AN EFFICIENT METHOD FOR HOP SELECTION AND CAPACITY ENHANCEMENT IN MULTI-HOP WIRELESS AD-HOC NETWORKS

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ABSTRACT

In this paper we propose an approach with reduced complexity for the optimization of a multi-hop transmission path through a wireless network. Based on the assumption that nodes use successive interference cancellation (SIC) and maximal ratio combining (MRC) to deal with mutual interference and to utilize all the received signal energy, we first derive the signal-to-interference-plus-noise ratio (SINR) expression for each node. Then we show that under mild conditions the maximum capacity for an H-hop path can be found by a max-min optimization on the line connecting the source and destination nodes. A simple algorithm is then proposed to select the best hop nodes for enhancing capacity. The complexity of the algorithm is controllable, and can be made low enough for large network sizes. Extensive simulations are conducted to demonstrate its performance.

1. INTRODUCTION

Multi-hop wireless networks formed by a large number of distributed nodes are one of the classes of emerging networks. Typical examples include wireless sensor networks [1], networked robotic systems and wireless ad-hoc networks. They have potentially wide applications in military, industry, and even future homes. What make them unique are their common characteristics, such as massively distributed yet redundant structure, coordinated information processing among nodes with limited individual bandwidth, energy and reliability, and large network size.

For such networks, network capacity is a critical concern not only because large networks generate massive information for communications, but also because the communications capacity per node reduces with the increase of number of nodes [2]. Differently from wired networks, nodes competition and cooperation in wireless networks make the hop selection and capacity optimization extremely difficult. Wireless nodes can interfere each other when transmitting, but can also help each other via relaying and cooperation. On the other hand, though interference in general degrades signal-to-interference-plus-noise ratio (SINR), receiving nodes may exploit some interference.

Capacity of wireless networks is still an open problem, with only some limited research results in the literature. Among them there are results about the scaling properties of large wireless networks with infinite size [2],[3],[4]. More detailed capacity region results are available for small networks with one or two hops and a few nodes only [5]. As a different approach, the method in [6] can calculate capacity regions for multi-hop wireless networks. However, because the complexity increases rapidly, its application is limited to small networks with less than 15 nodes. In fact, brute-forth exhaustive methods rapidly become prohibitive even for small networks.

Most wireless network routing protocols tend to avoid such special competition and cooperation issues [7], and thus can not provide optimal capacity and performance. There is another class of methods that depend on sophisticated simulation techniques for network optimization. In this direction, some of the new evolutionary computing techniques have been adopted for the optimization of wireless sensor networks [8], the throughput optimization of multi-hop wireless net-

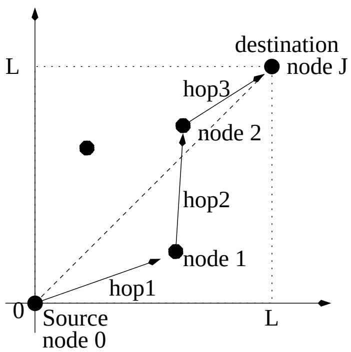

Fig. 1. A wireless network with a source node transmitting packets to destination node via a 3-hop relaying path.

Slot #	0	1	2	......	k	.....
Node 0	Tx Pk 0	Tx Pk 1	Tx Pk 2	......	Tx Pk k	.....
Node 1	Rx Pk 0	Tx Pk 0 Rx Pk 1	Tx Pk 1 Rx Pk 2	......	Tx Pk k Rx Pk k+1	.....
⋮	⋮					⋮
Node j				......	Tx Pk k−j Rx Pk k−j+1	.....
	⋮					⋮

Fig. 2. Transmission and receiving slots for the nodes in the relaying path.

works [9], optimal resource allocation for wireless ATM networks [10], and optimizing wireless network layouts [11].

Considering that the problem of hop selection and capacity optimization is still open but is critical for multi-hop wireless network development and performance analysis, in this paper we develop a new method that can efficiently optimize hop selection for enhancing the capacity of a multi-hop transmission path. It permits path capacity calculation in even large wireless networks.

The organization of this paper is as follows. In Section 2, we give the multi-hop wireless network model. Then in Section 3, we develop the new method. Extensive simulations are conducted in Section 4. Conclusions are then given in Section 5.

2. MULTI-HOP WIRELESS NETWORK MODEL

We consider a wireless network with $J + 1$ nodes. Without loss of generality, we let the nodes distribute uniformly within a square of $L \times L$ meters, as shown in Fig. 1. For simplicity, we consider only one transmission path from a source node, which we denote as node 0 with a position $(0, 0)$, to a destination node, which we denote as node J with a position (L, L). Any of the other $J - 1$ nodes, which we denote as node 1 to node $J - 1$, may participate in the relaying. In Fig. 1, a 3-hop transmission path is illustrated.

Let the distance between node i and node j be d_{ij}, and let each node have a transmission power p if participating in transmission. If node i transmits with a power p, then the received signal power at node j is pg_{ij}, where $g_{ij} = d_{ij}^{-\alpha}$ with the path loss exponent α. We do not consider other small scale fading in this paper for simplicity. Obviously, we need to limit the minimum distance among the nodes to be no less than 1 to avoid the impractical problem of receiving power higher than transmission power.

The problem considered in this paper is the optimization of hop selection in case H-hop relaying is required. The objective of such optimization is to maximize the transmission capacity of H-hop path. In an H-hop transmission path, the participation nodes are denoted as node $0, 1, \cdots, H$, where $H = J$, and each node i, for $1 \leq i \leq H - 1$, is chosen from the rest $J - 1$ nodes without repetition inside the network.

The transmission of packets follows a slotted structure, as shown in Fig. 2. Specifically, in slot 1, node 0 transmits a packet to node 1. Note that all the nodes (1 to H) can hear the transmission, but just having different received signal power because of the different distances to node 0. In general, the received signal power of node i from this transmission can be described as $pg_{0,i} = pd_{0,i}^{-\alpha}$, where $1 \leq i \leq H$. Without loss of generality, we assume that node 1 have the strongest received signal power. If the SINR of node 1 is large enough, then node 1 can successfully decode the packet and retransmit it in slot 2. Meanwhile, simultaneously the node 0 transmits a new packet in slot 2. This means that node 1 (and any other node) needs to receive and decode a new packet while transmitting the current packet. This simultaneous transmission and receiving assumption can greatly simplify our SINR and capacity analysis, because otherwise we have to consider endlessly many different slot transmission schemes. Later we will see that the simultaneous transmission does not cause theoretical problems for the node to receive a new packet.

Therefore, node 0 begins transmitting packet 0 in slot 0, and transmits one new packet in each subsequent slot. Node 1 begins transmission of packet 0 in slot 1 while detecting the packet 1. It detects one new packet and transmits one old packet simultaneously in each subsequent slot. So do all the other nodes 2 to $H-1$, except that the node i begins transmission of packet 0 in slot i. The destination node $H = J$ conducts receiving and decoding in all slots.

Although we assume that a node can conduct transmission and receiving at the same slot, it can not transmit and receive/decode the *same* packet simultaneously. Instead, a packet can be transmitted only after it was decoded during the previous slots. This guarantees proper multi-hop relaying delays, i.e., the larger the hop count H, the larger the delay, which is another important feature of wireless multi-hop networks besides path capacity.

Referring to Fig. 2, the received signal of node j in slot k can be described as

$$x_j(k) = \sum_{i=0}^{H-1} \sqrt{pg_{i,j}} e^{j\theta_{i,j}} u(k-i) + \sqrt{N} v_j(k), \quad (1)$$

where $u(k)$ denotes the signal of packet k, $\theta_{i,j}$ denotes the channel phase of the propagation path from node i to node j, and $v_j(k)$ denotes the noise received by node j in slot k. We assume that all the nodes have the same receiving noise power N for simplicity, which means $E[|v_j(k)|^2] = 1$. In (1), $u(k-i)$ means that the node i transmits packet $k-i$ in slot k. We have assumed that each node applies the same encoding and modulation schemes for the same packet, and the packet signal $u(k)$ have unit norm $E[|u(k)|^2] = 1$.

From (1) we see that while node j is receiving signal $x_j(k)$ in slot k, it also transmits a packet $u(k-j)$. Obviously, in order to support continuous operation, i.e., the node j transmits the packet $u(k-j+1)$ during slot $k+1$, we need to guarantee that the node j can detect the packet $u(k-j+1)$ in slot k using the received signal $x_j(\ell)$ for all $\ell = 0, \cdots, k$. Note that the signal $x_j(k)$ is a composition of H packets transmitted by the H nodes ($H-1$ relaying nodes and the source node), which means that the packet $u(k-j+1)$ is also contained in previously received signals. Specifically, the packet $u(k-j+1)$ is transmitted by nodes 0 to $j-1$ during slots $k-j+1$ to k, respectively. As a result, a better strategy for the node j is to utilize received signals $x_j(\ell)$ for $k-j+1 \leq \ell \leq k$ in order to detect the packet $u(k-j+1)$.

From the description of the signal model (1), we see that we have considered the two special properties of wireless networks: mutual interference among nodes and cooperation among nodes. However, a packet is transmitted by only one node in each slot, which means that we do not consider some more sophisticated cooperation strategies, such as the simultaneous transmission of a packet by multiple nodes [12], [13]. In addition, we consider only decode-and-forward relaying, not amplify-and-forward or others. Each node in the H-hop path needs to be able to decode each packet successfully. Based on this fact, we will derive the SINR of each node, from which the capacity of this node is available, and the capacity of this H-hop path can be derived as the minimum capacity among the H receiving nodes. The problem of hop selection and capacity optimization thus becomes a max-min optimization.

3. SINR ANALYSIS AND OPTIMIZATION

In this section, we first derive the SINR expression for each node, then propose a method to find the optimal hop nodes for an H-hop relaying path.

3.1. SINR analysis for each node

Recall that the node j needs to be able to decode the packet $u(k-j+1)$ in slot k using the received signals $x_j(\ell)$ for $k-j+1 \leq \ell \leq k$. Nevertheless, the signals $x_j(\ell)$ can be further simplified.

From (1), we see that in slot k, the node $j-1$ transmits the packet $u(k-j+1)$ to the node j, while all other nodes' signals are looked as interference by the node j. Among the H packets that contained in $x_j(k)$ in (1), the node j has already decoded and thus knows $H-j$ of them. In fact, the node j has already transmitted or is transmitting these $H-j$ packets. Specifically, the packets transmitted by any node $i >= j$, including the node j itself, are known to the node j. Only the packages transmitted by nodes $i < j$ are new to the node j. Therefore, using successive interference cancellation (SIC) and knowledge of all channels, the node j can remove the known packets from (1) and reduce $x_j(k)$ to

$$\hat{x}_j(k) = \sum_{i=0}^{j-1} \sqrt{pg_{i,j}} e^{j\theta_{i,j}} u(k-i) + \sqrt{N} v_j(k). \quad (2)$$

For this signal, the signal-to-interference plus noise ratio (SINR) is

$$s_j(k) = \frac{pg_{j-1,j}}{\sum_{i=0}^{j-2} pg_{i,j} + N}. \quad (3)$$

However, this is not the only signal that the node j has for the detection of packet $u(k-j+1)$, and thus this SINR can be improve by other signals. In the slot $k-1$, the node j has obtained a signal similar to (2), which is

$$x_j(k-1) = \sum_{i=0}^{j-1} \sqrt{pg_{i,j}} e^{j\theta_{i,j}} u(k-1-i) + \sqrt{N} v_j(k-1). \quad (4)$$

In the slot $k-1$, it should have decoded and thus known the packet $u(k-j)$. Then it can now remove this packet and reduce (4) to

$$\hat{x}_j(k-1) = \sum_{i=0}^{j-2} \sqrt{pg_{i,j}} e^{j\theta_{i,j}} u(k-1-i) + \sqrt{N} v_j(k-1), \quad (5)$$

which contains information about the package $u(k-j+1)$ as well. The SINR for the signal (5) is

$$s_j(k-1) = \frac{pg_{j-2,j}}{\sum_{i=0}^{j-3} pg_{i,j} + N}. \quad (6)$$

The above procedure can be easily extended to reducing all signals that contains the packet $u(k-j+1)$. Specifically, the node j can exploit its received and processed signals in slots $k-j+1, \cdots, k-1, k$, which have the general form as

$$\hat{x}_j(k-\ell) = \sum_{i=0}^{j-\ell-1} \sqrt{pg_{i,j}} e^{j\theta_{i,j}} u(k-\ell-i) + \sqrt{N} v_j(k-\ell), \quad (7)$$

where $\ell = j-1, \cdots, 0$, to detect the packet $u(k-j+1)$. The SINR for $\hat{x}_j(k-\ell)$ is

$$s_j(k-\ell) = \frac{pg_{j-\ell-1,j}}{\sum_{i=0}^{j-\ell-2} pg_{i,j} + N}. \quad (8)$$

Now the node j has j received signals to detect a packet $u(k-j+1)$, which needs to be optimally combined to maximize the SINR. One of the ways for optimal combining is maximal ratio combining (MRC). In order to derive MRC, we first need to normalize the signals $\hat{x}_j(k-\ell)$ by their corresponding interference plus noise power. Specifically, the interference plus noise power of the signal $\hat{x}_j(k-\ell)$ is

$$I_j(k-\ell) = \sum_{i=0}^{j-\ell-2} pg_{i,j} + N, \quad (9)$$

which is exactly the denominator of (8). Then the signals can be normalized as

$$\tilde{x}_j(k-\ell) = \frac{1}{\sqrt{I_j(k-\ell)}} \hat{x}_j(k-\ell). \quad (10)$$

Note that after normalization, the SINR for $\tilde{x}_j(k-\ell)$ is still (8).

Then the MRC is conducted as

$$y_j(k) = \sum_{\ell=0}^{j-1} a_\ell \tilde{x}_j(k-\ell), \quad (11)$$

with combining weights a_ℓ. The optimization objective is to maximize the SINR of $y_j(k)$, which we denote as s_j.

Proposition 1. With the optimal MRC coefficients

$$a_\ell = \sqrt{s_j(k-\ell)} e^{-j\theta_{j-\ell-1,j}}, \quad (12)$$

the SINR of $y_j(k)$ in (11) is maximized and equals to the summation of individual SINR in (8), i.e.,

$$s_j = \sum_{\ell=0}^{j-1} s_j(k-\ell). \quad (13)$$

Outline of Proof. This is a standard MRC optimization and can be proved by using Schwartz inequality. The key point is to locate the signal part in $\tilde{x}_j(k-\ell)$, which is $\sqrt{s_j(k-\ell)} e^{j\theta_{j-\ell-1,j}} u(k-j+1)$. The interference and noise components are mutually independent due to both the random channel phases and the random signals. □

Based on Proposition 1 and (13), we can calculate the SINR for a node j in an H-hop relaying path when detecting packets as

$$s_j = \sum_{\ell=0}^{j-1} \frac{pg_{j-\ell-1,j}}{\sum_{i=0}^{j-\ell-2} pg_{i,j} + N}, \quad (14)$$

for any node $j = 1, \cdots, H$.

A interesting property is that the nodes $i > j$ (after the node j) in the hop-chain do not play a role in the SINR of the node j. In contrast, the nodes $i < j$ (before the node j) in the hop-chain both contribute interference to reduce the SINR and contribute useful signal to increase the SINR of the node j.

For the H-hop relaying path with node SINR s_j, where $j = 1, \cdots, H$, the transmission capacity is

$$C_{1,\cdots,H}(H) = \min_{1 \le j \le H} \log_2(1 + s_j). \qquad (15)$$

Furthermore, in a network with $J + 1$ nodes, in order to find the highest H-hop transmission capacity from node 0 to node J, we need to select the best $H-1$ nodes to form an H-hop transmission path that has the highest capacity. This can be configured as a max-min optimization problem

$$C(H) = \max_{\text{nodes } \{1,\cdots,H-1\} \subset \{1,J-1\}} C_{1,\cdots,H}(H). \qquad (16)$$

Unfortunately, exhaustive search of all possible node combinations becomes prohibitive even for small J. Therefore we need to look for new methods with reduced complexity.

3.2. Hop optimization in source-destination line and node selection

From the SINR expression (14), we have seen the complex relationship among the nodes. For simplification, we consider the case that the first term in s_j (with $\ell = 0$) is the dominating one, i.e.,

$$\frac{pg_{j-1,j}}{\sum_{i=0}^{j-2} pg_{i,j} + N} \gg \sum_{\ell=1}^{j-1} \frac{pg_{j-\ell-1,j}}{\sum_{i=0}^{j-\ell-2} pg_{i,j} + N}. \qquad (17)$$

Intuitively, this means that the transmission of the node $j-1$ has a dominating contribution to the received signal of the node j. Obviously, this is a reasonable assumption for a fixed H hop count. Otherwise, if the first term is in-significant, then the transmission of nodes 0 to $j-2$ is even stronger than node $j-1$ to the node j. This means that the node $j-1$ in fact wastes its transmission power, and this path can not have the highest capacity among the H-hop paths. Therefore, there is no loss to avoid considering such cases.

Under the assumption (17), we can derive a simple way for selecting the hop nodes to enhance the transmission capacity.

Proposition 2. For any H-hop relaying path, there exists a corresponding H-hop relaying path along the line connecting the source and the destination that has larger transmission capacity, if the nodes can be put in corresponding places on this line.

Remark: Intuitively, this proposition means that shorter transmission distance has higher transmission capacity under the same total transmission power.

Outline of Proof. Let us consider an arbitrary H-hop relaying path as illustrated in Fig. 1. Corresponding to the node 1, we can find a point on the source-destination line that has equal distance to the source as the node 1 (which is $d_{0,1}$). Then if we place a relay in this point, it will have the same SINR as node 1.

Next, corresponding to node 2, we can place another relay on the line so that it has a distance to the first node (on the line) that is the same as $d_{1,2}$. Now because $d_{0,2}$ is smaller than the distance of the new node on the line to the source, this new node suffers smaller interference, and thus has higher SINR than the node 2. This can be shown easily under (17).

Similarly, we can show that for any other nodes in the path, we can find a corresponding point on the line that has larger SINR, up to the destination. □

The significance of Proposition 2 is that the upper bound of H-hop path capacity can be found by a max-min optimization along the source-destination line. This max-min optimization can be conducted relatively more efficiently. Specifically, in order to find the highest capacity of H-hop relaying, we just need to find $H-1$ positions in the line that gives the highest SINR.

Let the parameter d_k, $k = 0, \cdots, H-1$, denote the distance between the node k and the node $k+1$, respectively. Then the max-min optimization is formulated as a constrained optimization

$$\max_{\{d_k\}} \min_{1 \le j \le H} \sum_{\ell=0}^{j-1} \frac{p \sum_{m=j-\ell-1}^{j} d_m^{-\alpha}}{\sum_{i=0}^{j-\ell-2} p \sum_{m=i}^{j} d_m^{-\alpha} + N}, \qquad (18)$$

under the constraint $\sum_{k=0}^{H-1} d_k = d_{0,J}$. We may also need the constraints $d_k \ge 1$ for $k = 0, \cdots, H-1$ to avoid the impractical case that small d_k makes received power larger than transmission power.

The max-min optimization (18) usually needs good initial conditions for global convergence. Fortunately, for each hop count H, we just need to find one set of optimal d_k and the corresponding capacity. Importantly, the optimization is not related to the network size, or the total number of nodes $J+1$. As a result, it is feasible find the optimal solutions by numerical evaluations. Some numerical results are shown in Section 4.

Based on the pre-calculated optimal hop distances $\{d_k\}$, we can then develop an efficient algorithm to look for the best hop nodes for arbitrary wireless networks. The algorithm is outlined below.

Without loss of generality, let us assume that we need to determine an H-hop relaying path in a network of $J+1$ nodes from node 0 to node J. First, corresponding to each of the optimal hop distance d_k, we can locate a point on the source-destination line. Then centered around this point, we can find M nodes (among the $J-1$ arbitrary nodes) that are closest to this point. This step is repeated for each distance d_k. Finally, after we find $H-1$ sets of M nodes, we can do an exhaustive search of M^{H-1} possible paths to find the one that gives the highest capacity.

An intuitive explanation of the above procedure is that in a dense wireless network, there are nodes close to the optimal relay locations with high probability. The complexity of the exhaustive search in the final step is controllable via the pre-determined parameter M. The larger the M, the better the optimization results, but also the higher the complexity. Because of limited space in this paper, we next give some simulation results to show the superior performance of this algorithm.

4. SIMULATIONS

In this section, we use numerical optimization to evaluate (18) and use Monte-Carlo simulations to verify the proposed method. We assume $L = 100$ meters. For each hop count H, we can solve (18) and find the optimal relay locations. The corresponding path capacity can also be calculated by (15)-(16). We normalize the path capacity by the direct source to destination transmission capacity as $C(H)/C(1)$. The capacity based on numerical evaluation of (18) is shown in Fig. 4, where we denote the numerical results as "analysis" results.

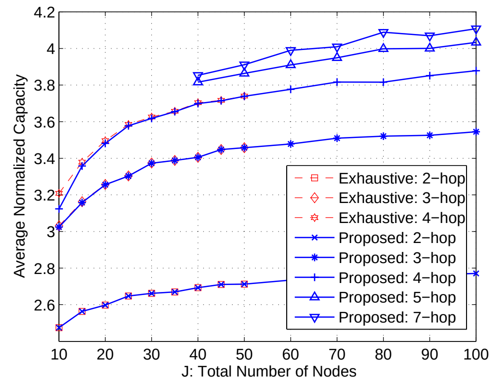

Fig. 3. Average capacity as function of hop count H and node amount J.

In Monte-Carlo simulations, for various node number J, we randomly place the nodes. Then we simulate both the complete exhaustive search with complexity $(J-1)\times(J-2)\times\cdots\times(J-H)$ and the proposed algorithm with a complexity M^H. We denote them as "Exhaustive" and "Proposed" results in the figures.

In Fig. 3, we clearly see that the proposed method fits very well with the complete exhaustive search. The error is very small, especially when the number of nodes is not very small. In addition, the proposed method works for extremely large number of nodes and long hops, where the exhaustive search method becomes computationally prohibitive. The average capacity increases when the node number J increases, or when the hop count H increases.

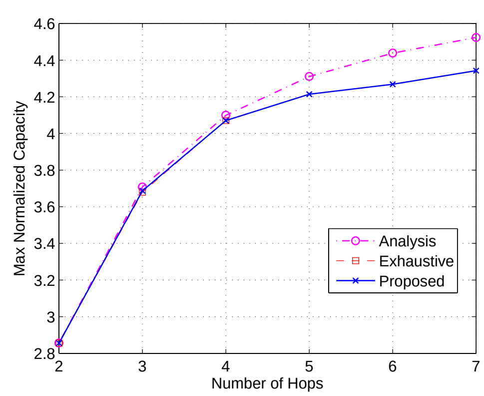

Fig. 4. Maximum capacity as function of hop count H.

In Fig. 4, we see that the maximum capacity obtained by the three ways fits very well. When hop account is small, the analysis results and the results of the proposed method are both almost identical to exhaustive search results. When hop count H increases, however, the proposed method gives results smaller than the analysis results, which is because the number of simulation iterations was limited so we could not encounter those optimal node placements. In Fig. 5, we further see that the maximum capacity found by our proposed method fits well with the exhaustive search method.

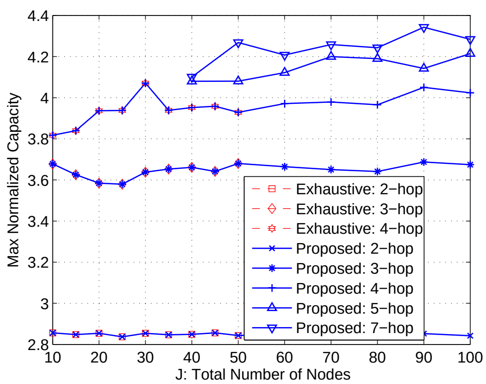

Fig. 5. Maximum capacity as function of hop count H and node number J.

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