ABSTRACT

In this paper we develop a new spectrum sharing scheme that uses compressive sensing to support the coexistence of the sporadic machine-to-machine (M2M) communications and the persistent conventional communications such as the 5G cellular transmissions within the same channel. The redundancy in the transmitted signals, such as training symbols, pilots, MAC overheads and correlated data, is exploited to create a sparse signal model. Compressive sensing techniques are then used to detect jointly all the transmitted signals from the mixture. The performance of the new scheme is analyzed. An M2M communication scenario in smart grid is simulated to verify the sparse signal model and the spectrum sharing scheme.

Index Terms— Coexistence, spectrum sharing, machine to machine communications, 5G, compressive sensing, sparsity

1. INTRODUCTION

There is an increasing interest in machine to machine (M2M) communications due to their wide applications in internet of things (IoT), sensor networks, smart meters, smart health, etc [1]. In contrast to human generated conventional communications, M2M involves communications between autonomous devices without human interaction. Most M2M communications share some common properties such as large number of devices, extremely low data rate, and highly sporadic transmissions.

Support of M2M communications will be an important function for communication systems, in particular the 5G cellular systems [2]-[4]. Nevertheless, integrating M2M communications into today’s cellular systems such as Long Term Evolution (LTE) is inefficient because the sporadic transmissions from a large number of M2M devices can easily lead to severe signaling overhead and channel shortage, even though the overall M2M traffic amount is not high.

Networking architectures to support M2M communications have been an active research topic [5]. Schemes for scheduling the sporadic transmissions in massive M2M communications are proposed in [6]. A random access scheme for M2M communications is developed in [7]. Exploiting the sporadic property, [8] models M2M communications with a sparse model and uses compressive sensing to support random access of a large number of M2M devices. Similarly, compressive sensing is used in [9] to address the multiple access of M2M devices.

In this paper, we consider the problem where a large number of sporadic M2M devices share the same channel with a conventional communication user such as a cellular user. While the conventional user transmits persistently, we allow the M2M devices to conduct their transmissions in this channel directly without too much channel scheduling and handshaking overhead. We will show that the redundancy in the transmitted signals can be exploited to separate the mixtures with compressive sensing techniques. As a unique contribution, the exploitation of redundancy differs this paper from [8]-[10].

The organization of this paper is as follows. In Section 2, we give the system model and the transmission scheme. In Section 3, we develop a new receiving algorithm. Simulations are presented in Section 4, and conclusions are given in Section 5.

2. SYSTEM MODEL

We consider a system consisting of a conventional user and $M$ M2M devices who share a common channel. The conventional user transmits its signal persistently while each M2M device transmits extremely sporadically due to both the low duty cycle and the low data rate of M2M applications. In particular, consider the case that the conventional user needs to transmit a symbol sequence $a_0(n)$, $n = 0, 1, \cdots$, via OFDM modulation. This symbol sequence is subdivided into a sequence of $N$-symbol OFDM blocks (or OFDM symbols) $a_0(mN+n)$, where $n = 0, \cdots, N-1$ and $m = 0, 1, \cdots$.

We put the symbols of the $m$th block into an $N \times 1$ symbol vector $a_0(m) = [a_0(mN), \cdots, a_0(mN+N-1)]^T$, where $(\cdot)^T$ denotes matrix/vector transpose. For the purpose of compressive sensing, the conventional user needs to conduct a transformation of the symbol vector. In this paper, the transformation is conducted as follows

$$s_0(m) = U_0 a_0(m),$$

where the $N \times N$ transformation matrix $U_0$ can be an arbitrary unitary matrix. The vector $s_0(m) = [s_0(mN), \cdots, s_0(mN+N-1)]^T$ is then modulated and transmitted by OFDM.

The purpose of the transformation (1) is to provide an overcomplete representation of the conventional user’s signal, which is needed during compressive sensing procedure at the receiving side. Although both $a_0(m)$ and $s_0(m)$ have $N$ symbols, the redundancy in $a_0(m)$ can be exploited to reduce the dimension of $a_0(m)$ and thus guarantee the over-complete representation. Some of the symbols in $a_0(m)$ are known as training or pilots. For example, most OFDM transmissions have over 20% symbols designated for training in each OFDM block, and have some subcarriers left either unmodulated or modulated with fixed default symbols. The information of the MAC head will also make some symbols known a priori. Note that such redundancy are conventionally used only for channel estimation, synchronization, etc [11], rather than supporting compressive sensing, spectrum sharing and coexistence.
The receiver can subtract all the known symbols from $a_0(m)$ so as to reduce the dimension of $a_0(m)$. This changes $a_0(m)$ to $x_0(m)$, where $x_0(m)$ has dimension $K_0$ and $K_0 < N$. By this procedure, (1) is effectively changed to the over-complete representation model

$$\hat{s}_0(m) = \hat{U}_0 a_0(m),$$

where the $N \times K_0$ matrix $\hat{U}_0$ is the corresponding submatrix of $U_0$. This over-complete representation model can in fact be obtained by simply calculating

$$\hat{s}_0(m) = s_0(m) - U_0 a_0(m - 1) = U_0 (a_0(m) - a_0(m - 1))$$

because adjacent OFDM blocks usually have the same known symbols.

In addition, if the modulation level is low, e.g. QPSK, we can also subtract $a_0(m)$ by a constant vector of an arbitrary QPSK symbol to further increase the number of 0 elements and reduce $K_0$. On the other hand, if such redundancy is unavailable, we can reduce the block length of $a_0(m)$ from $N$ to $K_0$. In this case, each block $a_0(m)$ has $K_0$ symbols, and is over-complete represented by the $N$ symbols in $s_0(m)$. The difference is that the former techniques exploit the redundancy of the communication signals for spectrum sharing, which does not reduce spectrum efficiency, while the last technique comes at the cost of extra degradation of bandwidth efficiency.

For the M2M devices, we have the similar modulation and transmission procedure. For notational simplicity, we assume that the M2M devices also use OFDM modulation with $N$ symbols per OFDM block. Specifically, if the M2M device $i$, where $i = 1, \cdots, M$, has data $a_i(n)$ for transmission, we construct $N$-symbol blocks $a_i(m) = [a_i(mN), \cdots, a_i(mN + N - 1)]^T$ and apply the transformation

$$s_i(m) = U_i a_i(m),$$

where $s_i(m) = [s_i(mN), \cdots, s_i(mN + N - 1)]^T$ and the transformation matrix $U_i$ is an $N \times N$ arbitrary unitary matrix. The symbol blocks $s_i(m)$ are OFDM modulated and transmitted.

After OFDM modulation and demodulation, the receiver obtains the received signal vector $r(m) = [r(mN), \cdots, r(mN + N - 1)]^T$, and

$$r(m) = \sum_{i=0}^{M} I_i(m) G_i s_i(m) + v(m)$$

where the vector $v(m)$ consists of additive white Gaussian noise with zero mean and variance $\sigma_v^2$, and the matrices $G_i$ are the frequency-domain channel matrices for the conventional user and M2M devices. The indicator function $I_i(m) = 1$ means the user/device $i$ transmits during the OFDM block $m$, while $I_i(m) = 0$ means the device $i$ does not transmit. Each channel matrix $G_i$ is an $N \times N$ diagonal matrix with frequency-domain channel coefficient $g_i(n)$, where $n = 0, \cdots, N - 1$, i.e.,

$$G_i = \begin{bmatrix} g_i(0) & & \\ & \ddots & \\ & & g_i(N - 1) \end{bmatrix}.\quad(6)$$

With the received signal $r(m)$, the receiver needs to detect the signal vectors $a_i(m)$, $i = 0, \cdots, M$, for all $i$ such that $I_i(m) = 1$.

In this paper, we assume that all the transformation matrices $U_i$ are known a priori since they are determined during system design. We also assume that all the channels $G_i$ have already been estimated. With the presence of training, it is standard for channel estimation in multi-user OFDM.

3. READER AND ITS PERFORMANCE

3.1. Signal detection via compressive sensing

We need to develop a compressive sensing based scheme for the receiver to detect the transmitted signals of the conventional user as well as the M2M devices from the mixture (5). First, we can use the redundancy of the M2M signals to change (5) into a sparse signal model. Specifically, for each M2M device $i$, in each block $a_i(m)$, all the known symbols can be subtracted to create 0 elements. The number of non-zero elements in $a_i(m)$, if it is small, is defined as the sparsity.

Nevertheless, subtracting the known symbols alone is usually not enough to reach desirable sparsity. Fortunately, a special property of M2M communications is that most M2M data packets are similar or highly correlated to their adjacent packets. This is because the sampled data of a sensor at a sampling time instant is very likely to be similar or highly correlated with the sampled data at the next sampling time instant. This property can be exploited to further increase sparsity.

In addition, in most M2M applications, such as IoT, sensor networks, smart meters, etc, the number of sampled data of a M2M device is usually limited. Therefore, the length of each data packet is usually short. We can assume that each OFDM block of the M2M devices contains all the data of one data packet. Considering the similarity of the data packets, the nearby OFDM blocks are also similar. Then, we can simply subtract $a_i(m - 1)$ from $a_i(m)$ to exploit all such similarities to enhance sparsity. Let

$$x_i(m) = a_i(m) - a_i(m - 1), \quad i = 1, \cdots, M.\quad(7)$$

Note that (7) is mainly for notational simplification. In general, we may need to apply various symbol position information to subtract the corresponding similar or identical systems.

The redundancy in the M2M blocks are usually significant. For example, the measurements or sampling values are highly correlated, or very similar to each other. The short data packet includes many identical contents, such as the IP addresses of the senders and receivers as well as the device ID. Over 20% of symbols are designed as training symbols, and the training symbols are the same among the OFDM blocks. Therefore, it is reasonable to assume that with high probability $x_i(m)$ is a sparse vector with just a small number of non-zero elements. Let the sparsity of $x_i(m)$ be $K_i$.

Considering the OFDM block $m$ where we have already detected all the blocks $a_i(\ell), \ell \leq m - 1$, and need to detect the blocks $a_i(m), i = 0, \cdots, M$. We subtract the detected signals from $r(m)$, which gives

$$y(m) = r(m) - \sum_{i=0}^{M} I_i(m) G_i U_i a_i(m - 1)$$

$$= G_0 \hat{U}_0 x_0(m) + \sum_{i=1}^{M} I_i(m) G_i U_i x_i(m) + v(m).\quad(8)$$

The conventional user’s signal is due to the over-complete representation model (2) after subtracting $a_0(m)$, while the M2M device’s signal is due to the sparsity model (7) after subtracting $a_i(m - 1)$. Note that certain symbol detection error does not degrade the sparsity too much.

If the sparsity $K_i$, i.e., the number of zero elements in $x_i(m)$, is desirable, we can use the compressive sensing technique to estimate
where $L$ denotes Hermitian. Note that $N$ denotes identity matrix in (12). The M2M device blocks can be found by solving the convex optimization

$$
\arg \min_{\mathbf{x}_i} \left\| \mathbf{y}(m) - \sum_{i=1}^{M} I_i(m) G_i U_i x_i \right\|_1
$$

(10)

Following [12]-[14], we can find the solution to (10) as

$$
\hat{x}_i(m) = (\hat{U}_0^H \hat{U}_0)^{-1} \hat{U}_0^H \hat{G}_0^{-1} \left( \mathbf{y}(m) - \sum_{i=1}^{M} I_i(m) G_i U_i \hat{x}_i(m) \right)
$$

(11)

where $(\cdot)^H$ denotes Hermitian. Note that $U_0$ is unitary and $\hat{U}_0^H \hat{U}_0 = I$.

The M2M device blocks can be found by solving the convex optimization

$$
\{ \hat{x}_i(m) \} = \arg \min_{\mathbf{x}_i} \left\| \mathbf{y}(m) - \sum_{i=1}^{M} I_i(m) G_i U_i x_i \right\|_1
$$

$$
- G_0 \hat{U}_0 \hat{x}_i(m) \right\|_1 + \sum_{i=1}^{M} \lambda_i I_i(m) \left\| x_i \right\|_1
$$

(12)

Note that $I_N$ is the $N \times N$ identity matrix in (12).

After obtaining $\hat{x}_i(m)$, we can estimate the packet $a_i(m)$ as

$$
a_i(m) = \hat{x}_i(m) + a_i(m-1).
$$

The procedure is outlined in the following algorithm.

### 3.2. Performance of compressive sensing based optimization

To simplify the analysis, let us consider the following variation of the optimization (12) with the $\ell_0$ norm constraint on sparsity

$$
\left\{ \arg \min_{\mathbf{x}_i} \left\| \mathbf{L} \left( \mathbf{y}(m) - \sum_{i=1}^{M} I_i(m) G_i U_i x_i \right) \right\|_0 \right. 
$$

s.t. $\sum_{i=1}^{M} I_i(m) \left\| x_i \right\|_0 \leq \frac{N-K_0}{2}
$$

(14)

where $\mathbf{L} = I_N - G_0 \hat{U}_0 \hat{U}_0^H G_0^{-1}$.

### Compressive sensing based spectrum sharing

**i) Transmission:**

1. Conventional user: use $U_0$ to represent $a_0(m)$ as $\hat{s}_0(m)$
2. M2M devices: use $U_i$ to represent $a_i(m)$ as $\hat{s}_i(m)$
3. All users: OFDM modulation and transmission of $s_i(m)$

**ii) Receiving:**

1. OFDM demodulation to get $r(m)$
2. Subtract the detected blocks from $r(m)$ to get $y(m)$
3. Estimate $\hat{x}_i(m)$ and $\hat{x}_0(m)$ jointly from $y(m)$
4. Calculate $\hat{a}_i(m)$.

Output: $a_i(m)$ for $i = 0, \cdots, M$.

**Proposition 1.** Under the M2M signal sparsity constraint

$$
\sum_{i=1}^{M} I_i(m) \left\| x_i(m) \right\|_0 < \frac{N-K_0}{2},
$$

(15)

the optimal solution to (14) satisfies $\lim_{\sigma \to 0} \hat{x}_i(m) = x_i(m)$.

**Proof:** From (8), we have

$$
\mathbf{y}(m) - \sum_{i=1}^{M} I_i(m) G_i U_i x_i =
$$

$$
G_0 \hat{U}_0 \hat{x}_0(m) + \mathbf{v}(m) + \sum_{i=1}^{M} I_i(m) G_i U_i \Delta x_i,
$$

(16)

where $\Delta x_i = x_i(m) - x_i$ is the residue error of the M2M signal subtraction. Since the matrix $\mathbf{L}$ is idempotent, we have $\mathbf{L}(\mathbf{y}(m) - \sum_{i=1}^{M} I_i(m) G_i U_i \hat{x}_i(m)) = \mathbf{L}(\mathbf{v}(m) + \sum_{i=1}^{M} I_i(m) G_i U_i \Delta x_i)$. Consider the singular value decomposition $\mathbf{L} = \mathbf{D}_L \mathbf{U}_L \mathbf{G}_0^{-1}$, where $\mathbf{D}_L$ is the diagonal singular value matrix with all the non-zero singular values only and $\mathbf{U}_L$ is the singular vector matrix. Since $K_0$ is the maximum rank of $\hat{U}_0$, the dimension (and the rank) of the matrix $\mathbf{D}_L$ is no larger than $N - K_0$. The minimization problem (14) is equivalent to

$$
\arg \min_{\mathbf{x}_i} \left\| \mathbf{D}_L \mathbf{U}_L^H \left( \mathbf{v}(m) + \sum_{i=1}^{M} I_i(m) G_i U_i \Delta x_i \right) \right\|_1.
$$

(17)

When $\sigma \to 0$, the optimization (17) is reduced to

$$
\arg \min_{\mathbf{x}_i} \left\| \mathbf{D}_L \mathbf{U}_L^H \sum_{i=1}^{M} I_i(m) G_i U_i \Delta x_i \right\|_1.
$$

(18)

If $\sum_{i=1}^{M} I_i(m) \left\| x_i(m) \right\|_0 < (N - K_0)/2$ and $\sum_{i=1}^{M} I_i(m) \left\| x_i \right\|_0 < (N - K_0)/2$, then we have $\sum_{i=1}^{M} I_i(m) \left\| \Delta x_i \right\|_0 < N - K_0$. Therefore, (17) is an over-determined linear equation system. This leads to $\lim_{\sigma \to 0} \Delta x_i = 0$, which means $\lim_{\sigma \to 0} \hat{x}_i(m) = x_i(m)$.

Proposition 1 shows that at high signal to noise ratio (SNR), if the sparsity condition (15) can be satisfied, all the M2M device signals can be estimated correctly. The sparsity condition (15) explains the importance of the over-complete representation (2) over the conventional user signal $a_0(m)$. Large $N - K_0$ (i.e., higher over-complete representation) provides more room for us to detect less sparse M2M signals. High sparsity of the M2M signals (i.e., small $K_i$ and small $\left\| x_i(m) \right\|_0$) makes the signal detection easier. Simultaneous transmission of many M2M signals will invalidate (15), which will prevent the signal detection.

Compressive sensing techniques in [12]-[14] were developed for data processing problems in signal processing or machine learning.
One of the major challenges for adapting them into communications is signal detection, the noise and residue error. In our case, equation (11) is used for detecting the conventional user’s signal, which suffers from the residue error of the estimation of \( \hat{x}_i(m) \). From the proof of Proposition 1, especially (17)-(18), we can see that the detection of the M2M signal \( x_i(m) \) is interfered by the noise \( v(m) \) only, not the conventional user signal \( x_0(m) \). In other words, the SNR of the M2M signal to the noise determines the performance. On the other hand, variance of the residue error in \( \Delta x_i(m) = x_i(m) - \hat{x}_i(m) \), denoted as \( \sigma_x^2 \), is comparable in size to noise variance \( \sigma_v^2 \). Furthermore, the optimization (12) with \( \ell_1 \) norm, rather than the \( \ell_0 \) norm, introduces such residue error to all the elements of \( \hat{x}_i(m) \). The estimator (11) thus suffers from a noise-plus-residue-error with variance \( \sigma_x^2 + \sigma_v^2 \).

There are two ways to mitigate this noise amplification effect. First, we can let

\[
\hat{x}_i(mN + n) = 0, \quad \text{if} \quad |\hat{x}_i(mN + n)| < \alpha \sigma_v
\]

for some constant \( \alpha \) (e.g., \( \alpha = 2 \)), and replace (11) by the weighted least squares estimator

\[
\hat{x}_0(m) = \hat{U}_0^H G_0^{-1} W \left( y(m) - \sum_{i=1}^{M} I_i(m) G_i U_i \hat{x}_i(m) \right) \tag{20}
\]

where \( W = \text{diag}[w_1, \ldots, w_N] \) with \( w_0 = 1 \) if \( \hat{x}_i(mN + n) = 0 \) and \( w_n = \sigma_x^2/\sigma_v^2 + \sum_{i=1}^{M} I_i(m) \sigma_x^2 \) otherwise.

The second way is to exploit the fact that there are usually channel coding in the transmissions which can be used to reduce detection errors. After obtaining the M2M signals \( \hat{x}_i(m) \) and \( \hat{a}_i(m) \), we exploit the channel decoding in the M2M signals to further reduce the error. Then, using the decoded sequence, we reconstruct \( \hat{x}_i(m) \) and calculate (11) again to estimate the conventional user signal.

### 4. SIMULATIONS

For M2M communications, we simulated the PMU (phasor measurement unit) communications in smart grids. Specifically, we simulated a 16-machine power system model and 8 PMUs were placed in the system. We simulated the happening of a fault. The 8 PMUs transmitted their sampled data to the data collector at 60 packets per second for each PMU. Each data packet consisted of the following sampling values: time, voltage magnitude, voltage angle, voltage frequency, and current of a neighboring line. Therefore we had altogether 12 bytes of data in a data packet. We used 16-bit A/D to quantify the data, and then converted the bit sequences into QPSK symbol sequences.

We considered UDP packet transmission, where in the head of UDP there were identical (but randomly generated) IP addresses as well as random fields. The packet head was also converted into QPSK symbols for modulation and transmission. Then, we added 20% training symbols into the sequence. With all these operations, we created symbol vectors \( a_i(m) \) with dimension \( N = 100 \) for each PMU \( i \) at each sampling time instant \( m \).

The sparsity was evaluated by calculating \( ||a_i(m) - a_i(m - 1)||_0/N \). The sparsity of the symbol sequence generated from the first PMU’s signals is shown in Fig. 1. We find that the maximum sparsity was less than 0.3, which was obtained when the fault occurred. The sparsity had an average value of 0.088 and standard deviation 0.03. We can see that the sparsity is well within the capability of our compressive sensing based transmission scheme.

Next, we simulated the transmission of the PMU signals using the channel of a conventional cell user. Due to the low duty cycle of the PMU data packets, for 8 PMUs, their transmission duration was only 24% of the transmission duration of the conventional user.

The detection BER is shown in Fig. 2. We can see that the compressive sensing based scheme successfully detected all the signals. The performance was only slightly worse than the idea case when each user or device fully occupied the channel exclusively. Note that we used one channel to support the transmissions of a conventional user and 8 PMU devices in this case.

### 5. CONCLUSIONS

In this paper we develop a compressive sensing based scheme for M2M devices to share the same spectrum with the conventional users so as to resolve the challenges involved in M2M communications. The redundancy in the transmitted signals is exploited to create the over-complete representation model and the sparse signal model. Compressive sensing techniques are then exploited for signal detection. Performance analysis and simulations are conducted to demonstrate this promising way of exploiting the inevitable signal redundancy to enhance the spectrum sharing efficiency.
6. REFERENCES


