SECONDARY TRANSMISSION POWER OF COGNITIVE RADIOS FOR DYNAMIC SPECTRUM ACCESS

Xiaohua Li[1]

[1]Department of ECE
State University of New York at Binghamton
Binghamton, NY 13902, USA
{xli,jhwu1}@binghamton.edu

Jinying Chen[2], *Juite Hwu*[1]

[2]Department of Communication Engineering
College of Information Engineering
Chengdu University of technology
Chengdu 610059, CHINA
chjy@cdut.edu.cn

ABSTRACT

In this paper, we analyze the allowable transmission power of cognitive radios when used as secondary users in a dynamic spectrum access network. Closed-form upper and lower bounds are derived, while accurate solutions to transmission power are available through numerical evaluation. The transmission power depends on the distance between primary and secondary transmitters as well as the number of randomly distributed primary receivers. The results show that at the cost of a small SINR redundancy of primary users, cognitive radios can use a significant level of transmission power in certain conditions.

***Index Terms*—** cognitive radio, dynamic spectrum access, transmission power, signal to interference and noise ratio (SINR)

1. INTRODUCTION

Cognitive radio (CR) has attracted great attention recently as a potential way of realizing dynamic spectrum access (DSA). CR-based DSA can help resolve the shortage problem of the overly crowded wireless communication spectrums [1]. It has been finding many practical applications, such as using the TV band for secondary spectrum access, or in certain military applications for both spectrum efficiency and security.

In DSA networks, there are various ways to support secondary spectrum access. One of the ways is for secondary users to utilize the spectrum hole which the primary users do not use during some time period and in some place. Another way is to allow the secondary users to utilize the same spectrum at the same time and the same place with the primary users. In order to mitigate the interference to primary users, secondary transmitters may use an underlay approach for spectrum access, such as the ultra-wideband (UWB) transmission. An alternative approach is overlay, in which secondary transmitters can have larger transmission power. In this case, in order to limit the interference to the primary users, the secondary transmitters either schedule their transmission power so that their interference to primary users is limited to an acceptable level [2], or exploit special coding techniques such as dirty paper coding so that they can use a portion of the transmission power to help the primary users while the rest of the power to transmit their own information [3].

In this paper, we focus on an approach similar to [2] where secondary spectrum access is allowable as long as the interference to primary users is within a certain threshold. We will analyze the signal-to-interference-and-noise-ratio (SINR) and find the secondary transmitters' allowable transmission power. In contrast to [2] which studies fixed users (both number and location), we derive the average transmission power by considering uniformly distributed primary receivers in the network. Some preliminary results along this line has been reported in [4], but without the closed-form bounds being derived.

The organization of this paper is as follows. In Section 2, we give the system model. Then in Section 3, we analyze the transmission power by a geometric method for a single secondary transmitter. Simulations are conducted in Section 4. Conclusions are then given in Section 5.

2. DSA SYSTEM MODEL

We consider a cellular-like system, where in a cell there is a base station that communicates with multiple mobile users. We denote the base station as primary transmitter T0 and the mobile users as primary receivers. In addition, there is a secondary transmitter (denoted as T1) and the corresponding secondary receivers. Both the number and the positions of the primary receivers are unknown to the secondary users. We put the base station T0 in the center of a cell with radius r_0,

Li was partly supported by US AFRL under a subcontract to Grant FA8750-07-2-0046.

Chen was partly supported by the Grant # 2006BAC13B04-0302 from the "11-5" Scientific Projects of China.

and let the distance between T0 and T1 be d. We assume there are M primary receivers that are uniformly distributed inside the cell.

The secondary transmitter T1 may transmit at the same time and the same frequency as the primary user T0. The transmission power of T1 should be determined appropriately so that all the primary receivers can still work. In other words, while secondary transmission degrades the primary receivers' SINR, such a degradation should be smaller than certain threshold. For this purpose, the primary system should have been designed with certain redundancy in SINR, i.e., the worst case SINR of the primary receivers is larger than the minimum required SINR Γ_0 when there is no secondary spectrum access.

Let the redundancy be described by a factor $\Delta\Gamma_0$. In case of without secondary transmission, primary receivers have SINR no less than

$$\frac{KP_0 r_0^{-\alpha}}{N} \geq \Gamma_0 + \Delta\Gamma_0, \tag{1}$$

where P_0 is the transmission power of the base station T0, N is the AWGN noise power at the receiver which we assume identical for all the receivers, the parameter α is the path-loss exponent, and K is the constant that includes all other propagation effects such as antenna gains and carrier wavelength. In case of secondary spectrum access, we just need to assure the SINR γ_0 of any primary receiver to satisfy

$$\gamma_0 \geq \Gamma_0. \tag{2}$$

3. SECONDARY TRANSMISSION POWER

If there are primary receivers close to T1, then the transmission power of T1 has to be small in order to avoid introducing excessive interference. The transmission power of T1 depends on the position of the primary receivers. Considering the random distribution of primary receivers and the fact that T1 does not have detailed information about the locations of the primary receivers, we evaluate the expected transmission power of T1 in this section.

3.1. Upper bound of secondary transmission power

To derive the upper bound of the secondary transmission power, let us consider first the case that all the primary receivers are located outside of a circle of radius x around T0. We can model the cummulative distribution of this case as

$$F_1(x) = \left(1 - \frac{\pi x^2 - A_0}{\pi r_0^2 - A_0}\right)^M, \tag{3}$$

where $x_0 \leq x \leq r_0$ and $A_0 = \pi x_0^2$. Note that the parameters x_0 and A_0 are used in order to include the general case where the primary receivers have a distance at least x_0 away from the primary transmitter T0. This distance may be due to the far-field effect of antenna transmissions, or due to some other

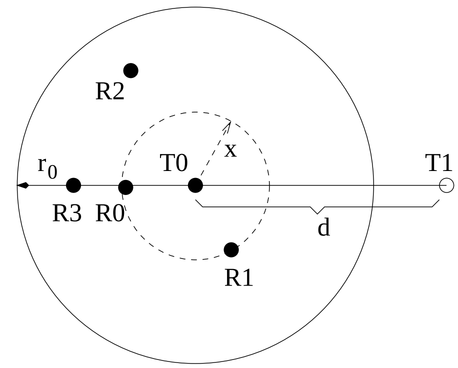

Fig. 1. A cell with a primary transmitter T0 and all the primary receivers being outside of the circle with radius x. T1 is the secondary transmitter. The primary receiver R0 has the highest SINR, which is exploited to derive the upper bound of T1's transmission power.

reasons. Then the probability density that there are some primary receivers with distance x to T0 but no primary receivers closer to T0 than x is

$$f_1(x) = -\frac{dF_1(x)}{dx} = \frac{2\pi M x}{\pi r_0^2 - A_0}\left(1 - \frac{\pi x^2 - A_0}{\pi r_0^2 - A_0}\right)^{M-1}. \tag{4}$$

Note that the negative sign in (4) is to guarantee a positive density.

Proposition 1. If the minimum distance between T0 and primary receivers is x, then the transmission power of T1 satisfies

$$P_1(x) \leq (x+d)^\alpha \left(\frac{P_0}{\Gamma_0} x^{-\alpha} - \frac{N}{K}\right). \tag{5}$$

The equality is achieved when the SINR Γ_0 is actually achieved by some primary receivers.

Outline of Proof. To save space, we only outline the proof. Details will be reported elsewhere. We can prove (5) by considering the two circles in Fig. 1, where the bigger one is the circle of T0 with radius r_0 whereas the smaller one is the circle with radius x. We can show that if there is a primary receiver R0 lies in the intersection of the small circle and the line T0-T1, then R0 has the highest SINR among all the primary receivers. The transmission power of T1 derived from this SINR is an upper bound of the secondary transmission power. R0's SINR is

$$\gamma_{\mathrm{R0}}(x) \quad = \quad \frac{KP_0 x^{-\alpha}}{KP_1(x)(x+d)^{-\alpha} + N},$$

from which (5) can be obtained. □

The upper bound of the expected secondary transmission power can then be derived from (5). When evaluating of average power, however, it might be better to use its decibel value. Therefore, we change (5) first into

$$\overline{P_1(x)}(\mathrm{dB}) = 10\log_{10}\left[(x+d)^\alpha \left(\frac{P_0}{\Gamma 0} x^{-\alpha} - \frac{N}{K}\right)\right]. \tag{6}$$

where $\overline{P_1(x)}(\text{dB})$ denotes the upper bound for this special case. The upper bound of the expected secondary transmission power is then evaluated as

$$\overline{P_1}(\text{dB}) \quad = \quad \int_0^{r_0} \overline{P_1(x)}(\text{dB}) f_1(x) dx \qquad (7)$$

From (7), we can evaluate $\overline{P_1}(\text{dB})$ numerically.

Nevertheless, to analyze the connections between the transmission power and the transmission parameters, a closed-form solution is more desirable. In order to derive closed-form solution to (7), we have to take some simplifications. First, we let $x_0 = 0$, so $A_0 = 0$. Then, we consider only the noiseless case with $N = 0$. After some tedious but straight-forward integration deduction, we have

$$\begin{aligned}
&\overline{P_{1,N=0}}(\text{dB}) = \\
&\quad 10\log_{10}\frac{P_0}{\Gamma_0}\left(\frac{r_0}{d}\right)^{-\alpha} + 5[\psi(M+1)-\psi(1)]\log_{10} e^{\alpha} \\
&\quad +10\left[\frac{\sqrt{\pi}M\Gamma(M)r_0}{2\Gamma\left(M+\frac{3}{2}\right)d}\,{}_2F_1\left(\frac{1}{2},1,M+\frac{3}{2},\frac{r_0^2}{d^2}\right)\right. \\
&\quad \left. -\frac{r_0^2}{2(M+1)d^2}\,{}_2F_1\left(1,1,M+2,\frac{r_0^2}{d^2}\right)\right]. \qquad (8)
\end{aligned}$$

Note that $\Gamma(\cdot)$, ${}_2F_1(\cdot)$ and $\psi(\cdot)$ denote Gamma function, Hypergeometric function, and PolyGamma function, respectively. From (8), we can readily see that the secondary transmission power increases when the ratio d/r_0 becomes large, or the number of primary receivers M reduces. A big path-loss exponent α is helpful for secondary spectrum access.

3.2. Lower bound of secondary transmission power

The evaluation of the lower bound of secondary transmission power needs to take a different approach. In contrast to the approach in Section 3.1, we consider the scenario that all the primary receivers are away from T1 with a distance x. Such a scenario gives a circle of radius x centered around T1, as shown in Fig. 2. Depending on the position of T1, the range of x is different. Specifically, if $d \le r_0$, i.e., T1 is within the primary cell, then $x_0 \le x \le r_0 + d$. Otherwise, if $d > r_0$, then the valid range of x is $\max\{x_0, d - r_0\} \le x \le d + r_0$.

The cummulative distribution of the case that there are no primary receivers in the circle of T1 depends on the area $A(x)$ of the cross section of the two circles in Fig. 2. Specifically, the cummulative distribution can be found as

$$F_2(x) = \left(1 - \frac{A(x) - A_0}{\pi r_0^2 - A_0}\right)^M, \qquad (9)$$

where $\max\{x_0, d-r_0\} \le x \le d+r_0$. If the radius x is small so that $x_0 \le x \le r_0 - d$, as shown in Fig. 1(a), then the area of the cross section is

$$A(x) = \pi x^2, \quad 0 \le x \le r_0 - d. \qquad (10)$$

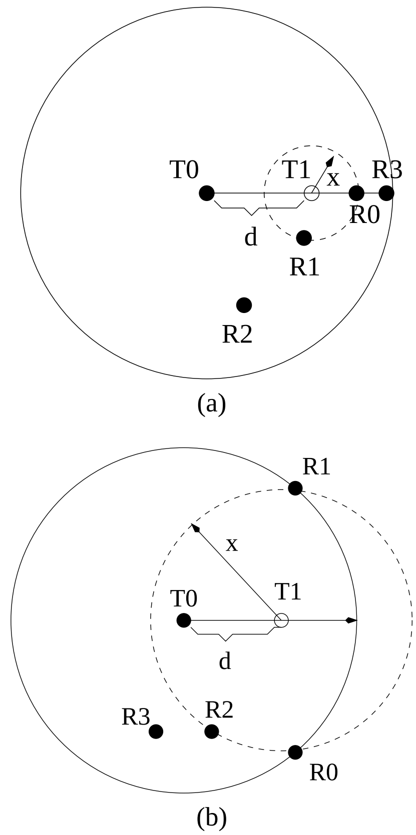

Fig. 2. Circles of primary transmitter T0 and secondary transmitter T1. (a) For $0 \le x \le r_0 - d$, primary receiver R0 has the smallest SINR. (b) For $r_0 - d < x \le r_0 + d$, primary receivers R0 and R1 have the same smallest SINR. Such smallest SINR are used to derive the lower bound of the secondary transmission power.

Otherwise, if the radius x is larger so that $r_0 - d < x \le r_0 + d$, as shown in Fig. 1(b), then the area of the intersection is

$$A(x) = x^2\eta + r_0^2\phi - dr_0\sin(\phi), \quad r_0 - d \le x \le r_0 + d, \quad (11)$$

where

$$\eta = \cos^{-1}\left(\frac{d^2 + x^2 - r_0^2}{2xd}\right), \quad \phi = \cos^{-1}\left(\frac{r_0^2 + d^2 - x^2}{2r_0 d}\right). \qquad (12)$$

For the case of $d > r_0$, it can be easily verified that the area is

$$A(x) = \frac{r_0^2}{2}[2\phi - \sin(2\phi)] + \frac{x^2}{2}[2\eta - \sin(2\eta)], \qquad (13)$$

where $d - r_0 \le x \le d + r_0$. Considering that $F_2(x)$ in (9) is a decreasing function of x, probability density function of x should be

$$f_2(x) = -\frac{dF_2(x)}{dx}. \qquad (14)$$

Proposition 2. Let all primary receivers lie outside of the circle of T1 with radius x, and the noise power be negligibly small (i.e., $N \ll KP_1(x)(r_0 + d)^{-\alpha}$). If $d \le r_0$, the secondary transmission power satisfies

$$P_1(x) \leq \begin{cases} x^\alpha \left[\frac{P_0}{\Gamma_0}(d+x)^{-\alpha} - \frac{N}{K}\right], & \text{if } x_0 \leq x \leq r_0 - d \\ x^\alpha \left(\frac{P_0}{\Gamma_0} r_0^{-\alpha} - \frac{N}{K}\right), & \text{if } r_0 - d \leq x \leq r_0 + d. \end{cases} \tag{15}$$

If $d > r_0$, then

$$P_1(x) \leq x^\alpha \left(\frac{P_0}{\Gamma_0} r_0^{-\alpha} - \frac{N}{K}\right) \tag{16}$$

if $\max\{x_0, d - r_0\} \leq x \leq r_0 + d$. The equality can be achieved in some special primary receiver distributions.

Outline of Proof. Considering first the two circles in Fig. 2 (a), where the bigger one is the circle of T0 with radius r_0 while the smaller one is the circle of T1 with radius x, we can show that the primary receiver R0 has the lowest SINR, based on which we can drive the first equation in (15). The other part of (15) can be proved similarly by considering Fig. 2 (b), and so does (16). □

Note that although we have "$\leq$" in (15)-(16), they are derived from the primary users with the lowest SINR. Therefore, we can define the lower bound of $P_1(x)$ as $\underline{P_1(x)}$ which equals to the right hand side of (15) and (16). The meaning of "lower bound" is that the secondary transmission power can always be larger than this lower bound without causing interference problem. Obviously, the secondary transmission power can be smaller than this value as well, in which case it just means the secondary transmitter does not fully utilize the transmission capacity.

The lower bound of the expected transmission power of the secondary transmitter T1 can thus be obtained by evaluating the expectation of $\underline{P_1(x)}$ over the probability density $f_2(x)$,

$$\underline{P_1}(\text{dB}) = \int 10 \log_{10}[\underline{P_1(x)}] f_2(x) dx. \tag{17}$$

If the distance of a secondary transmitter T1 to a primary transmitter T0 is known, then the lower bound of the average transmission power of T1 can be determined from (17) numerically.

The closed-from solution to (17) is even more difficult to derive than the upper bound case, mainly because the density function is nontrivial. Again, we have to adopt some approximations. First, the lower bound is usually extremely small when $d < r_0$, i.e., when T1 lies within the cell. Therefore, we consider only the $d \geq r_0$ case for closed-form solutions. Next, to deal with the major problem of the non-trivial density function, we approximate the cummulative distribution function $F_2(x)$ by

$$\tilde{F}_2(x) = \left(1 - \frac{(x + r_0 - d)^2}{4r_0^2}\right)^M, \quad d - r_0 \leq x \leq d + r_0, \tag{18}$$

which means we reduce the cross section area to a circle with radius $(x + r_0 - d)/2$. Note that we have also let $x_0 = 0$. The density function can thus be approximated as

$$\tilde{f}_2(x) = \frac{M}{2r_0^2}(x + r_0 - d)\left(1 - \frac{(x + r_0 - d)^2}{4r_0^2}\right)^{M-1}. \tag{19}$$

With the above simplifications, the lower bound of the secondary transmission power is approximated as

$$\underline{P_1}(\text{dB}) \approx \int_{d-r_0}^{d+r_0} 10 \log_{10}\left(\frac{P_0 x^\alpha}{\Gamma_0 r_0^\alpha}\right) \tilde{f}_2(x) dx. \tag{20}$$

If we consider the special case that T1 is near the cell boundary, i.e., $d/r_0 \to 1$, then we can reduce (20) into

$$\underline{P_{1,d/r_0 \to 1}}(\text{dB}) \approx 10 \log_{10} \frac{2^\alpha P_0}{\Gamma_0} - 5[\psi(M+1) - \psi(1)] \log_{10} e^\alpha. \tag{21}$$

In this case, (21) shows that the secondary transmission power reduces with the number of primary receivers when the secondary transmitter is near cell boundary.

To evaluate (20) in more general case, we can further apply an approximation $\log(x - r_0 + d) \geq x/(x - r_0 + d) + \log(d - r_0)$, where $\log(\cdot)$ is natural logarithm. Then we can reduce (20) into

$$\begin{aligned} \underline{P_1}(\text{dB}) \approx\ & 10 \log_{10} \frac{P_0}{\Gamma_0}\left(\frac{d - r_0}{d}\right)^\alpha + \\ & \frac{10\alpha r_0}{(d-r_0)\log(10)}\left[\frac{M\sqrt{\pi}\Gamma(M)}{\Gamma(M+\frac{3}{2})}\ {}_2F_1\left(1, \tfrac{3}{2}, M + \tfrac{3}{2}, \tfrac{4r_0^2}{(d-r_0)^2}\right)\right. \\ & \left. - \frac{4r_0}{(M+1)(d-r_0)}\ {}_2F_1\left(1, 2, M+2, \tfrac{4r_0^2}{(d-r_0)^2}\right)\right]. \end{aligned} \tag{22}$$

On the other hand, if we use the approximation $\log(x - r_0 + d) \leq x/(d - r_0) + \log(d - r_0)$ instead, then we can get a relatively looser but more succinct approximation as

$$\underline{P_1}(\text{dB}) \approx 10 \log_{10}\left(\frac{P_0(d-r_0)^\alpha}{\Gamma_0 d^\alpha}\right) + \frac{10\alpha r_0}{(d-r_0)\log(10)} \frac{M\sqrt{\pi}\Gamma(M)}{\Gamma(M+\frac{3}{2})} \tag{23}$$

Both (22) and (23) indicate that the lower bound of expected secondary transmission power is an increasing function of d/r_0. Especially, if M is large enough, then the lower bound increases with $\log_{10}(d/r_0 - 1)^\alpha$.

4. SIMULATIONS

In this section, we compare numerically the bounds and the exact values of the secondary transmission power. We assume the transmission power of T0 be 100 watts, the AWGN noise power be $N = 5 \times 10^{-10}$ watts. The gains of the transmission antenna and the receiving antenna are all 1. We have effective primary transmission range $r_0 \approx 1000$ meters. We set the

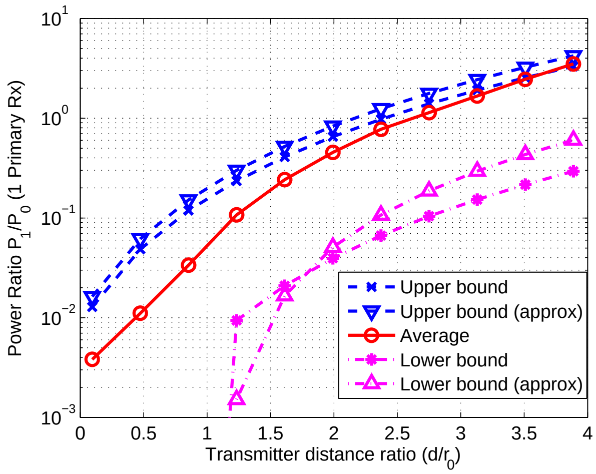

Fig. 3. Secondary transmission power as functions of primary/secondary transmitters distance ratio.

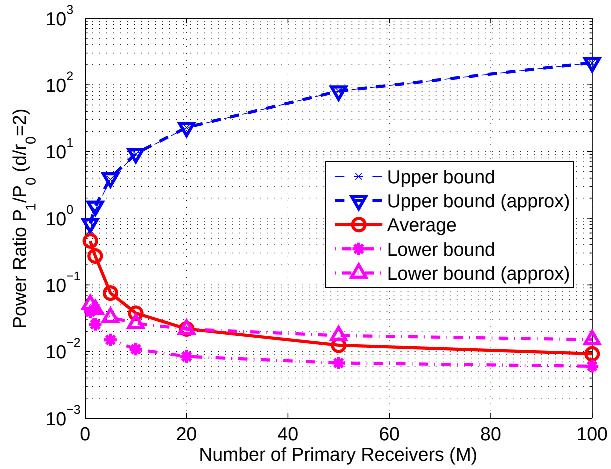

Fig. 4. Secondary transmission power as functions of number of primary receivers.

path loss exponent as $\alpha = 3$ to simulate an urban cellular radio environment. $\Gamma_0 = 20$ dB is the primary receiver's SINR requirement in case of without secondary transmissions. An SINR redundancy of 3 dB is simulated.

For the bounds of the secondary transmission power, we directly evaluate both the integration equations and the close-form solutions. The latter is indicated as "approx" in simulation figures. For the exact values of the secondary transmission power, which are denoted as "average" in the simulation figures, we use Monte-Carlo simulations, where in each run we randomly generate M primary receivers, and then calculate the transmission power based on the assumption that the secondary transmitter T1 knows all their positions.

In Fig. 3, we can see that the bounds evaluated from numerical integrations are very close to the bounds calculated from closed-form expressions. In addition, the exact values in general lie between the upper bounds and the lower bounds.

In Fig. 4, we can see that the exact values of the secondary transmission power are close to upper bound when M is small, and are close to the lower bound when M becomes large. In addition, the closed-form expressions again fit tightly to the numerical integrations.

5. CONCLUSIONS

In this paper, we analyzed the allowable transmission power of cognitive radios when used for secondary spectrum access purpose. Transmission power of a single secondary cognitive radios are derived in the form of integration equations, and closed-form solutions are obtained under certain simplifications. Simulations are conducted to show their effectiveness.

6. REFERENCES

[1] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Singal Processing Mag.*, vol. 24, no. 3, pp. 79-89, May 2007.

[2] M. Gastpar, "On capacity under receive and spatial spectrum-sharing constraints," *IEEE Trans. Info. Theory*, vol. 53, no. 2, pp. 471-487, Feb. 2007.

[3] N. Devroye, P. Mitran and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Info. Theory*, vol. 52, no. 5, pp. 1813-1827, May 2006.

[4] X. Li, J. Hwu and N. Fan, "Transmission power and capacity of secondary users in a dynamic spectrum access network," *IEEE Military Communications Conference (MILCOM'2007)*, Orlando, FL, Oct. 29-31, 2007.