Minimum Cost Optimization of Multicast Wireless Networks with Network Coding

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***Abstract*— Minimum cost optimization for multicast with network coding has attracted great research interests. In this paper, based on an information model that differentiates intermediate nodes in a multicast network into network coding, routing or replicating nodes, a method is developed to solve the minimum energy cost multicast problem in wireless networks. Besides transmission energy, many other important parameters can be conveniently optimized as well, such as the number of packets undergoing network coding, the number of network coding nodes, as well as node classification. Some simulations results are shown to verify that the proposed method is promising, especially since network coding may be expensive or impossible for certain wireless nodes.**

***Index Terms*— network optimization, wireless network, multicast network, network coding.**

I. INTRODUCTION

The basic idea of network coding is to make messages sent on a node's output link to be some function of messages that arrived earlier on the node's input links. An example of such functions is the XOR function, which is the addition operation (linear) in finite field GF (2^n). The capacity of multicast networks with network coding has been shown in [1] as $\min_{t\in T}\mathrm{Mincut}(s,t)$, which is the upper bound of the multicast rate. It has also been shown that linear network coding is sufficient to achieve the multicast capacity [2-3] and the coding coefficients necessary to achieve the capacity can be computed in polynomial time [4-5].

Minimum cost optimization for multicast with network coding is one of the special subjects in network coding and has attracted great interests. When using network coding, there are two types of minimum cost optimization problems for multicast: i) Find the optimal subgraph to code over, and ii) determine the code to use over the subgraph [6]. In this paper, we focus on the first problem. The optimization of single multicast and multiple multicast connections were formulated in [7] and the minimum energy cost problem in wireless network was discussed. A decentralized algorithm was proposed in [8] with which the intermediated nodes can determine network coding coefficients according to local information.

Nevertheless, the information model used in [7-8] is based on the assumption that each of the nodes in the network has the ability to conduct network coding. In practice, some nodes may not be so powerful to conduct network coding, especially in wireless networks where nodes are limited by battery power, computing power as well as communication capability.

Therefore, a more reasonable model should take node differentiation into consideration. An interesting node differentiation scheme was considered in [9], where the nodes are classified into three different types: routing, replicating or network coding. Nevertheless, only wire-line network is considered in [9]. In this paper, by exploiting the information model with node classification, we will set up an optimization framework to solve the minimum energy cost multicast problem in wireless networks.

For convenience, Table 1 lists all the notations to be used in the rest of this paper.

II. INFORMATION MODEL WITH NODE DIFFERENTIATION

Consider network (V,E), where V is the set of nodes (vertices), E is the set of edges (directed links). Let c_{ij} be the non-negative capacity of the edge $(i,j)\in E$. In this paper, we consider only the single source multicast problem over a network. Note that multiple independent source multicast problems can be converted into single source multicast problems [1]. Multicast session (s,T) stands for a multicast session which has a sender $s\in V$ and a set of receivers $T\subseteq V$. The number of receivers is $|T|\in[1,|V|-1]$, where $|V|$ is the number of nodes. If $|T|$ is equal to 1, then the problem becomes a unicast problem. On the other hand, if $|T|$ is equal to $|V|-1$, it becomes a broadcast problem.

To model all the possible receiver sets in the multicast problem, let us define P as the power set of T (except the empty set) and Q as a set containing all collections of two or more disjoint sets in P. We can order the elements in P and Q such that P_l is the l-th element in P while Q_m is the m-th element in Q. If the receiver set T includes K receivers, then there are 2^{K-1} non-emepty sets P_l, $l=1,\cdots,2^K-1$. For example, if there are 3 receivers $\{1,2,3\}$ for this multicast flow, then

$$P=\{\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,3\},\{1,2,3\}\},$$
$$Q=\{\{\{1\},\{2\}\},\{\{1\},\{3\}\},\{\{2\},\{3\}\},\{\{1\},\{2,3\}\},\{\{2\},\{1,3\}\},\{\{3\},\{1,2\}\},\{\{1\},\{2\},\{3\}\}\}.$$

Corresponding to all the sets P_l, for the flows in each

TABLE I

NOTATIONS USED IN THE PAPER

(V, E)	A network, V is the set of nodes, E is the set of edges (directed links)
c_{ij}	Non-negative capacity of edge $(i,j) \in E$
a_{ij}	Non-negative cost per unit flow pass through the edge $(i,j) \in E$
d_{ij}	Distance between wireless nodes i and j
(s, T)	A single source multicast session which has a sender $s \in V$ and a set of receivers $T \subseteq V$
K	Number of receivers in T
P	Power set of T (except the empty set), P_l is its l-th ordered element
Q	A set containing all collections of two or more disjoint sets in P, Q_m is its m-th ordered element
X_{ij}	A $2^K - 1$ dimensional information flow vector associated with the edge (i,j). $x_{ij}(P_l)$ is its l-th element which represents the information flow common to and only common to the receivers in the set P_l
R	Transmission rate
r_m^i	A routing/replication variable associated with the set Q_m at node i, which is the flow for each receiver in the set $\cup_{Q \in Q_m} Q$ being replicated with each copy meant for a set in Q_m
n_m^i	A network coding variable associated with the set Q_m at node i, which is the flow for each set of receivers $Q \in Q_m$ that merges to form one flow that reaches all the receivers in the set $\cup_{Q \in Q_m} Q$
z_{ij}	Actual information flow in each edge $(i,j) \in E$

edge (i,j), we define X_{ij} as a $2^K - 1$ dimentional information flow vector, where the l-th element $x_{ij}(P_l)$ represents the information flow common to and only common to the receivers in the set P_l that passes through the edge (i,j). Note that X_{ij} is the variable that we want to optimize. Note also that $\sum_{l:t \in P_l} x_{ij}(P_l)$ is the overall information flow that passes through the edge (i,j) and will be received by the receiver t.

There are two basic constraints for the multicast network optimization problem:

A. Edge constraint

It means that the amount of information common to all the sets $P_l \in P$ that passes through the edge (i,j) can not be higher than the capacity of the edge. We can set up the edge constrain as $\sum_{P_l \in P} x_{ij}(P_l) \le c_{ij}, \forall (i,j) \in E$.

B. Node constraint

For the sender s, the in-link flow is 0, while the out-link flow is R. For the receiver t, the in-link flow is R, while the out-link flow is 0. For any intermediate node, the in-link flow must be equal to the out-link flow. The rate R is the transmission rate (or the value of $s-T$ flow where s is the sender and T is the set of receivers). In many other research of the max-flow problem, R has been served as the objective function for maximization. In our case, R is a constraint, i.e., the transmission rate that we want to meet at minimum energy cost.

While the rate constraints for the sender and the receivers are straight-forward, the rate constraints for intermediate nodes are not so clear yet considering the differentiation of the intermediate node. To match the different functions of these nodes, let us define two extra variables r_m^i and n_m^i first. The former will be used to describe routing/replicating nodes, while the latter will be used to describe network coding nodes.

The variable r_m^i is a routing/replication variable associated with the set Q_m at node i. It stands for the flow to each receiver in the set $\cup_{Q \in Q_m} Q$ that will be replicated into multiple copies, with each copy meant for a set in Q_m. If the node is a routing node, then $r_m^i = 0$. On the other hand, if the node is a replicating node, then r_m^i is equal to the rate of the input flow. Let $|Q_m|$ be the number of sets in Q_m, then we need to replicate each flow by $|Q_m| - 1$ times to get $|Q_m|$ flows.

The variable n_m^i is a network coding variable associated with the set Q_m at node i. It stands for the flows, each for a set of receivers $Q \in Q_m$, which merge into a single flow that will reach all the receivers in the set $\cup_{Q \in Q_m} Q$. If the node i is a network coding node, the n_m^i is equal to the rate of the input flow and $|Q_m|$ flows will be network coded to a single flow.

Now let us define the rate constraints for the three types of intermediate nodes. For routing nodes, nothing happens to the information flow. For replicating nodes, it replicates the packets and each copy of the packet in the out-link has to reach nodes in a set $P_l \in Q_m$. For example, for node i, if there is one in-link flow common to a receiver set P_3={1,2,3}, and there are two out-links to the receiver {1} and the receiver set {2,3}, respectively, then $Q_m = \{\{1\},\{2,3\}\}$ which is the set of receiver sets. In this case, $|Q_m| = 2$ and we need to copy the packet $|Q_m| - 1 = 1$ time. Now we have two copies of the packet, one for $P_1 = \{1\}$, and the other for $P_2 = \{2,3\}$. Then for P_1 we have flow constraint

$$\sum_{(i,j)\in E} x_{ij}(P_1) = \sum_{(j,i)\in E} x_{ji}(P_1) + r_m^i.$$

Similarly for P_2 we have

$$\sum_{(i,j)\in E} x_{ij}(P_2) = \sum_{(j,i)\in E} x_{ji}(P_2) - r_m^i.$$

Combining these two cases, for the replicating node, we have node constraint

$$\sum_{(i,j)\in E} x_{ij}(P_l) = \sum_{(j,i)\in E} x_{ji}(P_l) + \sum_{m:P_l \in Q_m} r_m^i - \sum_{m:P_l = \cup_{Q\in Q_m} Q} r_m^i$$

For network coding nodes, two or more flows will be combined together by coding. For example, for node i, if there are two in-links common to the receiver {1} and the receiver set {2,3}, and there is one out-link common to the receiver set P_3={1,2,3}, then $Q_m = \{\{1\},\{2,3\}\}$. In this case, $|Q_m| = 2$ and the two flows merge into a single one. The constraint for P_1 becomes

$$\sum_{(i,j)\in E} x_{ij}(P_1) = \sum_{(j,i)\in E} x_{ji}(P_1) - n_m^i,$$

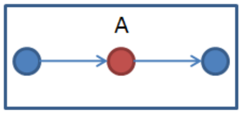
whereas the constraint for P_2 is

$$\sum_{(i,j)\in E} x_{ij}(P_3) = \sum_{(j,i)\in E} x_{ji}(P_3) + n_m^i.$$

Combining these two cases, for the network coding node, we have node constraint

$$\sum_{(i,j)\in E} x_{ij}(P_l) = \sum_{(j,i)\in E} x_{ji}(P_l) - \sum_{m:P_l \in Q_m} n_m^i + \sum_{m:P_l = \cup_{Q\in Q_m} Q} n_m^i$$

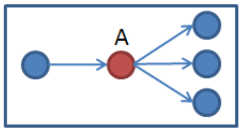
Type 1: Routing nodes:



$\sum_{(i,j)\in E} x_{ij}(P_l) = \sum_{(j,i)\in E} x_{ji}(P_l)$

$r_m^i = 0; n_m^i = 0.$

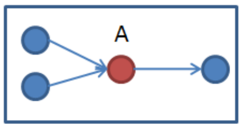
Type 2: Replicating nodes:



$\sum_{(i,j)\in E} x_{ij}(P_l) = \sum_{(j,i)\in E} x_{ji}(P_l) + \sum_{m:P_l\in Q_m} r_m^i - \sum_{m:P_l=\cup_{Q\in Q_m} Q} r_m^i$

$n_m^i = 0.$

Type 3: network coding nodes:



$\sum_{(i,j)\in E} x_{ij}(P_l) = \sum_{(j,i)\in E} x_{ji}(P_l) - \sum_{m:P_l\in Q_m} n_m^i + \sum_{m:P_l=\cup_{Q\in Q_m} Q} n_m^i$

$r_m^i = 0.$

Fig. 1 List of three types of intermediate nodes and the associated node constraints.

Now, we can develop the node constraints in general.

For the sender s, we have

$$\sum_{l:t\in P_l} \sum_{(s,j)\in E} x_{sj}(P_l) = R_1, \ \forall t \in T,$$

which means that all the information common to the receiver t passing through all out-links of the sender s is R_1. It is the rate of the information flow that we want to support. Obviously, R_1 can not be larger than the value of max flow rate R.

For the receiver t, we have

$$\sum_{l:t\in P_l} \sum_{(j,t)\in E} x_{jt}(P_l) = R_1, \ \forall t \in T,$$

which means that all the information common to the receiver t passing through all in-links of the receiver t is R_1.

For each intermediate node, we have

$$\sum_{(i,j)\in E} x_{ij}(P_l) = \sum_{(j,i)\in E} x_{ji}(P_l) + \sum_{m:P_l\in Q_m} (r_m^i - n_m^i) - \sum_{m:P_l=\cup_{Q\in Q_m} Q} (r_m^i - n_m^i), \ \forall P_l \in P.$$

The above results are listed in Fig. 1.

A node can be routing/replicating/network coding node at the same time. For example, consider an intermediate node has four in-links: one for receiver {1}, one for receiver {2}, one for receiver {3}, one for receiver set {4,5}; and four out-links: one for receiver {1}, one for receiver set {2,3}, one for receiver {4}, one for receiver {5}. Then the intermediate node is a routing node for the receiver {1}, a network coding node for the receiver set {2,3} and a replicating node for the receiver set {4,5}.

III. MINIMUM COST MULTICAST PROBLEM

We assume that the total cost of using an edge is proportional to the flow on it and a_{ij} is the non-negative cost per unit flow pass through the edge $(i,j) \in E$. Considering the edge constraints and node constraints, it is straightforward to optimize the flow allocations for minimizing the cost under the following optimization framework:

$$minimize \quad \sum_{(i,j)\in E} a_{ij} z_{ij}$$

subject to

$$c_{ij} \ge z_{ij} \ge \sum_{l:t\in P_l} x_{ij}(P_l), \qquad \forall (i,j) \in E, \forall t \in T$$
$$x_{ij}(P_l) \ge 0, \ \forall P_l \in P, \ \forall (i,j) \in E,$$
$$r_m^i \ge 0, n_m^i \ge 0, \ \forall m, \qquad \forall i \in V$$

Edge Constraints:

$$\sum_{P_l\in P} x_{ij}(P_l) \le c_{ij}, \forall (i,j) \in E \tag{1}$$

Node Constraints:

$$\sum_{l:t\in P_l} \sum_{(s,j)\in E} x_{sj}(P_l) = R_1, \qquad \forall t \in T$$
$$\sum_{l:t\in P_l} \sum_{(j,t)\in E} x_{jt}(P_l) = R_1, \qquad \forall t \in T$$
$$\sum_{(i,j)\in E} x_{ij}(P_l) = \sum_{(j,i)\in E} x_{ji}(P_l) + \sum_{m:P_l\in Q_m} (r_m^i - n_m^i) - \sum_{m:P_l=\cup_{Q\in Q_m} Q} (r_m^i - n_m^i)$$
$$\forall P_l \in P, \forall i \in V - \{s,T\}$$

Note that we have introduced variables z_{ij} corresponding to the actual rate of information flow on each edge $(i,j) \in E$. It is limited by the maximum flow rate to any receiver in the edge $(i,j) \in E$. We must also make sure that it is no more than the edge capacity c_{ij}. The objective function is the sum of z_{ij} weighted by the unit cost of transmission the flow. As a result, the summation result is the cost to transmit data to all receivers at rate R_1.

The edge constraints $\sum_{P_l\in P} x_{ij}(P_l) \le c_{ij}$ and $c_{ij} \ge z_{ij} \ge \sum_{l:t\in P_l} x_{ik}(P_l)$ can be omitted if the cost variables a_{ij} are set to extremely large values. In this case, the optimization problem may be easier to solve. We have not specify the cost. In fact, many natural cost criteria can be used, such as the transmission power, transmission energy, number of network coding node (i.e., $\sum_i n(i)$, where $n(i) = 1$ if $n_m^i > 0$ for some m, and $n(i) = 0$, otherwise), or the number of network coding operations (i.e., $\sum_i n_m^i$) [9].

IV. MINIMUM ENERGY MULTICAST OPTIMIZATION IN WIRELESS NETWORKS

Wireless networks are a special type of networks that has "multicast advantage", which means that if data is transmitted from node i to node j, then all nodes whose distance from i is smaller than j can receive this data for free. This is due to the broadcasting nature of wireless transmissions, and the transmitted signal strength attenuates rapidly along with transmission distance. This broadcasting property of wireless transmissions makes the set of out-links from a node i is a set that include all the wireless nodes within a certain fixed radius of the node i. As shown in [7], the networking optimization model in previous sections can be conveniently modified to taking the broadcasting nature of the wireless transmissions into consideration.

Recall that the variable z_{ij} is defined as the actual information flow rate of the edge (i,j). If another node k is farther away from the node i than the node j, i.e., $d_{ik} \ge d_{ij}$, then the unit cost parameters $a_{ik} \geq a_{ij}$. Due to the broadcasting nature of wireless transmissions, the transmission on the edge (i,k) can also be received by the node j. Therefore, the flow from the node i to the node j just need to satisfy the constraint

$$z_{ij} + \sum_{\{k|(i,k)\in E, a_{ik}\geq a_{ij}\}\setminus\{j\}} \left(z_{ik} - \sum\nolimits_{l:t\in P_l} x_{ik}(P_l) \right) \geq \sum\nolimits_{l:t\in P_l} x_{ij}(P_l), \qquad \forall t \in T$$

which is equivalent to

$$\sum\nolimits_{\{k|(i,k)\in E, a_{ik}\geq a_{ij}\}} \left(z_{ik} - \sum\nolimits_{l:t\in P_l} x_{ik}(P_l) \right) \geq 0, \ \forall t \in T.$$

Considering that wireless nodes are usually primarily limited in energy supply, we choose the energy usage as the optimization objective. In this case, the energy cost per unit flow of the edge (i,j) is a_{ij}, which is proportional to power of $d_{ij}^{-\alpha}$, where α is path loss exponential. As a matter of fact, there are at least two ways to define the energy-related parameters and formulate the energy-related optimization framework.

The first approach is the make edge capacity c_{ij} constant, which means the received signal power is constant, but the transmission power various according to the transmission distance. In this case, the parameters a_{ij} are determined according to distances d_{ij} .

In contrast, an alternative approach is to make the transmission power to be constant, while making the edge capacity varying according to the transmission distances. In our simulations, we will simulate both approaches.

Based on the general optimization framework (1), we can formulate minimum energy cost optimization for wireless networks as follows.

$$\begin{aligned}
&\textit{minimize} \quad \sum\nolimits_{(i,j)\in E} a_{ij} z_{ij} \\
&\textit{subject to} \\
&\qquad \sum\nolimits_{\{k|(i,k)\in E, a_{ik}\geq a_{ij}\}} \left(z_{ik} - \sum\nolimits_{l:t\in P_l} x_{ik}(P_l) \right) \geq 0, \\
&\qquad\qquad \forall (i,j) \in E', \forall t \in T \\
&\qquad x_{ij}(P_l) \geq 0, \ \forall P_l \in P, \ \forall (i,j) \in E \\
&\qquad r_m^i \geq 0, n_m^i \geq 0, \forall m, \forall i \in V
\end{aligned}$$

<u>Edge Constraints:</u>

$$\sum\nolimits_{P_l\in P} x_{ij}(P_l) \leq c_{ij}, \ \forall (i,j) \in E \qquad (2)$$

<u>Node Constraints:</u>

$$\sum\nolimits_{l:t\in P_l} \sum\nolimits_{(s,j)\in E} x_{sj}(P_l) = R_1, \qquad \forall t \in T$$

$$\sum\nolimits_{l:t\in P_l} \sum\nolimits_{(j,t)\in E} x_{jt}(P_l) = R_1, \qquad \forall t \in T$$

$$\sum\nolimits_{(i,j)\in E} x_{ij}(P_l) = \sum\nolimits_{(j,i)\in E} x_{ji}(P_l) + \sum\nolimits_{m:P_l\in Q_m} (r_m^i - n_m^i) - \sum\nolimits_{m:P_l=\cup_{Q\in Q_m} Q} (r_m^i - n_m^i) \qquad \forall P_l \in P, \forall i \in V - \{s,T\}$$

Note that E' is a subset of E, where the major difference is that if $a_{ik} = a_{ij}$ then E' just includes one of two edges (i,k) and (i,j).

For applying the optimization framework (2), we can easily add other constraints as well, such as those associated with the node types, e.g., specifying that the node i is not a network coding node by $n_m^i = 0, \forall m$, or specifying that the node i is a routing node by $r_m^i = 0, n_m^i = 0, \forall m$. We may also add constraints with respect to other special wireless transmission properties besides the broadcasting nature addressed in this paper. However, such study will be reported elsewhere.

V. SIMULATION

In this section, we use simulation to study the performance of the minimum energy cost optimization framework. We consider the classical butterfly network with wireless transmissions instead of wireline transmissions. The network is a single-source multicast network with one sender S and two receivers Y and Z. The wireless transmission ranges of the network nodes are shown in Fig. 2, where each dashed line indicates the broadcast circle of the node in the center. All the other nodes (except the one in the center point) inside a broadcast circle can receive the data sent by the center node.

Considering the two multicast receivers Y and Z, we have $P = \{\{Y\},\{Z\},\{Y,Z\}\}$ which has three elements $P_1 = \{Y\}$, $P_2 = \{Z\}$, $P_3 = \{Y,Z\}$. We have the set $Q = \{\{\{Y\},\{Z\}\}\}$ which consists of only one element $Q_1 = \{\{1\},\{2\}\}$. In the optimization problem, the optimization variables are the information flow rate vector X_{ij} associated with each edge (i,j). Each vector X_{ij} has 3 elements corresponding to the receiver sets in P, respectively. Note that the wireless network is treated as a special wireline network with broadcasting, so we can still use the "edge" concept for simplification.

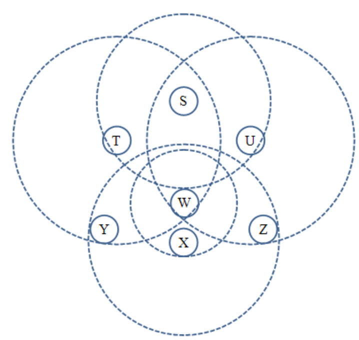


Fig. 2. Wireless network with the classical butterfly network topology.

In the first series of experiments in our simulations, we set the energy cost coefficients a_{ij} to 1. We also set the edge capacity c_{ij} to be unit.

Without node type constraint, the information flow rate vectors X_{ij} after the optimization procedure are annotated in Fig. 3. It clearly shows that the upper bound of the multicast rate, $min_{t\in T} Mincut(s,t)$, which is 2 unit flow per unit time, can be achieved. Specifically, the two date packets b_1 and b_2 can be transmitted to the two receivers at unit time. To explain in details, for the edge ST, [0,0,1] means the information flow commons to the receivers set $P_3 = \{Y,Z\}$ is 1 because only the third item is 1. Node W is a network coding node, because the information flows on the edge TW

(which is common to P_2) and on the edge UW (which is common to P_1) merge into a single information flow on the edge WX (which is common to P_3). Similarly, we can find that the nodes T, U and X are replicating nodes.

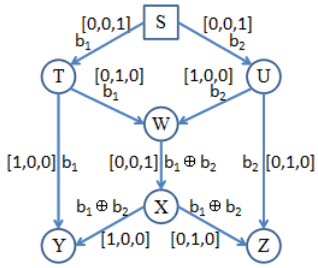


Fig. 3 . Information flow vectors of the network optimized without node type constraint. The node W becomes network coding node.

In contrast, with node type constraint (more specifically, the node W does not have the ability of conducting network coding), the information flow vectors after optimization are shown in Fig. 4. In this case, it is well known that the upper bound of the multicast rate cannot be achieved. Instead, the maximum data rate achievable is 1.5. In other words, three data packets b_1, b_2, b_3 can be transmitted to the destinations Y and Z by using two units of time.

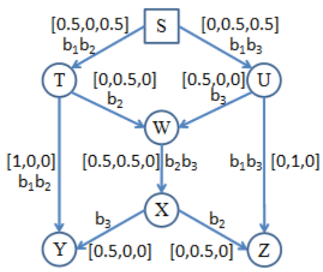


Fig. 4. Information flow vectors of the network optimized with node type constraint. Specifically, the node W can not conduct network coding.

For the above two cases, the actual rates z_{ij} for all edges (i,j) are shown in the first two rows in Table II. Specifically, the case 1 is corresponding to Fig. 3, whereas the case 2 is corresponding to Fig. 4. The "multicast advantage" created by the broadcasting property of the wireless transmissions are clearly seen. For example, the edge TW has rate 0.5 instead of 1 because the node W can also received the transmission from T to Y. By using such a lower rate, more energy can be saved.

Next, we simulated the cases with non-unit a_{ij}. We just made some nodes movable so as to change the distances between some nodes. Due to the importance of the node W, we moved W closer to T, which made the values of the energy cost coefficients a_{TW}, a_{UW}, a_{WX} changing to $0.5, 1.5, 1.8$, respectively. The case with W as network coding node is listed as case 3 in Table II, whereas the case with W being no network coding node is listed as case 4. Both cases gives the same overall rate as case 1 and 2, but with quite different edge rates. In addition, the uneven rates due to the broadcasting property become even more phenomenon. Specifically, some edges (such as TW) have rate 0, which means some transmissions can be avoided. Note that in this case the receiving node (such as W) can still receive information because they can hear others' transmissions. This type of ceasing transmissions surely can save more energy.

The above simulations were conducted with the first approach (i.e., fixing edge capacity to be unit) discussed in Section IV. As the second approach, we also conducted simulation by letting edge capacities c_{TW}, c_{UW}, c_{WX} be $1.5, 0.5, 1.8$, respectively. We also let $a_{ij} = 1/c_{ij}$ for all edges. Some simulation results are listed in Table II as the case 5, which had node constraints.

TABLE II. SIMULATION RESULTS OF FIVE DIFFERENT CASES

Cases			z_{ij}								
No.	R	Cost	ST	SU	TW	UW	TY	WX	UZ	XY	XZ
1	2	6	1	1	0.5	0.5	0.5	1	0.5	0.5	0.5
2	1.5	4.5	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.25	0.25
3	2	7.3	1	1	0	1	1	1	0	0.5	0.5
4	1.5	5.15	0.75	0.75	0	0.5	1	0.5	0.5	0.25	0.25
5	1.5	4.78	0.75	0.75	0	0.5	1	0.5	0.5	0.25	0.25

VI. CONCLUSIONS

In this paper, by classifying the intermediate nodes in a multicast network into network coding/replicating/routing nodes, a method is presented to formulate and solve the minimum energy cost multicast problem in wireless communication networks with network coding. Both the node differentiation and the wireless broadcasting property are addressed. The resulted optimization framework may be convenient to include more wireless transmission properties different from the conventional wire-line transmissions. This will be our on-going work. The simulation result shows that the method is useful to find the subgraph to achieve the minimum energy cost and maximum data rate in the wireless networks in case certain nodes do not have the ability to do network coding.

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