Blind Adaptive Equalization of OFDM Transmission with Insufficient Cyclic Prefix

Taewoo Han and Xiaohua (Edward) Li

Abstract - **We create a new blind equalization method with both batch and adaptive algorithms developed for OFDM systems with insufficient cyclic prefix. Special correlation property of cyclic prefix is exploited for constraint estimation. Then constrained minimum-output-energy (MOE) optimization is applied for blind equalizer estimation. Using shorter cyclic prefix, we can either reduce block size for low complexity or enhance bandwidth efficiency. Simulations demonstrate their superior performance.**

Keywords – **cyclic prefix, blind equalization, channel estimation, OFDM**

I. Introduction

For most receivers, the received signal may comprehend interference and noise. Among them, the intersymbol inference (ISI) is a great degradation factor for the system performance, and must be mitigated by equalizations before higher throughput can be obtained. Training based equalization methods are the most widely applied approaches in practice, where training sequences are sent for either channel or equalizer estimation. However, since training sequences reduce system throughput, blind equalization may be more promising. Blind equalization is effective to cross out the repeatedly transmitted training signals to improve system throughput, which certainly advances the ongoing, assiduous research to find an effective, robust and computationally efficient algorithms.

Orthogonal frequency division multiplexing (OFDM) [1] is a strong candidate for many digital communication systems. OFDM has already been used in some broadband systems such as wireless LAN, ADSL, DVB/DAB, etc. OFDM transmits symbols with a cyclic prefix through frequency selective channels which are converted into a parallel of flat fading channels with the help of IFFT. Then equalization can be performed by one-tap equalizers [3]. OFDM requires that the length of cyclic prefix be larger than the length of channel length. Consequently, the bandwidth efficiency drops down because of the overhead of cyclic prefix. To reduce cyclic prefix length for bandwidth efficiency, or to deal with insufficient cyclic prefix, channel shortening methods are widely studied [5, 6]. However, just as time-domain equalization of single-input-single-output (SISO) channels, reduction of SISO channels will introduce residual ISI which the subsequent OFDM demodulation procedure can not eliminate.

The goal of this paper is to generate a new blind equalization method for OFDM transmission with insufficient cyclic prefix. Taking advantage of the special correlation property of the cyclic prefix, we will show that both batch and adaptive algorithms for blind equalization can be successfully developed. They work in systems where the length of cyclic prefix is shorter than that of the channel. Therefore, shorter cyclic prefix length can be used to enhance bandwidth efficiency. On the other hand, they can also be used in systems in case of insufficient cyclic prefix.

For the adaptive algorithm, we are going to employ the efficient Frost's algorithm [2]. It is a well-known dynamic adaptation algorithm for constrained optimization. Our algorithm minimizes the power of the equalizer's output subject to a set of constraints. Note that constrained optimization methods for blind equalization have been proposed in [4] and in multiuser code division multiple access (CDMA) systems.

In this paper, the operator $(\cdot)^*$ denotes conjugation, $(\cdot)^T$ denotes transposition, and $(\cdot)^H$ denotes complex conjugate transposition.

This paper is organized as follows. In Section II, the OFDM system model is described. In Section III, we develop the new method for systems with insufficient cyclic prefix. In Section IV, we expound the batch and adaptive algorithms. In Section V, we put to use simulations to demonstrate their performance. Finally, conclusions are given in Section VI.

II. System Model

Fig. 1 shows the baseband OFDM system with cyclic prefix. Note that although we consider the OFDM system, our approach is not dependent on the IFFT/FFT. Traditionally, equalization is performed after FFT. In this paper, however, as shown in Fig 1, we perform minimum-output-energy (MOE) based equalization before FFT. In fact, as can be seen, the IFFT/FFT blocks are optional only. The input symbol sequence, $\{ s_n \}$, are grouped to form

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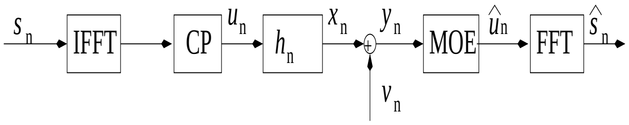

Fig. 1. Baseband OFDM transmitter and MOE receiver.

blocks. Each block has N symbols, and is processed by the optional IFFT. The data sequence, $\{u_n\}$, are obtained after IFFT and cyclic prefix. Let the length of the cyclic prefix be L, and $P = N + L$. We define the blocks of symbol and data sequences as $\mathbf{s}_N(nN) = [s_{nN}, \cdots, s_{nN-N+1}]^{\mathrm{T}}$ and $\mathbf{u}_P(nP) = [u_{nP}, \cdots, u_{nP-N+1}]^T$, respectively. Then

$$\mathbf{u}_P(nP) = \mathbf{F}^{-1}\mathbf{s}_N(nN), \quad (1)$$

where $\mathbf{F}^{-1}$ is the inverse discrete Fourier Transformation matrix. Cyclic prefix samples are added to $\{u_n\}$ as

$$u_{nP-N-i} = u_{nP-i} \quad , \quad i = 0, \cdots, L-1. \quad (2)$$

Then the data sequence $\{u_n\}$ are transmitted.

The received baseband signal is

$$y_n = x_n + v_n \triangleq \sum_{k=0}^{L_h} h_k u_{n-k} + v_n \;, \quad (3)$$

where x_n and v_n denote noiseless signal and AWGN, respectively. The channel $\mathbf{h} = [h_0, \cdots, h_{L_h}]^T$ is assumed to be FIR with length $L_h + 1$. The condition of sufficient cyclic prefix as in traditional OFDM is $L \ge L_h$. Note that large L value brings down transmission efficiency which is $N/(N+L)$.

Since we allow that the length of cyclic prefix L maybe shorter than the channel length L_h , for the flexibility of manipulating cyclic prefix correlations, we construct the $M \times 1$ dimensional received data vectors as

$$\mathbf{y}_M(k) = \mathbf{x}_M(k) + \mathbf{v}_M(k), \quad (4)$$

where $\mathbf{y}_M(k) = [y_k, \cdots, y_{k-M+1}]^T$, $\mathbf{x}_M(k) = [x_k, \cdots, x_{k-M+1}]^T$ and the AWGN $\mathbf{v}_M(k) = [v_k, \cdots, v_{k-M+1}]^T$. The noiseless received signal vector is thus

$$\mathbf{x}_M(k) = H\,\mathbf{u}(k) \triangleq \begin{bmatrix} h_0 & \cdots & h_{L_h} & & \\ & \ddots & & \ddots & \\ & & h_0 & \cdots & h_{L_h} \end{bmatrix} \begin{bmatrix} u_k \\ \vdots \\ u_{k-M-L_h+1} \end{bmatrix}. \quad (5)$$

where the channel matrix H is with dimension $M \times (M + L_h)$.

As a special case, if $L_h \le L$, we can choose $M = N$ and construct

$$\mathbf{x}_N(nP) = \mathbf{H}\mathbf{u}_P(nP) \triangleq \begin{bmatrix} h_0 & \cdots & h_{L_h} & & \\ & \ddots & & \ddots & \\ & & \ddots & \cdots & h_{L_h} \\ \vdots & & & \ddots & \vdots \\ h_1 & \cdots & h_{L_h} & & h_0 \end{bmatrix} \begin{bmatrix} u_{nP} \\ \vdots \\ u_{nP-N+1} \end{bmatrix} \quad (6)$$

where $\mathbf{H}$ is an $N \times N$ circulant matrix similarly as traditional OFDM system. Note that in case of sufficient cyclic prefix, we can directly use Eq. (6) in our methods for further simplification.

III. Blind Equalization with Insufficient Cyclic Prefix

III-1. Constraint parameter estimation

In this section, we consider the insufficient case where the cyclic prefix length L is less than the channel length L_h. In this case, the channel matrix can not be circular, and the traditional FFT-based OFDM demodulation method fails.

We first consider the noiseless system. In order to apply the MOE optimization, we need to find a proper constraint, which can be some columns in the channel matrix H in Eq. (5). It is therefore equivalent to blind channel estimation.

We construct data sample vectors as per (4) and calculate the following correlation matrices

$$\mathbf{R}_K(l) = E\{\mathbf{x}_K(nP+l)\mathbf{x}_K^H(nP+l)\}, \quad (7)$$

where $E\{\cdot\}$ denotes expectation, and the vector dimension K can be chosen as $K \ge L_h + 1$.

For $l = 0, \cdots, K + L_h - L$, we have

$$\mathbf{R}_K(l) = H \begin{bmatrix} \mathbf{O}_l & & \\ & \mathbf{I}_L & \\ & & \mathbf{O}_{K+L_h-L-l} \end{bmatrix} H^H, \quad (8)$$

where $\mathbf{O}_l$ is an $l \times l$ zero matrix and $\mathbf{I}_L$ is $L \times L$ identity matrix. On the other hand, if $K + L_h - 1 \le l \le K + L_h$, we have

$$\mathbf{R}_K(l) = H \begin{bmatrix} \mathbf{O}_l & \\ & \mathbf{I}_{K+L_h-l} \end{bmatrix} H^H. \quad (9)$$

Then we evaluate the summation of a sequence of correlation matrices as

$$\mathbf{R}_K = \sum_{l=0}^{Q} [\mathbf{R}_K(L_h + lL) - \mathbf{R}_K(L_h + 1 + lL)], \quad (10)$$

where $Q = \lfloor (K-1)/L \rfloor$. It can be shown that the correlation matrix $\mathbf{R}_K$ satisfies $\mathbf{R}_K = H\,\mathrm{diag}\{\mathbf{O}_{L_h}, 1, \mathbf{O}_{K-1}\}H^H$. Hence, the channel coefficients can be estimated as the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_K$. This gives channel estimation $\hat{\mathbf{h}} = \alpha [h_L, \cdots, h_0, 0, \cdots, 0]^T$, where α is a phase ambiguity inherent for blind methods. The constraints will be constructed from the estimated coefficients in $\hat{\mathbf{h}}$.

III-2. MOE blind equalizer

Consider the channel matrix H in (5). Define $m = \lfloor (M + L_h)/P \rfloor$. Because of the L cyclic prefix data in every block of P transmitted data u_k, there are at most $M + L_h - mL$ columns corresponding to different data u_k if we combine those columns in H that are corresponding to the same data. Denote the combined channel matrix as $\tilde{H}$.

Proposition. With proper M and $\lfloor (M + L_h)/P \rfloor > (L_h / L)$, the corresponding channel matrix $\tilde{H}$ is full column rank.

Proof. The matrix $\tilde{H}$ has M rows and at most $M + L_h - mL$ columns. In addition, considering the Toeplitz structure of the channel matrix H, it is easy to find that the matrix $\tilde{H}$ can be arranged in a block upper-triangular form with h_0 on the main diagonal. Therefore, it is full column rank. □

We construct the data sample vector $\mathbf{x}_M(nP)$ according to Eq. (4) with properly chosen dimension M, and find the correlation matrix

$$\mathbf{R}_M(0) = E\{\mathbf{x}_M(nP)\mathbf{x}_M^H(nP)\} = \tilde{H}\tilde{H}^H. \qquad (11)$$

In addition, we can select any adjacent N columns in $\tilde{H}$, e.g., the $(D, \cdots, D + N - 1)th$ columns with some proper delay parameter D. We denote them as columns $\mathbf{c}_d$, $d = 0, \cdots, N - 1$. Note that they can be represented by the estimated channel coefficients in the previous section $\hat{\mathbf{h}} = \alpha[h_L, \cdots, h_0, 0, \cdots, 0]^T$.

Then we define a bank of N constrained MOE optimization problems as

$$\mathbf{f}_d = \arg\min \mathbf{f}_d^H \mathbf{R}_M(0)\mathbf{f}_d \quad \text{, s.t.,} \quad \mathbf{f}_d^H \mathbf{c}_d = 1, \qquad (12)$$

for all $d = 0, \cdots, N-1$. The solution of $\mathbf{f}_d$ is the generalized eigenvector of the matrix pencil $(\mathbf{R}_M(0), \mathbf{c}_d\mathbf{c}_d^H)$. Because $\tilde{H}$ is full column rank, $\mathbf{f}_d$ is zero-forcing in noiseless case, i.e., $\mathbf{f}_d^H \tilde{H} = \mathbf{e}_{D+d}$, $d = 0, \cdots, N - 1$, where $\mathbf{e}_d$ is a unit vector with value 1 in the $(D+1)th$ entry. Therefore, the block of N data can be estimated by the equalizer bank $\mathbf{f}_d$ as

$$\mathbf{f}_d^H \mathbf{x}_M(nP) = \hat{u}_{nP-D-d}, \quad d = 0, \cdots, N - 1. \qquad (13)$$

Then FFT can be used to recover the transmitted symbols s_n in OFDM systems.

IV. BLIND ALGORITHMS

We have developed the general idea of the new blind methods in section III with noiseless signal $\mathbf{x}_M(k)$. In this section we develop both the batch and the adaptive algorithms with noisy signal $\mathbf{y}_M(k)$.

IV-1. Batch algorithm

We need to estimate the correlation matrices $\mathbf{R}_K$ as per (10) for constraint estimation and $\mathbf{R}_M(0)$ as per (11) for equalizer estimation with the consideration of noise.

Algorithm 1

1. System design: select parameters K, M and find received sample vectors $\mathbf{y}_M(k)$ as per Eq (4).
2. Blind Channel estimation: calculate the matrix $\mathbf{R}_K$ as per (10), identify the channel as the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_K$.
3. Blind equalization: estimate the equalizers $\mathbf{f}_d$ as per (12) and estimate symbols as per (13).

IV-2. Adaptive algorithm

In order to track channel time variation and to reduce the computational complexity, we can adaptively implement the above algorithm. First, because $\mathbf{R}_K$ in (10) is a rank one matrix, channel estimation can be easily implemented as to calculate one column of $\mathbf{R}_K$,

$$\hat{\mathbf{h}}^{(n)} = \hat{\mathbf{h}}^{(n-1)} + \sum_{l=0}^{Q} [\mathbf{y}_K(nP + L_h + lL) y^*_{nP+L_h+lL-q} - \mathbf{y}_K(nP + L_h + 1 + lL) y^*_{nP+L_h+1+lL-q}], \qquad (14)$$

where q is a parameter for sufficiently large norm $\|\hat{\mathbf{h}}^{(n)}\|$.

The equalizers $\mathbf{f}_d$ can be adaptively implemented as

$$\mathbf{f}_d^{(n)} = \mathbf{f}_c + \mathbf{P}[\mathbf{f}_d^{(n-1)} \text{-} \mu z^*(n)\mathbf{y}_M(np)], \qquad (15)$$

where $\mathbf{P} = \mathbf{I} - \mathbf{c}_d(\mathbf{c}_d^H\mathbf{c}_d)^{-1}\mathbf{c}_d^H$, $\mathbf{f}_c = \mathbf{c}_d(\mathbf{c}_d^H\mathbf{c}_d)^{-1}\mathbf{c}_d^H$, $z(n) = \mathbf{f}_d^H(n)\mathbf{y}_M(nP)$ and the step size μ should be properly chosen for a tradeoff between convergence rate and misadjustment.

Algorithm 2

1. System design: refer to Step 1 of Algorithm 1.
2. Blind channel estimation: find q and $\hat{\mathbf{h}}^{(n)}$ (14).
3. Blind equalization: adaptively estimate $\mathbf{f}_d^{(n)}$ as per (15). Estimate symbols as per (13).

The complexity of this adaptive algorithm is low. Each linear equalizer needs $O(M)$ operation per iteration. Although N such equalizers need calculating, they can be implemented in parallel.

V. SIMULATIONS

We used simulations to study the performance of the proposed algorithms in section VI and to compare them with traditional OFDM receiver and an OFDM channel shortening technique [5] which we denote as SOFDM. We used bit-error-rate (BER) to evaluate their performance for OFDM transmission in case of insufficient cyclic prefix. We used a channel with length $L_h = 3$ and coefficients as $[-1.28 - j0.301, -0.282 + j0.562, 0.031 - j0.201, 0.106 + j1.164]$. We chose $N = 8$ for block length, L=2 for cyclic prefix, $K = 5$ for channel estimation, and $M = 28$ for equalization. 5000 OFDM symbols were used for constraint and equalizer estimation. The performance comparison is shown in Fig.2. When the SNR is below 5 dB, their results are not much different. However, when SNR is greater than 10 dB, our batch and adaptive algorithms achieve sufficiently lower BER than the other two. In particular, the traditional FFT-based OFDM receiver does not work.

VI. CONCLUSION

In this paper, we create a new blind equalization method for OFDM in case the cyclic prefix is shorter than channel length. Blind channel estimation is performed with the correlation property of cyclic prefix, and then the constrained minimum-output-energy optimization (MOE) method is used for equalization. The algorithm provides an effective way to mitigate the problem of insufficient cyclic prefix, or to improve bandwidth efficiency with shorter cyclic prefix length. Our batch and adaptive algorithms are compared with some other OFDM demodulation methods.

REFERENCES

[1] A. Bahai and B. Saltzberg, *Multi-carrier Digital Communications*, Kluwer Academic / Plenum Publishers, 1999.

[2] D. H. Johnson and D. E. Dudgeon, *Array Signal Processing*, Prentice hall, Upper Saddle River, NJ, 1993.

[3] H. Bolckei, P. Duhamel and R. Hleiss, "A subspace-based approach to blind channel identification in pulse shaping OFDM/OQAM system," *IEEE Trans. Signal Processing*, vol.49, no.7, pp. 1594-1598, July 2001.

[4] M. Tsatsanis and Z. Xu, "Constrained optimization methods for direct blind equalization," *IEEE J. Select. area Commun.*, vol. 17, no. 3, pp. 424-433, Mar. 1999.

[5] R. K. Martin and J. Balakrishnan, "Blind adaptive channel shortening for multicarrier systems," 36*th* *Asilom. Conf. Signals, Systems, Computers,* vol. 1 , pp. 372 -376, Pacific Grove, CA, Nov. 2002.

[6] P. Melsa, R. Younce and C. Rohrs, "Impulse response shortening for discrete multitone transceivers," *IEEE Trans. Commun.,* vol. 44, no. 12, pp. 1662-1672, Dec. 1996.

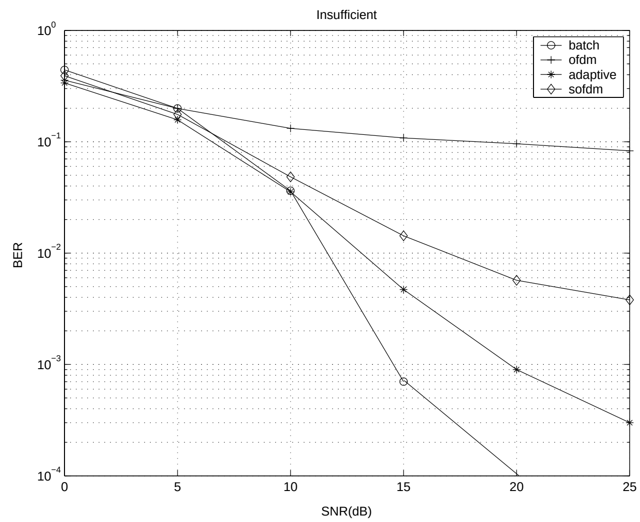

Fig.2 Performance in OFDM system with insufficient cyclic prefix.