

# Using Cyclic Prefix to Mitigate Carrier Frequency and Timing Asynchronism in Cooperative OFDM Transmissions

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**Abstract**—While OFDM is a good modulation scheme for cooperative transmissions, the difficulty of synchronizing carrier frequencies of distributed transmitters presents one of the primary challenges. In this paper we show that cyclic prefix (CP) can be used not only to resolve the timing asynchronism, which is well-known, but also to mitigate the carrier frequency offsets (CFO) among the transmitters. Depending on the CP length, CFO can be mitigated or removed completely, which indicates an interesting trade-off between bandwidth efficiency and cooperation overhead.

## I. INTRODUCTION

Cooperative transmissions have attracted great attention recently. By sharing the antennas of multiple distributed transmitters or receivers to create virtual antenna arrays, cooperative transmissions have been shown to enhance bandwidth efficiency, power efficiency, reliability, etc [1]. An important form of cooperative transmissions is to adapt the existing antenna array techniques, such as space-time block codes (STBC) [2], into the distributed environment. This has great importance in practical wireless networks considering that small wireless nodes may not be able to have physical antenna arrays, while antenna array techniques are viable to them.

As far as the distributed implementation is concerned, one of the major issues is the synchronization of the cooperative transmitters. The “synchronization” in this paper refers specifically to the synchronization of the carrier frequency and arrival timing of all cooperative transmitters, i.e., their signals should have the same carrier frequency and timing when arriving at a receiver. Using the receiver’s local carrier and timing as references, perfect synchronization means zero carrier frequency offset (CFO) and zero timing-phase offset (TPO). Without such a perfect synchronization, many existing antenna array techniques such as STBC can not be directly used in cooperative transmissions [3].

Orthogonal frequency division multiplexing (OFDM) transmission technique is desirable for combating the loss of timing-phase synchronization, since any limited propagation delay (or timing-phase) difference among the signals of cooperative transmitters can be tolerated by simply increasing

the length of cyclic prefix (CP) [4], [5]. Because of this, it may find wide applications in cooperative transmissions, similarly as it flourishes in conventional antenna array systems where it provides a major advantage in simplifying the channel dispersion problem. Nevertheless, OFDM suffers critically from the loss of carrier frequency synchronization, in which case the CFO incurs inter-carrier interference (ICI) [6]. This CFO problem becomes even worse in multi-transmitter OFDM systems because of the increase in inter-transmitter interference, not only ICI [5].

While the CFO problem is still mostly open for research in cooperative OFDM systems, it is an extensively studied subject in single-user OFDM systems [6], [7] and multi-user OFDM systems [8], [9]. One may argue that cooperative OFDM systems are similar to multi-user OFDM systems (such as OFDMA and MC-CDMA [10]). However, this also means that both of them have multiple different CFOs so that complete CFO cancellation is difficult. More important, the decentralized operation nature of cooperative transmissions makes the existing CFO mitigation techniques of the multi-user OFDM systems not suitable for the cooperative OFDM systems [11].

In this paper, we present a novel approach for CFO mitigation or even complete cancellation. Our basic idea is to utilize the redundancy of the long CP. A unique feature of our approach is that it is implemented purely as a “pre-processing” procedure, independent from cooperative encoding/decoding details. In other words, it simply makes the CFO problem transparent to the cooperative OFDM transmission designs.

Some important notations are listed below:  $(\cdot)^T$ ,  $(\cdot)^H$ ,  $(\cdot)^+$  denote matrix transpose, Hermitian and pseudo-inverse;  $[\cdot]_m$  denotes the  $m^{\text{th}}$  element of a vector and  $[\cdot]_{m,n}$  denotes the  $(m, n)^{\text{th}}$  element of a matrix, where  $m, n$  are counted from 0;  $\text{diag}(\mathbf{x})$  denotes a diagonal matrix with diagonal entries listed in the vector  $\mathbf{x}$ ;  $\mathbf{0}_m$  is zero vector of dimension  $m$ ,  $\mathbf{0}_{M \times N}$  is  $M \times N$  zero matrix, and  $\mathbf{I}_N$  is  $N \times N$  identity matrix;  $x|N$  denotes  $x \bmod N$ .

The rest of the paper is organized as follows. In Section II, we give the cooperative OFDM transmission model. In Section III, we describe our CFO mitigation algorithm. Then

<sup>0</sup>This work was supported by US AFRL under grant FA 8750-06-2-0167.

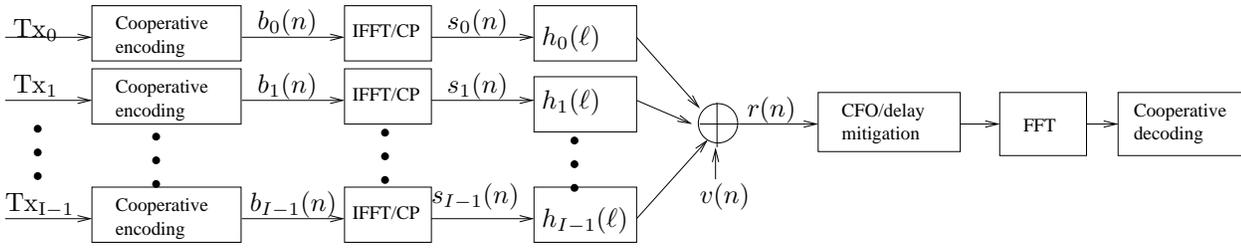


Fig. 1. Multi-transmitter cooperative OFDM transmission and receiving block diagram.

we conduct simulations in Section IV and conclude this paper in Section V.

## II. SYSTEM MODEL

Consider a cooperative transmission system with  $I$  cooperative transmitters and one receiver as shown in Fig. 1. All the  $I$  cooperative transmitters are assumed to have the same data packet that is to be encoded and transmitted, using some predefined cooperative encoding schemes such as cooperative STBC [3]. The encoder output  $b_i(n)$ ,  $i = 0, \dots, I-1$ ,  $n = 0, 1, \dots$ , are then OFDM modulated, which gives the OFDM signal  $s_i(n)$ . Note that each transmitter may use all or a portion of the OFDM sub-carriers depending on the predefined cooperation schemes [4], [5] that we do not need to specify (because our proposed method is independent of them).

The discrete baseband channel from the  $i^{\text{th}}$  transmitter to the receiver is assumed frequency selective fading with coefficients  $h_i(\ell)$ ,  $\ell = 0, \dots, L$ . Without loss of generality, we let all the channels have the same order  $L$ . From the received signal  $r(n)$ , the receiver mitigates the asynchronism in carrier frequency and timing using our proposed method, after which conventional OFDM demodulation and cooperative decoding techniques such as [4] are applied.

With the consideration of asynchronous transmitters, the signal of each transmitter  $i$  may have a propagation delay  $d_i$  and a CFO  $\epsilon_i$  (relative to a reference timing and a reference local carrier) when received at the receiver. We assume  $d_i$  to be integer (with symbol interval as unit) since the fractional portion of the delay contributes nothing but some extra channel dispersion which can be assimilated into the dispersive channel model. The CFO  $\epsilon_i$  is derived as the residual carrier frequency normalized by the OFDM sub-carrier frequency separation [9]. Both  $d_i$  and  $\epsilon_i$  are assumed non-negative with some known upper bounds. In order to simplify the problem, we assume  $\epsilon_i \neq \epsilon_j$  for all  $i \neq j$ . As will be clear after Section III, if  $\epsilon_i = \epsilon_j$ , we only need to consider one of them, which is equivalent to reducing the total number of transmitters by 1.

The transmitted signal  $s_i(n)$  is derived from the Inverse Fast Fourier Transform (IFFT) of the encoded symbol  $b_i(n)$ . Since there is no inter-block interference (IBI) thanks to cyclic prefix, we consider one OFDM block for notational simplicity. Then the  $i^{\text{th}}$  transmitter's signal  $s_i(n)$  can be written as

$$s_i(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} b_i(k) e^{j2\pi nk/N}, \quad -N_g \leq n \leq N-1 \quad (1)$$

where  $N_g$  is the length of the CP and  $N$  is the IFFT block length (we also define it as OFDM block length). Obviously,  $N_g \geq L + \max_{0 \leq i \leq I-1} d_i$  should be satisfied in order to avoid IBI. In addition, we assume  $N > L + \max_{0 \leq i \leq I-1} d_i$ , which is usually a reality in practical systems.

The noiseless signal from the  $i^{\text{th}}$  transmitter is

$$x_i(n) = \sum_{\ell=0}^L h_i(\ell) s_i(n-\ell), \quad (2)$$

based on which the composite signal received by the receiver, with delay  $d_i$  and CFO  $\epsilon_i$  considered, is

$$r(n) = \sum_{i=0}^{I-1} x_i(n-d_i) e^{j(\epsilon_i n + \phi_i)} + v(n), \quad (3)$$

where  $\phi_i$  is the initial phase, i.e., the phase of the residual carrier of the  $i^{\text{th}}$  transmitter's signal in the symbol interval  $n = 0$ . The AWGN  $v(n)$  is assumed with zero-mean and variance  $\sigma_v^2$ .

From the received composite signal, a conventional OFDM demodulator would remove CP and consider the sample vector  $\mathbf{r}(0) = [r(0), \dots, r(N-1)]^T$ . In our case, from (2)-(3) this gives

$$\mathbf{r}(0) = \sum_{i=0}^{I-1} e^{j\phi_i} \mathbf{E}_i(0) \mathbf{H}_i(0) \mathbf{s}_i(d_i) + \mathbf{v}(0), \quad (4)$$

where the symbol vector  $\mathbf{s}_i(d_i) = [s_i(-d_i), \dots, s_i(N-1-d_i)]^T$ , and the channel matrix  $\mathbf{H}_i(0)$  is  $N \times N$  circulant. The  $N \times N$  diagonal matrix  $\mathbf{E}_i(0) = \text{diag}\{1, e^{j\epsilon_i}, \dots, e^{j\epsilon_i(N-1)}\}$  is defined as the CFO matrix. The AWGN vector is  $\mathbf{v}(0) = [v(0), \dots, v(N-1)]^T$ .

To remove the negative indices in  $\mathbf{s}_i(d_i)$ , we substitute all the negative indices with the equivalent positive ones by CP, which leads to  $\mathbf{s}_i(d_i) = [s_i(N-d_i), \dots, s_i(N-1), s_i(0), \dots, s_i(N-1-d_i)]^T$ . Then, we rearrange the order of the entries of  $\mathbf{s}_i(d_i)$  to get  $\mathbf{s}_i = [s_i(0), \dots, s_i(N-1)]^T$ . By switching correspondingly the columns of  $\mathbf{H}_i(0)$ , we can change (4) into

$$\mathbf{r}(0) = \sum_{i=0}^{I-1} e^{j\phi_i} \mathbf{E}_i(0) \mathbf{H}_i \mathbf{s}_i + \mathbf{v}(0), \quad (5)$$

where

$$\mathbf{H}_i = \begin{bmatrix} \mathbf{0}_{N-L-d_i} & h_i(L) & \cdots & h_i(0) & \mathbf{0}_{d_i-1} \\ \mathbf{0}_{N-L-d_i+1} & h_i(L) & \cdots & h_i(0) & \mathbf{0}_{d_i-2} \\ \vdots & & & & \vdots \\ \mathbf{0}_{N-L-d_i-1} & h_i(L) & \cdots & h_i(0) & \mathbf{0}_{d_i} \end{bmatrix} \quad (6)$$

is  $N \times N$  circulant with right cyclic-shifted rows. One of the interesting characteristics of the model (5)-(6) is that the delay  $d_i$  is contained in  $\mathbf{H}_i$  only, whereas the CFO  $\epsilon_i$  is contained in the CFO matrix  $\mathbf{E}_i(0)$  only. This property permits us to mitigate CFO  $\epsilon_i$  independently from  $d_i$ .

If there is no CFO, i.e.,  $\mathbf{E}_i(0) = \mathbf{I}_N$ , then performing FFT on  $\mathbf{r}(0)$  leads to the conventional cooperative OFDM demodulation [4]. The situation is different with CFO, where the major problem is that  $\mathbf{E}_i(0)$  prevents the diagonalizing of  $\mathbf{H}_i$ , but instead causes ICI as well as multi-transmitter interference, if directly conducting FFT. Therefore, we need to look for ways to reduce or remove all the  $I$  CFO matrices  $\mathbf{E}_i(0)$ .

### III. CFO MITIGATION AND CANCELLATION

#### A. Using Redundant CP

Our basic idea is to exploit the redundancy of the CP based on the structure of the signal model (5). If the CP length  $N_g$  is longer than  $L + \max_{0 \leq i \leq I-1} d_i$ , then in addition to those in  $\mathbf{r}(0)$ , we have more IBI-free samples  $r(-m)$ ,  $0 < m \leq N_g - L - \max_{0 \leq i \leq I-1} d_i$ , with which we can construct new sample vectors  $\mathbf{r}(m) = [r(-m), \dots, r(N-1-m)]^T$ , and we have

$$\mathbf{r}(m) = \sum_{i=0}^{I-1} e^{j\phi_i} \tilde{\mathbf{E}}_i(m) \mathbf{H}_i(0) \mathbf{s}_i(d_i + m) + \mathbf{v}(m), \quad (7)$$

where  $\tilde{\mathbf{E}}_i(m) = \text{diag}\{e^{j\epsilon_i(-m)}, \dots, e^{j\epsilon_i(N-1-m)}\}$ , the symbol vector  $\mathbf{s}_i(d_i + m) = [s_i(-d_i - m), \dots, s_i(N-1-d_i - m)]^T$ , and the channel matrix  $\mathbf{H}_i(0)$  is the same as that in (4). It is easy to see that  $\mathbf{s}_i(d_i + m) = [s_i((-d_i - m)|N), \dots, s_i(N-1), s_i(0), \dots, s_i((N-1-d_i - m)|N)]^T$ , where we use modulo  $N$  operations in order to cope with extremely large  $m$  (since we may use long CP  $N_g > N$ ). Next, we re-order the entries of  $\mathbf{s}_i(d_i + m)$  to change it into the vector  $\mathbf{s}_i$ , and switch the corresponding columns in  $\mathbf{H}_i(0)$  similarly as what we did in (5). The result is that (7) is changed to

$$\mathbf{r}(m) = \sum_{i=0}^{I-1} e^{j\phi_i} \tilde{\mathbf{E}}_i(m) \mathbf{H}_i(m) \mathbf{s}_i + \mathbf{v}(m), \quad (8)$$

where  $\mathbf{H}_i(m)$  is an  $N \times N$  circulant matrix. Its first row is  $[\mathbf{0}_{(-d_i-m)|N-L}, h_i(L), \dots, h_i(0), \mathbf{0}_{N-1-(-d_i-m)|N}]$ , and its rest rows are the right cyclic shifts of the first row.

Comparing  $\mathbf{H}_i(m)$  with  $\mathbf{H}_i$  in (6), we see that if we move the first  $N - d_i - (-d_i - m)|N$  rows of  $\mathbf{H}_i(m)$  to the end of this matrix, then we can change  $\mathbf{H}_i(m)$  into  $\mathbf{H}_i$ . Taking this adjustment, and changing the columns of  $\tilde{\mathbf{E}}_i(m)$  correspondingly, we obtain from (8) an expression similar to

(5), i.e.,

$$\mathbf{r}(m) = \sum_{i=0}^{I-1} e^{j\phi_i} \mathbf{E}_i(m) \mathbf{H}_i \mathbf{s}_i + \mathbf{v}(m), \quad (9)$$

where

$$\mathbf{E}_i(m) = \begin{bmatrix} \mathbf{0}_{(m|N) \times (N-m|N)} & \mathbf{E}_a \\ \mathbf{E}_b & \mathbf{0}_{(N-m|N) \times (m|N)} \end{bmatrix}, \quad (10)$$

and  $\mathbf{E}_a = \text{diag}\{e^{j\epsilon_i(-m)}, \dots, e^{j\epsilon_i(-m-1+m|N)}\}$ ,  $\mathbf{E}_b = \text{diag}\{e^{j\epsilon_i(-m+m|N)}, \dots, e^{j\epsilon_i(N-m-1)}\}$ . Note that we have used  $d_i < N$  and  $N - d_i - (-d_i - m)|N = m|N$  when deriving (10).

Noticing that (9) and (5) contain the same  $\mathbf{H}_i$  and  $\mathbf{s}_i$  but have different CFO matrices, we can stacking together all available vectors  $\mathbf{r}(m)$ ,  $0 \leq m < M \triangleq N_g - L - \max_{0 \leq i \leq I-1} d_i + 1$ , to get  $\mathbf{y} = [\mathbf{r}^T(0), \dots, \mathbf{r}^T(M-1)]^T$ . Then we have

$$\begin{aligned} \mathbf{y} &= \sum_{i=0}^{I-1} e^{j\phi_i} \begin{bmatrix} \mathbf{E}_i(0) \\ \vdots \\ \mathbf{E}_i(M-1) \end{bmatrix} \mathbf{H}_i \mathbf{s}_i + \mathbf{u} \\ &\triangleq \sum_{i=0}^{I-1} e^{j\phi_i} \mathbf{A}_i \mathbf{H}_i \mathbf{s}_i + \mathbf{u}, \end{aligned} \quad (11)$$

where  $\mathbf{u} = [\mathbf{v}^T(0), \dots, \mathbf{v}^T(M-1)]^T$ . The dimensions of  $\mathbf{y}$  and  $\mathbf{A}_i$  are  $MN \times 1$  and  $MN \times N$ , respectively.

Our basic idea is thus to design an  $N \times MN$  CFO mitigation matrix  $\mathbf{X}$  such that

$$\mathbf{X} \mathbf{A}_i = \mathbf{I}_N \quad (12)$$

for all  $i = 0, \dots, I-1$ . If  $\mathbf{X}$  is available for (12), then CFO can be mitigated via

$$\mathbf{z} = \mathbf{X} \mathbf{y}. \quad (13)$$

Note that a straightforward solution for  $\mathbf{X}$  is

$$\mathbf{X} = [\mathbf{I}_N \ \cdots \ \mathbf{I}_N] \begin{bmatrix} \mathbf{E}_0(0) & \cdots & \mathbf{E}_{I-1}(0) \\ \vdots & & \vdots \\ \mathbf{E}_0(M-1) & \cdots & \mathbf{E}_{I-1}(M-1) \end{bmatrix}^+ \quad (14)$$

If (12) can be satisfied perfectly, then we have  $\mathbf{z} = \sum_{i=0}^{I-1} e^{j\phi_i} \mathbf{H}_i \mathbf{s}_i + \mathbf{X} \mathbf{u}$ , which is a conventional CFO-free OFDM sample vector after removing the CP. Note that the scalar  $e^{j\phi_i}$  is nothing more than a phase factor of the channel  $\mathbf{H}_i$ . With the vector  $\mathbf{z}$ , conventional OFDM demodulation can be applied to detect symbols  $b_i(k)$ .

#### B. Element-wise derivation of the CFO mitigation matrix

One of the major problems is whether (12) has accurate solutions. Another problem is the computational complexity of solving (12) for the solution. The way of using (14) is clearly not desirable considering its high complexity. To address both problems, we conduct an element-wise analysis of  $\mathbf{E}_i(m)$  and  $\mathbf{X}$ , which will lead to more efficient algorithms.

Considering the structure of the CFO matrices (10), with some tedious but straightforward verification, we can see that

each CFO matrix  $\mathbf{E}_i(m)$ ,  $0 \leq i \leq I-1$ ,  $0 \leq m \leq M-1$ , has non-zero element  $e^{j\epsilon_i[(\ell+m)|N-m]}$  only in the  $[(\ell+m)|N]^{th}$  row and the  $\ell^{th}$  column, which means that (10) can be described element-wise as

$$[\mathbf{E}_i(m)]_{p,\ell} = \begin{cases} e^{j\epsilon_i(p-m)}, & \text{if } p = (\ell+m)|N \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

where  $0 \leq p \leq N-1$ ,  $0 \leq \ell \leq N-1$ .

Since not all  $\mathbf{r}(m)$  have to be used, we choose  $Q$  vectors from them, which we define as  $\mathbf{r}(m_0), \mathbf{r}(m_1), \dots, \mathbf{r}(m_{Q-1})$ , where the integer indices satisfy

$$0 \leq m_0 \leq m_1 \leq \dots \leq m_{Q-1} \leq M-1. \quad (16)$$

Note that the corresponding CFO matrices are  $\mathbf{E}_i(m_0), \dots, \mathbf{E}_i(m_{Q-1})$ , respectively, for  $0 \leq i \leq I-1$ . Then (12) is changed to looking for an  $N \times NQ$  CFO mitigation matrix  $\mathbf{X}$  such that

$$\mathbf{X} \begin{bmatrix} \mathbf{E}_i(m_0) \\ \vdots \\ \mathbf{E}_i(m_{Q-1}) \end{bmatrix} = \mathbf{I}_N, \quad 0 \leq i \leq I-1. \quad (17)$$

Let the  $k^{th}$  row of  $\mathbf{X}$ ,  $0 \leq k \leq N-1$ , be

$$\mathbf{x}_k = [\mathbf{x}_k(m_0), \mathbf{x}_k(m_1), \dots, \mathbf{x}_k(m_{Q-1})], \quad (18)$$

where each  $\mathbf{x}_k(m)$  is a  $1 \times N$  vector. Using  $[\mathbf{x}_k(m)]_p$  to denote the  $p^{th}$  element, (17) is equivalent to an element-wise representation

$$\sum_{q=0}^{Q-1} \sum_{p=0}^{N-1} [\mathbf{x}_k(m_q)]_p [\mathbf{E}_i(m_q)]_{p,\ell} = \begin{cases} 1, & \text{for } \ell = k \\ 0, & \text{for } \ell \neq k \end{cases} \quad (19)$$

for all  $\ell = 0, \dots, N-1$ .

Let us consider the  $\ell = k$  case of (19) first. Due to (15), we can reduce (19) into

$$\sum_{q=0}^{Q-1} [\mathbf{x}_k(m_q)]_{(k+m_q)|N} [\mathbf{E}_i(m_q)]_{(k+m_q)|N,k} = 1. \quad (20)$$

Applying the element value of (15) into (20), we obtain

$$\sum_{q=0}^{Q-1} [\mathbf{x}_k(m_q)]_{(k+m_q)|N} e^{j\epsilon_i[(k+m_q)|N-m_q]} = 1. \quad (21)$$

Because the same set of  $Q$  variables  $\{[\mathbf{x}_k(m_q)]_{(k+m_q)|N}, 0 \leq q \leq Q-1\}$  need to satisfy (21) for all  $0 \leq i \leq I-1$ , we can find them by solving

$$\mathbf{B}_k \mathbf{z}_k = \mathbf{b}, \quad (22)$$

where  $\mathbf{b} = [1, \dots, 1]^T$  is an  $N \times 1$  vector, the matrix

$$\mathbf{B}_k = \begin{bmatrix} e^{j\epsilon_0[(k+m_0)|N-m_0]} & \dots & e^{j\epsilon_0[(k+m_{Q-1})|N-m_{Q-1}]} \\ \vdots & & \vdots \\ e^{j\epsilon_{I-1}[(k+m_0)|N-m_0]} & \dots & e^{j\epsilon_{I-1}[(k+m_{Q-1})|N-m_{Q-1}]} \end{bmatrix} \quad (23)$$

has dimension  $I \times Q$ , and  $\mathbf{z}_k$  is the  $Q \times 1$  variable vector

$$\mathbf{z}_k = \begin{bmatrix} [\mathbf{x}_k(m_0)]_{(k+m_0)|N} \\ \vdots \\ [\mathbf{x}_k(m_{Q-1})]_{(k+m_{Q-1})|N} \end{bmatrix}. \quad (24)$$

Obviously, in order for (22) to have solutions, in general we need

$$Q \geq I \quad (25)$$

which means the number of sample vectors  $\mathbf{r}(m)$  should be no less than the number of transmitters. Considering that the matrix  $\mathbf{B}_k$  may not be square or full rank, the solution of (22) can be written as

$$\mathbf{z}_k = \mathbf{B}_k^+ \mathbf{b}, \quad (26)$$

and we need to calculate (26) for all  $0 \leq k \leq N-1$ . Note that although the matrix inverse is still involved, (26) has a complexity much lower than (14) because the matrix dimension is reduced by orders.

The  $k^{th}$  row of  $\mathbf{X}$  has  $NQ$  variables (c.f. (18)), but only  $Q$  of them are determined in (26). Fortunately, thanks to the special structure of the CFO matrices, the rest of the  $N(Q-1)$  variables do not play any role in (20), and can be simply set as zeros. This zero-setting is in fact not an option but a must when considering (19) for the case  $\ell \neq k$ , which is

$$\sum_{q=0}^{Q-1} [\mathbf{x}_k(m_q)]_{(\ell+m_q)|N} [\mathbf{E}_i(m_q)]_{(\ell+m_q)|N,\ell} = 0. \quad (27)$$

From the range of  $\ell, k$ , i.e.,  $0 \leq \ell \leq N-1$  and  $0 \leq k \leq N-1$ , we see that  $\ell \neq k$  means

$$(\ell+m_q)|N \neq (k+m_q)|N. \quad (28)$$

As a result, the variables  $[\mathbf{x}_k(m_q)]_{(\ell+m_q)|N}$  in (27) are different from the variables  $[\mathbf{x}_k(m_q)]_{(k+m_q)|N}$  in (21)-(24), so we can simply let the former be zeros for (27), i.e.,

$$[\mathbf{x}_k(m_q)]_p = 0, \quad \forall p \neq (k+m_q)|N, \quad 0 \leq p \leq N-1. \quad (29)$$

From (26) and (29) all the  $NQ$  variables of the  $k^{th}$  row of  $\mathbf{X}$  are determined. Repeating this procedure for each of the  $N$  rows, the matrix  $\mathbf{X}$  is thus available.

Note that  $\mathbf{X}$  need only be calculated once if the CFOs are constant. The calculation of (13) can also take the advantage of the sparse structure of  $\mathbf{X}$ . Details about complexity analysis and conditions for complete CFO cancellation can be found in [11].

#### IV. SIMULATIONS

In order to evaluate the performance of our algorithm, we simulated a system with two cooperative transmitters and one receiver, using Alamouti STBC [1], [2]. We used  $N = 32$ , QPSK. The integer delays  $d_i$ , the CFOs  $\epsilon_i$ , and the channels (with order  $L = 3$ ) were all randomly generated for each transmitter during each run of the simulation. We used 10,000 runs of the simulations to derive the average symbol error rate (SER) under various signal-to-noise ratio (SNR) or various

CFO.

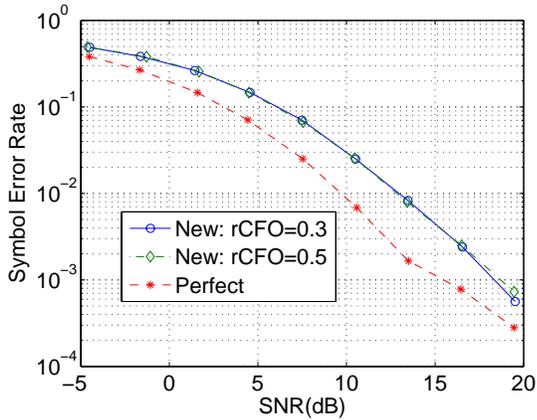


Fig. 2. CFO mitigation capability of our “New” algorithm, simulated with  $|d_2 - d_1| = 1$ , and rCFO 0.3 or 0.5.

In Fig. 2, we used the sample vectors  $\mathbf{r}(0)$  and  $\mathbf{r}(32)$ , and the results show that our algorithm has good performance in combating CFO, even when the relative CFO (rCFO)  $|\epsilon_1 - \epsilon_0|$  is large. The performance is less than 3 dB worse compared with the “perfect” OFDM.

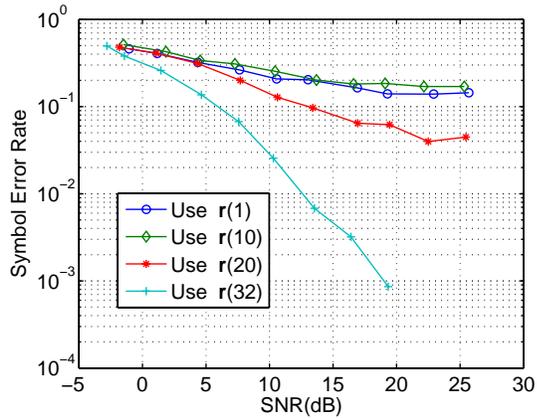


Fig. 3. CFO mitigation performance of our “New” algorithm increases with longer CP. rCFO= 0.2,  $|d_1 - d_0| = 1$ .

Fig. 3 shows the results of the trade-off between the CP length and the CFO mitigation performance. It can be seen clearly that the CFO mitigation performance increases with longer CP, up to a perfect CFO cancellation when  $\mathbf{r}(32)$  is used.

In Fig. 4, we varied the rCFO over a wide range from 0.1 to 0.9, and compared the performance of our algorithm to the conventional OFDM receiver and the “HL” CFO mitigation scheme [9]. Note that for the conventional OFDM receiver, we simply estimated the CFO at the middle of each OFDM block and used it to achieve a certain level of CFO compensation. As shown in Fig. 4, the conventional method did not resolve the CFO problem, neither did the “HL” scheme when the

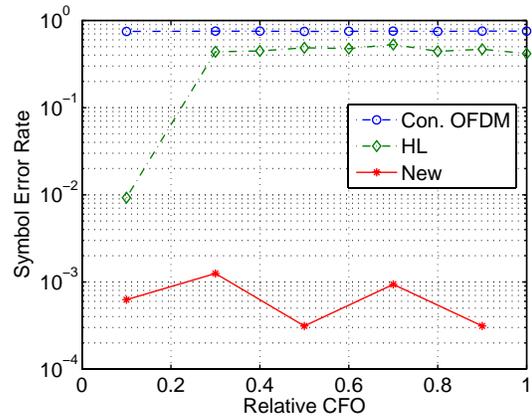


Fig. 4. Performance comparison of our “New” CFO mitigation algorithm with the conventional OFDM receiver and the CFO mitigation algorithm “HL” [9]. SNR 20 dB.

rCFO was not very small. In contrast, our new method showed almost constant performance under various rCFO.

## V. CONCLUSIONS

In this paper, we proposed a new algorithm for multi-transmitter cooperative OFDM transmissions, which can mitigate or cancel completely CFO using redundant CP. The algorithm is formulated as a computationally efficient preprocessing procedure independently from the cooperative encoding/decoding details, and may thus have ubiquitous applications in cooperative OFDM transmissions.

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