OPTIMAL RELAY SELECTIONS IN TWO-WAY AMPLIFY-AND-FORWARD NETWORKS

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ABSTRACT

Two-way amplify-and-forward (AF) relay network with both cooperative communication and analog network coding (ANC) has attracted great attention. In this paper, we formulate the fundamental issue, i.e., the optimal selection of multiple AF relays in a two-way dual-hop cooperative network, into linear fractional programming (LFP) problems. Firstly, in case of channel reciprocity, by minimizing the sum of the inverse of signal-to-noise ratios (SNR), the AF relays can be selected optimally by the LFP algorithms. Moreover, we derive a closed-form solution to this case, and prove that a node either participates relaying with full transmission power, or ceases relaying. Then, for the general (possibly channel non-reciprocity) case, by maximizing the weighted sum SNR, we show that the relay selection can be optimized by the sum-of-ratios LFP algorithms. Simulations are conducted to verify the proposed results.

Index Terms— Cooperative communications, two-way relaying, analog network coding, linear fractional programming, sum-of-ratios linear fractional programming, signal-to-noise ratio

1. INTRODUCTION

Two-way cooperative relaying has attracted great attention in the last few years because of its advantages of exploiting redundant nodes as relays and of using the analog (physical-layer) network coding (ANC) to boost transmission performance. One of the common protocols for the two-way relaying is that first all sources broadcast their transmitted signals simultaneously. These signals combine in the air before reaching the relaying nodes, which is called analog network coding. The relaying nodes then broadcast the received signals back to the source nodes. With the knowledge of their own transmitted signals, the source nodes will decode the ANC-coded signals and demodulate the signals transmitted from other sources.

While two-way relaying has been an active research area for years, a fundamental issue, i.e., how to select relays optimally from all available redundant nodes, remains open and challenging. This is true even for the one-way cooperative transmissions, where the relay selection problem can be classified into single-relay selection and multiple-relay selection [1]. While the single-relay selection has been studied extensively [2], the latter is more challenging [3]. Some suboptimal or heuristic methods have thus been proposed, e.g. selecting relays closer to the source node, or selecting relays with signal-to-noise ratio (SNR) above a certain heuristic threshold [4]. Obviously, the relaying nodes selection and optimization problem is even more challenging for two-way and ANC-coded cooperative transmissions. It is in fact still a mostly open problem [5]-[7]. In the recent and also representative work [5], the closed-form expressions to the optimal relay selection were developed for a special channel-reciprocal case. However, only approximate solutions to the general (possibly channel-nonreciprocal) case were derived.

A popular relaying strategy for two-way ANC-coded cooperative networks is the amplify-and-forward (AF) relaying [4], where each relay node simply retransmits the signal it has received, with appropriate amplification only for desirable transmission power. This strategy has attracted great practical interests because it can simplify relay design, reduce processing energy, and reduce processing delay [5][8]. In [9], we derived a closed-form solution to the problem of multiple-relay selection for maximum destination SNR in a dual-hop AF cooperative network. Nevertheless, [9] studied one-way relay network only without ANC.

In this paper, following our research in one-way AF relay network [9], we derive the optimal relay selection in two-way AF ANC-coded cooperative networks. We will show that the problem can be solved by the sum-of-ratios linear fractional programming, for which many standard and effective algorithms exist. Moreover, if reciprocity between the source-relay channels and the relay-source channels can be assumed, we can formulate a much simpler linear fractional programming for the optimal relay selection. A surprisingly simple closed-form solution can also be derived, which shows many interesting properties of the optimal relay selection.

The organization of this paper is as follows. In Section 2, we give the system model. In Section 3, we develop the optimal relay selection and propose the algorithms. Simulations will be conducted in Section 4 and conclusions will be given in Section 5.

2. SYSTEM MODEL

Consider a wireless ad-hoc two-way AF relaying network with two source nodes (S_1 and S_2), and N other nodes that can potentially work as relays, as illustrated in Fig. 1. The forward channels from S_1 and S_2 to the relay node i, $i = 1, \cdots, N$, are denoted as $h_{1,i}$ and $h_{2,i}$, respectively. We use $g_{1,i}$ and $g_{2,i}$ to denote their corresponding backward channels from the relay node i to the source nodes S_1 and S_2, respectively.

We assume that the relays use the so-called multiple access broadcast channel (MABC) relaying scheme [6], which is a dual-slot relaying scheme with analog network coding [10]. As shown in Fig. 2, during the first time slot T_1, the source nodes broadcast the signals $s_1(n)$ and $s_2(n)$ to all the other nodes simultaneously. The signal received by the node i is thus

$$x_i(n) = \sqrt{P_{S_1}} h_{1,i} s_1(n) + \sqrt{P_{S_2}} h_{2,i} s_2(n) + v_i(n), \quad (1)$$

where P_{S_1} and P_{S_2} are the transmission powers of the source nodes S_1 and S_2, respectively. We denote the maximum available trans-

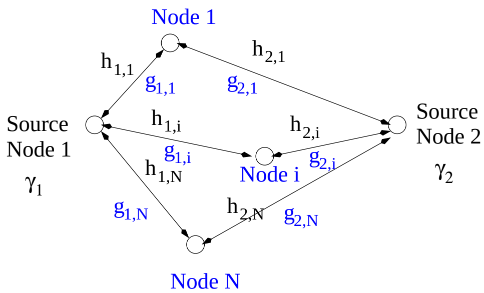

Fig. 1. Dual-hop two-way cooperative wireless network with N candidate relay nodes. The forward channels are $h_{1,i}$ and $h_{2,i}$, while the backward channels are $g_{1,i}$ and $g_{2,i}$, $i = 1, \cdots, N$. The SNRs of the sources nodes are γ_1 and γ_2, respectively.

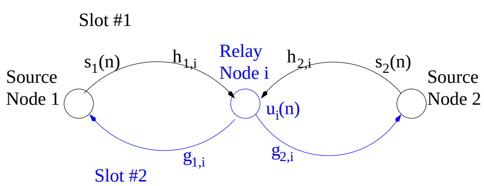

Fig. 2. The dual-hop MABC relaying scheme, with the associated transmission signals and channels.

mission power of the two source nodes as $\bar{P}_{S_1}$ and $\bar{P}_{S_2}$. The variable $v_i(n)$ is the additive white Gaussian noise (AWGN) at the receiving node i.

During the second time slot T_2, each relay node i amplifies the received signal $x_i(n)$ and re-broadcasts the received signal with certain desirable transmission power P_i. The transmitted signal is

$$u_i(n) = \sqrt{\frac{P_i}{E[|x_i(n)|^2]}} x_i(n). \tag{2}$$

where $E[\cdot]$ denotes expectation and

$$E[|x_i(n)|^2] = P_{S_1}|h_{1,i}|^2 + P_{S_2}|h_{2,i}|^2 + \sigma_{v_i}^2. \tag{3}$$

Note that without loss of generality, we assume the transmitted signals $s_1(n)$ and $s_2(n)$ both have unit power. Let $\bar{P}_i$ be the maximum available transmission power of the relay node i. Then $0 \le P_i \le \bar{P}_i$. $P_i = 0$ means that the node i is not selected as relay.

The received signals at the source nodes S_1 and S_2 are then, respectively,

$$y_1(n) = \sum_{i=1}^{N} g_{1,i} u_i(n) + v_{S_1}(n) \tag{4}$$

and

$$y_2(n) = \sum_{i=1}^{N} g_{2,i} u_i(n) + v_{S_2}(n), \tag{5}$$

where $v_{S_1}(n)$ and $v_{S_2}(n)$ are the AWGN at the source nodes S_1 and S_2. We assume that all AWGNs are independent from each other and from the source nodes' signals, and have zero mean and variances $\sigma_{v_i}^2$ at each relay node i. Specifically, $\sigma_{v_{S_1}}^2$ and $\sigma_{v_{S_2}}^2$ denote the noise variance at the source nodes S_1 and S_2, respectively.

As in the typical analog network coding setting, each of the two source nodes decodes the signal received from the relays by exploiting the knowledge of its own transmitted signals. In addition, within our AF transmission, we assume that the relaying nodes do not need to estimate channels or synchronize timing. This is in contrast to the transmit-beamforming requirement in [7] and [5], where each relay had to know the phases of both its receiving channel and its transmitting channel, and had to guarantee perfect timing synchronization with other relays. Although it can enhance destination SNR, the cost of acquiring perfect channel information and synchronization may well outweigh many relays' contribution.

In the sequel, we will derive each source node's SNR, and formulate the optimal relay selection problem as maximizing the weighted sum of these SNRs.

Without loss of generality, consider the source node S_1. Its received signal (4) can be changed to

$$\begin{aligned} y_1(n) \quad &= \quad \sum_{i=1}^{N} \sqrt{\frac{P_i}{E[|x_i(n)|^2]}} g_{1,i} \sqrt{P_{S_2}} h_{2,i} s_2(n) \\ &\quad + \sum_{i=1}^{N} \sqrt{\frac{P_i}{E[|x_i(n)|^2]}} g_{1,i} v_i(n) + v_{S_1}(n), \end{aligned} \tag{6}$$

because the signal component $s_1(n)$ is known to this node and can thus be removed. The SNR of (6) can be derived as

$$\gamma_1 = \frac{\sum_{i=1}^{N} \frac{P_i}{E[|x_i(n)|^2]} |g_{1,i}|^2 P_{S_2} |h_{2,i}|^2}{\sum_{i=1}^{N} \frac{P_i}{E[|x_i(n)|^2]} |g_{1,i}|^2 \sigma_{v_i}^2 + \sigma_{S_1}^2}. \tag{7}$$

Assume the source nodes transmit with full power, i.e., $P_{S_1} = \bar{P}_{S_1}$ and $P_{S_2} = \bar{P}_{S_2}$. Define the ratio of the transmission power for each node i as

$$z_i = \frac{P_i}{\bar{P}_i}, \quad i = 1, \cdots, N, \tag{8}$$

and define the nominal edge SNR (i.e., the SNR of the receiving node when no other nodes transmit but just the node on the other hand of the edge and the node transmits with full transmission power)

$$\begin{aligned} \alpha_{1,i} &= \frac{\bar{P}_{S_1}|h_{1,i}|^2}{\sigma_{v_i}^2}, \qquad \alpha_{2,i} = \frac{\bar{P}_{S_2}|h_{2,i}|^2}{\sigma_{v_i}^2}; \\ \beta_{1,i} &= \frac{\bar{P}_i|g_{1,i}|^2}{\sigma_{S_1}^2}, \qquad \beta_{2,i} = \frac{\bar{P}_i|g_{2,i}|^2}{\sigma_{S_2}^2}. \end{aligned} \tag{9}$$

Then the SNR expression (7) can be simplified to

$$\gamma_1 = \frac{\sum_{i=1}^{N} \frac{\alpha_{2,i}\beta_{1,i}}{\alpha_{1,i}+\alpha_{2,i}+1} z_i}{1 + \sum_{i=1}^{N} \frac{\beta_{1,i}}{\alpha_{1,i}+\alpha_{2,i}+1} z_i} \tag{10}$$

Similarly, the SNR of the source node S_2 can be derived as

$$\gamma_2 = \frac{\sum_{i=1}^{N} \frac{\alpha_{1,i}\beta_{2,i}}{\alpha_{1,i}+\alpha_{2,i}+1} z_i}{1 + \sum_{i=1}^{N} \frac{\beta_{2,i}}{\alpha_{1,i}+\alpha_{2,i}+1} z_i} \tag{11}$$

For more succinct notations, we define $N \times 1$ vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$, whose ith elements are, respectively,

$$\begin{aligned} a_i &= \alpha_{2,i}, \qquad b_i = \frac{\beta_{1,i}}{\alpha_{1,i} + \alpha_{2,i} + 1} \\ c_i &= \alpha_{1,i}, \qquad d_i = \frac{\beta_{2,i}}{\alpha_{1,i} + \alpha_{2,i} + 1} \end{aligned} \tag{12}$$

In addition, define the $N \times 1$ vector $\mathbf{z} = [z_1, \cdots, z_N]^T$, where $[\cdot]^T$ denotes transportation. Then, we can rewrite (10) and (11) as

$$\gamma_1 = \frac{(\mathbf{a} \odot \mathbf{b})^T \mathbf{z}}{1 + \mathbf{b}^T \mathbf{z}}, \quad \gamma_2 = \frac{(\mathbf{c} \odot \mathbf{d})^T \mathbf{z}}{1 + \mathbf{d}^T \mathbf{z}}, \tag{13}$$

where $\odot$ denotes element-wise vector/matrix product (Hadamard product).

Note that we have assumed that each source node knows its own transmitted signal and can thus remove (subtract) such signal from the mixture. Nevertheless, differently from most existing work in ANC-coded two-way relay networks, we do not assume perfect synchronization among the relaying nodes, nor between the two source nodes. We do not assume that the relaying nodes have perfect knowledge about the backward and forward channels.

Based on (13), many different optimization objective functions can be defined to optimize the relay node selection. In this paper, we propose to maximize the weighted sum of SNRs, i.e.,

$$\arg \max_{0\le z_i \le 1, i=1,\cdots,N} \quad \lambda\gamma_1 + (1-\lambda)\gamma_2 \tag{14}$$

where $\lambda \in [0,1]$ is any arbitrary constant to define the tradeoff between the two source nodes. The optimization (13)-(14) is in the form of the sum-of-ratios linear fractional programming (LFP). Note that our case is in fact a special, and thus simpler, case of the general sum-of-ratios LFP because we have only two linear fractional ratios. The general sum-of-ratios LPF has posed significant theoretical and computational challenges, which has stimulated the development of many successful algorithms to solving this type of optimization [11]-[15]. This is in contrast to the perfectly synchronized transmit-beamforming case [5] where the optimization could be conducted approximately only.

In next section, we first consider the special case with channel reciprocity, for which we can derive even a closed-form solution. Then we address the general case of channel non-reciprocity.

3. OPTIMAL SELECTION OF RELAYS

3.1. Linear fractional programming in channel reciprocity case

We first consider a special case where the channels are reciprocal, i.e., the backward channels equal to the forward channels. As shown in Fig. 3, we have

$$g_{1,i} = h_{1,i}, \quad g_{2,i} = h_{2,i}. \tag{15}$$

This situation holds, e.g., in time-division-duplex (TDD) relaying systems.

Similarly to [5], we can use the following optimization criteria rather than (14),

$$\arg\min_{0\le z_i\le 1, i=1,\cdots,N} \quad f(\mathbf{z}) = \frac{\lambda}{\gamma_1} + \frac{1-\lambda}{\gamma_2} \tag{16}$$

It was proved in [5] that (16) and (14) share the same capacity region, although their optimization results may be different for each specific λ.

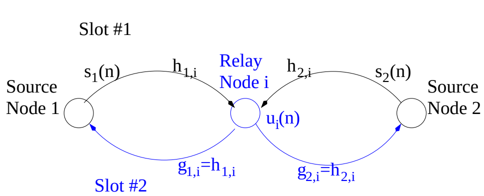

Fig. 3. System model of the two-way relaying network in reciprocal channel case.

Due to channel reciprocity, we have $\alpha_{1,i} = \beta_{1,i}$, $\alpha_{2,i} = \beta_{2,i}$, $i = 1,\cdots,N$. Then we have $\mathbf{d}\odot\mathbf{c} = \mathbf{b}\odot\mathbf{a}$. Define

$$\mathbf{e} = \mathbf{b}\odot\mathbf{a}. \tag{17}$$

Then the optimization (16) becomes

$$\arg\min_{0\le z_i\le 1, i=1,\cdots,N} \quad f(\mathbf{z}) = \frac{1+[\lambda\mathbf{b}^T + (1-\lambda)\mathbf{d}^T]\mathbf{z}}{\mathbf{e}^T\mathbf{z}} \tag{18}$$

This minimization problem is the linear fractional programming (LFP), and can be solved by many standard LPF algorithms [16]. One popular algorithm is to adopt the Charnes-Cooper transformation

$$\mathbf{y} = \frac{1}{\mathbf{e}^T\mathbf{z}}\mathbf{z}, \quad t = \frac{1}{\mathbf{e}^T\mathbf{z}}, \tag{19}$$

to change the LFP (18) into the following linear programming

$$\begin{aligned} \min \quad f(\mathbf{y},t) \quad &= \quad [\lambda\mathbf{b}^T + (1-\lambda)\mathbf{d}^T]\mathbf{y} + t \\ s.t., \quad & \quad \left\{ \begin{array}{l} \mathbf{e}^T\mathbf{y} = 1 \\ \mathbf{y} \le t[1, \; \cdots, \; 1]^T \\ t\ge 0, \quad \mathbf{y} \ge \mathbf{0} \end{array} \right. \end{aligned} \tag{20}$$

Note that the constraints are due to the relationship between $\mathbf{y}$ and t defined in (19), as well as the fact that $0 \le z_i \le 1$. Obviously, the optimization (20) is a linear programming [16]. After finding the optimal $\mathbf{y}^*$ and t^*, the optimal relay vector can be derived as

$$\mathbf{z}^* = \frac{\mathbf{y}^*}{t^*}. \tag{21}$$

The optimal SNR is thus

$$\lambda\gamma_1 + \lambda\gamma_2 = \lambda\frac{\mathbf{e}^T\mathbf{z}^*}{1+\mathbf{b}^T\mathbf{z}^*} + (1-\lambda)\frac{\mathbf{e}^T\mathbf{z}^*}{1+\mathbf{d}^T\mathbf{z}^*}. \tag{22}$$

3.2. Closed-form solution to the channel reciprocity case

It is more desirable to derive closed-form solution to (16). Similar to [9], the closed-form solution for relay node selection can be derived as follows.

Proposition 1. For each node i, $i = 1,\cdots,N$, we have the optimal solution to (16) as

$$z_i^* = \left\{ \begin{array}{ll} 1, & \text{if } \sum_{j=1,j\ne i}^N \left[\lambda b_j\left(\frac{\alpha_{2,j}}{\alpha_{2,i}} - 1\right)\right. \\ & \quad \left. +(1-\lambda)d_j\left(\frac{\alpha_{1,j}}{\alpha_{1,i}}-1\right)\right]^+ < 1 \\ 0, & \text{else} \end{array} \right. \tag{23}$$

where the non-negative function

$$[x]^+ = \left\{ \begin{array}{ll} x, & \text{if } \; x > 0 \\ 0, & \text{else} \end{array} \right. \tag{24}$$

Proof. From the $f(\mathbf{z})$ in (18), we can take the derivative of $f(\mathbf{z})$ with respect to each variable z_i as

$$\begin{aligned} \frac{\partial f(\mathbf{z})}{\partial z_i} \quad &= \quad \frac{1}{(\mathbf{e}^T\mathbf{z})^2}\Big\{ \sum_{j=1,j\ne i}^N [\lambda b_i + (1-\lambda)d_i]\, a_j b_j z_j \\ & \qquad - [\lambda b_j + (1-\lambda)d_j]\, z_j a_i b_i - a_i b_i \Big\}. \end{aligned} \tag{25}$$

Obviously, $\partial f(\mathbf{z})/\partial z_i$ being positive or negative is independent of z_i. If $\partial f(\mathbf{z})/\partial z_i \geq 0$, then the optimal value $z_i = 0$. Otherwise, the optimal value is $z_i = 1$. So we just need to consider the condition

$$\sum_{j=1,j\neq i}^{N} \{[\lambda b_i + (1-\lambda)d_i]a_j b_j - [\lambda b_j + (1-\lambda)d_j]a_i b_i\} z_j \gtrless a_i b_i. \quad (26)$$

With some deduction, (26) can be changed to

$$\sum_{j=1,j\neq i} [\lambda \alpha_{1,i} b_j(\alpha_{2,j} - \alpha_{2,i}) + (1-\lambda)\alpha_{2,i} d_j(\alpha_{1,j} - \alpha_{1,i})] z_j \gtrless \alpha_{1,i}\alpha_{2,i}. \quad (27)$$

Then, following the proof of [9], we can readily prove (23). Details are skipped due to page limit, and will be reported elsewhere. □

The Proposition 1 indicates that a node i either participates in relaying with full (maximum) transmission power, or does not participate in relaying at all. This phenomenon is similar to the one-way AF relaying network studied in [9]. It is especially desirable for efficient multiple relay selection and optimization, since there is no need of determining the transmission power of each relay node.

3.3. Sum-of-ratios linear fractional programming in channel non-reciprocity case

In the general case that includes both the channel reciprocity case and the channel non-reciprocity case, we need to deal with the sum-of-ratios linear fractional programming (14). Note that channel non-reciprocity may happen when the frequency-division-duplex (FDD) is employed [5].

Sum-of-ratios LPF has been found in many applications, which stimulated decades of research. Although the problem in the general setting (with arbitrarily large number of ratios, number of variables, as well as number of constraints) may still be challenging and deserve more investigation, there are many sophisticated algorithms that can be used to solve the special case in our setting.

One of the relatively simple algorithm is developed in [15], which exploits the mapping from the N-dimensional $\mathbf{z}$-space to the two-dimensional ratio space. Such a mapping fundamentally reduces the complexity of the algorithm. Another popular algorithm [13][14] is based on the parametric simplex algorithm. By applying some appropriate transformations including the Charnes-Cooper transformation we used in Section 3.2, the sum-of-ratios LFP can be converted into parametric linear programming, which can thus be solved by the parametric simplex algorithms. In addition, in a more recent algorithm [11][12], the sum-of-ratios LFP is converted into an equivalent concave minimization problem, which can then be solved via a branch-and-bound search.

We have implemented the algorithm in [15] to solve the sum-of-ratios LFP, as shown in simulations.

4. SIMULATIONS

In this section, we report our implementation of the proposed optimization algorithms (for both the channel reciprocal case and the channel non-reciprocal cases) and the application of them in the simulation of a random wireless ad hoc network. We simulated a random wireless ad hoc network of $N+2$ nodes with N relay candidate nodes. The nodes' positions were randomly generated within a square of 1000×1000 meters. The nominal edge SNRs $\alpha_{i,j}$ and $\beta_{i,j}$ were calculated as $10^8 d_{ij}^{-2.6}$ where d_{ij} is the propagation distance. Source and destination nodes were fixed with distance $d_{0,N+1} = 1000$ meters.

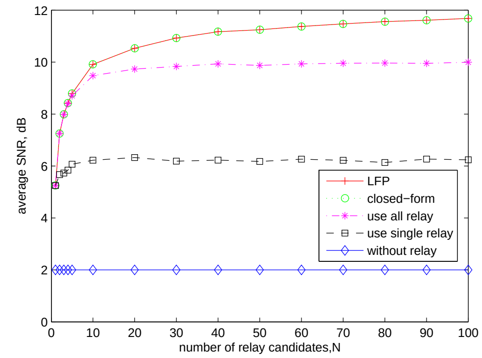

Fig. 4. Average sum SNR as functions of number of relay candidates.

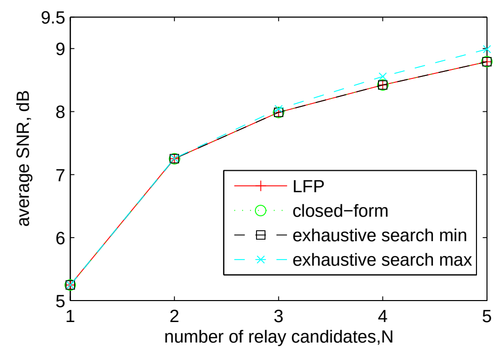

Fig. 5. Comparison of the proposed algorithms to the exhaustive search.

We simulated our new algorithms in Sections 3.1 and 3.3, as well as our optimal analytical results in Section 3.2. We compared them with the schemes using a single relay, or using all the N relay nodes, or the direct transmission without relaying. $10,000$ runs of the simulations were conducted to find the average sum SNR in each case.

First, we simulated the transmissions with the relay nodes selected optimally from all available relay candidates by the LFP algorithm in Section 3.1 and the closed-form expression in Section 3.2. The results were then compared to the cases of without optimal relay selections, either using all the N relay candidates or using one relay with similar distances from both sources. We also compared our optimal relaying scheme with the baseline transmission without any relay. Simulation results in Fig. 4 show that the closed-form expression for optimal relay selection (23) had the identical performance as the optimal results obtained by LFP, which indicates the derivation is correct and the results are optimal. They both achieved the sum SNR that is much better than other cases.

Then, in order to verify that our optimal algorithms indeed achieves the optimal relay selection, we compared the average sum SNR achieved by our optimal relay selection algorithms to an exhaustive search based power control scheme. Note that the exhaustive search of the optimal transmission power for each relay

N	1	2	3	4	5
LFP	5.246	7.251	7.987	8.421	8.792
Closed-form	5.246	7.251	7.987	8.421	8.792
Exhaustive min	5.246	7.251	7.987	8.421	8.792
Exhaustive max	5.246	7.256	8.042	8.556	8.991

Table 1. Compare average sum SNR obtained under various optimization methods.

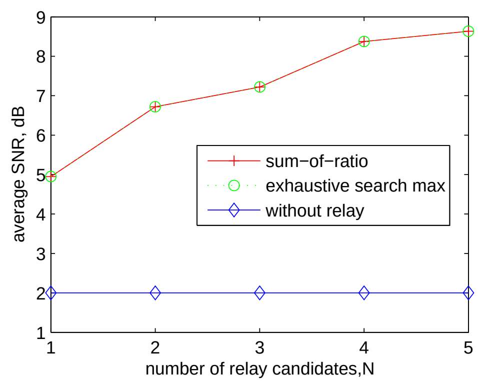

Fig. 6. Optimal relation selection for max sum SNR in channel non-reciprocal case.

candidate can only be conducted approximately in a finite grid. Specifically, while z_i could be any value between 0 and 1 in the power control scheme, we assume that each relay node has ten power levels only and exhaustively search $\min_{0\leq z_i\leq 1} f(\mathbf{z})$ in (18) or $\max_{0\leq z_i\leq 1} \lambda\gamma_1 + (1-\lambda)\gamma_2$ in (14). Note that when the transmission power is zero, the relay is not selected. Unfortunately, the exhaustive search of the power control scheme can only be used in small networks due to the complexity. The simulation results are shown in Fig. 5 and Table 1, which clearly shows that our proposed algorithms indeed achieves the optimal performance. Note that in "Exhaustive max" has higher SNR because it directly maximize sum SNR, while the other three minimizes the sum of inverse SNR instead.

For the simulation of nonreciprocal channel cases, we implemented the sum-of-ratios linear fractional programming algorithm of [15] to maximize the sum of SNRs (14). To simulate the channel non-reciprocity, we changed randomly the channel coefficients $g_{1,i}$ and $g_{2,i}$ after obtaining them according to the distance and fading rules described before. We compared the optimization results with the exhaustive search results. As shown in Fig. 6, we can clearly see that the two approaches give identical results.

5. CONCLUSION

For a dual-hop amplify-and-forward two-way cooperative network with analog network coding, we formulate a linear fractional programming solution to the optimal selection of all possible relays if channel reciprocity is assumed. Closed-form results are also obtained, which indicates that each relay candidate either participates in relaying with full transmission power or ceases relaying. For the general case including the channel non-reciprocity case, we formulate the problem into a sum-of-ratios linear fractional programming, which can be solved by many standard optimization algorithms. Simulations are conducted to verify the proposed optimization algorithms.

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