Cooperative STBC-OFDM Transmissions with Imperfect Synchronization in Time and Frequency

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Abstract—In this paper, we study the cooperative STBC-OFDM transmissions when the cooperative transmitters are not perfectly synchronized in time and carrier frequency. While OFDM can effectively resolve the limited delay asynchronism among the transmitters, the asynchronism in carrier frequency presents a major hurdle. Based on an approximate channel model, we analyze the performance of STBC-OFDM cooperative transmissions, compare them with non-cooperative ones, and propose a joint ICI mitigation and STBC decoding algorithm.

I. INTRODUCTION

Cooperative transmissions have attracted great attention recently. Along with the rapid development of wireless communications and the reduced cost of communication devices, wireless networks have become denser and denser while bandwidth efficiency becomes more and more important. It is thus natural to consider that multiple communication devices conduct transmission and reception cooperatively in a distributed manner. The advantage will be the more optimized bandwidth/energy efficiency and the more enhanced communication reliability.

Cooperative transmission has been proposed by many investigators [3]-[6]. Cooperative transmission shares antennas of source(S) and relay (R) to create a virtual antenna array. The relay can be either amplify-and-forward (AF) or decode-and-forward (DF). Detail analysis of this two schemes can be found in [5], [6]. Analysis of cooperative transmission indicates higher bandwidth efficiency, higher system capacity, as well as many other benefits. It is widely believed that the objective of cooperative transmissions is to utilize multiple distributed transmitters/receivers to simulate antenna array processing. As a typical example, space-time block codes (STBC) [1] have been widely used in cooperative transmissions. This has great practical importance because small wireless nodes usually can not be equipped with physical antenna arrays. In addition, since network relays have to support higher traffic loads, the advantage of cooperative communications is more important for wireless multi-hop networks.

One of the major hurdle to cooperative transmissions is the synchronization and coordination among the cooperative nodes. Traditional antenna array technology are usually developed based on perfect synchronization. However, for cooperative array, synchronization becomes either a difficult problem or requires extra cost in order to be achieved. We define "coordination" as the networking-level activity for the nodes to get tuned, for which much research has been conducted in the networking and above layers. In contrast, "synchronization" is for the cooperative nodes to achieve identical timing and frequency. This usually has to be conducted in the physical-layer.

In cooperative transmission, all transmitters should have identical clock, carrier frequency and symbol timing in order to directly use existing STBC-encoded transmission and decoding techniques. Such synchronization is hard to achieve in cooperative networks where transmitters are distributed distanty.

The synchronization problem has been addressed by many including us [7]. Poor synchronization in time will introduce intersymbol interference (ISI), while poor synchronization in frequency will introduce residual carrier which makes channels time-varying. If the system is STBC encoded, then the former destroys the required orthogonal structure of STBC and thus makes the traditional efficient STBC decoding method fail. We have shown in [7] that if the asynchronism in time is not so large (which we call quasi-orthogonal), then cooperative STBC can still be used with some guard intervals, where the residual carrier and time-varying channels may be dealt with by adaptive equalization techniques. Nevertheless, the ISI introduced requires complex equalization which may introduces performance reduction.

An alternative approach is to use OFDM transmission. OFDM has shown in [2] to tolerate small timing/delay asynchronism. Such asynchronism just introduces a phase shift to the frequency-domain flat channels so that traditional OFDM receiver can still work. However, a big problem is the residual carrier. In OFDM, residual carrier not only makes channels time-varying, but also introduces inter-carrier interference (ICI). The latter may be another major performance degradation factor.

In this paper, we consider the STBC-encoded cooperative transmissions with OFDM modulation when there are limited asynchronism in timing and in carrier frequency. We derive the system models, and then examine especially the effect of ICI. A new way is proposed to mitigate the ICI.

The organization of this paper is as follows. In Section II, we give the STBC-OFDM system model and analyze ICI. In Section III, we propose a new approach to mitigate the ICI.
II. COOPERATIVE STBC-ENCODED OFDM TRANSMISSION

A. STBC-OFDM system model

We consider cooperative transmission as shown in Fig. 1 where \( J \) transmitters transmit a data packet to a receiving node. In some intermediate hops of a multi-hop path, these \( J \) nodes can have the same data packet when the data packets are forwarded to them from the previous hop. On the other hand, one of the nodes who has the data packet can broadcast the data packet to the other cooperative nodes. The former does not require extra cost, while the latter requires some extra bandwidth and energy for the broadcasting.

Once all the \( J \) nodes have the same data packet, then they can conduct distributed STBC encoded transmission. We consider the distributed OFDM transmission shown in Fig. 2. Let the original shared data packet be \( \{ x_n \} \). Each node can conduct its own (partial) STBC encoding to obtain its data sequence \( \{ x_{n,j} \} \), where \( j \) is the index of cooperating nodes and \( n \) is the index of data. Of course, each node knows its position (or the index \( j \)) in a \( J \)-node STBC transmission scheme. Therefore, each node only generates its own portion of the signal. For example, if \( J = 2 \), then the first node will only generate a sequence \( x_{2n,1} = x_{2n}, x_{2n+1,1} = x_{2n+1} \), whereas the second node will generate \( x_{2n,2} = -x_{2n+1}^* \) and \( x_{2n+1,2} = x_{2n+1}^* \).

After generating the \( J \) sequences, the \( J \) nodes then transmit each of the sequence by OFDM modulation. Each node first conducts an IFFT with its own signal, and then adds cyclic prefix. The cyclic prefixed signal is denoted as \( s_{n,j} \) which is to be transmitted.

The receiver receives the summation of the signals transmitted by the \( J \) transmitters, and conduct cyclic removal and FFT. In case the \( J \) transmitters are perfect synchronized (just as a traditional antenna array-based OFDM transmissions), the received samples can be written as

\[
    r_n = \sum_{j=1}^{J} [h_{0,j} \cdots h_{L,j}] \left[ \begin{array}{c} s_{n,j} \\ \vdots \\ s_{n-L,j} \end{array} \right] + v_n. \tag{1}
\]

From (1), after OFDM demodulation, we have

\[
    y_{n,i} = \sum_{j=1}^{J} H_{j,i} x_{n,i,j} + w_{n,i}, \tag{2}
\]

where \( y_{n,i} \) denotes the sample in the \( n^{th} \) OFDM block and the \( i^{th} \) frequency bin. \( H_{j,m} \) denotes the frequency-domain channel coefficient from the \( j^{th} \) node and the \( i^{th} \) frequency bin, and \( x_{n,i,j} \) denotes the transmitted symbol from the \( j^{th} \) node in the \( i^{th} \) frequency bin of the \( n^{th} \) OFDM block.

Then the receiver can conduct STBC decoding according to the encoding method. With \( J \) transmitting nodes, an STBC block include \( J \) frequency bins of an OFDM block across \( M \) OFDM blocks where \( M \) depends on the STBC encoding method. For example, if \( J = 2 \), then every 2 bins across \( M = 2 \) OFDM blocks form a STBC decoding block. If the FFT/IFFT block length is \( N = 8 \), then \( N/J = 4 \) STBC blocks are transmitted parallelly in different bins, and are decoded independently.

B. STBC-OFDM with delay and frequency asynchronism

In order to address fractional delay and frequency offsets, we consider the continuous-time OFDM signal expression. Let \( T_s \) denote OFDM symbol length (or the effective IFFT interval), \( T_{cp} \) denote cyclic prefix length, and \( T = T_s + T_{cp} \). Note that \( T_s = NT_b \) where \( N \) denotes the number of symbols in each OFDM symbol (block), and \( T_b \) is the symbol interval.

Let the \( j^{th} \) transmitter transmit \( s_j(t) \) which is obtained as a continuous-time IFFT and cyclic prefix of \( x_{n,i,j} \), i.e.,

\[
    s_{n,j}(t) = \sum_{i=-\infty}^{\infty} x_{n,i,j} e^{j2\pi it/T_s},
\]

\[
    s_j(t) = \sum_{n=-\infty}^{\infty} s_{n,j}(t-nT), \tag{3}
\]

where the continuous time variable \( t \) of \( s_{n,j}(t) \) has range \([-T_{cp} \leq t \leq T_s \). The subscript \( n \) denotes the \( n^{th} \) OFDM symbol (block), and \( i \) denotes the \( i^{th} \) frequency bin (subcarrier) of the OFDM symbol (block). \( x_{n,i,j} \) is the symbol transmitted by the \( j^{th} \) node during the \( n^{th} \) OFDM block with the \( i^{th} \) subcarrier. Note that the time-division of the STBC can be across either \( n \) or \( i \). However, the former is better if residue carrier is unavoidable.

Each of the transmitter \( j \) gives a contribution of \( r_j(t) \) to the receiver at the antenna front end, where

\[
    r_j(t) = \int_{\tau=0}^{\infty} h_j(\tau)s_j(t-\tau)d\tau, \tag{4}
\]
which involves the convolution of the continuous channel \( h_j(t) \) and transmitted signal \( s_j(t) \).

The total received signal is the summation

\[
r(t) = \sum_{j=1}^{J} r_j(t - d_j) e^{j(2\pi f_j t + \theta_j)} + v(t),
\]

considering the different delay \( d_j \) and residue carrier frequency \( f_j \) for the \( j \)-th transmitter. \( \theta_j \) is the carrier phase. Note that \( d_j \) denotes the relative delay among the \( J \) transmitters, whereas \( f_j \) is the relative carrier frequency with respect to the nominal local carrier at the receivers. We will show that \( d_j \) and \( \theta_j \) just introduce a scalar multiplication factor to the channel, while \( f_j \) gives more complex interference.

The receiver then removes cyclic prefix and conducts FFT. The samples of the \( i \)-th FFT subcarrier of the \( n \)-th OFDM symbol (block) is

\[
y_{n,i} = \frac{1}{T_s} \int_{t=nT}^{(n+1)T} r(t) e^{-j2\pi n \frac{t-nT}{T_s}} dt.
\]

Note that the delay \( d_j \) and residue carrier \( f_j \) are contained in \( r_j(t) \) while the local receiver simply uses the nominal FFT expression (without any other delay or residual frequency involved).

Let \( u = t - nT \), and substitute (4) into (6) we have

\[
y_{n,i} = \frac{1}{T_s} \sum_{j=1}^{J} \sum_{i'=1}^{J} A_{j} \int_{u=rac{-nT}{2}}^{rac{nT}{2}} x_{n,i',j} G_{i',j} \int_{u=0}^{T_s} e^{j2\pi i' \frac{u}{T_s}} e^{j2\pi f_j u} du + v_{n,i},
\]

where \( v_{n,i} \) is the corresponding discrete noise.

By evaluating (7), we obtain

\[
y_{n,i} = \frac{1}{T_s} \sum_{j=1}^{J} A_{j} \sum_{i'=1}^{J} x_{n,i',j} G_{i',j} \int_{u=0}^{T_s} e^{j2\pi i' \frac{u}{T_s}} e^{j2\pi f_j u} du + v_{n,i},
\]

where \( A_j = e^{j2\pi f_j (nT + \theta_j)} \) is a phase rotation. Note that the phase rotation is time-varying with respect to \( n \). The new frequency domain channel coefficients are defined as

\[
G_{i',j} = \int_{\tau=0}^{\tau_{\text{max}}} h_j(\tau) e^{-j2\pi \frac{i' \tau}{T_s}} d\tau e^{-j2\pi f_j \frac{\tau}{T_s}} = H_{i',j} e^{-j2\pi f_j \frac{\tau}{T_s}}.
\]

In (8), for \( i' \neq i \) the integral will not equal to \( T_s \), which introduces inter-carrier interference (ICI). For the case of \( i' = i \), only a phase rotation was introduced.

\[
y_{n,i} = \frac{1}{T_s} \sum_{j=1}^{J} A_j x_{n,i,j} G_{i,j} \int_{u=0}^{T_s} e^{j2\pi f_j u} du + \frac{1}{T_s} \sum_{j=1}^{J} A_j \sum_{i'=1}^{J} x_{n,i',j} G_{i',j} \int_{u=0}^{T_s} e^{j2\pi \left( \frac{u}{T_s} + f_j \right)} du + v_{n,i}.
\]

Note that the first item is the desired one (with \( i' = i \) and no residue carrier), while the second item is ICI due to residue carrier \( f_j \).

### III. JOINT ICI MITIGATION AND STBC DECODING

A. Analysis of signal-to-interference plus noise ratio (SINR)

Assume that the receiver knows the carrier frequency of each of the transmitters, i.e., \( f_{c,j} \), \( j = 1, \ldots, J \). It can freely choose local carrier \( f_c \) such that the residue carrier is

\[
f_j = \begin{cases} f_c - f_{c,j} & \text{if } f_c \geq f_{c,j} \\ f_{c,j} - f_c & \text{if } f_c < f_{c,j} \end{cases}
\]

Note that one of the major differences between the cooperative transmission scenario and the traditional array transmission scenario is that \( f_{c,j} \) can be different for different transmitters \( j \). Therefore, it may be impossible to completely remove residue carrier even if the receiver can estimate accurately each transmitter’s carrier. On the other hand, the receiver can choose proper \( f_c \) to adjust \( f_j \). This unique property can be exploited to enhance the residue carrier mitigation capability for cooperative transmissions.

We also assume that the receiver knows the channels \( H_{i,j} \), for all \( i \) and \( j \), as well as the delays \( d_j \) and the initial phases \( \theta_j \). These parameters may be obtained by training or some other approaches. While the ways for parameter estimation in distributed environment is itself an interesting problem, we focus on analyzing the performance of cooperative transmissions and developing ways to enhance performance.

First, we analyze the signal part, i.e., the first quantity in (10). The equation (10) can be simplified to

\[
y_{n,i,j} = \sum_{j=1}^{J} A_j x_{n,i,j} G_{i,j} \frac{e^{j2\pi f_j u} - 1}{j2\pi T_s f_j} + I_{n,i} + v_{n,i,j}
\]

\[
= \sum_{j=1}^{J} A_j x_{n,i,j} G_{i,j} e^{j\pi f_j T_s} \sin(c(\pi f_j T_s)
\]

\[
+ I_{n,i} + v_{n,i,j},
\]

where we use \( I_{n,i} \) to denote the interference quantity. Note that due to residue carrier, even the desired signal part has an attenuation factor \( \sin(c(\pi f_j T_s)) \).

With the delay \( d_j \) explicitly included, we rewrite (12) as

\[
y_{n,i} = \sum_{j=1}^{J} x_{n,i,j} H_{i,j} \sin(c(\pi f_j T_s)) e^{j\Phi_{i,n,j}} + I_{n,i} + v_{n,i},
\]

where the phase rotation \( \Phi_{i,j} \) is

\[
\Phi_{i,j} = \theta_j + 2\pi f_j (nT + T_s) - 2\pi i d_j.
\]

As an example, consider the case where \( T_b = 10^{-6} \) sec (i.e., \( 10^6 \) symbols per second transmission rate). If the OFDM symbol (block) length \( N = 100 \sim 1000 \) and the residue carrier \( f_j = 5 \sim 10^5 \) Hz, then \( f_j T_s = 10^{-4} \sim 10^2 \). Note that \( f_j T_s \) is in the same range. Therefore, the channel will have an attenuation factor and time-varying phase. As we know, STBC requires that channels be almost constant within an encoding
block, e.g., with length $J \sim 2J$. Therefore, a large $f_jT$ will make channels violate such a requirement. Next, let us give a detailed analysis on the structure of ICI quantity $I_{n,i}$. From (10) and (12), we obtain
\[
I_{n,i} = \frac{1}{T_s} \sum_{j=1}^{J} A_j \sum_{i'=1}^{N-1} x_{n,i',j} G_{v',j} e^{j2\pi (u + f_j)u} du \\
= \frac{1}{T_s} \sum_{j=1}^{J} A_j \sum_{i'=1}^{N-1} x_{n,i',j} G_{v',j} \frac{1}{2\pi} e^{j2\pi (\frac{i'}{T_s} + f_j)} \times \left[ e^{j2\pi (\frac{i'}{T_s} + f_j)T_s} - 1 \right].
\]
Since $e^{j2\pi (i'-i+f_j)} = e^{j2\pi f_jT_s}$, we have
\[
I_{n,i} = \sum_{j=1}^{J} A_j e^{j2\pi f_jT_s} \sin(\pi f_jT_s) \sum_{i'=1}^{N-1} x_{n,i',j} G_{v',j} \left( \frac{T_s}{2\pi} \right) e^{j2\pi (\frac{i'}{T_s} + f_j)}.
\]

The ICI term given in (16) is the major hurdle to the cooperative OFDM-STBC scheme. The magnitude of ICI of each sub-carrier depends on all the other sub-carriers. In order to see the degradation of signal-to-interference plus noise ratio (SINR), we assume that symbols are i.i.d with zero mean. Then we have $E[I_{n,i}] = 0$. The SINR is
\[
\text{SINR} = \frac{E[|y_{n,i} - I_{n,i} - v_{n,i}|^2]}{E[|I_{n,i} + v_{n,i}|^2]} = \frac{\sigma_n^2 + \sigma_i^2 \sum_{j=1}^{J} |H_{n,i,j}|^2}{\sigma_n^2 + \sigma_i^2 \sum_{j=1}^{J} |H_{n,i,j}|^2}.
\]

Note that in most practical situations, $(i'-i)/(T_s f_j) > 1$. In particular, for large $i'-i$, the contribution of the item $|H_{n,i,j}|^2$ is very small. This gives us a way to consider only those major contributions for simplification.

In addition, the SINR expression is obtained when $E[x_{n,i,j}^2] = \sigma_e^2 \delta(i-j)\delta(j-k)$, i.e., symbols transmitted in each OFDM symbol (block) interval are not correlated. This means that if STBC is applied, the time-division encoding is better conducted across $n$, not $i$. Otherwise, more complex ICI pattern is involved.

### B. Joint ICI mitigation and STBC decoding

Let us first consider a simple example with the Alamouti STBC for $J = 2$ transmitters. Referring to the STBC encoding technique introduce in Section II, 2N symbols are encoded to produce 4N symbols which are transmitted by using 4 OFDM blocks in 2 time intervals. The received OFDM blocks in these two intervals can be constructed as $(r_1 + r_2, r_3 + r_4, \ldots, r_{N+1} + r_N)$ and $(-r_1^*, -r_2^* + r_3^*, \ldots, -r_N^* + r_{N+1}^*)$. From these samples we can estimate the symbols by performing STBC decoding while resolving ISI.

Consider the general expression (10) of the received signal. For notation simplicity, $n$ can be omitted from the equation. The signal part contains symbols associated with the channel coefficients (including phase rotation, channel gain, and frequency offset factor). Each component of the ICI part contains a summation of all the symbols with their own interference channel coefficients.

The channel coefficients of the signal and ICI parts are, respectively,
\[
C_{i,j} = A_j G_{i,j} \int_{u=0}^{T_s} e^{j2\pi u du},
\]
\[
C_{i',j} = A_j G_{i',j} \int_{u=0}^{T_s} e^{j2\pi u du}.
\]

These coefficients and symbols have linear relationship inside the $N$ received samples. Therefore we can construct $N$ linear equations, from which we can solve for the symbols and at the same time completely removing ICI. From (16), we can see that for the $i^{th}$ bin, the major ICI is contributed from the near-by several bins. As $i'$ goes further away from $i$, the factor $i'-i$ increases and ICI decreases. Therefore, for simplicity, we consider only two neighborhoods of $i$.

Let $C_i = [C_{i,1}, C_{i,2}]$, $x_i = [x_{i,1}, x_{i,2}]^T$. We have $N$ linear equations,
\[
\begin{bmatrix}
\vdots \\
y_{i-1} \\
y_i \\
y_{i+1} \\
\vdots
\end{bmatrix} =
\begin{bmatrix}
\vdots \\
C_{i-2} C_{i-1} C_i \ldots \\
\vdots \\
C_i C_{i+1} C_{i+2} \ldots \\
\vdots
\end{bmatrix}
\begin{bmatrix}
\vdots \\
x_{i-2} \\
x_{i-1} \\
x_i \\
x_{i+1} \\
x_{i+2} \\
\vdots
\end{bmatrix} + v.
\]

Note that we have to conjugate some received samples $y_n$ according to the STBC structure. Then the symbols can be estimated by solving the linear equation system.

### IV. Simulations

In this section, we first demonstrate the performance between cooperative and non-cooperative transmission schemes under delay asynchronism. Then we show the performance of our new method proposed in Section III, where neither delay nor frequency offset are perfectly synchronized.

The numerical studies were carried out on an cooperative based OFDM system model as described in Section II. The general setting is following. QPSK symbols were encoded by Alamouti STBC [1], with 2 transmitters and 1 receiver. Channels were modeled as 3-tap frequency-selective fading, and were assumed to be constant over every $J$ OFDM block. Additive white Gaussian Noise was added to the received signal.
Fig. 3. Performance comparison between cooperative and non-cooperative transmissions with delay asynchronism.

A. Delay asynchronism

In Fig. 3 we compared the cooperative and non-cooperative schemes with different delay asynchronism. In this simulation, each OFDM block consisted of 32 sub-carriers and had a CP length of 6. Each run of our simulation comprised of 100 of such OFDM blocks. We can see that there is about 10 dB gain by using cooperative scheme. Delay of 3 symbol intervals can be tolerated. As delay plus channel length increases above the CP length, the symbol error rate (SER) of cooperative transmission seems increasing faster than non-cooperative ones. This is mainly because STBC structure has been distorted by large delay. However, cooperative scheme with a delay 7 still outperform non-cooperative ones by 5dB.

B. Residue frequency-offset

In this experiment, each OFDM block consisted of 128 sub-carriers and 10% of CP. Different transmitters had different carrier frequency. Typically residue carrier frequency was a few hundred Hertz. We used $f_1, f_2$ to be 100Hz and 80Hz respectively. In addition, a delay of 2.5 was added. Simulation results shown in Fig. 4 indicate that our joint ICI mitigation and STBC decoding method outperforms the conventional cooperative schemes.

Fig. 4. Comparison between cooperative and non-cooperative transmissions with presence of frequency-offset and delay asynchronism.

Fig. 5. The effect of frequency offset on performance.

In Fig. 5, we studied the performance of our method under various residue carrier. Simulation shows that the performance is fairly robust.

V. CONCLUSIONS

In this paper, we studied the performance of OFDM cooperative transmissions when the cooperative transmitters can not be perfectly synchronized. Specifically, we analyzed the degradation of performance by the inter-carrier interference (ICI) due to the residue carrier frequency. A new algorithm is proposed to mitigate ICI, and its performance is demonstrated by simulations.

REFERENCES