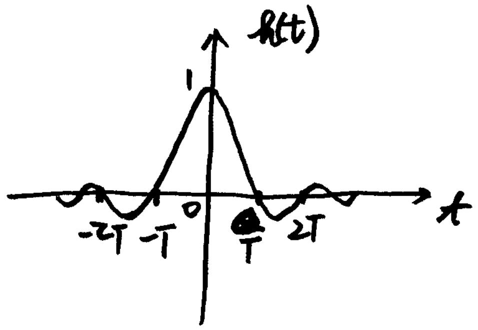
3.3.3. Pulse-shaping with Raised-Cosine filter

- Raised-cosine spectrum and impulse response

$$H(f) = \begin{cases} 1, & |f| < 2W_0 - W \\ \cos^2\left(\frac{\pi}{4}\frac{|f| + W - 2W_0}{W - W_0}\right), & 2W_0 - W < |f| < W \\ 0, & |f| > W. \end{cases}$$

$$h(t) =$$





- Bandwidth of signal:

$$\boxed{W = \frac{1+r}{2T}}$$

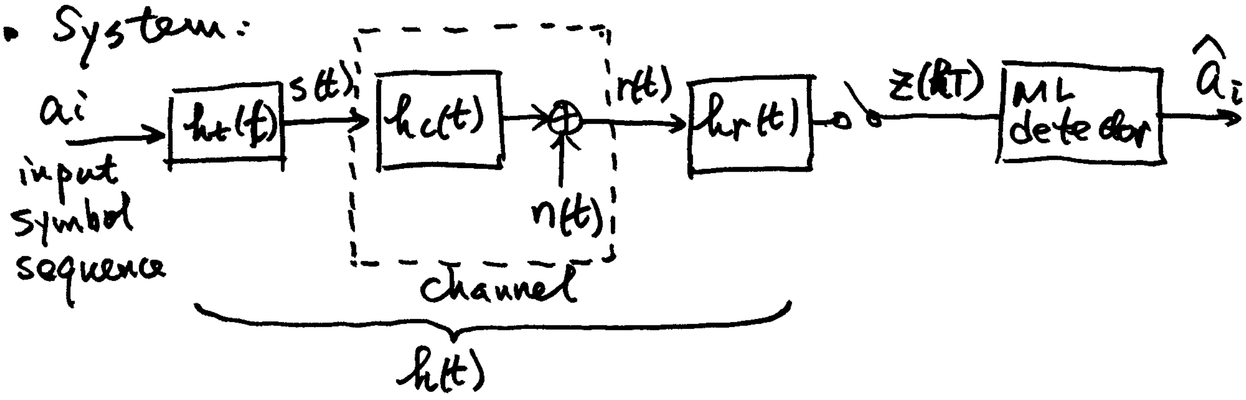
where r is raised-cosine ~~roll~~-off factor:

$$r = \frac{W - W_0}{W_0} = \frac{W - \frac{1}{2T}}{\frac{1}{2T}}, \quad \underline{0 \le r \le 1}$$

- In practice: use the truncated version, e.g., $h(t)$, $-3T \le t \le 3T$

3.3.4 System Response (why zero ISI)

- System:



- Signal analysis:

$$s(t) = \sum_{i=-\infty}^{\infty} a_i h_t(t - iT)$$

$$r(t) = \sum_{i=-\infty}^{\infty} a_i \, h'(t - iT) + n(t)$$

where $h'(t) = h_t(t) * h_c(t)$

$$z(t) = \sum_{i=-\infty}^{\infty} a_i \, h(t - iT) + v(t),$$

where $\begin{cases} h(t) = h_t(t) * h_c(t) * h_r(t) \\ \text{noise } v(t) = n(t) * h_r(t) \end{cases}$

$$z(kT) = \sum_{i=-\infty}^{\infty} a_i \, h((k-i)T) + v(kT)$$

$$= a_k h(0) + \sum_{i \neq k} a_i \, h((k-i)T) + v(kT)$$

$$= \underbrace{a_k h(0)}_{\text{desired signal part}} + \underbrace{v(mT)}_{\text{AWGN part.}}$$

- zero-ISI

3.4 Eye-pattern and equalization

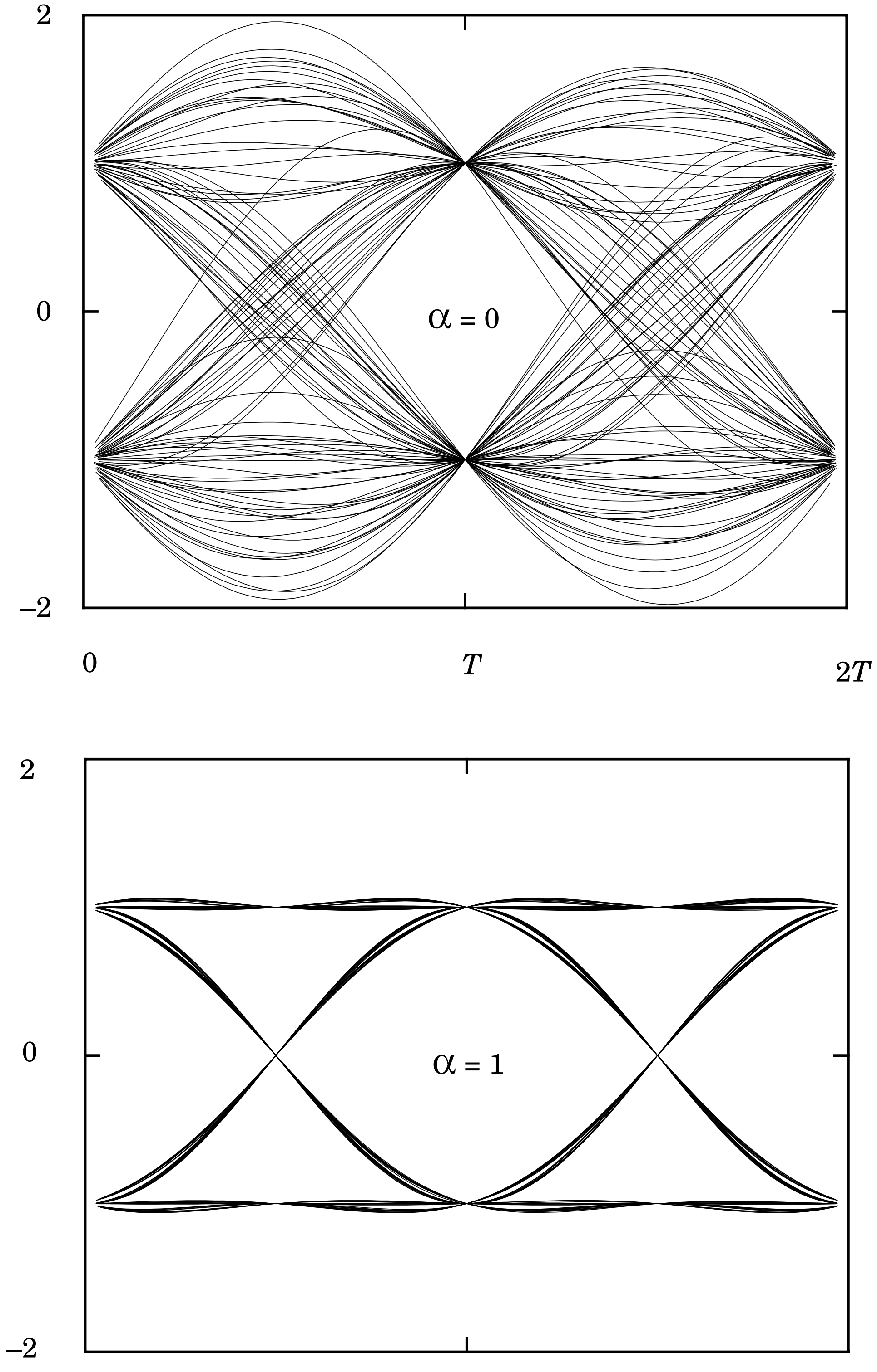
- Eye-pattern: measure ISI, or the reliability of symbol detection

  i) eye-open: small ISI, low BER, reliable

  ii) eye-closed: large ISI, high BER, unreliable

- How to generate eye-pattern?

  plot the received signal $r(t)$ periodically (with period progress of T) on the same time scale.



2
0
−2
α = 0
0
T
2T
2
0
−2
α = 1
0
T
2T

- Equalization:

i) when channel is unknown, or $h_t(t) * h_c(t) * h_r(t)$ has large ISI, use an equalizer to reduce ISI

ii) Equalizer can be implented as linear FIR / IIR filter, or non-linear filter (decision feedback)

iii) linear FIR filter

- zero-forcing equalizer
- MMSE equalizer

iv) Adaptive equalizer

ADAPTIVE EQUALIZATION using the LMS ALGORITHM

Equalizer output:

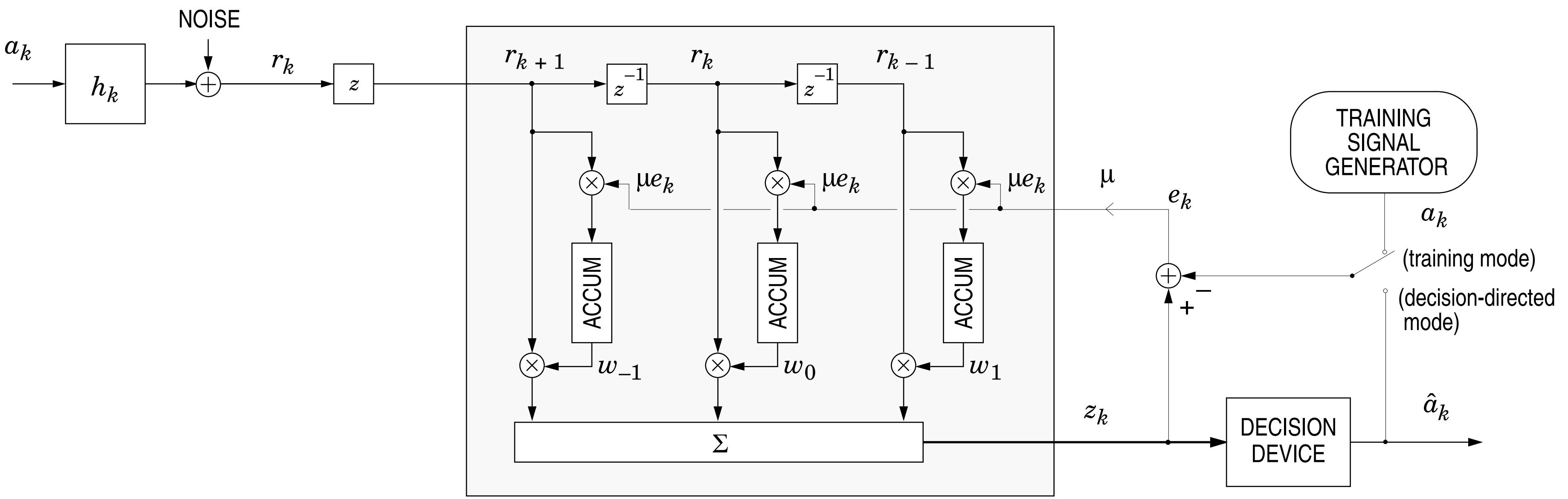
$$\begin{aligned} v_k &= w_k * r_k \\ &= w_{-1} r_{k+1} + w_0 r_k + w_1 r_{k-1} \\ &= [w_{-1}\ w_0\ w_1] \begin{bmatrix} r_{k+1} \\ r_k \\ r_{k-1} \end{bmatrix} = \boldsymbol{w}^T \boldsymbol{r}_k. \end{aligned}$$

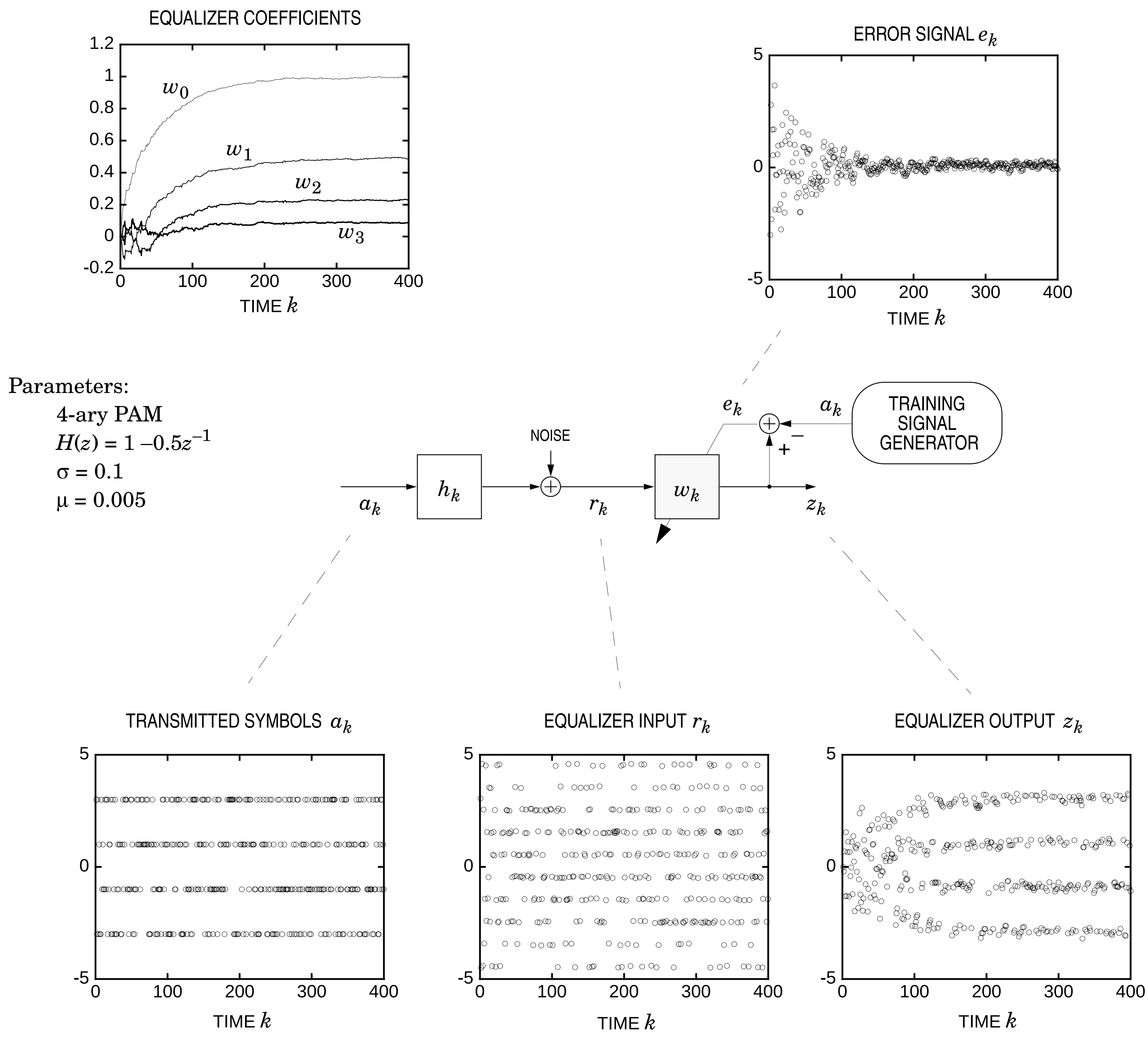
Error:

$$e_k = z_k - a_k.$$

LMS algorithm:

$$\boldsymbol{w}(k+1) = \boldsymbol{w}(k) - \mu e_k \boldsymbol{r}_k^{*}.$$





EQUALIZER COEFFICIENTS
w_0
w_1
w_2
w_3
TIME k
ERROR SIGNAL e_k
TIME k
Parameters:
4-ary PAM
$H(z) = 1 - 0.5z^{-1}$
$\sigma = 0.1$
$\mu = 0.005$
NOISE
a_k
h_k
r_k
w_k
z_k
e_k
TRAINING SIGNAL GENERATOR
TRANSMITTED SYMBOLS a_k
TIME k
EQUALIZER INPUT r_k
TIME k
EQUALIZER OUTPUT z_k
TIME k

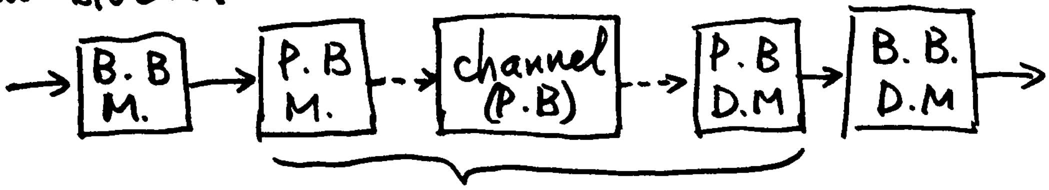
Chapter 4. Bandpass Modulation/Demodulation

4.1. Introduction

- Why use bandpass?

  transmission efficiency, frequency reuse

- System block:



- Modulation methods

  AM → ASK
  PM → PSK
  AM+PM → QAM
  FM → FSK

- Key points:

  i) Signal waveforms (modulator)
  ii) Signal-space representation
  iii) Optimal demodulator (coherent, noncoherent)
  iv) spectral and P_E.

4.2. General passband waveforms

- Carrier: $s(t) = A(t)\cos(\omega t)$.
$$= A(t)\cos(\omega_0 t + \phi(t)).$$

-

Modulation types	varying parameter	Demodulation
PSK	$\phi(t)$	coherent / noncoherent (phase estimation) (Differential encoding)
QAM	$\begin{cases} A(t) \\ \phi(t) \end{cases}$	same
FSK	$\omega_0(t)$	coherent: frequency estimation noncoherent: envelop detector

- Waveform illustration.

4.2.2 PSK

- **Waveform:**

$$S_i(t)=\sqrt{\frac{2E}{T}}\cos[w_0 t+\phi_i(t)],\quad 0\le t\le T,\quad i=1,\cdots,M$$

$$\phi_i(t)=\frac{2\pi i}{M}$$

(consider pulse shaping $g(t)$, we have:

$$S_i(t)=\sqrt{\frac{2E}{T}}\,g(t)\cos[w_0 t+\phi_i(t)]\ )$$

- BPSK: $M=2$, $\phi=\{0,\pi\}$,

  QPSK: $M=4$, $\phi=\{0,\frac{\pi}{2},\pi,\frac{3}{2}\pi\}$ or others

  8-PSK:

- Symbol energy: $E=\int_0^T S_i^2(t)\,dt$

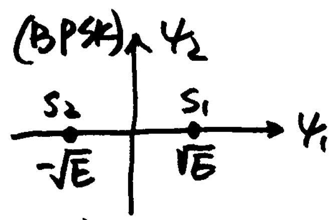
  Bandwidth: $B_W = 2$ Baseband Bandwidth

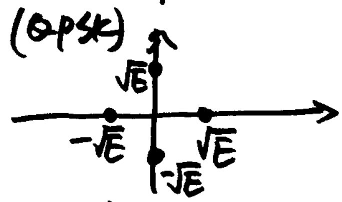
- Signal space representation:

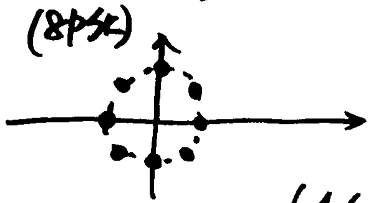
$$\vec{S}_i=[\sqrt{E}\cos\frac{2\pi i}{M},\ \sqrt{E}\sin\frac{2\pi i}{M}],$$

$$i=1,\cdots,M$$

basis:
$$\begin{cases}\psi_1(t)=\sqrt{\frac{2}{T}}\,g(t)\cos(w_0 t)\\ \psi_2(t)=\sqrt{\frac{2}{T}}\,g(t)\sin(w_0 t)\end{cases}$$

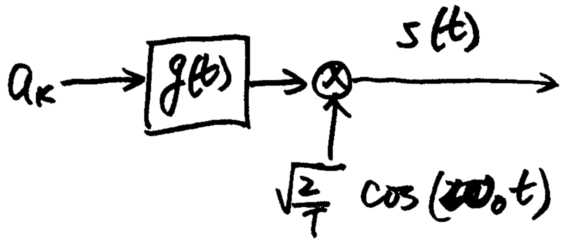




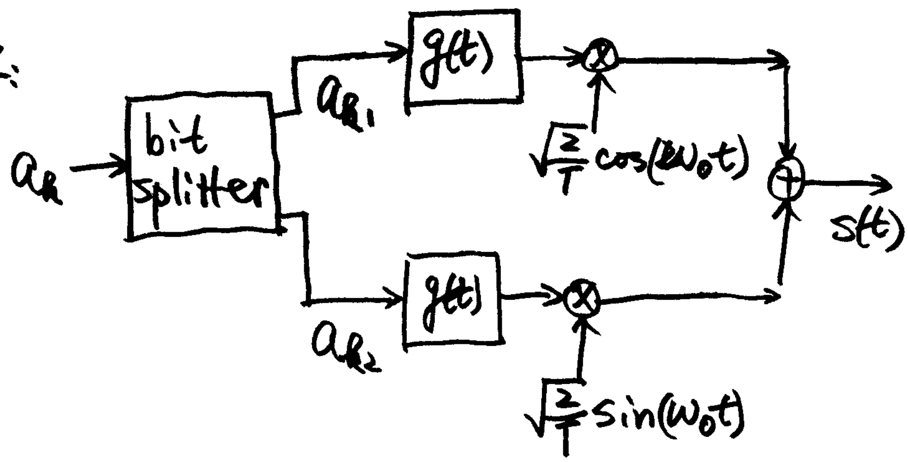


- **Modulator:**

BPSK:



M-PSK:



$$s(t) = a_{k1}\, g(t)\sqrt{\frac{2}{T}}\cos(\omega_0 t) + a_{k2}\, g(t)\sqrt{\frac{2}{T}}\sin(\omega_0 t)$$

- Represented in the complex domain?

$$s(t) = \mathrm{Re}\left\{ a_k\, g(t)\sqrt{\frac{2}{T}}\, e^{j\omega_0 t} \right\}$$

$$a_k = a_{k1} + i\, a_{k2}$$

4.2.3 QAM. (Quadrature Amplitude Modulation)

- Signal waveform

$$S_i(t) = \sqrt{\frac{2E_i(t)}{T}}\, g(t) \cos[\omega_0 t + \phi_i(t)], \quad 0 \le t \le T, \; i = 1, \cdots, M$$

$$= A_i\, g(t) \sqrt{\frac{2}{T}} \cos(\omega_0 t) + B_i\, g(t) \sqrt{\frac{2}{T}} \sin(\omega_0 t)$$

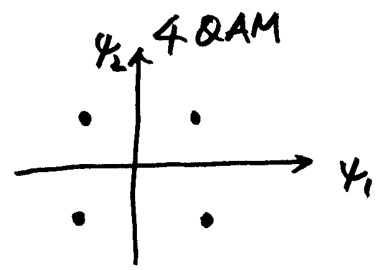
- Eg: 4 QAM,
  16 QAM

- Bandwidth: 2 × Baseband bandwidth

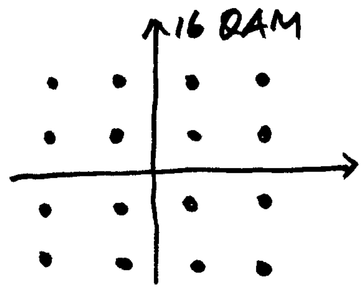
- Signal space representation

$$\vec{S_i} = \left[\sqrt{E_i(t)} \cos\frac{2\pi i}{M}, \sqrt{E_i(t)} \sin\frac{2\pi i}{M}\right]$$

$i = 1, \cdots, M$



basis: $\begin{cases} \psi_1(t) = \sqrt{\frac{2}{T}}\, g(t) \cos(\omega_0 t) \\ \psi_2(t) = \sqrt{\frac{2}{T}}\, g(t) \sin(\omega_0 t) \end{cases}$



- Modulator:

  Similar to M-PSK
  different in magnitude.

4.4. Coherent demodulation/detection

- <u>General rule:</u>

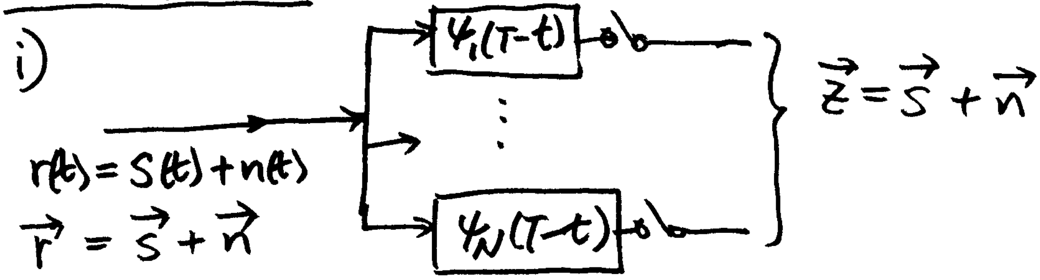
  use basis as MF, output samples are signal-space point value

- <u>General requirement:</u>

  carrier frequency and phase synchronization

- <u>For PSK:</u>

i)

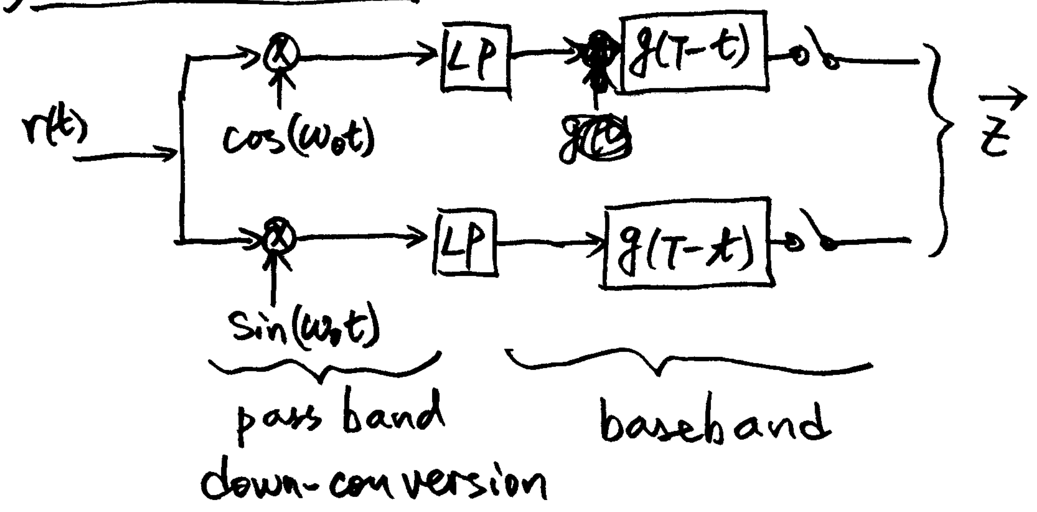


$$\vec{z} = \left[\sqrt{E}\cos\frac{2\pi i}{M}, \sqrt{E}\sin\frac{2\pi i}{M}\right] + [n_1, n_2]$$

where n_1, n_2 are AWGN, zero mean, variance $\frac{N_0}{2}$, independent

ii) Detection: $\min\limits_{\{S_m\}} \|\vec{z} - \vec{S}_m\|^2$

iii) Second view of the demodulator



• **QAM:** similar to PSK

• **FSK:**