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Reliability and Feedback of Multiple Hop Wireless Networks

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Abstract: This paper analyzes fault-tolerance over the entire design life of a class of multiple-hop wireless networks, where cooperative transmission schemes are used. The networks are subject to both node failure and random channel fading. A node lifetime distribution is modeled with an increasing failure rate, where the node power consumption level enters the parameters of the distribution. A method for assessing both link and network reliabilities projected at the network's design life is developed. Link reliability is enhanced through use of redundant nodes. The number of redundant nodes is restricted by the cooperative transmission scheme used. The link reliability is then used to establish a re-transmission control policy that minimizes an expected cost involving power, bandwidth expenditures, and packet loss. The benefit and cost of feedback in network operations are examined. The results of a simulation study under specific node processing times are presented. The study quantifies the effect of loop closure frequency, acknowledgment deadline, and nodes' storage capacity on the performance of the network in terms of network lifetime, packet loss rate, and false alarm rate. The study concludes that in a network where energy is severely constrained, feedback must be applied judiciously.

Keywords: Fault-tolerance, network reliability, energy efficiency, packet loss rate, discrete event simulation, feedback, optimal control.

1 Introduction

1.1 Objectives and scope

The class of wireless networks under consideration is the class of multiple-hop, distributed networks of abundant nodes in clusters. Each node has a limited energy supply that cannot be replenished, and is capable of packet transmission, reception, and processing that involves detection, fusion, coding and decoding. Our main objectives are to maximize the *network reliability*[1] at its *design life*[2], and to quantify the effect of loop closure frequency on the performance of the network in terms of network lifetime, and packet loss rate.

A crucial step to achieving the objectives is to develop a power covariate network reliability model. As a result, the network reliability can become the overarching measure that encompasses aspects of symbol error rate, energy efficiency, bandwidth efficiency, the effect of clustering, and the effect of feedback.

Many algorithms have been developed for the computation of node-pair reliability of networks, which is the probability that at least one route exists between a source node and a terminal node[1]. Unlike any other networks, however, each route in our network itself forms a sub-network of a nested structure bound by the cooperative transmission scheme used. The nested structure is a result of modeling both node life and channel fading. Therefore, we confine ourselves to the sub-network of a K-cluster route through which packets hop from cluster 1 to cluster K. The restriction to the single-route problem is due to our intention to capitalize on some new physical layer transmission schemes[2−4]. Our interest is not in devising routing protocols[5] that enhance the network connectivity evaluated using the knowledge of the spatial distribution of the wireless nodes[6], or prolong network lifetime assessed using the deterministic knowledge of energy expenditure at each node[7]. Instead, we are seeking to understand and to optimize the temporal evolution of network reliability, and to use this information to help determine the level of redundancy and level of supervisory activity in the network operation.

With proper formulation of a cooperative transmission problem employing multiple nodes, transmission diversity can be provided to combat deep-fading suffered by the near-ground communications[8, 9]. The existing cooperative diversity schemes, though efficient in transmission power, increase the circuit energy consumption associated with, for example, static current in transceivers and encoding/decoding circuitry, when multiple nodes must be kept on for listening and reception[10]. The authors have recently developed cooperative transmission schemes that can address simultaneously power efficiency, bandwidth efficiency, and fault-tolerance. Our simulation with a new cooperative transmission scheme[2] indicated a 6-fold reduction in power consumption at an enhanced level of network reliability with a two-node cluster that achieves a 15 dB signal to noise ratio at the receiving cluster. This scheme can be implemented using a space-time block coding technique[3, 4]

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[1] The network reliability $R^{net}(t)$ is defined as the probability that the network performs successfully its required function over a period of t time units under the stated operating conditions.

[2] A design life T_D is defined as the maximum time by which a prescribed network reliability is maintained.

without loss of bandwidth efficiency.

Built on our work of cooperative transmission, the first idea to be explored in this paper is to determine the level of redundancy appropriate for the cooperative transmission that also maximizes the network reliability at the network design life. Of particular interest is the question on how much feedback is appropriate at a certain level of redundancy usage for a prescribed network design life and a prescribed packet loss rate. The answer is sought numerically in this paper under the constraints that 1) processing time of a packet by a node is finite, 2) supervisory activities consume lifetimes of the nodes, and 3) a specific supervisory protocol is assumed. A general-purpose simulation tool Arena[11, 12] is used for this purpose. Though Arena does not have the network-oriented convenience afforded by specialized tools for networks, it offers a greater flexibility to model and to scrutinize aspects of a network and its performance for the stated unique objectives of this work.

The paper is organized as follows. In the next subsection, a re-transmission chain is formed that serves to motivate the quest for understanding the impact of loop-closure on the network reliability. Section 2 introduces link reliability through modeling a power-covariate lifetime distribution of a node, based on which network level reliability is also derived. The link reliabilities are then applied to optimally assign participating nodes to clusters. The section also solves a simple version of a re-transmission problem formulated as a Markov decision problem with partial information feedback. Section 3 describes a particular acknowledgment protocol for packet reception for a 5-hop network modeled with Arena. The section analyzes network lifetime, packet loss rate, and false alarm as they relate to loop closure rate based on data generated through simulations. Section 4 discusses the implications on network design based on our analytical and simulation study, limitations of our work, and areas to be investigated in the future.

1.2 A motivating example

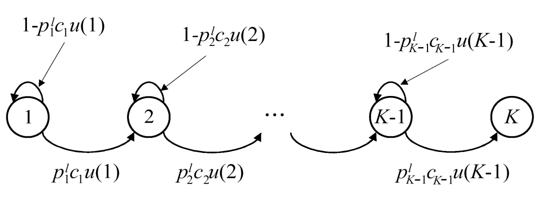


Fig. 1 Packet transmission in a K-cluster route

The simple Markov chain in Fig. 1 describes a K-cluster packet transmission process, where state name i stands for the i^{th} cluster through which a packet hopping from the source through the network to the destination must pass, where link reliability p_i^l, to be derived in the next section, is the probability that a packet reaching the i^{th} cluster is successfully relayed to the $i+1^{\text{th}}$ cluster with a required power level. c_i is the conditional probability (called a supervisory coverage)[13] that upon the failure of the first transmission attempt, a re-transmission command is successfully issued to cluster i. In an unsupervised environment, c_i is set to 1 for the initial transmission attempt, and to 0 for all re-transmissions. In a supervised environment, on the other hand, $0 < c_i < 1$. The factors affecting c_i include lack of observability of state, erroneous state estimation, failure of a supervising nodes and channels involved in supervisory activities, and collision of packets involved in such activities. Therefore, it is reasonable to assume $c_i \leq p_i^l$.

With the Markov chain established, the state probability p_i^c, i.e., the probability that a packet reaches cluster i, can be calculated by solving for $\pi(k) = [p_1^c \ \cdots \ p_K^c]$ recursively from $\pi(k+1) = \pi(k)M(k, k+1)$, where $M(k, k+1)$ is called a probability transition matrix[14] with entries depending on $p_i^l c_i$.

Assume each transmission attempt consumes power P_i. The average number of transmissions needed to reach state $i+1$ can be shown to be $\bar{N}_i = (p_i^l c_i)^{-1}$, $i = 1, \cdots, K$. Then the power usage per packet transmission through the network can be estimated by $\bar{P} = \sum_{i=1}^{K} p_i^c \bar{N}_i P_i$. Then $E = \sum_{t=0}^{T_D} \bar{P}$ is an estimate of the network energy efficiency over its lifetime. Here the notion of network age t is specialized to the number of packet transmissions that the network has carried out so far with the assumption that all clusters age uniformly and the number of redundant nodes in each cluster is large.

Let us consider two simple but representative cases. In the first case there are no feedback and no supervisory activity, i.e., $c_i = 0$ for all re-transmissions, while the cluster transmission with multiple nodes is used. In the second case a supervisory scheme is in place to issue re-transmission whenever needed, while only a single node in a cluster is used at a time for each transmission attempt.

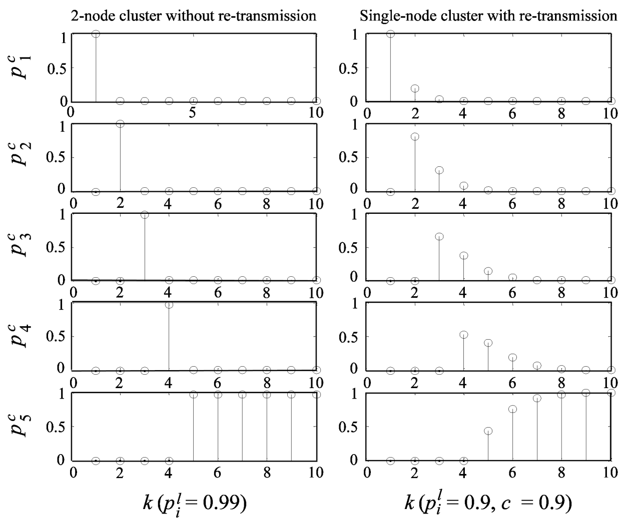


Fig. 2 2-node w/o feedback v.s. 1-node w/ feedback

Fig. 2 shows 10 snapshots of state probabilities for a 5-cluster route when a packet transmission is initiated at $k = 1$ for the above mentioned two representative cases. The five rows of the plots are p_1^c through p_5^c, indexed at consecutive time instants of packet transmissions. The left column of plots is for the unsupervised case, where the link reliability $p_i^l = 0.99$ for all i at the current network age, resulting from a 2-node cooperative transmission with a reliability of 0.9 for each node. For the moment perfect channels are assumed, in which case a link reliability is the same as a cluster reliability. The right column of plots is for the supervised case, where the link reliability $p_i^l = 0.9$ for all i, resulting from a 1-node/transmission scheme with a node reliability also 0.9, and a supervisory coverage $c_i = 0.9$.

The following can be observed. 1) Without feedback, the network reliability $\prod_{i=1}^{K} p_i^l$ depends solely on the individual link reliabilities. Therefore, high link reliability is crucial, especially for a route with a large number of hops. Given the limited standalone node reliability and channel fading phenomena, high link reliability is not possible without using a multiple-node cooperative transmission scheme. 2) Feedback enables the network to eventually settle in its absorbing state at the expense of power and bandwidth expenditures. More specifically, it takes an average of 1.23 transmissions to send a packet to the next cluster in this example, which leads to less power efficiency, and more delay. In conclusion, it is most desirable to have a supervisory scheme that is, however, rarely called for under high coverage and high link reliability conditions.

Our remaining tasks have become obvious: to assess and maximize link reliabilities, to devise a transmission stopping rule that abandons a route when it becomes a liability to the network, and to quantify the cost and benefit of feedback.

2 Optimization of network fault-tolerance

2.1 Network reliability

2.1.1 Node and channel reliability models

Due to dependence on power consumption, time to failure distribution of a node must be of increasing failure rate (IFR), i.e., a node that is found to be good after some usage must have a shorter residual life than a brand new node. Weibull IFR[3] distribution

$$F^n(t) = 1 - r^n(t) = 1 - e^{-\left(\frac{t}{\theta(P(J))}\right)^{\beta(P(J))}} \tag{1}$$

is used in this paper, where $F^n(t)$ is the distribution of node time to failure, $r^n(t)$ is the node reliability, $\beta(P(J)) > 1$ is called a shape parameter, and $\theta(P(J)) > 0$ is called a characteristic life. This particular Weibull model is deemed power-covariate because of the explicit dependence of its parameters on power $P(J)$ joules/packet/node involving a J-node cooperative transmission. For simplicity $P(J)$ will be suppressed in the following discussion. t is now defined with the number of packets the node has relayed. The characteristic life can be scaled by $\bar{N}_i^{-1}$ to reflect the additional life expenditure due to the need of re-transmission at the i^{th} cluster.

For a given type of node and a family of distributions, the parameters of the distribution can be determined statistically[15]. Suppose at a fixed power level, an m-unit concurrent test is performed. The test terminates at the arrival of the s^{th} node failure, i.e., upon the observation of failure times $\{t_1, \cdots, t_s\}$. The maximum likelihood estimates of the Weibull parameters can be solved from

$$\frac{m}{\hat\beta} + \sum_{i=1}^{s} \log t_i - \frac{1}{\hat\theta} \sum_{i=1}^{s} t_i^{\hat\beta} \log t_i + (m-s) t_s^{\hat\beta} \log t_s = 0$$
$$\frac{m}{\hat\beta} + \frac{1}{\hat\theta^2} \sum_{i=1}^{s} t_i^{\hat\beta} + (m-s) t_s^{\hat\beta} = 0.$$

In addition, Mann's two parameter F-test can be performed to determine whether to reject the hypothesized Weibull with a specified significance level[16]. The empirical dependence of $\hat\beta$ and $\hat\theta$ on $P(J)$ can be established by repeating the experiments for many power levels.

Let T_{lc} denote the period of loop closure, indicating how often a node is checked out to determine whether it has failed. Assuming a uniform aging process, the residual life distribution $F_k(t) \equiv P\left[T \le t | T > (k-1)T_{lc}\right]$ of a node follows

$$F_k(t) = 1 - \frac{r_i^n(t)}{r_i^n((k-1)T_{lc})}, \quad t \ge (k-1)T_{lc}, \quad k = 1, 2, \cdots$$

Channel failure distributions are assumed to be time independent, independent and identical for all channels in the network. Therefore, it suffices to describe them with a constant reliability value $r_i^c = r^c$. Randomness is associated with the fading phenomena[17].

2.1.2 Link and network reliability models

Suppose the i^{th} cluster of the K-cluster network contains a total of I_i nodes. Suppose for every sequence of I_i requests of packet transmission that arrive at the i^{th} cluster, a node responds to a set of J_i consecutive requests. Let us call such an arrangement a participating/non-participating protocol, the burden of packet transmission for every node is effectively reduced to a fraction J_i/I_i, and the single node characteristic life θ_i is increased effectively to $\theta_i I_i/J_i$. Note again that the current age of a node is the number of packet transmissions the node has carried out so far. This protocol evens out node ages across a cluster.

The example in Fig. 3 (a) depicts a portion of an interconnection containing two nodes in each cluster, where S_j^i denotes the j^{th} node in the i^{th} cluster, and C_{jk}^i denotes the channel linking the j^{th} node in the i^{th} cluster to the k^{th} node in the $i+1^{\text{th}}$ cluster. Consideration of channel failures in addition to node failures turns the interconnection into a nested structure rather than a cascade structure.

[3] IFR stands for increasing failure rate.

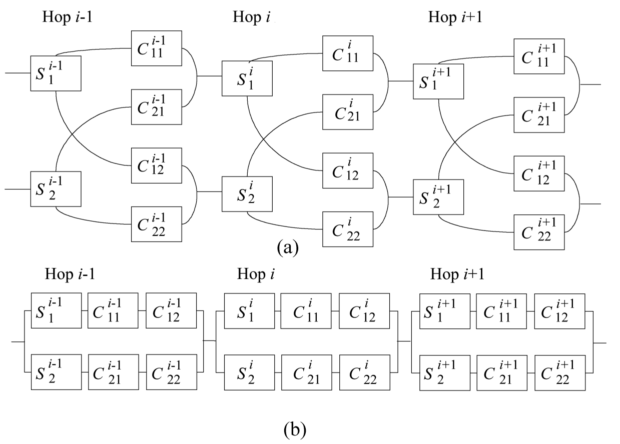


Fig. 3 (a) dependence diagram, (b) conservative simplification

The nested structure in Fig. 3 (a) can be decomposed into logic stages for which the output signal availability can be computed when conditioned on the input signal availability using a combinatorial method. More specifically, one may write for the i^{th} hop in Fig. 3 (a)

$$y_1^i = C_{11}^i S_1^i u_1^i + C_{21}^i S_2^i u_2^i, \ \ y_2^i = C_{12}^i S_1^i u_1^i + C_{22}^i S_2^i u_2^i$$

for which 16 conditional probabilities of the form

$$P[y_1^i y_2^i = ab | u_1^i u_2^i = cd], \ \ a, b, c, d \in \{0, 1\} \tag{2}$$

can be computed, where a '1' stands for the presence of a signal and a '0' absence of a signal. For example, with t suppressed, it can be shown that[18]

$$P[y_1^i y_2^i = 00 | u_1^i u_2^i = 11] = \\ (1 - r_i^n)^2 + 2(1 - r^c)^2 r_i^n (1 - r_i^n) + (1 - r^c)^4 (r_i^n)^2.$$

The stages are linked by $u_1^{i+1} = y_1^i$, $u_1^i = y_1^{i-1}$, $u_2^{i+1} = y_2^i$, and $u_2^i = y_2^{i-1}$.

Extension of the above result from a 2-node clusters to a J_i-node cluster is straightforward, and can be carried out in a systematic manner. Nevertheless, reliability evaluation of the nested structure is a major hurdle for optimization, especially in real-time. It is therefore desirable to work with simpler network reliability models that provide bounds on the nested network reliability. For example, with a k_i-out-of-J_i[16] requirement based on cooperative transmission considerations, where k_i is the required minimal number of operative nodes and J_i is the number participating nodes in the i^{th} cluster, network reliability R^{net} is bounded below by[18]

$$\prod_{i=1}^{K} \underline{R}_i^{J_i}, \ \underline{R}_i^{J_i} = \sum_{s=k_i}^{J_i} \binom{J_i}{s} [(r^c)^{J_{i+1}} r_i^n]^s [1 - (r^c)^{J_{i+1}} r_i^n]^{J_i - s}. \tag{3}$$

Let $R_i^{J_i}$ be the probability that a packet reaches at least k_{i+1} nodes among the J_{i+1} participating nodes in cluster $i+1$ with the required power level, given that the packet is transmitted at cluster i from at least k_i nodes among J_i participating nodes. It can be easily shown that $0 < R_i^{J_i} - \underline{R}_i^{J_i} < 1 - (r^c)^{J_{i+1}}$. The error bound is thus tight as long as channel reliability is sufficiently high. Therefore, link reliability $p_i^l \cong \underline{R}_i^{J_i}$, in which case the composite network reliability R^{net} can be replaced by its lower bound $p_1^l \times \cdots \times p_K^l$. Now the participating node allocation problem is amendable to solutions using dynamic programming[19, 20].

2.2 Optimization and control

2.2.1 Participating node allocation

The purpose of participating node allocation is to determine $J_1, \cdots, J_K$, the number of participating nodes at each cluster so that the network reliability is the highest at its design life T_D without violating a bandwidth constraint. In cluster i, $J_{i,\min}$ is imposed by the particular transmission scheme, whereas $J_{i,\max}$ ($\leq I_i$) is mainly imposed by the available bandwidth.

Bounding condition (3) converts the network level decision into a series of coupled cluster level decisions. In this case, channel failures introduce only local coupling which can be resolved by an ordered selection process starting from J_K, ending at J_1. The reader is referred to [18] for an example of participating node allocation using dynamic programming for a network operating unsupervised. The solution $J_1^*, \cdots, J_K^*$ can then be inserted to the staged conditional probability formulae (2) to calculate the true network reliability.

To illustrate the basic idea, consider a 3-cluster network with 10 nodes in each cluster. A tree structure shown in Fig. 4 can be created to represent all possible solutions at T_D, where all branches violating the constraints have been trimmed. Constrains particular to the cooperative transmission scheme[2] are $\sum_{i=1}^{3} J_i \leq 12$ and $J_{i,\min} = 2$. Each joint of the tree at a given cluster index represents a possible cumulative number of nodes. Each branch leading to the joint carries a cost equal to $\underline{R}_i^{J_i}(T_D)$ for a particular J_i. The accumulated reliability for each passage from the root to a leaf can be computed using Bellman's principle of optimality[19]. The principle is applied at every unit index i by comparing all the accumulated reliabilities leading to the same joint. Only the solution of the highest reliability is retained at each joint, and the rest are removed. Once the set $\{J_1^*, \cdots, J_K^*\}$ is obtained, the link reliabilities are set to $p_i^l = \underline{R}_i^{J_i^*}, i = 1, \cdots, K-1$. Suppose unit reliabilities $\underline{R}_i^2(T_D)$ through $\underline{R}_i^8(T_D)$ have been found to be 0.85, 0.90, 0.95, 0.99, 0.995, 0.999, and 0.999 5, respectively, for the network in Fig. 4, the optimal node allocation derived using dynamic programming is: $J_i = 4$ for $i = 1, 2, 3$.

Note that unit reliability is a complex function of J_i, which is determined by the methods discussed in Sections 2.1.1 and 2.1.2. The optimization in this section is carried out under the assumption that network is operating unsupervised.

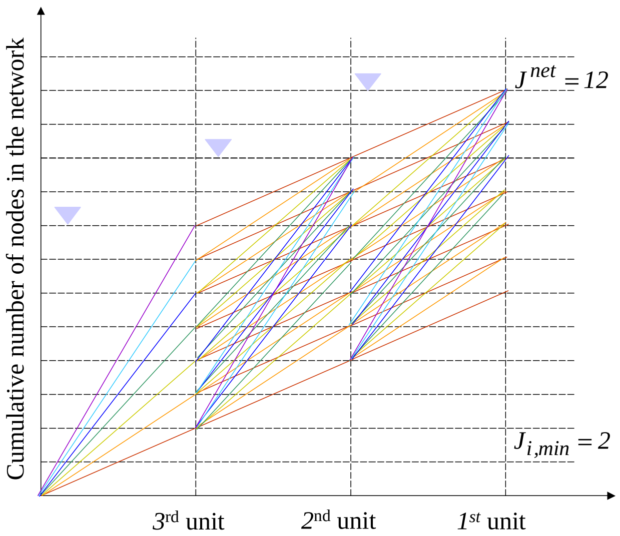


Fig. 4 Trellis diagram for node allocation

2.2.2 Retransmission control

This subsection revisits the re-transmission chain introduced in Section 1.2. It is now assumed that the network is supervised to the extent that it can detect a cluster transmission failure but not the state of the nodes and channels. In addition, the participating/nonparticipating protocol is effective to manage the large number of nodes available at each cluster. The decision regarding re-transmission in each of the clusters upon the detection of a cluster transmission failure can be made based on the solution of a Markov decision problem. The main purpose is to be able to terminate the service of the K-cluster route so that it does not turn into a black hole in the network.

Let $X_k \in \{1, 2, \cdots, K\}$ denote the random state variable at $t = k$ in the chain. Control action $u(x_k) = 1$ (or 0) indicates the network's decision to (or not to) re-transmit a packet. Let $C(x_k, u_k)$ be the cost incurred when control action u_k is taken based on x_k. Our goal is to determine a retransmission policy π to minimize the total expected cost $V_\pi(i) = E_\pi \sum_k C(X_k, u_k)$ for any initial state i. It has been shown that under the condition $0 \leq C(j, u) < \infty$ for all j and all u that belongs to some finite admissible sets U_j, the minimal cost $V^*(i)$ satisfies the following optimality equation[14, 20]

$$V(i) = \min_{u \in U_i} \{C(i, u) + \sum_{j=1}^{K} p_{i,j} V(j)\}.$$

In addition, policy π^* is optimal if and only if it yields $V^*(i)$ for all i.

Referring to the Markov chain in Fig. 1, the optimality equation can be specialized to the following form

$$\begin{aligned} V(i) = \min\nolimits_{u \in U_i} \{ & \underbrace{u(i)T_i + [1 - u(i)]\, L_i +}_{C(i,u(i))} \\ & u(i)\, \underbrace{[p_{i,i} V(i) + p_{i,i+1} V(i+1)]\}}_{\sum_{j=1}^{K} p_{i,j} V(j)} \end{aligned} \tag{4}$$

where $p_{i,i} = 1 - p_i^l c_i$, $p_{i,i+1} = p_i^l c_i$, T_i is the power and bandwidth cost incurred when the network chooses to re-transmit the packet, L_i is the packet loss cost incurred when the network chooses not to re-transmit, and network age t is suppressed. Equation (4) can be expressed as $V(i) = \min\{T_i + p_{i,i}V(i) + p_{i,i+1}V(i+1),\ L_i\}$.

To gain some insight into the optimal policy, assume $T_i = T$, $L_i = L$, $p_{i,i} = 1 - \rho$, and $p_{i,i+1} = \rho$, for $i = 1, \cdots, K$. Since

$$(1 - \rho)V(j) + \rho V(j + 1) < (1 - \rho)V(i) + \rho V(i + 1)$$

as long as $j > i$, the optimal policy is of the threshold type [14, 20] with some threshold i^*, i.e.,

$$V(i) = \begin{cases} T/\rho + V(i+1), & i > i^*, \quad (u(i) = 1) \\ L, & i \leq i^*, \quad (u(i) = 0) \end{cases}.$$

Given that $V(K) = 0$, $V(i)$ can be solved

$$V(i) = \begin{cases} (K - i)T/\rho, & i > i^*, \quad (u(i) = 1) \\ L, & i \leq i^*, \quad (u(i) = 0) \end{cases}$$

from which the threshold is obtained

$$i^* = \left\lceil K - \frac{\rho L}{T} \right\rceil \tag{5}$$

where $\lceil \cdot \rceil$ denotes the smallest nonnegative integer greater than $K - \rho L/T$. It can be seen that the optimal policy favors a re-transmission when a packet is near the end of the K-cluster route (large i), when a cluster is young (large ρ), when the cost of a packet loss is large (large L), when power & bandwidth are cheap (small T), when a route is short (small K).

A study without the simplifying assumptions along this direction is being conducted.

3 Effect of acknowledgement on performance

3.1 Acknowledgment protocol and network modeling

The objective of this section is to quantify the effect of loop closure frequency and the nodes' storage capacity on the performance of the network in terms of network lifetime, packet loss rate, and false alarm rate. The constraints considered are node's finite packet processing time, node's life expenditure while performing supervisory activities, and a specific acknowledgment protocol. As a result of the added complexity, it becomes necessary to resort to numerical means in order to achieve our objective.

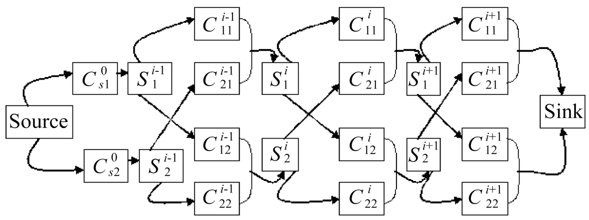


Fig. 5 A 5-hop wireless network including a source and a sink

Suppose in the nested configuration of Fig. 5, each receiving node at the next hop transmits an acknowledgement (ACK) to a transmitting node of the previous hop after receiving a sequence of N packets. N is referred to as the loop closure period. This transmission of acknowledgement to the previous node acts as a feedback. The transmitting node of the previous cluster inhibits the packet transmission to the next sensor if it does not receive the acknowledgement within a certain deadline (DL). This deadline forces the transmitting node to wait for acknowledgement beyond the loop closure period in case some packets are lost to announce that the receiving cluster has failed. To address the issue of how acknowledgements process is handled, consider the case where packets numbered $1, 2, 3, \cdots, N$ have been transmitted. The transmitting node waits for an acknowledgement from the receiving nodes until it receives an acknowledgement upon which it resets its counter, or until the deadline by which it discontinues sending packets and declares the end of network life. A false alarm is said to have occurred if all transmitting nodes cease to transmit packets at the end of the deadline even though a minimum required number of receiving nodes are still operative at each cluster.

The additional constraints described above rule out the possibility of analytical approach for the intended performance analysis. For this reason, the wireless model in Fig. 5 is constructed with Arena[11, 12]. The model construction involves using different modules in Arena that are arranged into a number of templates such as ‘Basic Process’, ‘Advanced Process’ and ‘Advanced Transfer’[12]. The Basic Process template contains modules that are used in modeling packet arrival and packet departure, assigning attributes to packets, channel random fading, and node processing. The Advanced Process panel comprises specific logical functions such as a fictitious control logic unit that is used to match the incoming packets based on their attributes, to duplicate, and to merge packets. Finally, the Route module in Advanced Transfer template is used to transfer the packets to specified stations. Independent replications are performed for each simulation of the wireless system model and the simulation results are stored and reported.

In the 5-hop wireless network of Fig. 5, the data packets are generated at a source of the network with a Poisson rate. They pass through channels and nodes, and are delivered to a sink. For simplicity, the source and the sink are assumed to never fail. The cluster reliability requirement is 1-out-of-2. In addition, there is no transport time for passing through channels. The processing time at each node is fixed at T units of time per packet. A packet can be lost through a faded channel, in a failed receiving node, in a failed transmitting node, or in a collision. The channels have independent failure probabilities. The nodes have lifetimes that follow independent Weibull distributions, as discussed in Section 2.1.1.

3.2 Performance analysis via simulation

This subsection investigates the dependence of network performance on frequency of acknowledgements from the receiving nodes and on the storage capacity of the nodes. In a transmitting node that is responsible for re-transmission, a copy of a transmitted packet is stored for as long as the set deadline for the reception of an acknowledgement of that packet. Since this part of study does not deal with re-transmission which has been generally examined in the previous section, only a counter is needed. As a receiving node, a received packet may be allowed to wait in a buffer for its turn to be processed at a node while a previously received packet is being processed. Performance consideration in this section includes time to network failure, packet loss rate, and false alarm rate. The wireless network is simulated both with and without feedback for a varying packet generation rate. The importance of selection of appropriate deadline, buffer size, and loop closure period to the network performance is delineated.

Designing and analyzing simulation experiment depend on the type of simulation[21, 22]. The performance measures of interest in this study necessitate terminating simulations. The terminating condition for the wireless network materializes when the network fails, which occurs when there is no longer passage of packets from source to sink. This could be due to the loss of the required minimum number of nodes in a cluster, or due to a false alarm that occurs when an acknowledgment deadline is passed even though there are still enough surviving nodes in a receiving cluster.

For a given scenario, n independent replications of a terminating simulation are run where each replication is terminated as soon as a network failure is declared, and is begun with the same initial condition of an empty and fully operative network. The behavior of the network is studied based on apposite data collected in the course of simulation and the performance measures of interest are estimated using the data. As the number of collected data sample n increases, that is as $n \to \infty$, the sample mean of a measured performance from the multiple independent replications converges almost surely to the true mean of the underlying distribution of the performance measure, based on the Strong Law of Large Numbers[15].

The packet inter-arrival time is exponentially distributed with mean time varied at 0.1, 0.25 and 0.35 s. The service time of each node is fixed at $T = 0.02$ s. The channels are opted to have an independent failure probability of 0.01.

The failure time distribution of a node is Weibull. The mean of Weibull distribution is given by $(\beta/\alpha)\Gamma(1/\alpha)$, where Γ is the complete gamma function. For chosen parameter values of $\beta = 50$, $\alpha = 3$ in the 5-hop wireless network, the mean failure of the node occurs after serving about 2 233 packets.

Time to network failure (TNF) is defined as the expected number of packets received at the sink before the network fails. Time to network failure is estimated and is characterized as the average of total number of packets received at the sink before the terminating condition occurs over multiple replications. Suppose N_i^R is the total number of packets that reach the sink when the simulation terminates for the $i^{\rm th}$ single run of the network. The simulation is run n times with the same initial conditions. Data $N_1^R, N_2^R, \cdots, N_n^R$ are collected and are used to obtain the time to network failure as a point estimate of the form

$$\widehat{\rm TNF}_n = \frac{1}{n}\sum_{i=1}^{n} N_i^R. \tag{6}$$

The $(1-\alpha)$ confidence interval of the estimate is given by $\left[\widehat{\rm TNF}_n - t_{n-1,\alpha/2}\sqrt{S_n^2/n},\ \widehat{\rm TNF}_n + t_{n-1,\alpha/2}\sqrt{S_n^2/n}\right]$, where $\widehat{\rm TNF}_n - E\{{\rm TNF}\}/\sqrt{S_n^2/n}$ follows a Student's t distribution of $n-1$ degrees of freedom, and the sample variance is given by $S_n^2 = \sum_{i=1}^{n}(N_i^R - \widehat{\rm TNF}_n)^2/(n-1)$. This method is called the method of independent replications[12].

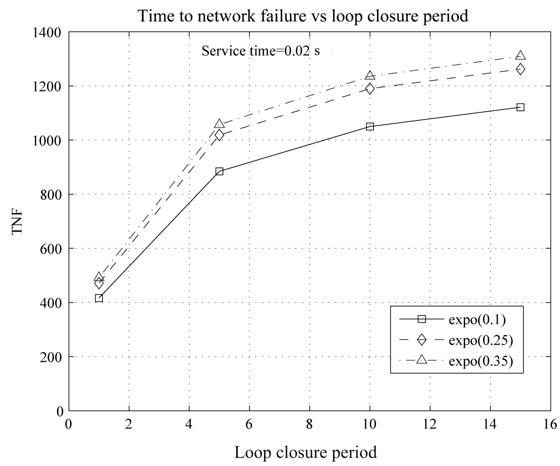
Consider the case when the feedback is applied to the wireless network, with $N = 5$, DL= 25, inter-arrival time of an exponential distribution of mean time 0.1 s, and the rest of the specifications remain the same. Then, with $n = 30$, the time to network failure is estimated as $\widehat{\rm TNF}_{30} = \sum_{i=1}^{30} N_i^R/30 = 884.5$ packets, and the half width of 95% confidence interval is determined as $t_{n-1,\alpha/2}\sqrt{S_n^2/n} = 105.29$. Thus, the 95% confidence interval for TNF is $779.21 \leq {\rm TNF} \leq 989.79$.

To assure that the selected number of simulation runs is sufficiently large to uphold the central limit theorem, a simple test can be performed to confirm whether the data resembles a normal distribution. If not, more replications are required.

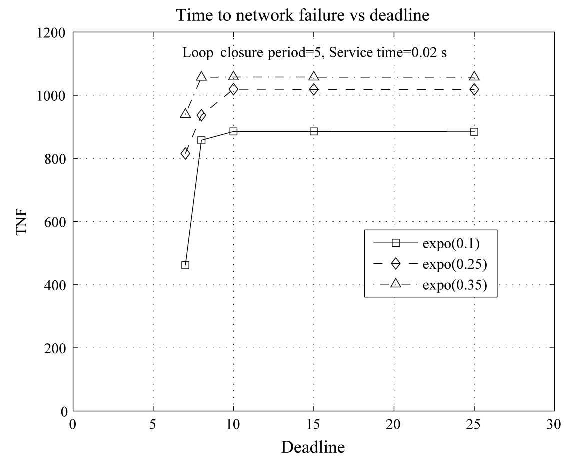
Simulation analysis of the 5-hop network inclusive of the feedback mechanism indicates, as shown in Fig. 6, that the time to network failure increases with increasing period of loop closure (N) for a given deadline, and with increasing deadline for a given loop closure period. The former is attributed to the increased life expenditure associated with more frequent supervisory activities that consume extra power. The latter has to do with reduced false alarm rate as the deadline increases. A transmitting node assumes that a receiving node has failed if it does not receive acknowledgement from the receiving node by deadline. Hence, false alarm rate increases with a shorter deadline.

Packet loss rate is often used as a performance measure in wireless networks. To estimate the packet loss rate, two sets of data are collected. The 1st set collected from the $i^{\rm th}$ replication is N_i^S, the total packets generated in the source by the time the termination condition is met. The 2nd set collected from the $i^{\rm th}$ replication is $(N_i^S - N_i^R)$, which is the difference between the total number of packets created in the source and that received in the sink in the $i^{\rm th}$ replication by the time the termination condition is met. The sample is therefore $(N_i^S - N_i^R)/N_i^S$, which represents the percentage loss with respect to total packets generated in the $i^{\rm th}$ replication. The point estimate of packet loss rate can be obtained as

$$\widehat{\rm PLR} = \frac{1}{n}\sum_{i=1}^{n}\frac{N_i^S - N_i^R}{N_i^S}. \tag{7}$$

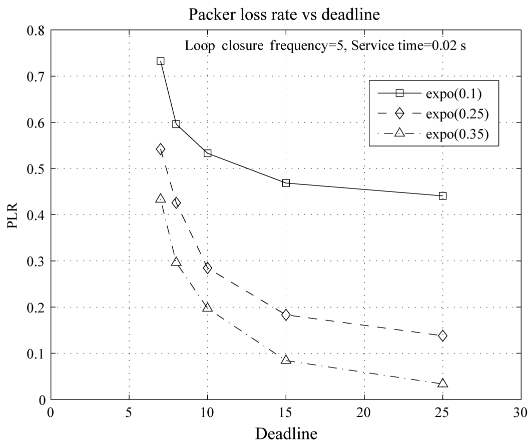


(a)

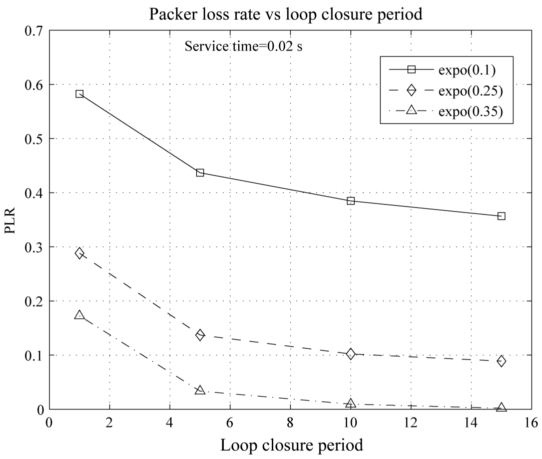


(b)

Fig. 6 Time to network failure with respect to (a) loop closure period, (b) acknowledgement deadline



(a)



(b)

Fig. 7 Packet loss rate with respect to (a) loop closure period and (b) deadline

From Fig. 7, it is evident that the packet loss rate decreases with increasing loop closure period, and increasing deadline for all inter arrival rates. This is because a larger loop-closure period implies less node power expenditures in supervisory activities and hence more likely success in transmission, and an extended deadline implies a lowered false alarm rates and hence an effectively longer network life. As inter-arrival time increases, the PLR decreases, because the chance of packet collision decreases.

Also of interest is the estimate of false alarm rate (FAR), for which the sample function is defined in terms of an indicator function as follows

$$I_i = \begin{cases} 1, \text{ if false alarm occurs in the } i^{\text{th}} \text{ replication} \\ 0, \text{ otherwise.} \end{cases}$$

Recall that a false alarm occurs when a transmitting node does not receive acknowledgement from any of the receiving nodes at the time of a deadline while some of the receiving nodes are still alive. Then the point estimate for the false alarm rate is defined as

$$\widehat{\text{FAR}} = \frac{1}{n}\sum_{i=1}^{n} I_i. \tag{8}$$

It is obvious that false alarm rate decreases with increasing deadline until there is no more benefit with further extension of deadline beyond which network fails almost surely before false alarm occurs, as shown in Fig. 8.

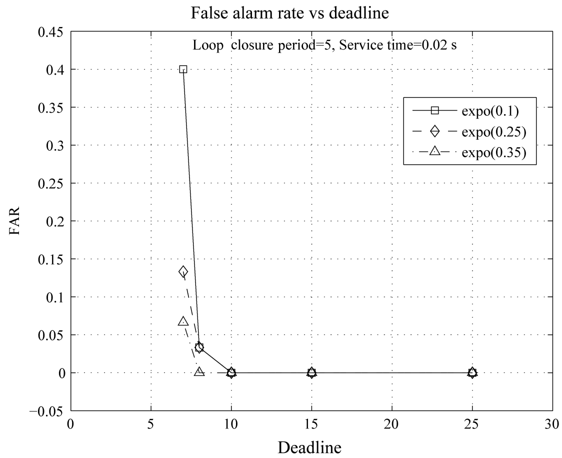


Fig. 8 False alarm rate with respect to deadline

The packet loss rate (PLR) is also examined against the buffer size of a node. For the given range of arrival rate, buffer size of 1 is determined to be sufficient, as shown in Fig. 9. The reduction in packet loss rate when a sufficiently large buffer is in place is mainly attributed to the effective avoidance of collision.

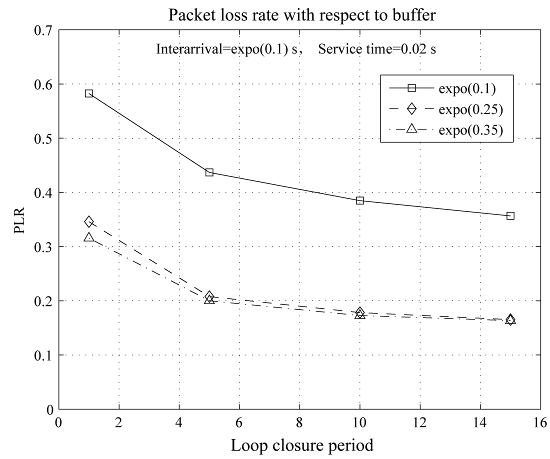


Fig. 9 Packet loss rate with respect to loop closure period with buffer size as a parameter

4 Discussion

This section summarizes some of the implications based on the results of our study on the K-cluster wireless network. The network is subject to both node failures and channel fading, which cause the network structure to become nested.

The high packet loss rate observed is largely attributed to the lack of reliability of the nodes and of the channels. The multiple-hop environment accentuates unreliability.

High link reliabilities required for a network with a large number of hops can be achieved by using clustered nodes to perform cooperative communications between clusters. Careful selection of participating nodes is necessary for optimized reliability at the network's design life.

The need for acknowledgment arises when the nodes benefit from knowing when to stop transmitting. Acknowledgment, or generally any supervisory activity, always harms instantaneous reliability and incurs a cost in terms of node lifetime. One way to reduce the expenditure of lifetime is to keep a sufficiently low loop closure rate.

A sufficiently large buffer size can effectively reduce the chance of packet collision, which in turn reduces packet loss rate. In our study, however, the utilization of individual nodes is low, which explains the low demand on buffer size.

Re-transmission of packets considerably complicates the analysis on the effect of feedback. In that case a lower loop closure rate implies a longer delay for a packet to pass the network. Therefore, one must consider trading off delay against extra power consumption and bandwidth contention that come with a more frequent loop closure rate. In this case, however, as long as the storage capacity is sufficient, packet loss can be reduced to practically none, at least in the early life of the network. Again optimal level of redundancy should be sought with respect to all competing interests to either prolong the network life, or equivalently, to maximize the network reliability at its design life.

Extensions of this work along three directions are being considered.

1) Real time re-allocation of participating nodes as the network ages based on network reliability projected at its design life. (2.2.1)

2) Derivation of solutions to re-transmission control with relaxed assumptions. (2.2.2)

3) Simulation of networks that involve retransmission, which is conceivably more complex. (3.2)

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