Carrier Frequency Offset Mitigation in Asynchronous Cooperative OFDM Transmissions

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***Abstract*—Carrier frequency offset (CFO) mitigation is critical for orthogonal frequency-division multiplexing (OFDM)-based cooperative transmissions because even small CFO per transmitter may lead to severe performance loss, especially when the number of cooperative transmitters is large. In this paper, we show that cyclic prefix (CP) can be exploited to mitigate or even remove completely the CFO. The mitigation performance increases along with the CP length. In particular, long CP with length proportional to NI, where N is the fast Fourier transform (FFT) block length and I is the number of cooperative transmitters, can guarantee a complete CFO removal. While this comes with a reduction in bandwidth efficiency, the long CP in the proposed scheme is exploited to enhance transmission power efficiency in a way similar to spread-spectrum systems, and thus is different from conventional CP that degrades both bandwidth and power efficiency. An efficient CFO-mitigation algorithm is developed that has complexity at most $O(NI^2)$, or even linear in N approximately in some cases. Implemented as a preprocessing procedure independently from cooperative encoding/decoding details, this algorithm makes the CFO problem effectively transparent to and thus has general applications in OFDM-based transmissions.**

***Index Terms*—Carrier frequency offset (CFO), cooperative transmissions, orthogonal frequency-division multiplexing (OFDM), synchronization.**

I. Introduction

COOPERATIVE transmissions have attracted great attention recently. By sharing the antennas of multiple distributed transmitters or receivers to create virtual antenna arrays, cooperative transmissions have been shown to enhance bandwidth efficiency, power efficiency, reliability, etc. [1]–[3]. An important form of cooperative transmissions is to adapt the existing antenna array techniques, such as space-time block codes (STBC) [4], into the distributed environment [3]. This has great importance in practical wireless networks considering that small wireless nodes may not be able to have physical antenna arrays, while antenna array techniques are viable to their performance.

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As far as the distributed implementation is concerned, one of the major issues is the synchronization of the cooperative transmitters. The "synchronization" in this paper refers specifically to the synchronization of the carrier frequency and arrival timing of all cooperative transmitters, i.e., their signals should have the same carrier frequency and timing when arriving at a receiver. Using the receiver's local carrier and timing as references, perfect synchronization means zero carrier frequency offset (CFO) and zero timing-phase offset (TPO) [5]. Without such a perfect synchronization, many existing antenna array techniques such as STBC cannot be directly used in cooperative transmissions [6]. Unfortunately, in distributed environment it is difficult to guarantee perfect synchronization because clock drifting, oscillator parameter drifting, propagation distance, Doppler shifting, etc., may be different among the transmitters and may be randomly time varying.

Orthogonal-frequency-division-multiplexing (OFDM) transmission techniques [7] are desirable for combating the loss of timing-phase synchronization, since any limited propagation delay (or timing-phase) difference among the signals of cooperative transmitters can be tolerated by simply increasing the length of cyclic prefix (CP) [8], [9]. Because of this, they may find wide applications in cooperative transmissions, similarly as their flourish in conventional antenna array systems where they provide a major advantage in simplifying the channel dispersion problem. Nevertheless, OFDM suffers critically from the loss of carrier frequency synchronization where the CFO incurs intercarrier interference (ICI) [10]. This CFO problem becomes even worse in multitransmitter OFDM systems because of the increase in intertransmitter interference, not only ICI [9].

While the CFO problem is still mostly open for research in cooperative OFDM systems, it is an extensively studied subject in either single-user OFDM systems [10]–[15] or multi-user OFDM systems [16]–[24]. One of the ways for avoiding the CFO problem in practice is for the receiver to feedback the estimated CFO to the transmitters so that the latter can adjust their carriers for perfect synchronization [16], [17]. However, this approach has extra costs in both bandwidth and power [23]. For OFDMA systems, CFO can be mitigated by exploiting the fact that different transmitters are assigned with different OFDM subcarriers so that their signals can be easily separated [19]. For general multiuser OFDM systems, some iterative interference cancellation schemes have been developed, including [21]–[23]. Based on the fact that the fast Fourier transform (FFT) operation conducted by the receiver reduces the CFO-induced interference to some extent, the interference cancellation approach can often satisfactorily mitigate CFO. Nevertheless, their performance is limited by the signal-to-interference ratio of the post-FFT signals [23], which means the performance may in particular de-

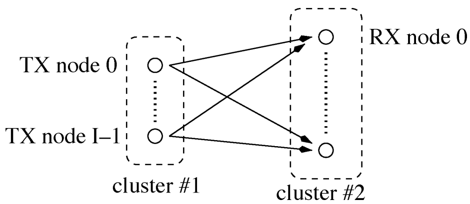

Fig. 1. Node-cluster-based cooperative transmission where I transmit (TX) nodes cooperatively transmit to several receiving (RX) nodes. If the Doppler shifts of the transmission paths are all different (e.g., due to different node movement), it is impossible for the transmitters to synchronize their carrier frequencies toward all the receivers simultaneously.

grade when more transmitters are involved, or when larger CFO is encountered, or when the same subcarrier is shared by different transmitters simultaneously which is typical in cooperative transmissions.

Though some of the aforementioned approaches may be adapted to cooperative OFDM systems, the CFO problem in cooperative OFDM systems has some unique characteristics. In some cases, feedback cannot resolve the CFO problem. As an example, for the cluster-based cooperative transmission illustrated in Fig. 1, if the moving direction of the transmitters are different with respect to each receiver, then their Doppler shifting are also different. This makes it impossible to synchronize the carriers toward all the receivers simultaneously, even if the Doppler shifts are assumed known. In general, the lack of centralized controller makes distributed synchronization more difficult and costly, which means receiver-based CFO mitigation techniques are quite necessary for distributed cooperative communications.

Considering that many existing methods may not be directly applicable (such as the OFDMA specific approach [19]) or may suffer performance degradation (such as [21]–[23]) for cooperative OFDM systems with subcarrier sharing or large CFO, we present a novel approach which can guarantee a complete CFO cancellation, no matter how many transmitters there are and how large the CFO is. Our basic idea is to utilize the redundancy of the long CP for CFO mitigation or cancellation. Another unique feature of our approach is that it is implemented purely as a "preprocessing" procedure, independently from cooperative encoding/decoding details. In other words, it simply makes the CFO problem transparent to the cooperative OFDM transmission design. Note that our approach may be applicable to many centralized OFDM systems as well although it is developed in this paper in a cooperative communications setting.

To avoid lengthy derivation, we assume that the receiver has already estimated the timing, the CFO, and the channel of each of the cooperative transmitters [19], [25]. The effect of CFO estimation error will be investigated by simulations.

Some important notations are listed below: $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^+$ denote matrix transpose, Hermitian and pseudoinverse; $[\cdot]_m$ denotes the mth element of a vector and $[\cdot]_{m,n}$ denotes the (m,n)th element of a matrix, where m, n are counted from 0; $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix with diagonal entries listed in the vector $\mathbf{x}$; $\mathbf{0}_m$ is zero vector of dimension m, $\mathbf{0}_{M\times N}$ is $M \times N$ zero matrix, and $\mathbf{I}_N$ is $N \times N$ identity matrix; $x|N$ denotes x mod N.

The rest of the paper is organized as follows. In Section II, we give the cooperative OFDM transmission model. In Section III, we describe our CFO mitigation algorithm. In Section IV, we analyze the performance of the algorithm for CFO mitigation or complete cancellation. Then, we conduct simulations in Section V and conclude this paper in Section VI.

II. System Model

Based on the cooperative communication system illustrated in Fig. 1, we consider a cooperative transmission scheme with I cooperative transmitters and one receiver. As shown in Fig. 2, all the I cooperative transmitters are assumed to have the same data packet that is to be encoded and transmitted, using some predefined cooperative encoding schemes such as cooperative STBC [6]. The encoder output $b_i(n), i = 0, \cdots, I-1, n = 0, 1, \cdots$, are then OFDM modulated, which gives the OFDM signal $s_i(n)$. Note that each transmitter may use all or a portion of the OFDM subcarriers depending on the predefined cooperation schemes [8], [9] that we do not need to specify (because our proposed method is independent of them).

The discrete baseband channel from the ith transmitter to the receiver is assumed frequency selective fading with coefficients $h_i(\ell)$, $\ell = 0, \cdots, L$. Without loss of generality, we let all the channels have the same order L. We also assume that channels are time-invariant during the transmission of one OFDM block (including information symbols and CP), but may be randomly time varying between blocks. Since we need longer CP, the time-invariant assumption is stronger. However, this assumption is reasonable in practice because the time-variation factors, such as Doppler-shifting and residue carrier, are included in CFO, not in the channel $h_i(\ell)$.

From the received signal $r(n)$, the receiver mitigates the asynchronism in carrier frequency and timing using our proposed method, after which conventional OFDM demodulation and cooperative decoding techniques such as [8] are applied.

With the consideration of asynchronous transmitters, the signal of each transmitter i may have a propagation delay d_i and a CFO ϵ_i (relative to a reference timing and a reference local carrier) when received at the receiver. We assume d_i to be integer (with symbol interval as unit) since the fractional portion of the delay contributes nothing but some extra channel dispersion which can be assimilated into the dispersive channel model. The CFO ϵ_i is derived as the residual carrier frequency normalized by the OFDM subcarrier frequency separation [15]. Both d_i and ϵ_i are assumed non-negative with some known upper bounds. In order to simplify the problem, we assume $\epsilon_i \neq \epsilon_j$ for all $i \neq j$. As will be clear after Section III, if $\epsilon_i = \epsilon_j$, we only need to consider one of them, which is equivalent to reducing the total number of transmitters by 1.

The transmitted signal $s_i(n)$ is derived from the inverse fast Fourier transform (IFFT) of the encoded symbol $b_i(n)$. Since there is no interblock interference (IBI) thanks to the cyclic prefix [20], we consider one OFDM block for notational simplicity. Then, the ith transmitter's signal $s_i(n)$ can be written as

$$s_i(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} b_i(k) e^{j2\pi nk/N}, \quad -N_g \leq n \leq N-1 \quad (1)$$

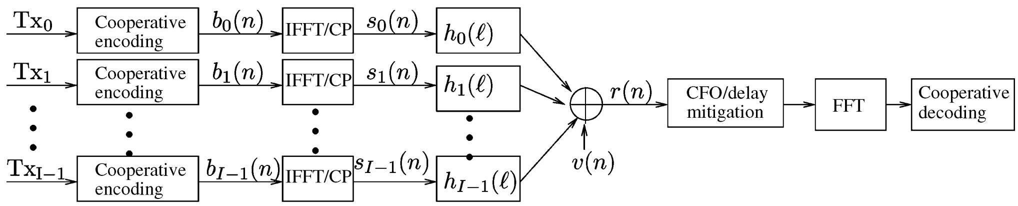

Fig. 2. Multi-transmitter cooperative OFDM transmission and receiving block diagram.

where N_g is the length of the CP and N is the IFFT block length (we also define it as OFDM block length). Obviously, $N_g \geq L + \max_{0\leq i\leq I-1} d_i$ should be satisfied in order to avoid IBI [18]. In addition, we assume $N > L + \max_{0\leq i\leq I-1} d_i$, which is usually a fact in practical systems.

The noiseless signal from the ith transmitter is

$$x_i(n) = \sum_{\ell=0}^{L} h_i(\ell)s_i(n-\ell) \tag{2}$$

based on which the composite signal received by the receiver, with delay d_i and CFO ϵ_i considered, is

$$r(n) = \sum_{i=0}^{I-1} x_i(n-d_i)e^{j(\epsilon_i n+\phi_i)} + v(n) \tag{3}$$

where ϕ_i is the initial phase, i.e., the phase of the residual carrier of the ith transmitter's signal in the symbol interval $n=0$. The additive white Gaussian noise (AWGN) $v(n)$ is assumed with zero-mean and variance σ_v^2.

From the received composite signal, a conventional OFDM demodulator would remove CP and consider the sample vector $\mathbf{r}(0) = [r(0),\cdots,r(N-1)]^T$. In our case, from (2), (3) this gives

$$\mathbf{r}(0) = \sum_{i=0}^{I-1} e^{j\phi_i}\mathbf{E}_i(0)\begin{bmatrix} h_i(L) & \cdots & h_i(0) & & \\ & \ddots & & \ddots & \\ & & h_i(L) & \cdots & h_i(0)\end{bmatrix} \times \begin{bmatrix} s_i(-L-d_i) \\ \vdots \\ s_i(N-1-d_i)\end{bmatrix} + \mathbf{v}(0) \tag{4}$$

where the $N \times N$ diagonal matrix $\mathbf{E}_i(0) = \mathrm{diag}\{1, e^{j\epsilon_i},\cdots,e^{j\epsilon_i(N-1)}\}$ is defined as the CFO matrix, and the AWGN vector $\mathbf{v}(0) = [v(0),\cdots,v(N-1)]^T$. The CP in (1) means that the first N_g symbols $s_i(n), n=-N_g,\cdots,-1$, just repeat the last N_g symbols $s_i(n), n=N-N_g,\cdots,N-1$. Therefore, we have $s_i(-\ell-d_i) = s_i(N-\ell-d_i), 1\leq \ell \leq L$, from which (4) can be rewritten as

$$\mathbf{r}(0) = \sum_{i=0}^{I-1} e^{j\phi_i}\mathbf{E}_i(0)\mathbf{H}_i(0)\mathbf{s}_i(d_i) + \mathbf{v}(0), \tag{5}$$

where the symbol vector $\mathbf{s}_i(d_i) = [s_i(-d_i),\cdots,s_i(N-1-d_i)]^T$, and the channel matrix $\mathbf{H}_i(0)$ is $N\times N$ circulant. Note that the first row (row $k=0$) of $\mathbf{H}_i(0)$ is $[h_i(0), \mathbf{0}_{N-L-1}, h_i(L),\cdots,h_i(1)]$, whereas each subsequent kth row is the $(k-1)$-step right cyclic shift of the first row. For example, the second row $(k=1)$ is $[h_i(1), h_i(0), \mathbf{0}_{N-L-1}, h_i(L),\cdots,h_i(2)]$.

To remove the negative indexes in $\mathbf{s}_i(d_i)$, we substitute all the negative indexes with the equivalent positive ones according to CP, which leads to $\mathbf{s}_i(d_i) = [s_i(N-d_i),\cdots,s_i(N-1), s_i(0),\cdots,s_i(N-1-d_i)]^T$. Then, we rearrange the order of the entries of $\mathbf{s}_i(d_i)$ to get $\mathbf{s}_i = [s_i(0),\cdots,s_i(N-1)]^T$. By switching correspondingly the columns of $\mathbf{H}_i(0)$, we can change (5) into

$$\mathbf{r}(0) = \sum_{i=0}^{I-1} e^{j\phi_i}\mathbf{E}_i(0)\mathbf{H}_i\mathbf{s}_i + \mathbf{v}(0) \tag{6}$$

where

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{0}_{N-L-d_i} & h_i(L)\cdots h_i(0) & \mathbf{0}_{d_i-1} \\ \mathbf{0}_{N-L-d_i+1} & h_i(L)\cdots h_i(0) & \mathbf{0}_{d_i-2} \\ \vdots & & \vdots \\ \mathbf{0}_{N-L-d_i-1} & h_i(L)\cdots h_i(0) & \mathbf{0}_{d_i}\end{bmatrix} \tag{7}$$

is $N\times N$ circulant with right cyclic-shifted rows. Note that if $d_i = 0$, then the first row of $\mathbf{H}_i$ should be $[h_i(0), \mathbf{0}_{N-L-1}, h_i(L),\cdots,h_i(1)]$. One of the interesting characteristics of the model (6), (7) is that the delay d_i is contained in $\mathbf{H}_i$ only, whereas the CFO ϵ_i is contained in the CFO matrix $\mathbf{E}_i(0)$ only. This property permits us to mitigate CFO ϵ_i independently from d_i.

If there is no CFO, i.e., $\mathbf{E}_i(0) = \mathbf{I}_N$, then performing FFT on $\mathbf{r}(0)$ leads to the conventional cooperative OFDM demodulation [8]. The situation is different with CFO, where the major problem is that $\mathbf{E}_i(0)$ prevents the diagonalization of $\mathbf{H}_i$, but instead causes ICI as well as multitransmitter interference, if directly conducting FFT. Therefore, we need to look for ways to reduce or remove all the I CFO matrices $\mathbf{E}_i(0)$.

III. CFO Mitigation and Cancellation

A. Using Redundant CP

Our basic idea is to exploit the redundancy of the CP based on the structure of the signal model (6). If the CP length N_g is longer than $L + \max_{0\le i\le I-1} d_i$, then in addition to those in $\mathbf{r}(0)$, we have more IBI-free samples $r(-m)$, $0 < m \le N_g - L - \max_{0\le i\le I-1} d_i$, with which we can construct new sample vectors $\mathbf{r}(m) = [r(-m), \cdots, r(N-1-m)]^T$. Similarly to (4), we have

$$\mathbf{r}(m) = \sum_{i=0}^{I-1} e^{j\phi_i} \tilde{\mathbf{E}}_i(m) \begin{bmatrix} h_i(L) & \cdots & h_i(0) & & \\ & \ddots & & \ddots & \\ & & h_i(L) & \cdots & h_i(0) \end{bmatrix} \times \begin{bmatrix} s_i(-L-d_i-m) \\ \vdots \\ s_i(N-1-d_i-m) \end{bmatrix} + \mathbf{v}(m) \quad (8)$$

where $\tilde{\mathbf{E}}_i(m) = \operatorname{diag}\{e^{j\epsilon_i(-m)}, \cdots, e^{j\epsilon_i(N-1-m)}\}$ and $\mathbf{v}(m) = [v(-m), \cdots, v(N-1-m)]^T$.

By utilizing CP, we can change (8) into

$$\mathbf{r}(m) = \sum_{i=0}^{I-1} e^{j\phi_i} \tilde{\mathbf{E}}_i(m) \mathbf{H}_i(0) \mathbf{s}_i(d_i+m) + \mathbf{v}(m) \quad (9)$$

where the symbol vector $\mathbf{s}_i(d_i+m) = [s_i(-d_i-m), \cdots, s_i(N-1-d_i-m)]^T$, and the channel matrix $\mathbf{H}_i(0)$ is the same as that in (5). It is easy to see that $\mathbf{s}_i(d_i+m) = [s_i((-d_i-m)|N), \cdots, s(N-1), s(0), \cdots, s_i((N-1-d_i-m)|N)]^T$, where we use modulo N operations in order to cope with extremely large m (since we may use long CP $N_g > N$, as shown in Section IV-B). Next, we reorder the entries of $\mathbf{s}_i(d_i+m)$ to change it into the vector $\mathbf{s}_i$, and switch the corresponding columns in $\mathbf{H}_i(0)$ similarly as what we did in (6). The result is that (9) is changed to

$$\mathbf{r}(m) = \sum_{i=0}^{I-1} e^{j\phi_i} \tilde{\mathbf{E}}_i(m) \mathbf{H}_i(m) \mathbf{s}_i + \mathbf{v}(m) \quad (10)$$

where $\mathbf{H}_i(m)$ is an $N \times N$ circulant matrix. Its first row is $[\mathbf{0}_{(-d_i-m)|N-L}, h_i(L), \cdots, h_i(0), \mathbf{0}_{N-1-(-d_i-m)|N}]$, and its rest rows are the right cyclic shifts of the first row.

Comparing $\mathbf{H}_i(m)$ with $\mathbf{H}_i$ in (7), we see that if we move the first $N - d_i - (-d_i - m)|N$ rows of $\mathbf{H}_i(m)$ to the end of this matrix, then we can change $\mathbf{H}_i(m)$ into $\mathbf{H}_i$. Taking this adjustment, and changing the columns of $\tilde{\mathbf{E}}_i(m)$ correspondingly, we obtain from (10) an expression similar to (6), i.e.,

$$\mathbf{r}(m) = \sum_{i=0}^{I-1} e^{j\phi_i} \mathbf{E}_i(m) \mathbf{H}_i \mathbf{s}_i + \mathbf{v}(m) \quad (11)$$

where [see (12), shown at the bottom of the page]. Note that we have used $d_i < N$ and $N - d_i - (-d_i - m)|N = m|N$ when deriving (12).

Noticing that (11) and (6) contain the same $\mathbf{H}_i$ and $\mathbf{s}_i$ but have different CFO matrices, we can stack together all available vectors $\mathbf{r}(m)$, $0 \le m < M \triangleq N_g - L - \max_{0\le i\le I-1} d_i + 1$,

$$\begin{bmatrix} \mathbf{r}(0) \\ \vdots \\ \mathbf{r}(M-1) \end{bmatrix} = \sum_{i=0}^{I-1} e^{j\phi_i} \begin{bmatrix} \mathbf{E}_i(0) \\ \vdots \\ \mathbf{E}_i(M-1) \end{bmatrix} \mathbf{H}_i \mathbf{s}_i + \begin{bmatrix} \mathbf{v}(0) \\ \vdots \\ \mathbf{v}(M-1) \end{bmatrix}. \quad (13)$$

For notational simplicity, we define

$$\mathbf{y} = [\mathbf{r}^T(0), \cdots, \mathbf{r}^T(M-1)]^T$$

and

$$\mathbf{A}_i = [\mathbf{E}_i^T(0), \cdots, \mathbf{E}_i^T(M-1)]^T.$$

Then, (13) can be denoted as

$$\mathbf{y} = \sum_{i=0}^{I-1} e^{j\phi_i} \mathbf{A}_i \mathbf{H}_i \mathbf{s}_i + \mathbf{u}. \quad (14)$$

The dimensions of $\mathbf{y}$ and $\mathbf{A}_i$ are $MN \times 1$ and $MN \times N$, respectively.

Our basic idea is thus to design an $N \times MN$ CFO mitigation matrix $\mathbf{X}$ such that

$$\mathbf{X}\mathbf{A}_i = \mathbf{I}_N \quad (15)$$

for all $i = 0, \cdots, I-1$. If $\mathbf{X}$ is available for (15), then CFO can be mitigated via

$$\mathbf{z} = \mathbf{X}\mathbf{y}. \quad (16)$$

Note that a straightforward solution for $\mathbf{X}$ is

$$\mathbf{X} = [\mathbf{I}_N \cdots \mathbf{I}_N] \begin{bmatrix} \mathbf{E}_0(0) & \cdots & \mathbf{E}_{I-1}(0) \\ \vdots & & \vdots \\ \mathbf{E}_0(M-1) & \cdots & \mathbf{E}_{I-1}(M-1) \end{bmatrix}^+. \quad (17)$$

If (15) can be satisfied perfectly, then we have $\mathbf{z} = \sum_{i=0}^{I-1} e^{j\phi_i} \mathbf{H}_i \mathbf{s}_i + \mathbf{X}\mathbf{u}$, which is a conventional CFO-free OFDM sample vector after removing the CP. Note that the scalar $e^{j\phi_i}$ is nothing more than a phase factor of the channel

$$\mathbf{E}_i(m) = \begin{bmatrix} \mathbf{0}_{(m|N)\times(N-m|N)} & \operatorname{diag}\left\{e^{j\epsilon_i(-m)}, \cdots, e^{j\epsilon_i(-m-1+m|N)}\right\} \\ \operatorname{diag}\left\{e^{j\epsilon_i(-m+m|N)}, \cdots, e^{j\epsilon_i(N-m-1)}\right\} & \mathbf{0}_{(N-m|N)\times(m|N)} \end{bmatrix}. \quad (12)$$

$\mathbf{H}_i$. With the vector $\mathbf{z}$, conventional OFDM demodulation can be applied to detect symbols $b_i(k)$.

B. Simple Example

One of the major problems is whether (15) has accurate solutions. Another problem is the computational complexity of solving (15) for the solution. The way of using (17) is clearly not desirable considering its high complexity. To address both problems, it is helpful to examine $\mathbf{E}_i(m)$ for a better understanding of its structure. Due to the complexity of (12), we consider a simple but illustrative example with $N = 3$ and $I = 2$ in this section.

In this case, we may use $\mathbf{r}(m)$, $m = 0, 1$, for CFO mitigation. According to (12), the CFO matrices of the signal from the transmitter $i = 0$ are

$$\mathbf{E}_0(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{j\epsilon_0} & 0 \\ 0 & 0 & e^{j2\epsilon_0} \end{bmatrix}, \quad \mathbf{E}_0(1) = \begin{bmatrix} 0 & 0 & e^{-j\epsilon_0} \\ 1 & 0 & 0 \\ 0 & e^{j\epsilon_0} & 0 \end{bmatrix}. \tag{18}$$

The CFO matrices for the transmitter $i = 1$ are identical to (18) except having ϵ_1 instead of ϵ_0. The matrix $\mathbf{X}$, with dimension 3×6, should satisfy [c.f., (15)]

$$\mathbf{X}\begin{bmatrix} \mathbf{E}_0(0) \\ \mathbf{E}_0(1) \end{bmatrix} = \mathbf{I}_3, \quad \mathbf{X}\begin{bmatrix} \mathbf{E}_1(0) \\ \mathbf{E}_1(1) \end{bmatrix} = \mathbf{I}_3. \tag{19}$$

From (18) and (19), it is easy to see that each row of $\mathbf{X}$ can have only two nonzero entries. Specifically, the first row $[x_{00}, \cdots, x_{05}]$ of $\mathbf{X}$ needs only to satisfy $x_{00} + x_{04} = 1$, whose solution always exists. On the other hand, the second row $[x_{10}, \cdots, x_{15}]$ has to satisfy

$$\begin{cases} x_{11}e^{j\epsilon_0} + x_{15}e^{j\epsilon_0} = 1 \\ x_{11}e^{j\epsilon_1} + x_{15}e^{j\epsilon_1} = 1 \end{cases} \tag{20}$$

which unfortunately has no exact solutions. Instead, we can only optimize x_{11} and x_{15} to minimize $|x_{11} + x_{15} - e^{-j\epsilon_0}|^2 + |x_{11} + x_{15} - e^{-j\epsilon_1}|^2$. In other words, we can only mitigate, but not cancel, CFO.

Next, let us consider the case that the CP length N_g is long enough for us to use $\mathbf{r}(m)$ with $m = 0$ and $m = 2$. Then, we also have $\mathbf{E}_0(0)$ as in (18), but instead of $\mathbf{E}_0(1)$ we have a new CFO matrix

$$\mathbf{E}_0(2) = \begin{bmatrix} 0 & e^{-j2\epsilon_0} & 0 \\ 0 & 0 & e^{-j\epsilon_0} \\ 1 & 0 & 0 \end{bmatrix} \tag{21}$$

as do $E_1(0)$ and $E_1(2)$ for the other transmitter. In this case, (15) reduces to the following two linear equation systems:

$$\begin{cases} x_{00} + x_{05} = 1, \\ x_{11}e^{j\epsilon_i} + x_{13}e^{-j2\epsilon_i} = 1, \\ x_{22}e^{j2\epsilon_i} + x_{24}e^{-j\epsilon_i} = 1, \end{cases} \quad \text{for } i = 0, 1. \tag{22}$$

In fact, (22) can be broken down further into three 2×2 linear equation systems, e.g., one of which is

$$\begin{bmatrix} e^{j\epsilon_0} & e^{-j2\epsilon_0} \\ e^{j\epsilon_1} & e^{-j2\epsilon_1} \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{23}$$

It is easy to verify that (22) has exact solutions, which means that the CFO can be completely cancelled.

From this simple example, we have several helpful observations:

1) the CFO can be completely removed only if CP is long enough and appropriate sample vectors $\mathbf{r}(m)$ are used;
2) not all available $\mathbf{r}(m)$ need to be used, and in fact, using less $\mathbf{r}(m)$ leads to reduced complexity;
3) the inverse of the big matrix in (17) can be avoided by exploiting the special structure of $\mathbf{E}_i(m)$.

Observation 3) motivates us to conduct an elementwise analysis of (15) for more efficient algorithms (Section III-C), whereas the first two observations give us clues in the performance analysis (Section IV).

C. Elementwise Derivation of the CFO Mitigation Matrix

Considering the structure of the CFO matrices (12), with some tedious but straightforward verification, we can see that each CFO matrix $\mathbf{E}_i(m)$, $0 \le i \le I-1$, $0 \le m \le M-1$, has nonzero element $e^{j\epsilon_i[(\ell+m)|N-m]}$ only in the $[(\ell + m)|N]$th row and the ℓth column, which means that (12) can be described element-wise as

$$[\mathbf{E}_i(m)]_{p,\ell} = \begin{cases} e^{j\epsilon_i(p-m)}, & \text{if } p = (\ell + m)|N \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

where $0 \le p \le N-1$, $0 \le \ell \le N-1$.

Considering that not all $\mathbf{r}(m)$ have to be used, we choose Q vectors from them, which we define as $\mathbf{r}(m_0), \mathbf{r}(m_1), \cdots, \mathbf{r}(m_{Q-1})$, where the integer indexes satisfy

$$0 \le m_0 \le m_1 \le \cdots \le m_{Q-1} \le M-1. \tag{25}$$

Note that the corresponding CFO matrices are $\mathbf{E}_i(m_0), \cdots, \mathbf{E}_i(m_{Q-1})$, respectively, for $0 \le i \le I-1$. Then (15) is changed to looking for an $N \times NQ$ CFO mitigation matrix $\mathbf{X}$ such that

$$\mathbf{X}\begin{bmatrix} \mathbf{E}_i(m_0) \\ \vdots \\ \mathbf{E}_i(m_{Q-1}) \end{bmatrix} = \mathbf{I}_N, \quad 0 \le i \le I-1. \tag{26}$$

Consider the kth row of $\mathbf{X}$, $0 \le k \le N-1$, which we define as

$$\mathbf{x}_k = [\mathbf{x}_k(m_0), \mathbf{x}_k(m_1), \cdots, \mathbf{x}_k(m_{Q-1})] \tag{27}$$

where each $\mathbf{x}_k(m)$ is a $1 \times N$ vector. Using $[\mathbf{x}_k(m)]_p$ to denote the pth element, (26) is equivalent to an elementwise representation

$$\sum_{q=0}^{Q-1}\sum_{p=0}^{N-1}[\mathbf{x}_k(m_q)]_p\,[\mathbf{E}_i(m_q)]_{p,\ell}=\begin{cases}1, & \text{for } \ell=k\\ 0, & \text{for } \ell\neq k\end{cases} \quad (28)$$

for all $\ell = 0, \cdots, N-1$.

Let us consider the $\ell = k$ case of (28) first. Considering (24), we can reduce (28) into

$$\sum_{q=0}^{Q-1}[\mathbf{x}_k(m_q)]_{(k+m_q)|N}\,[\mathbf{E}_i(m_q)]_{(k+m_q)|N,k}=1. \quad (29)$$

Applying the element value of (24) into (29), we obtain

$$\sum_{q=0}^{Q-1}[\mathbf{x}_k(m_q)]_{(k+m_q)|N}\,e^{j\epsilon_i[(k+m_q)|N-m_q]}=1. \quad (30)$$

Because the same set of Q variables $\{[\mathbf{x}_k(m_q)]_{(k+m_q)|N}, 0 \leq q \leq Q-1\}$ need to satisfy (30) for all $0 \leq i \leq I-1$, we can find them by solving

$$\mathbf{B}_k\mathbf{z}_k=\mathbf{b} \quad (31)$$

where $\mathbf{b} = [1, \cdots, 1]^T$ is an $N \times 1$ vector, the matrix

$$\mathbf{B}_k=\begin{bmatrix} e^{j\epsilon_0[(k+m_0)|N-m_0]} & \cdots & e^{j\epsilon_0[(k+m_{Q-1})|N-m_{Q-1}]}\\ \vdots & & \vdots\\ e^{j\epsilon_{I-1}[(k+m_0)|N-m_0]} & \cdots & e^{j\epsilon_{I-1}[(k+m_{Q-1})|N-m_{Q-1}]}\end{bmatrix} \quad (32)$$

has dimension $I \times Q$, and $\mathbf{z}_k$ is the $Q \times 1$ variable vector

$$\mathbf{z}_k=\begin{bmatrix}[\mathbf{x}_k(m_0)]_{(k+m_0)|N}\\ \vdots\\ [\mathbf{x}_k(m_{Q-1})]_{(k+m_{Q-1})|N}\end{bmatrix}. \quad (33)$$

Obviously, in order for (31) to have solutions, in general we need

$$Q \geq I \quad (34)$$

which means the number of sample vectors $\mathbf{r}(m)$ should be no less than the number of transmitters. Considering that the matrix $\mathbf{B}_k$ may not be square or full rank, the solution of (31) can be written as

$$\mathbf{z}_k=\mathbf{B}_k^{+}\mathbf{b} \quad (35)$$

and we need to calculate (35) for all $0 \leq k \leq N-1$. Note that although the matrix inverse is still involved, (35) has a complexity much lower than (17) because the matrix dimension is reduced by orders.

The kth row of $\mathbf{X}$ has NQ variables [c.f. (27)], but only Q of them are determined in (35). Fortunately, thanks to the special structure of the CFO matrices, the rest of the $N(Q-1)$ variables do not play any role in (29), and can be simply set as zeros. This zero-setting is in fact not an option but a must when considering (28) for the case $\ell \neq k$, which is

$$\sum_{q=0}^{Q-1}[\mathbf{x}_k(m_q)]_{(\ell+m_q)|N}\,[\mathbf{E}_i(m_q)]_{(\ell+m_q)|N,\ell}=0. \quad (36)$$

From the range of ℓ, k, i.e., $0 \leq \ell \leq N-1$ and $0 \leq k \leq N-1$, we see that $\ell \neq k$ means

$$(\ell+m_q)|N \neq (k+m_q)|N. \quad (37)$$

As a result, the variables $[\mathbf{x}_k(m_q)]_{(\ell+m_q)|N}$ in (36) are different from the variables $[\mathbf{x}_k(m_q)]_{(k+m_q)|N}$ in (30)–(33), so we can simply let the former be zeros for (36), i.e.,

$$[\mathbf{x}_k(m_q)]_p=0,\quad \forall p\neq(k+m_q)|N,\ \ 0\leq p\leq N-1. \quad (38)$$

From (35) and (38), all the NQ variables of the kth row of $\mathbf{X}$ are determined. Repeating this procedure for each of the N rows, the matrix $\mathbf{X}$ is thus available.

D. Efficient Algorithm Implementation

In Section III-C, we have shown that although the matrix $\mathbf{X}$ is large with dimension $N \times NQ$, there are only Q nonzero entries in each row. In other words, there is only one nonzero entry, which is $[\mathbf{x}_k(m_q)]_{(k+m_q)|N}$, in each $1 \times N$ subvector $\mathbf{x}_k(m_q)$. These nonzero entries are obtained by solving (35). After obtaining $\mathbf{X}$, we can use it for CFO mitigation. This procedure is summarized below.

Algorithm 1: Preprocessing for CFO Mitigation

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1. Select proper parameters $m_0, m_1, \cdots, m_{Q-1}$ as per (25). With the knowledge of CFO ϵ_i, calculate $\mathbf{X}$ as per (35) and (38).
2. Construct sample vector $\mathbf{y} = [\mathbf{r}^T(m_0), \cdots, \mathbf{r}^T(m_{Q-1})]^T$ for each OFDM block, as per (13) and (14).
3. Mitigate CFO by $\mathbf{z} = \mathbf{X}\mathbf{y}$ as per (16). Repeat steps 2 and 3 for all OFDM blocks.

After the preprocessing specified in Algorithm 1, conventional OFDM demodulator and cooperative decoder such as [8] can then be applied based on the output $\mathbf{z}$. The only difference is the scalar phase $e^{j\phi_i}$ that needs to be updated along with each new OFDM block, which is trivial.

The computational complexity consists of two parts. The first part is the calculation of $\mathbf{X}$ (step 1), where the good news is that $\mathbf{X}$ needs to be calculated only once (for all OFDM blocks) if ϵ_i is not time varying. In this case, the complexity is in the order of

$O(NQ^3)$, or as low as $O(NI^3)$ since we can usually use $Q = I$. The second part is the calculation of $\mathbf{Xy}$ for each OFDM block, where the complexity is $O(NQ)$ or $O(NI)$ since the majority of $\mathbf{X}$ entries are zeros. Considering that the first part happens only once and I (the number of cooperative transmitters) is usually much smaller than N, the proposed algorithm has complexity almost linear in N, which is very efficient.

IV. Performance of the CFO Mitigation Algorithm

In this section, we show that the performance of the proposed algorithm depends on the length of CP. Short CP length can guarantee a certain level of CFO mitigation only, which is briefly analyzed in Section IV-A. Our main focus is the condition of complete CFO cancellation using long CP, which is given in Section IV-B.

A. CFO Mitigation Capability Under Short CP

Consider (31) and the structure of the matrix $\mathbf{B}_k$ in (32). When (34) is satisfied, the choice of the parameters $m_0, \cdots, m_{Q-1}$ determines the level of CFO mitigation. To show this, let us consider the special case of $k = 0$ (corresponding to the 0th subcarrier) first. In this case, the CFO can always be completely cancelled, because $\mathbf{B}_0$ is $I \times Q$ with value

$$\mathbf{B}_0 = \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{bmatrix} \tag{39}$$

and $\mathbf{z}_0 = 1/I[1, \cdots, 1]^T$ is always a solution to (31). Note that such a $\mathbf{z}_0$ can suppress noise as well while removing CFO.

Unfortunately, when the CP is short (m_{Q-1} is too small), for many other subcarriers k, the CFO cannot be cancelled completely. But rather, we can only mitigate the CFO to some extent. For example, for k such that $0 \le k + m_{Q-1} \le N - 1$, from (32) we have

$$\mathbf{B}_k = \begin{bmatrix} e^{j\epsilon_0 k} & \cdots & e^{j\epsilon_0 k} \\ \vdots & & \vdots \\ e^{j\epsilon_{I-1} k} & \cdots & e^{j\epsilon_{I-1} k} \end{bmatrix} = \begin{bmatrix} e^{j\epsilon_0 k} & & \\ & \ddots & \\ & & e^{j\epsilon_{I-1} k} \end{bmatrix} \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{bmatrix}. \tag{40}$$

In this case, under our assumption $\epsilon_0 \ne \cdots \ne \epsilon_{I-1}$, we cannot find $\mathbf{z}_k$ to satisfy (31), which means we do not achieve complete CFO cancellation. In fact, the solution to (31) with $\mathbf{B}_k$ in (40) becomes the optimization

$$\min_{\alpha_k} \left\| \alpha_k \left[e^{j\epsilon_0 k}, \cdots, e^{j\epsilon_{I-1} k} \right]^T - \mathbf{b} \right\|^2 \tag{41}$$

where α_k is the summation of the elements of $\mathbf{z}_k$. Taking the derivative of (41) with respect to α_k and letting it be zero, we can derive the optimal solution

$$\alpha_k = \frac{1}{I} \sum_{i=0}^{I-1} e^{-j\epsilon_i k}. \tag{42}$$

Then, the solution to (31) can be simply written as

$$\mathbf{z}_k = \frac{1}{I} \alpha_k \mathbf{b}. \tag{43}$$

Because $\|\mathbf{z}_k\|^2 \le 1/I$, such a $\mathbf{z}_k$ reduces noise as well, although the overall signal-to-interference-and-noise ratio is complex to analyze.

If $k + m_{Q-1} \ge N$, the modulo operations in (32) increase the rank of $\mathbf{B}_k$, so (31) becomes closer to be satisfied, which means CFO can be mitigated better. Because $0 \le k \le N - 1$, a moderate increase of CP length N_g (and thus m_{Q-1}) can greatly enhance CFO mitigation capability. The analysis of such general cases, however, is mathematically involved. We skip those details, but instead focus on the more interesting scenario of complete CFO cancellation, as shown in the next section.

B. Complete CFO Cancellation Under Long CP

We have seen in Sections III-B and IV-A that longer CP improves the CFO mitigation capability, up to a complete cancellation. Though we do not know what the minimum m_{Q-1} (or the minimum CP length N_g) is for complete CFO cancellation, we have the following more relaxed but interesting result.

Proposition 1: Let the CP length be

$$N_g \ge (I-1)N + L + \max_{0 \le i \le I-1} d_i.$$

With parameters $m_q = qN$, where $q = 0, \cdots, I-1$, CFO can be completely cancelled by Algorithm 1 if $\epsilon_i - \epsilon_j \ne (2\pi/N)\ell$, for any $i \ne j$ and integer ℓ.

Proof: Considering $m_q = qN$ and $Q = I$, from (32), we have

$$\begin{aligned} \mathbf{B}_k &= \begin{bmatrix} e^{j\epsilon_0 k} & e^{j\epsilon_0 (k-N)} & \cdots & e^{j\epsilon_0 [k-(I-1)N]} \\ \vdots & \vdots & & \vdots \\ e^{j\epsilon_{I-1} k} & e^{j\epsilon_{I-1} (k-N)} & \cdots & e^{j\epsilon_{I-1} [k-(I-1)N]} \end{bmatrix} \\ &= \begin{bmatrix} e^{j\epsilon_0 k} & & \\ & \ddots & \\ & & e^{j\epsilon_{I-1} k} \end{bmatrix} \\ &\quad \times \begin{bmatrix} 1 & e^{-j\epsilon_0 N} & \cdots & e^{-j\epsilon_0 (I-1)N} \\ \vdots & \vdots & & \vdots \\ 1 & e^{-j\epsilon_{I-1} N} & \cdots & e^{-j\epsilon_{I-1} (I-1)N} \end{bmatrix} \\ &= \mathrm{diag}\{e^{j\epsilon_0 k}, \cdots, e^{j\epsilon_{I-1} k}\} \mathbf{V}_k \end{aligned} \tag{44}$$

for $k = 0, \cdots, N-1$. Since $\mathbf{V}_k$ is an $I \times I$ Vandermonde matrix, under the condition $\epsilon_i - \epsilon_j \ne (2\pi/N)\ell$ for any $i \ne j$ and integer ℓ, both matrices $\mathbf{V_k}$ and $\mathbf{B}_k$ are $I \times I$ square with full rank. Using (44), the (31) is changed to

$$\mathbf{V}_k \mathbf{z}_k = [e^{-j\epsilon_0 k}, \cdots, e^{-j\epsilon_{I-1} k}]^T \tag{45}$$

whose solution always exists. This means (31), and thus (26), can both be satisfied. Then, based on (14) and (16), we can use the matrix $\mathbf{X}$ to completely remove all the CFO matrices $\mathbf{E}_i(m)$ from $\mathbf{y}$. ■

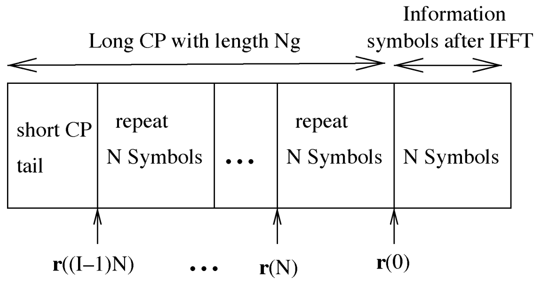

Fig. 3. Structure of OFDM signal with long CP for complete CFO cancellation. The CP consists of $I-1$ repetitions of the N information symbols plus a short tail of conventional cyclic prefix with length no less than $L+\max_{0\le i\le I-1} d_i$ (to combat channel and delay effects). The beginning points of the sample vectors $\mathbf{r}(qN)$ are shown.

Note that the condition $\epsilon_i-\epsilon_j \neq (2\pi/N)\ell$ is stronger than $\epsilon_i \neq \epsilon_j$. But even if $\epsilon_i-\epsilon_j = (2\pi/N)\ell$, we can use a value slightly different from ϵ_i (or ϵ_j) in (45) instead to avoid numerical problems. The resulting $\mathbf{X}$ can still approximately remove CFO.

The structure of the OFDM signal with the long CP specified in Proposition 1 is illustrated in Fig. 3. At first sight, it appears that we have to sacrifice too much bandwidth efficiency for complete CFO cancellation, e.g., even for $I=2$, with the CP length $N_g = N+L+\max_{0\le i\le I-1} d_i$, the bandwidth efficiency is lower than 50% of the convention OFDM. However, the point is that this scheme enhances transmission power efficiency by the long CP while guaranteeing complete CFO cancellation in a computationally efficient manner. Specifically, the transmission power of the long CP is automatically collected by Algorithm 1. In fact, this scheme works like spread-spectrum operations such as multicarrier direct-sequence code-division multiple access (MC-DS-CDMA) [26] though in our case the CFO coefficients in $\mathbf{V}_k$ work as spreading codes, and the procedure (16) becomes effectively a despreading procedure which combines the samples received from the repeated transmissions.

For example, if we consider the first row of the matrix $\mathbf{V}_k$ only (i.e., when $I=1$), then

$$\mathbf{z}_k = (1/I)e^{-j\epsilon_0 k}[1, e^{j\epsilon_0 N}, \cdots, e^{j\epsilon_0 (I-1)N}]^T$$

which not only cancels CFO ϵ_0 but also provides a processing gain I for noise and interference suppression (because $\|\mathbf{z}_k\|^2 = 1/I$). When considering multiple transmitters, i.e., when considering all the rows of $\mathbf{V}_k$, the solution $\mathbf{z}_k$ to (45) may not attain the full processing gain I anymore (since $\|\mathbf{z}_k\|^2$ may be larger than $1/I$), but it still guarantees a certain processing gain.

Therefore, the proposed algorithm is desirable for cooperative transmissions in ad hoc wireless networks, where the long CP (repeated transmissions like spectrum-spreading) is used for CFO cancellation, for high transmission power efficiency as well as for better noise/interference suppression. In addition, the proposed algorithm can also be adapted into existing MC-DS-CDMA systems that are potential choices for future multiple-access communication systems, where the repeated transmissions (with spreading codes) are used for multiple access [26].

Some other benefits of the implementation specified by the Proposition 1 comes from the Vandermonde matrix $\mathbf{V}_k$. Vandermonde equation systems such as (45) have very efficient algorithms to solve, with complexity $O(I^2)$ instead of $O(I^3)$ [27]. As a result, the complexity of calculating $\mathbf{X}$ becomes $O(NI^2)$, instead of $O(NI^3)$ in Section III-D.

Furthermore, Vandermonde system solver can usually give surprisingly accurate solutions, even for ill-conditioned matrix $\mathbf{V}_k$ [27]. This property is especially helpful in the case where some CFOs ϵ_i are close to each other.

The noise performance of $\mathbf{z}_k$ can also be readily analyzed. Since CFO (and ICI) is removed completely, the noise property of the proposed algorithm depends on $\|\mathbf{z}_k\|^2$. Using the property of Vandermonde matrix [27], we have

$$\|\mathbf{z}_k\|_\infty \le \left\|\mathbf{V}_k^{-1}\right\|_\infty \le \max_{0\le\ell\le I-1} \prod_{\substack{i=0\\ i\neq\ell}}^{I-1} \frac{2}{\left|1-e^{j(\varepsilon_i-\varepsilon_\ell)N}\right|}. \quad (46)$$

Therefore, if $\epsilon_i N$ and $\epsilon_\ell N$ are not too close, then the noise performance of our algorithm will be good. Note that the multiplication of N greatly enhances our algorithm's robustness to small CFO difference (i.e., $\varepsilon_i-\varepsilon_\ell$ small). This is partially demonstrated by simulations.

V. Simulations

In order to evaluate the performance of our algorithm, we simulated a system with two cooperative transmitters and one receiver, using Alamouti STBC [3], [4]. We compared the performance of our algorithm (denoted as "New") against the ideal cooperative transmissions with perfect synchronization ("Perfect"), the conventional OFDM receiver without CFO compensation ("Conv.RX"), as well as two OFDMA CFO mitigation schemes: [21] ("CLJL") and [22] ("HL"). Note that for the conventional OFDM receiver "Conv.RX", we simply estimated the CFO at the middle of each OFDM block and used it to compensate for the phase of this OFDM block.

The OFDM FFT block length is $N=32$, with QPSK symbols. The integer delays d_i, the CFOs ϵ_i, and the channels (with order $L=3$) were all randomly generated for each transmitter during each run of the simulation. We usually used 10 000 runs of the simulations to derive the average symbol error rate (SER) under various signal-to-noise ratio (SNR) or various CFO, but more simulation runs were conducted when necessary for extremely low SER.

A. *Performance of Our Algorithm*

First, we studied the performance of our algorithm in combating delay (timing) asynchronism. We set the relative delay of the signals of the two transmitters as 3,5,7 (i.e., $d_0=0$, $d_1=$3, 5, 7), and the relative CFO (rCFO) between them as 0.1. Note that rCFO is defined as the maximum absolute difference of the transmitters' CFOs, i.e.,

$$\text{rCFO} = \max_{\forall i,j} |\epsilon_i-\epsilon_j|.$$

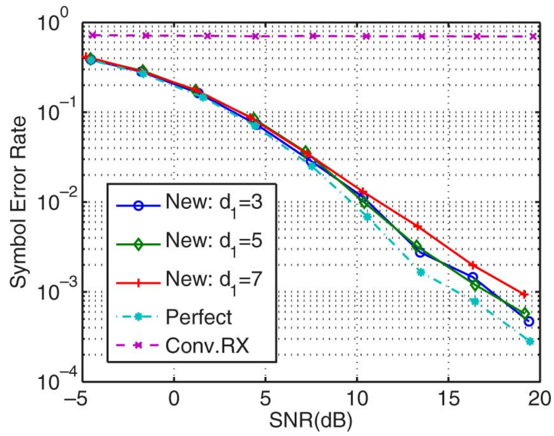

Fig. 4. Performance of our algorithm is independent of delays or TPO. For our "New" algorithm, signals of the two transmitters have delays $d_0 = 0$ and d_1, and CFOs $\epsilon_0 = 0.1$, $\epsilon_1 = 0.2$. The conventional OFDM receiver working under such conditions is denoted as "Conv.RX", while the "Perfect" OFDM works with $d_0 = d_1 = 0$ and $\epsilon_0 = \epsilon_1 = 0$.

In this specific experiment, we used $\epsilon_0 = 0.1$ and $\epsilon_1 = 0.2$. The CP length is $N_g = 40$. We used two sample vectors $\mathbf{r}(0)$ and $\mathbf{r}(32)$ for CFO mitigation in our algorithm. The results are shown in Fig. 4. As expected, our algorithm can work successfully in distributed transmissions with asynchronous delays and small CFO. We also noticed a small noise amplification of our algorithm, which degraded its performance by less than 3 dB compared with the "Perfect" case. Without CFO mitigation, conventional OFDM receiver "Conv.RX" could not work, even with such a long CP.

Next, we studied the performance of our algorithm in extremely large rCFO. The delay difference of the two transmitters was fixed to $|d_0 - d_1| = 1$, while the rCFO was fixed to $|\epsilon_0 - \epsilon_1| = 0.3$ or 0.5. The sample vectors $\mathbf{r}(0)$ and $\mathbf{r}(32)$ were used again. As we can see from Fig. 5, our algorithm has good performance in cancelling CFO, even when rCFO is large. The performance is less than 3 dB worse compared with the "Perfect" OFDM. The slight performance degradation may again be mainly due to the noise amplification effect of the linear CFO mitigation procedure. As expected, conventional OFDM receiver "Conv.RX" did not work here. We also simulated this case using three sample vectors $\mathbf{r}(0)$, $\mathbf{r}(20)$, and $\mathbf{r}(32)$, and the performance was almost identical to Fig. 5.

Fig. 6 shows the tradeoff between the CP length and the CFO mitigation performance. It can be seen clearly that the CFO mitigation performance increases with longer CP, up to a perfect CFO cancellation when $\mathbf{r}(32)$ is used. The parameters for complete CFO cancellation fit well with those in Proposition 1.

In order to evaluate the robustness of our algorithm to CFO estimation errors, we simulated the case when the receiver had CFO estimation error up to $\Delta\epsilon$. Specifically, if the CFO estimation error for the i^{th} transmitter's signal is up to $\Delta\epsilon$, then the estimated CFO $\epsilon_{i,e}$ is uniformly distributed in $[\epsilon_i - \Delta\epsilon, \epsilon_i + \Delta\epsilon]$. In our simulations, the receiver randomly generated the estimated CFO $\epsilon_{i,e}$ within this range, and used it to calculate the matrix $\mathbf{X}$ for CFO mitigation. The results are shown in Fig. 7, from which we can see that CFO estimation error degrades the performance of our algorithm. Nevertheless, at least for $\Delta\epsilon \leq 0.01$, our algorithm still has desirable CFO mitigation performance. Note that many CFO estimation algorithms have estimation accuracy well within this error range. For example, [25] reported CFO estimation accuracy at approximately 10^{-3} to 10^{-4} for the corresponding SNR, while [19] reported multiuser CFO estimation accuracy at approximately 0.004 to 0.0003. Therefore, when integrating our algorithm with these CFO estimation algorithms in practical implementations, the reliability of our algorithm can be guaranteed.

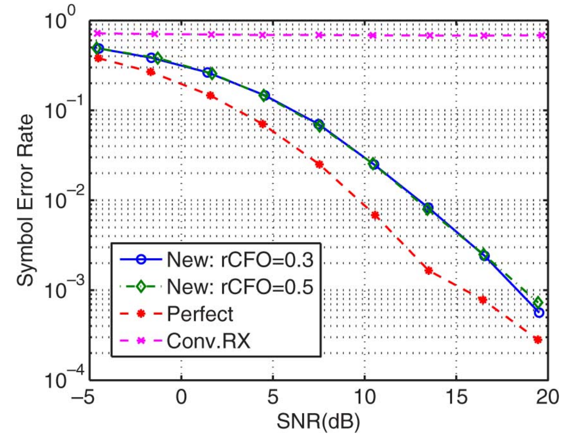

Fig. 5. Our "New" algorithm can mitigate extremely large CFOs. Simulated with $|d_0 - d_1| = 1$, and $\text{rCFO}|\epsilon_0 - \epsilon_1| = 0.3$ or 0.5.

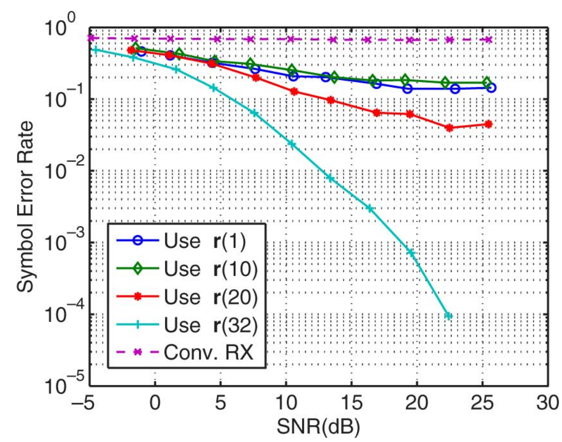

Fig. 6. CFO mitigation performance of our "New" algorithm increases when using longer CP. $\text{rCFO}|\epsilon_0 - \epsilon_1| = 0.1$, $|d_0 - d_1| = 1$.

B. Comparison With Other CFO Mitigation Algorithms

In this experiment, we compared our algorithm with two other CFO mitigation algorithms, specifically, "CLJL" [21] and "HL" [22]. The simulation parameters (such as N, L, etc) were set the same as the previous experiments, except $d_i = 0$. Note that simulations in [21] and [22] used 1/2 convolutional code which we did not implement. Instead, we changed their algorithm to use STBC. In addition, one iteration was used for [22]. A tricky problem was that our scheme had a longer CP, and thus had lower bandwidth efficiency. For $I = 2$, the OFDM block length in our scheme was $32 + 32 + 3 = 67$, while that for conventional OFDM was $32 + 3 = 35$. For a fair comparison, we tried

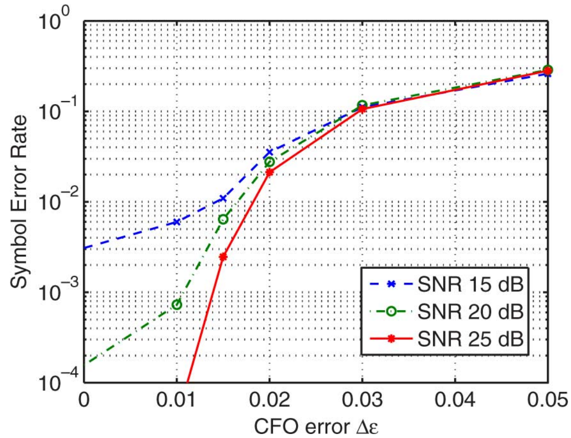

Fig. 7. Performance of our "New" algorithm under CFO estimation errors up to $\Delta\epsilon$, for SNR 15, 20, and 25 dB. $\Delta\epsilon = 0$ means perfect CFO knowledge for the receiver.

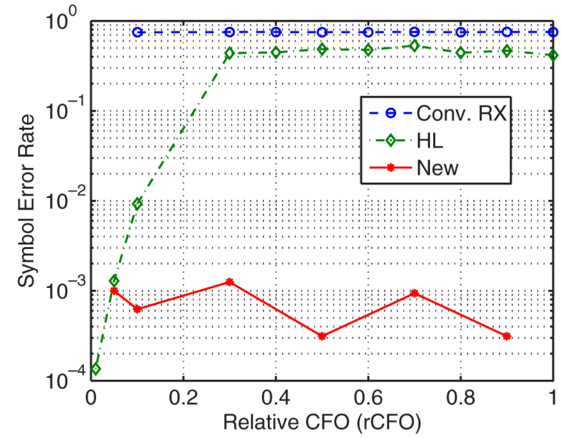

Fig. 9. Our "New" algorithm has a performance almost independent of the size of CFO, while "HL" works only when rCFO is small enough. SNR 20 dB.

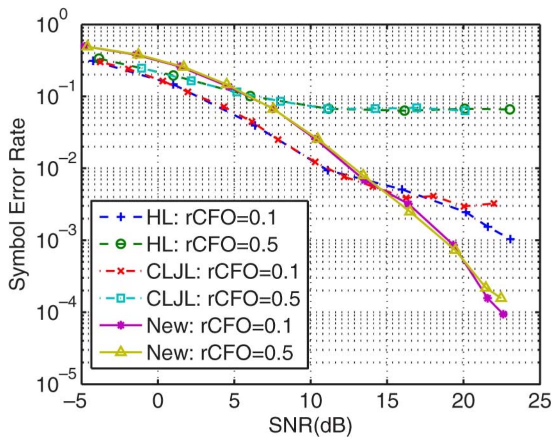

Fig. 8. Performance comparison of our "New" algorithm with HL [21] and CLJL [21] under rCFO $|\epsilon_0 - \epsilon_1| = 0.1$ and 0.5.

two ways: reduce the transmission power of our scheme by a factor 67/35, or change the modulation from QPSK to 16QAM. Nevertheless, these two ways gave a similar performance, so we just show the results obtained by the first way in Fig. 8 and Fig. 9. From Fig. 8, we can see that our algorithm has much better performance when SNR is not extremely low, and the advantage is even more significant for large rCFO. In particular, when rCFO is 0.5, "HL" and "CLJL" failed, but our algorithm had a performance almost independent of the size of rCFO. On the other hand, "HL" and "CLJL" worked reliably under small rCFO, such as $\text{rCFO} = 0.1$, and in this case they may outperform our scheme in low SNR.

From Fig. 9, we can see that our "New" algorithm has a performance almost independent of the size of CFO, which clearly demonstrates the advantage of complete CFO cancellation. For a wide range of rCFO from 0 to 1, our algorithm can successfully mitigate CFO. The slight variation in SER may be explained by (46), which shows that the noise amplification effect of our algorithm depends on $e^{j(\epsilon_i - \epsilon_\ell)N}$ which is a periodic function. From Fig. 9, we also see that the conventional OFDM receiver "Conv.RX" could not resolve the CFO problem, neither did the "HL" scheme when the rCFO was not very small. The "HL" worked when the rCFO was less than about 0.1, which was somewhat worse than what reported in [22]. The reason might be that we simulated STBC-encoded transmissions with subcarrier sharing, while [22] simulated OFDMA without subcarrier sharing, so the interference level in our simulation was higher, which can greatly degrade the performance of interference cancellation schemes like [21]–[23].

VI. Conclusion

In this paper, we proposed a new CFO mitigation algorithm for multi-transmitter cooperative OFDM transmissions. A unique feature is that it can completely cancel CFO when the cyclic prefix is long enough. In addition, the long CP can be exploited for transmission power efficiency because our algorithm provides processing gain to combat interference and noise. The algorithm is formulated as a computationally efficient preprocessing procedure independently from the cooperative encoding/decoding details, and may thus have ubiquitous applications in cooperative OFDM transmissions. On the other hand, while enhancing power efficiency, a major problem for the proposed algorithm is that in the case of a large number of cooperative transmitters, complete CFO cancellation comes with a rapid reduction of bandwidth efficiency. As a result, it remains as an interesting future research topic to develop complete CFO cancellation techniques without the loss of bandwidth efficiency.

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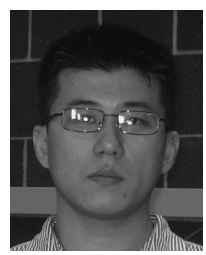

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