Blind Source Separation Using Joint Canonical Decomposition of Two Higher Order Tensors

Mingjian Zhang*‡, Xiaohua Li†, and Jun Peng*
Email: mingjianzhang@163.com xli@binghamton.edu pengj@csu.edu.cn
*School of Information Science and Engineering, Central South University, Changsha, Hunan 410083, China
†Department of ECE, State University of New York at Binghamton, Binghamton, NY 13902, USA
‡Department of Information Technology, Hunan Police Academy, Changsha, Hunan 410138, China

***Abstract*—A two-stage-type algorithm is presented for blind source separation in the overdetermined instantaneous mixture case. The algorithm accomplishes two tasks: blind identification for estimating the mixing matrix and source estimation for recovering the original source signals with the identified mixing matrix. In this paper, we focus on the former task. A new mixing matrix identification method, which is based on the joint canonical decomposition of two third-order tensors, is proposed. Generalized singular value decomposition technique is used to perform the joint canonical decomposition. The merits of the proposed algorithm include the robustness to noise and superior performance compared with the classical blind source separation algorithms. Simulations for speech source separation are conducted to demonstrate the effectiveness of the proposed algorithm.**

I. Introduction

The problem of blind source separation (BSS) is concerned with recovery of a number of unknown original sources from sensor outputs without knowing any prior knowledge of the transmission channel [1]. Over the past two decades, a large number of papers have addressed BSS problem. BSS has become an intensively researched and extensively used technique for data analysis and signal processing. BSS has numerous applications in a wide range of fields including biomedical signal processing (MEG, EEG, ECG, EMG, fMRI) [2]–[5], speech enhancement [6]–[8], radar/sonar systems [9], wireless communications [10], image restoration [11], [12], hyperspectral image processing [13], etc. When the number of sensor signals is greater than or equal to that of sources, BSS is called overdetermined BSS (OBSS). In contrast, when the number of sensor signals is less than that of sources, BSS is called underdetermined BSS (UBSS).

BSS algorithms can be categorized into two different types of methods: direct signal separation and indirect signal recovery. Direct signal separation methods estimate demixing matrix directly, and then use it to separate mixed source signals. Some OBSS algorithms [11], [14], [15] belong to the class of direct signal separation algorithms, while some [16], [17] belong to the class of indirect signal separation algorithms. Indirect signal recovery methods consist of two cascaded steps: blind identification step for estimating mixing matrix and source estimation step for estimating source signals. If the mixing matrix is full column rank, it is easy to convert indirect signal recovery to direct signal separation because the demixing matrix can be obtained by performing pseudoinverse computation of the identified mixing matrix. For the UBSS case, the mixing matrix is not full column rank any longer. Hence, UBSS algorithms have to rely on indirect signal recovery method to fulfill the task of signal separation.

Therefore, the mixing matrix identification plays a crucial role in indirect signal recovery algorithms [17]–[19]. Taking into account this fact, this paper aims at developing a generalized singular value decomposition-based algorithm for sequential blind identification of overdetermined mixtures (GSVD-SBIOM). Different from the simultaneous matrix diagonalization (SMD)-based second-order blind identification algorithm [20], [21], the proposed algorithm estimates the columns of the mixing matrix in a sequential manner. The sequential-type algorithms rely on the use of deflation. As pointed out in [22]–[24], the conventional deflation procedure suffers from growing estimation errors during the successive deflation stages, which we term "error accumulation". To overcome this disadvantage, in this paper, we exploit the deflation technique proposed in [25], with which error accumulation can be avoided.

The canonical decomposition (CANDECOMP) [26]–[31], also known as parallel factor (PARAFAC) model [32]–[40], of a higher order tensor is a useful tool for multi-linear algebra. CANDECOMP aims at representing a general tensor as a sum of rank-1 tensors. A rank-1 matrix can be written as the outer product of two vectors. Similarly, a third-order rank-1 tensor can be written as the outer product of three vectors. Uniqueness in CANDECOMP/PARAFAC was addressed in [27], [39], [40].

The algebraic methods for blind identification of mixing matrix implicitly or explicitly exploit CANDECOMP of a higher order tensor containing either second-order or higher order cumulants of the data. CANDECOMP is an advantageous tool for blind identification problem because it can be used to solve the problem for both OBSS and UBSS cases [25]–[27]. In this paper, blind identification is performed based on the joint CANDECOMP of two higher order tensors rather than only a single tensor as in [27].

The rest of this paper is organized as follows. Section II formulates the problem of BSS. In Section III, we propose a new GSVD-SBIOM-based blind source separation algorithm for the overdetermined mixtures case. The performance of the proposed algorithm is demonstrated in Section IV. Finally, conclusions and future works are considered in Section V.

II. Problem Formulation

Consider the following instantaneous mixing model:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{J\times R}$ is an unknown full column rank mixing matrix, $\mathbf{x}(t) = [x_1(t), \ldots, x_J(t)]^T$ is a vector of observed sensor signals, $\mathbf{s}(t) = [s_1(t), \ldots, s_R(t)]^T$ is a vector of original source signals. The sensor signals are corrupted by additive noise $\mathbf{n}(t)$. Throughout this paper, the superscript $(\cdot)^T$ denotes the transpose. For the separability of the mixing system with the proposed algorithm, we make the following assumptions about the sources $\mathbf{s}(t)$ and noise $\mathbf{n}(t)$: 1) source signals are zero-mean, spatially uncorrelated but temporally correlated; 2) the additive noise $\mathbf{n}(t)$ is modeled as a temporally white, stationary, zero-mean random process, which is independent of the sources.

The covariance matrix of $\mathbf{x}(t)$ is defined as $\mathbf{C}_i = E\{\mathbf{x}(t)\,\mathbf{x}^T(t+\tau_i)\}$, where τ_i denotes a time lag. Because the additive noise is assumed to be temporally white, the noise vector for nonzero time lag τ_i has no influence on the covariance matrix $\mathbf{C}_i$. Hence, for $\tau_i \neq 0$, the covariance matrix $\mathbf{C}_i$ takes the form $\mathbf{C}_i = \mathbf{A}E\{\mathbf{s}(t)\mathbf{s}^T(t+\tau_i)\}\mathbf{A}^T$.

The eigenvalue decomposition (EVD) of the covariance matrix $\mathbf{C}_0 = E\{\mathbf{x}(t)\mathbf{x}^T(t)\}$ is performed as

$$\mathbf{C}_0 = [\mathbf{U}_s, \mathbf{U}_n]\begin{bmatrix}\mathbf{\Lambda}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{\Lambda}_n\end{bmatrix}[\mathbf{U}_s, \mathbf{U}_n]^T \tag{2}$$

where $\mathbf{U}_s$ is an eigenvector matrix whose column vectors corresponding to the R principal eigenvalues of $\mathbf{\Lambda}_s = \text{diag}[\lambda_1, \ldots, \lambda_R]$, and $\mathbf{U}_n$ is an eigenvector matrix whose column vectors corresponding to the $(J-R)$ smallest eigenvalues of $\mathbf{\Lambda}_n = \text{diag}[\lambda_{R+1}, \ldots, \lambda_J]$.

With the eigenvectors contained in the matrix $\mathbf{U}_s$, we perform a transformation of the observation signals as follows:

$$\bar{\mathbf{x}}(t) = \mathbf{U}_s^T\mathbf{x}(t) \tag{3}$$

The covariance matrix of $\bar{\mathbf{x}}(t)$ is then written as

$$\mathbf{R}_i = E\{\bar{\mathbf{x}}(t)\bar{\mathbf{x}}^T(t+\tau_i)\} = \mathbf{B}\mathbf{D}_i\mathbf{B}^T \tag{4}$$

where $\mathbf{B} = \mathbf{U}_s^T\mathbf{A} \in \mathbb{R}^{R\times R}$ is a square matrix. Since the source signals are assumed to be spatially uncorrelated, the matrix $\mathbf{D}_i = E\{\mathbf{s}(t)\mathbf{s}^T(t+\tau_i)\} = \text{diag}[d_{i1}, \ldots, d_{iR}]$ is diagonal.

In the two-stage-type BSS, the aim of the first stage is to identify $\mathbf{B}$, and the aim of the second stage is to recover source signals as $\hat{\mathbf{s}}(t) = \hat{\mathbf{B}}^{-1}\bar{\mathbf{x}}(t)$, where $\hat{\mathbf{B}}^{-1}$ denotes the inverse of the estimate $\hat{\mathbf{B}}$ of $\mathbf{B}$.

III. GSVD-SBIOM-Based BSS Algorithm

A. Two Third-Order Tensors and Their Equivalent Matrix Format

Stack the matrices $\mathbf{R}_1, \ldots, \mathbf{R}_K$ in a tensor $\mathcal{C}^{(1)} \in \mathbb{R}^{R\times R\times K}$, and $\mathbf{R}_{K+1}, \ldots, \mathbf{R}_{K+N}$ in a tensor $\mathcal{C}^{(2)} \in \mathbb{R}^{R\times R\times N}$, respectively, as follows: $(\mathcal{C}^{(1)})_{ijk} \overset{\text{def}}{=} (\mathbf{R}_k)_{ij}$ and $(\mathcal{C}^{(2)})_{ijn} \overset{\text{def}}{=} (\mathbf{R}_{K+n})_{ij}$, $i = 1, \ldots, R, j = 1, \ldots, R, k = 1, \ldots, K, n = 1, \ldots, N$.

Define two matrices $\tilde{\mathbf{D}} \in \mathbb{R}^{K\times R}$ and $\bar{\mathbf{D}} \in \mathbb{R}^{N\times R}$ by $(\tilde{\mathbf{D}})_{kr} \overset{\text{def}}{=} (\mathbf{D}_k)_{rr}$ and $(\bar{\mathbf{D}})_{nr} \overset{\text{def}}{=} (\mathbf{D}_{K+n})_{rr}$, respectively, $k = 1, \ldots, K, n = 1, \ldots, N, r = 1, \ldots, R$.

Then, we have [27]

$$\mathcal{C}^{(1)} = \sum_{r=1}^{R}\mathbf{b}_r \circ \mathbf{b}_r \circ \tilde{\mathbf{d}}_r \tag{5}$$

$$\mathcal{C}^{(2)} = \sum_{r=1}^{R}\mathbf{b}_r \circ \mathbf{b}_r \circ \bar{\mathbf{d}}_r \tag{6}$$

where $\circ$ denotes the tensor outer product and in which $\{\mathbf{b}_r\}$, $\{\tilde{\mathbf{d}}_r\}$, and $\{\bar{\mathbf{d}}_r\}$ are the columns of $\mathbf{B}$, $\tilde{\mathbf{D}}$, and $\bar{\mathbf{D}}$, respectively. The joint CANDECOMP of $\mathcal{C}^{(1)}$ and $\mathcal{C}^{(2)}$ described in (5) and (6) can be written in matrix format, which will be discussed in detail in the remaining part of this subsection.

Stack the entries of the tensor $\mathcal{C}^{(1)}$ in the matrix $\tilde{\mathbf{C}} \in \mathbb{R}^{RK\times R}$ as follows [1]:

$$(\tilde{\mathbf{C}})_{(j-1)K+k,i} = (\mathcal{C}^{(1)})_{ijk}$$
$$i = 1, \ldots, R, j = 1, \ldots, R, k = 1, \ldots, K$$

Similarly, stack the entries of the tensor $\mathcal{C}^{(2)}$ in the matrix $\bar{\mathbf{C}} \in \mathbb{R}^{NR\times R}$ as follows:

$$(\bar{\mathbf{C}})_{(n-1)J+i,j} = (\mathcal{C}^{(2)})_{ijn}$$
$$i = 1, \ldots, R, j = 1, \ldots, R, n = 1, \ldots, N$$

Then, (5) and (6) can be written in a matrix format as

$$\tilde{\mathbf{C}} = (\mathbf{B} \odot \tilde{\mathbf{D}})\mathbf{B}^T \tag{7}$$

$$\bar{\mathbf{C}} = (\bar{\mathbf{D}} \odot \mathbf{B})\mathbf{B}^T \tag{8}$$

in which $\odot$ denotes the Khatri-Rao product. As we can see, the tensors $\mathcal{C}^{(1)}$ and $\mathcal{C}^{(2)}$ bridge the set of matrices $\mathbf{R}_1, \ldots, \mathbf{R}_K, \mathbf{R}_{K+1}, \ldots, \mathbf{R}_{K+N}$ and the matrix pencil $(\tilde{\mathbf{C}}, \bar{\mathbf{C}})$. In the next subsection, by using the equivalent matrix format (7) and (8) of the third-order tensors $\mathcal{C}^{(1)}$ and $\mathcal{C}^{(2)}$, we perform their joint CANDECOMP based on GSVD technique.

B. GSVD-Based Blind Identification

The GSVD and its applications were introduced in [41]–[44]. We perform the reduced-size GSVD of the matrix pencil $(\tilde{\mathbf{C}}, \bar{\mathbf{C}})$ as follows

$$\tilde{\mathbf{C}} = (\mathbf{B} \odot \tilde{\mathbf{D}})\mathbf{B}^T = \mathbf{U}\tilde{\mathbf{\Sigma}}\mathbf{Q} \tag{9}$$

$$\bar{\mathbf{C}} = (\bar{\mathbf{D}} \odot \mathbf{B})\mathbf{B}^T = \mathbf{V}\bar{\mathbf{\Sigma}}\mathbf{Q} \tag{10}$$

where $\mathbf{U} \in \mathbb{R}^{RK\times R}$ and $\mathbf{V} \in \mathbb{R}^{NR\times R}$ are columnwise orthonormal matrices, $\tilde{\mathbf{\Sigma}} = \text{diag}[\tilde{\sigma}_1, \ldots, \tilde{\sigma}_R]$ and $\bar{\mathbf{\Sigma}} = \text{diag}[\bar{\sigma}_1, \ldots, \bar{\sigma}_R]$ are nonnegative diagonal matrices of the generalized singular values, and $\mathbf{Q} \in \mathbb{R}^{R\times R}$ is a nonsingular matrix.

From (9) and (10), we have

$$(\mathbf{B} \odot \tilde{\mathbf{D}}) = \mathbf{U}\tilde{\mathbf{\Sigma}}\mathbf{Q}(\mathbf{B}^T)^{-1} \tag{11}$$

$$(\bar{\mathbf{D}} \odot \mathbf{B}) = \mathbf{V}\bar{\mathbf{\Sigma}}\mathbf{Q}(\mathbf{B}^T)^{-1} \tag{12}$$

where $(\mathbf{B}^T)^{-1}$ denotes the inverse of $\mathbf{B}^T$. (11) and (12) can be further rewritten in the form

$$(\mathbf{B} \odot \tilde{\mathbf{D}}) = \mathbf{G}\mathbf{F} \tag{13}$$

$$(\bar{\mathbf{D}} \odot \mathbf{B}) = \mathbf{H}\mathbf{F} \tag{14}$$

where $\mathbf{G} = \mathbf{U}\tilde{\mathbf{\Sigma}} \in \mathbb{R}^{RK\times R}$, $\mathbf{H} = \mathbf{V}\bar{\mathbf{\Sigma}} \in \mathbb{R}^{NR\times R}$, and the square matrix $\mathbf{F} = \mathbf{Q}(\mathbf{B}^T)^{-1} \in \mathbb{R}^{R\times R}$.

According to the definition of Khatri-Rao product, (13) and (14) can be rewritten as

$$\left[\mathbf{b}_1 \otimes \tilde{\mathbf{d}}_1, \ldots, \mathbf{b}_R \otimes \tilde{\mathbf{d}}_R\right] = \mathbf{G}\mathbf{F} \tag{15}$$

$$\left[\bar{\mathbf{d}}_1 \otimes \mathbf{b}_1, \ldots, \bar{\mathbf{d}}_R \otimes \mathbf{b}_R\right] = \mathbf{H}\mathbf{F} \tag{16}$$

[1]In [27], the stacking methods for $\bar{\mathbf{C}} = (\mathbf{D} \odot \mathbf{A})\mathbf{A}^H$ in Equation (6) and $\bar{\bar{\mathbf{C}}} = (\mathbf{A}^* \odot \mathbf{D})\mathbf{A}^T$ in Equation (7) are incorrect. The correct expressions should be $(\bar{\mathbf{C}})_{(k-1)J+i,j} = (\bar{\bar{\mathbf{C}}})_{(j-1)K+k,i} = c_{ijk}$, $i = 1, \ldots, J, j = 1, \ldots, J, k = 1, \ldots, K$.

where $\otimes$ denotes the Kronecker product.

Then, the blind identification can be formulated as follows: given the matrices $\mathbf{G}$ and $\mathbf{H}$, it is desired to estimate the matrix $\mathbf{F}$ and then identify $\mathbf{B}$.

Define an operator unvec$(\cdot)$ by $\mathbf{M} = \text{unvec}(\mathbf{m}) \Leftrightarrow (\mathbf{M})_{ij} = (\mathbf{m})_{(i-1)J+j}$, which stacks an IJ-dimensional vector $\mathbf{m}$ in an $(I \times J)$ matrix $\mathbf{M}$. Then, we have

$$\begin{aligned} \text{unvec}(\mathbf{G}\mathbf{f}_k) &= \text{unvec}(\mathbf{b}_k \otimes \tilde{\mathbf{d}}_k) \\ &= \mathbf{b}_k \tilde{\mathbf{d}}_k^T, \quad k = 1, \ldots, R \end{aligned} \tag{17}$$

where $\mathbf{b}_k$ denotes the kth column of $\mathbf{B}$, $\tilde{\mathbf{d}}_k$ denotes the kth column of $\tilde{\mathbf{D}}$, and $\mathbf{f}_k$ denotes the kth column of $\mathbf{F}$.

Similarly, we have

$$\begin{aligned} \text{unvec}(\mathbf{H}\mathbf{f}_k) &= \text{unvec}(\bar{\mathbf{d}}_k \otimes \mathbf{b}_k) \\ &= \bar{\mathbf{d}}_k \mathbf{b}_k^T, \quad k = 1, \ldots, R \end{aligned} \tag{18}$$

where $\bar{\mathbf{d}}_k$ denotes the kth column of $\bar{\mathbf{D}}$, $\mathbf{b}_k$ denotes the kth column of $\mathbf{B}$, and $\mathbf{f}_k$ denotes the kth column of $\mathbf{F}$.

Remark: According to (17), $\mathbf{b}_k$ and $\tilde{\mathbf{d}}_k$ are the dominant left singular vector and the dominant right singular vector of the matrix unvec$(\mathbf{G}\mathbf{f}_k)$ up to scale factors, respectively. Similarly, according to (18), $\bar{\mathbf{d}}_k$ and $\mathbf{b}_k$ are the dominant left singular vector and the dominant right singular vector of the matrix unvec$(\mathbf{H}\mathbf{f}_k)$ up to scale factors, respectively. The aim of blind identification is to estimate $\{\mathbf{b}_k\}$ rather than $\{\tilde{\mathbf{d}}_k\}$ and $\{\bar{\mathbf{d}}_k\}$. Taking into account this fact, during the procedure for the joint decomposition of $\tilde{\mathbf{C}}$ in (9) and $\bar{\mathbf{C}}$ in (10), we do not need to estimate $\{\tilde{\mathbf{d}}_k\}$ and $\{\bar{\mathbf{d}}_k\}$ for the purpose of blind identification. In this sense, the blind identification stage only exploits partial result of the joint CANDECOMP of the tensors $\mathcal{C}^{(1)}$ in (5) and $\mathcal{C}^{(2)}$ in (6).

It is obvious to see that the columns of unvec$(\mathbf{G}\mathbf{f}_k)$ and the rows of unvec$(\mathbf{H}\mathbf{f}_k)$ are all proportional to the vector $\mathbf{b}_k$. In other words, the columns of unvec$(\mathbf{G}\mathbf{f}_k)$ and the rows of unvec$(\mathbf{H}\mathbf{f}_k)$ are proportional.

The columns of the matrix $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_K] \in \mathbb{R}^{R \times K}$ and the rows of the matrix $\mathbf{Y} = [\mathbf{y}_1^T, \ldots, \mathbf{y}_N^T]^T \in \mathbb{R}^{N \times R}$ are proportional if and only if the ith column of $\mathbf{X}$ and the jth row of $\mathbf{Y}$ satisfy $\gamma_{i,j}\mathbf{x}_i = \gamma_{j,i}\mathbf{y}_j^T$, $1 \le i \le K, 1 \le j \le N$, where $\gamma_{i,j}$ and $\gamma_{j,i}$ are two scalars. The condition $\gamma_{i,j}\mathbf{x}_i = \gamma_{j,i}\mathbf{y}_j^T$ is equivalent to

$$x_{si}y_{tj} - x_{ti}y_{sj} = 0, \quad 1 \le i \le K, 1 \le j \le N, 1 \le s < t \le R. \tag{19}$$

Define $\mathbf{G}_k = \text{unvec}(\mathbf{g}_k) \in \mathbb{R}^{R \times K}, k = 1, \ldots, R$, where $\mathbf{g}_k$ denotes the kth column of $\mathbf{G}$. From the set of matrices $\{\mathbf{G}_k\}$, we construct matrices $\mathbf{W}_i \in \mathbb{R}^{R \times R}$ as

$$\mathbf{W}_i = \left[\mathbf{g}_i^1, \ldots, \mathbf{g}_i^R\right], \quad i = 1, \ldots, K \tag{20}$$

where $\mathbf{g}_i^k$ denotes the ith column of $\mathbf{G}_k$.

Define $\mathbf{H}_k = \text{unvec}(\mathbf{h}_k) \in \mathbb{R}^{N \times R}, k = 1, \ldots, R$, where $\mathbf{h}_k$ denotes the kth column of $\mathbf{H}$. From the set of matrices $\{\mathbf{H}_k\}$, we construct matrices $\mathbf{T}_j \in \mathbb{R}^{R \times R}$ as

$$\mathbf{T}_j = \left[(\mathbf{h}_j^1)^T, \ldots, (\mathbf{h}_j^R)^T\right], \quad j = 1, \ldots, N \tag{21}$$

where $(\mathbf{h}_j^k)^T$ denotes the transpose of the jth row of $\mathbf{H}_k$.

Applying (19) to unvec$(\mathbf{G}\mathbf{f}_k)$ and unvec$(\mathbf{H}\mathbf{f}_k)$, we have

$$\begin{aligned} &\left(\sum_{l=1}^{R} (\mathbf{W}_i)_{sl}(\mathbf{F})_{lk}\right)\left(\sum_{l=1}^{R} (\mathbf{T}_j)_{tl}(\mathbf{F})_{lk}\right) \\ &\quad - \left(\sum_{l=1}^{R} (\mathbf{W}_i)_{tl}(\mathbf{F})_{lk}\right)\left(\sum_{l=1}^{R} (\mathbf{T}_j)_{sl}(\mathbf{F})_{lk}\right) = 0. \end{aligned} \tag{22}$$

We denote by $\mathbf{w}_q^p$ the qth row of $\mathbf{W}_p$, and by $\mathbf{t}_q^p$ the qth row of $\mathbf{T}_p$. Then, (22) can be rewritten in vector notations

$$\left(\mathbf{w}_s^i \mathbf{f}_k \mathbf{t}_t^j - \mathbf{w}_t^i \mathbf{f}_k \mathbf{t}_s^j\right)\mathbf{f}_k = 0 \tag{23}$$

or

$$\left(\mathbf{t}_t^j \mathbf{f}_k \mathbf{w}_s^i - \mathbf{t}_s^j \mathbf{f}_k \mathbf{w}_t^i\right)\mathbf{f}_k = 0. \tag{24}$$

(23) and (24) can be further rewritten in matrix-vector notations

$$\mathbf{P}_k\mathbf{f}_k = \mathbf{0}, \quad k = 1, \ldots, R \tag{25}$$

where $\mathbf{P}_k \in \mathbb{R}^{M \times R}$, $M = \frac{KNR(R-1)}{2}$, and $\mathbf{0}$ is a zero vector. The M rows of $\mathbf{P}_k$ have the form $\mathbf{w}_s^i\mathbf{f}_k\mathbf{t}_t^j - \mathbf{w}_t^i\mathbf{f}_k\mathbf{t}_s^j + \mathbf{t}_t^j\mathbf{f}_k\mathbf{w}_s^i - \mathbf{t}_s^j\mathbf{f}_k\mathbf{w}_t^i, 1 \le i \le K, 1 \le j \le N, 1 \le s < t \le R$. Because (25) provides a criterion for detecting whether the identification of $\mathbf{f}_k$ is achieved or not, we term it "identification detecting device". Note that (25) is the starting point of the update rule for estimating $\mathbf{f}_k$'s that will be derived in the next subsection.

C. Update Rule for GSVD-SBIOM

Let us consider the constrained optimization problem as in [25]:

$$\begin{aligned} &\text{minimize} \;\; \mathcal{J}(\mathbf{z}) = (\mathbf{P}\mathbf{z})^T\mathbf{P}\mathbf{z} = \mathbf{z}^T(\mathbf{P}^T\mathbf{P})\mathbf{z} \\ &\text{subject to the constraint} \;\; \|\mathbf{z}\|_F = 1 \end{aligned}$$

where $\mathbf{z} \in \mathbb{R}^{R \times 1}$ and $\|\cdot\|_F$ denotes the Frobenius norm. The M rows of $\mathbf{P}$ have the form $\mathbf{w}_s^i\mathbf{z}\mathbf{t}_t^j - \mathbf{w}_t^i\mathbf{z}\mathbf{t}_s^j + \mathbf{t}_t^j\mathbf{z}\mathbf{w}_s^i - \mathbf{t}_s^j\mathbf{z}\mathbf{w}_t^i, 1 \le i \le K, 1 \le j \le N, 1 \le s < t \le R$. According to the previous subsection, the columns $\mathbf{f}_k$'s of the matrix $\mathbf{F}$ are the optimal solutions to the constrained optimization problem. To search for an optimal solution, we update $\mathbf{z}$ according to the following iterative update rule [25]:

$$\mathbf{z} \leftarrow \mathbf{z} + \mathbf{v}, \quad \mathbf{z} \leftarrow \frac{\mathbf{z}}{\|\mathbf{z}\|_F} \tag{26}$$

where $\mathbf{v}$ is the eigenvector of the matrix $\mathbf{P}^T\mathbf{P}$ corresponding to the smallest eigenvalue. It is worth noting that the matrix $\mathbf{P}$ should be updated as well with $\mathbf{z}$ that is estimated at the previous iteration. After convergence, $\mathbf{z}$ is an estimate $\hat{\mathbf{f}}_1$ of a column of $\mathbf{F}$.

An estimated column $\hat{\mathbf{b}}_1$ of $\mathbf{B}$ can be computed as the dominant left singular vector of the matrix unvec$(\mathbf{G}\hat{\mathbf{f}}_1)$, which corresponds to the largest singular value. Let us denote by $\hat{\mathbf{f}}_l$ the lth estimated column of the matrix $\mathbf{F}$ and $\hat{\mathbf{b}}_l$ the lth identified column of the matrix $\mathbf{B}$. The sequential-type identification process of $\mathbf{B}$ can be formulated as: given $\hat{\mathbf{f}}_1, \ldots, \hat{\mathbf{f}}_l$, $\mathbf{G}$, and $\mathbf{H}$, it is desired to estimate $\hat{\mathbf{f}}_{l+1}$ and then identify $\hat{\mathbf{b}}_{l+1}$.

Next, we consider a variable λ as a function of $\mathbf{z}$

$$\lambda = \frac{\mathbf{z}^T\left(\mathbf{P}^T\mathbf{P}\right)\mathbf{z}}{\mathbf{z}^T\left(\mathbf{u}\mathbf{u}^T\right)\mathbf{z}} \tag{27}$$

where $\mathbf{u} \in \mathbb{R}^{R\times 1}$, which can be computed as in [25], is orthogonal to $\hat{\mathbf{f}}_1, \ldots, \hat{\mathbf{f}}_l$. As pointed out in [25], the minimization of (27) forces $\mathbf{z}$ to be different from $\hat{\mathbf{f}}_1, \ldots, \hat{\mathbf{f}}_l$. It is clear that λ is the generalized eigenvalue of the matrix pencil $(\mathbf{P}^T\mathbf{P}, \mathbf{u}\mathbf{u}^T)$. Therefore, the sequential-type identification process uses the iterative update rule described in (26), in which the vector $\mathbf{v}$ is chosen to be the generalized eigenvector of $(\mathbf{P}^T\mathbf{P},\mathbf{u}\mathbf{u}^T)$ corresponding to the generalized eigenvalue that has the smallest magnitude.

After the convergence of the iterative update rule in (26) is achieved, $\mathbf{z}$ is the $(l+1)$th estimated column $\hat{\mathbf{f}}_{l+1}$ of the matrix $\mathbf{F}$. $\hat{\mathbf{b}}_{l+1}$ can be computed as the dominant left singular vector of the matrix unvec$(\mathbf{G}\hat{\mathbf{f}}_{l+1})$. We can continue the sequential-type identification process until all the columns of $\mathbf{B}$ are identified.

D. Source Estimation

As a kind of indirect blind signal recovery-type algorithm, the aim of the second stage of the proposed algorithm is to estimate original source signals with the estimated mixing matrix. By using $\hat{\mathbf{B}}$, the original source signals can be estimated as

$$\hat{\mathbf{s}}(t) = \hat{\mathbf{B}}^{-1}\bar{\mathbf{x}}(t) \tag{28}$$

where $\hat{\mathbf{B}}^{-1}$ is the inverse of the estimated matrix $\hat{\mathbf{B}}$. It is worth noting that $\hat{\mathbf{s}}(t) = [\hat{s}_1(t), \ldots, \hat{s}_R(t)]^T$ is the estimated version of the original source signals $\mathbf{s}(t) = [s_1(t), \ldots, s_R(t)]^T$ up to permutation and amplitude scaling. For more precise source estimation, minimum mean-square error (MMSE)-based approach [45] can be used.

IV. Computer Simulations

In this section, we illustrate the performance of the proposed GSVD-SBIOM-based BSS algorithm and compare it with the FastICA [46] and JADE [47] algorithms by computer simulations. $R = 3$ speech source signals are considered, each is 1.25 seconds long (see Fig. 1). They are the truncated versions of the sound signals obtained from [48]. They are mixed with a randomly generated 5×3 mixing matrix

$$\mathbf{A} = \begin{bmatrix} -0.3852 & 0.0559 & 0.4951 \\ 0.1577 & -0.0742 & -1.5148 \\ 1.0646 & -0.5558 & 1.1651 \\ 0.3244 & 0.2339 & 0.5071 \\ -0.0560 & -1.7841 & -0.6436 \end{bmatrix}.$$

The observed mixtures of speech source signals (SNR = 30 dB) are shown in Fig. 2.

In simulations, total of $K+N = 20$ covariance matrices are generated with the time lags $\tau_i = 1, 2, \ldots, 20$. White noise is added to introduce signal-to-noise ratios (SNRs) varying from -5 dB to 30 dB. The signal-to-noise ratio is defined as SNR $= 10\log_{10}\frac{E\{\|\mathbf{A}\mathbf{s}(t)\|^2\}}{E\{\|\mathbf{n}(t)\|^2\}}$. The vector $\mathbf{z}$ in (26) is initialized with a column of $\hat{\mathbf{F}}_0$, where $\hat{\mathbf{F}}_0 = \mathbf{Q}(\hat{\mathbf{B}}_0^T)^{-1}$ and $\hat{\mathbf{B}}_0$ is roughly estimated by the generalized eigenvalue decomposition of the matrix pencil $(\mathbf{R}_1, \mathbf{R}_2)$. Fig. 3 shows the estimated speech source signals up to amplitude scaling and permutation.

To measure the performance of the algorithms, we use the

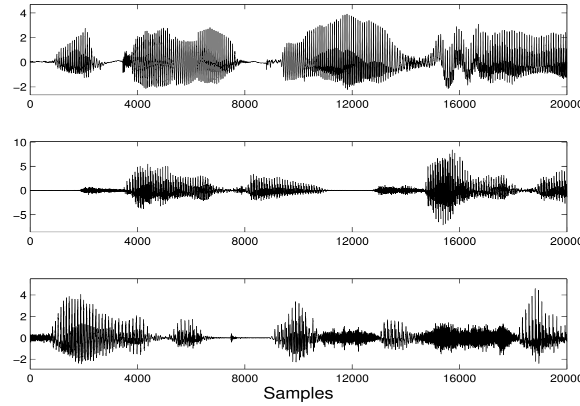

Fig. 1. Speech source signals.

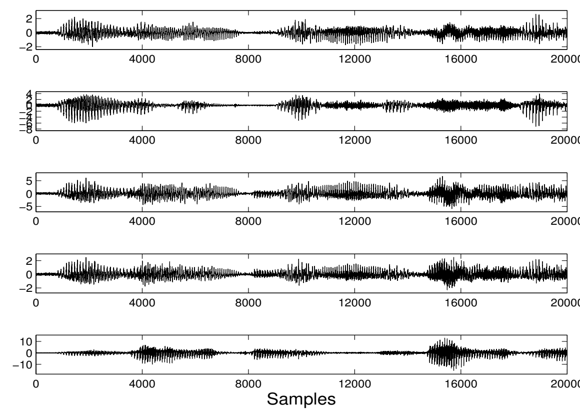

Fig. 2. Observed mixtures of speech source signals.

performance index [1]

$$\begin{aligned} \mathrm{PI} = \frac{1}{R(R-1)} \sum_{i=1}^{R} \Bigg\{ & \left(\sum_{k=1}^{R} \frac{|\hat{g}_{ik}|}{\max_j |\hat{g}_{ij}|} - 1 \right) \\ & + \left(\sum_{k=1}^{R} \frac{|\hat{g}_{ki}|}{\max_j |\hat{g}_{ji}|} - 1 \right) \Bigg\} \end{aligned} \tag{29}$$

where $\hat{g}_{ij}$ is the (i,j)-element of the estimated global mixing-separating matrix $\hat{\mathbf{G}} = \hat{\mathbf{B}}^{-1}\mathbf{U}_s^T\mathbf{A}$. The performance index (PI) measures to what extent the estimated global mixing-separating matrix is close to a generalized permutation matrix. Obviously, the smaller the value of PI, the better the source separation performance.

We used the MATLAB code from the website `http://www.cis.hut.fi/projects/ica/fastica/` for FastICA simulation. 100 independent runs are conducted for our proposed GSVD-SBIOM algorithm, FastICA, and JADE to calculate the average PI. Simulation results are illustrated in Fig. 4.

It can be seen that our proposed GSVD-SBIOM algorithm achieves better performance than FastICA and JADE. The performance of FastICA and JADE is much worse than the proposed algorithm, especially when the SNR is not too high (SNR $\in [-5,\ 10]$ dB). This means that our proposed GSVD-SBIOM algorithm is more robust to noise.

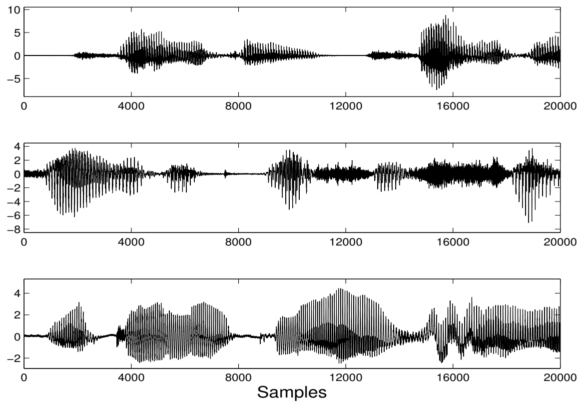

Fig. 3. Estimated speech source signals.

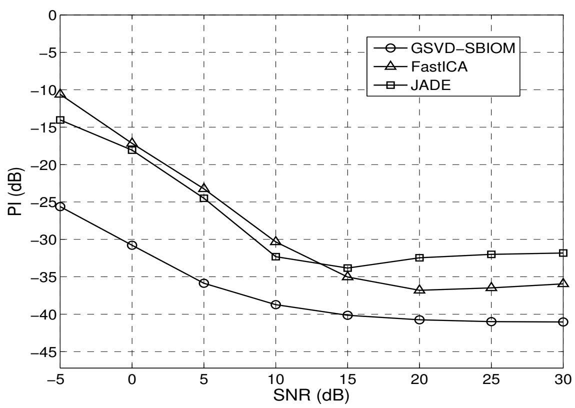

Fig. 4. Performance index (PI) versus SNR. $J = 5, R = 3, K + N = 20$.

V. Conclusion

We have developed a new two-stage-type algorithm based on the joint CANDECOMP of two higher order tensors for blind source separation from instantaneous mixtures. We use GSVD technique to perform the joint CANDECOMP. Using speech source separation as example, simulations have shown that our proposed algorithm has superior performance over two typical BSS algorithms FastICA and JADE. Future investigation may concern the application of the proposed algorithm to UBSS problem.

Acknowledgment

This work was supported in part by the National Science Foundation under Grant CNS-1443885, the National Natural Science Foundation of China under Grant 61471169, the National Social Science Foundation of China under Grant 15BYY097, the Humanities and Social Science Project of Ministry of Education of China under Grant 13YJCZH250, the Hunan Social Science Foundation under Grant 14YBA147, the Project of Hunan Province Department of Education under Grant 13C278, the Hunan Science and Technology Plan Project under Grant 2013GK2013, and the Teaching Reform Project of Hunan Province under Grant 2015(609).

References

[1] A. Cichocki and S. Amari, *Adaptive Blind Signal and Image Processing: Learning Algorithms and Applications*. New York: Wiley, 2002.

[2] V. Zarzoso and P. Comon, “Robust independent component analysis by iterative maximization of the kurtosis contrast with algebraic optimal step size,” *IEEE Trans. Neural Netw.*, vol. 21, no. 2, pp. 248–261, Feb. 2010.

[3] T. Puoliväli, F. Cong, V. Alluri, Q. H. Lin, P. Toiviainen, A. K. Nandi, E. Brattico, and T. Ristaniemi, “Semi-blind independent component analysis of functional MRI elicited by continuous listening to music,” in *Proc. 38th IEEE Int. Conf. Acoust. Speech Signal Process.*, May 2013, pp. 1310–1314.

[4] O. Debals, M. V. Barel, and L. D. Lathauwer, “Löwner-based blind signal separation of rational functions with applications,” *IEEE Trans. Signal Process.*, vol. 64, no. 8, pp. 1909–1918, Apr. 2016.

[5] R. Martín-Clemente, J. L. Camargo-Olivares, S. Hornillo-Mellado, M. Elena, and I. Román, “Fast technique for noninvasive fetal ECG extraction,” *IEEE Trans. Biomed. Eng.*, vol. 58, no. 2, pp. 227–230, Feb. 2011.

[6] A. Sarmiento, I. Durán-Díaz, A. Cichocki, and S. Cruces, “A contrast function based on generalized divergences for solving the permutation problem in convolved speech mixtures,” *IEEE/ACM Trans. Audio Speech, Lang. Process.*, vol. 23, no. 11, pp. 1713–1726, Nov. 2015.

[7] J. Traa, D. Wingate, N. D. Stein, and P. Smaragdis, “Robust source localization and enhancement with a probabilistic steered response power model,” *IEEE/ACM Trans. Audio Speech, Lang. Process.*, vol. 24, no. 3, pp. 493–503, Mar. 2016.

[8] X. Jaureguiberry, E. Vincent, and G. Richard, “Fusion methods for speech enhancement and audio source separation,” *IEEE/ACM Trans. Audio Speech, Lang. Process.*, vol. 24, no. 7, pp. 1266–1279, Jul. 2016.

[9] S. Redif, S. Weiss, and J. G. McWhirter, “Relevance of polynomial matrix decompositions to broadband blind signal separation,” *Signal Process.*, vol. 134, pp. 76–86, 2017.

[10] Y. Fadlallah, A. Aissa-El-Bey, K. Abed-Meraim, K. Amis, and R. Pyndiah, “Semi-blind source separation in a multi-user transmission system with interference alignment,” *IEEE Wireless Commun. Lett.*, vol. 2, no. 5, pp. 551–554, Oct. 2013.

[11] D. Liu, S. Hu, and H. Zhang, “Simultaneous blind separation of instantaneous mixtures with arbitrary rank,” *IEEE Trans. Circuits Syst. I: Regular Papers*, vol. 53, no. 10, pp. 2287–2298, Oct. 2006.

[12] L. Drumetz, M. A. Veganzones, S. Henrot, R. Phlypo, J. Chanussot, and C. Jutten, “Blind hyperspectral unmixing using an extended linear mixing model to address spectral variability,” *IEEE Trans. Image Process.*, vol. 25, no. 8, pp. 3890–3905, Aug. 2016.

[13] N. Falco, J. A. Benediktsson, and L. Bruzzone, “Spectral and spatial classification of hyperspectral images based on ICA and reduced morphological attribute profiles,” *IEEE Trans. Geosci. Remote Sens.*, vol. 53, no. 11, pp. 6223–6240, Nov. 2015.

[14] S. Dégerine and E. Kane, “A comparative study of approximate joint diagonalization algorithms for blind source separation in presence of additive noise,” *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 3022–3031, Jun. 2007.

[15] M. J. Zhang, “Blind separation of mixed-kurtosis signals using soft switch activation function,” *Electron. Lett.*, vol. 43, no. 17, pp. 954–955, Aug. 2007.

[16] A. Yeredor, “Non-orthogonal joint diagonalization in the least-squares sense with application in blind source separation,” *IEEE Trans. Signal Process.*, vol. 50, no. 7, pp. 1545–1553, Jul. 2002.

[17] S. Cruces, “Bounded component analysis of noisy underdetermined and overdetermined mixtures,” *IEEE Trans. Signal Process.*, vol. 63, no. 9, pp. 2279–2294, May 2015.

[18] S. Saito, K. Oishi, and T. Furukawa, “Convolutive blind source separation using an iterative least-squares algorithm for non-orthogonal approximate joint diagonalization,” *IEEE/ACM Trans. Audio Speech, Lang. Process.*, vol. 23, no. 12, pp. 2434–2448, Dec. 2015.

[19] Z. Koldovský, P. Tichavský, A. H. Phan, and A. Cichocki, “A two-stage MMSE beamformer for underdetermined signal separation,” *IEEE Signal Process. Lett.*, vol. 20, no. 12, pp. 1227–1230, Dec. 2013.

[20] A. Belouchrani, K. Abed-Meraim, J. F. Cardoso, and E. Moulines, “A blind source separation technique using second-order statistics,” *IEEE Trans. Signal Process.*, vol. 45, no. 2, pp. 434–444, Feb. 1997.

[21] G. Chabriel, J. Barrère, N. Thirion-Moreau, and E. Moreau, “Algebraic

joint zero-diagonalization and blind sources separation," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 980–989, Mar. 2008.

[22] E. Ollila, "The deflation-based FastICA estimator: Statistical analysis revisited," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1527–1541, Mar. 2010.

[23] S. C. Douglas, M. Gupta, H. Sawada, and S. Makino, "Spatio-temporal FastICA algorithms for the blind separation of convolutive mixtures," *IEEE Trans. Audio Speech, Lang. Process.*, vol. 15, no. 5, pp. 1511–1520, Jul. 2007.

[24] L. T. Duarte, B. Rivet, and C. Jutten, "Blind extraction of smooth signals based on a second-order frequency identification algorithm," *IEEE Signal Process. Lett.*, vol. 17, no. 1, pp. 79–82, Jan. 2010.

[25] M. Zhang, S. Yu, and G. Wei, "Sequential blind identification of underdetermined mixtures using a novel deflation scheme," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 24, no. 9, pp. 1503–1509, Sep. 2013.

[26] A. Karfoul, L. Albera, and G. Birot, "Blind underdetermined mixture identification by joint canonical decomposition of HO cumulants," *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 638–649, Feb. 2010.

[27] L. D. Lathauwer and J. Castaing, "Blind identification of underdetermined mixtures by simultaneous matrix diagonalization," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 1096–1105, Mar. 2008.

[28] P. Comon, "Tensors : A brief introduction," *IEEE Signal Process. Mag.*, vol. 31, no. 3, pp. 44–53, May 2014.

[29] S. Pouryazdian, S. Beheshti, and S. Krishnan, "CANDECOMP/PARAFAC model order selection based on reconstruction error in the presence of kronecker structured colored noise," *Digital Signal Process.*, vol. 48, pp. 12–26, 2016.

[30] A. H. Phan, P. Tichavský, and A. Cichocki, "Tensor deflation for CANDECOMP/PARAFAC–part I: Alternating subspace update algorithm," *IEEE Trans. Signal Process.*, vol. 63, no. 22, pp. 5924–5938, Nov. 2015.

[31] ——, "Tensor deflation for CANDECOMP/PARAFAC–part II: Initialization and error analysis," *IEEE Trans. Signal Process.*, vol. 63, no. 22, pp. 5939–5950, Nov. 2015.

[32] A. Sadhu, B. Hazra, and S. Narasimhan, "Decentralized modal identification of structures using parallel factor decomposition and sparse blind source separation," *Mech. Syst. Signal Process.*, vol. 41, no. 1-2, pp. 396–419, Dec. 2013.

[33] D. Nion and N. D. Sidiropoulos, "Adaptive algorithms to track the PARAFAC decomposition of a third-order tensor," *IEEE Trans. Signal Process.*, vol. 57, no. 6, pp. 2299–2310, Jun. 2009.

[34] T. Yokota, Q. Zhao, and A. Cichocki, "Smooth PARAFAC decomposition for tensor completion," *IEEE Trans. Signal Process.*, vol. 64, no. 20, pp. 5423–5436, Oct. 2016.

[35] P. R. Singh, Y. Wang, and P. Charg, "Bistatic MIMO radar for near field source localisation using PARAFAC," *Electron. Lett.*, vol. 52, no. 12, pp. 1060–1061, Jun. 2016.

[36] G. Tomasi and R. Bro, "A comparison of algorithms for fitting the PARAFAC model," *Comput. Stat. Data Anal.*, vol. 50, no. 7, pp. 1700–1734, Apr. 2006.

[37] L. R. Ximenes, G. Favier, and A. L. F. de Almeida, "Semi-blind receivers for non-regenerative cooperative MIMO communications based on nested PARAFAC modeling," *IEEE Trans. Signal Process.*, vol. 63, no. 18, pp. 4985–4998, Sep. 2015.

[38] R. Bro, "PARAFAC. tutorial and applications," *Chemometrics Intell. Lab. Syst.*, vol. 38, no. 2, pp. 149–171, 1997.

[39] A. Stegeman, J. M. F. T. Berge, and L. D. Lathauwer, "Sufficient conditions for uniqueness in Candecomp/Parafac and Indscal with random component matrices," *Psychometrika*, vol. 71, no. 2, pp. 219–229, 2006.

[40] J. M. F. ten Berge and N. D. Sidiropoulos, "On uniqueness in CANDECOMP/PARAFAC," *Psychometrika*, vol. 67, no. 3, pp. 399–409, 2002.

[41] C. F. V. Loan, "Generalizing the singular value decomposition," *SIAM J. Numer. Anal.*, vol. 13, no. 1, pp. 76–83, 1976.

[42] G. H. Ju and L. S. Lee, "A perceptually constrained GSVD-based approach for enhancing speech corrupted by colored noise," *IEEE Trans. Audio Speech, Lang. Process.*, vol. 15, no. 1, pp. 119–134, Jan. 2007.

[43] C. C. Paige and M. A. Saunders, "Towards a generalized singular value decomposition," *SIAM J. Numer. Anal.*, vol. 18, no. 3, pp. 398–405, 1981.

[44] P. Howland and H. Park, "Generalizing discriminant analysis using the generalized singular value decomposition," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 26, no. 8, pp. 995–1006, Aug. 2004.

[45] J. Cho, J. Choi, and C. D. Yoo, "Underdetermined convolutive blind source separation using a novel mixing matrix estimation and MMSE-based source estimation," in *Proc. 2011 IEEE Int. Workshop Mach. Learn. Signal Process.*, Sep. 2011, pp. 1–6.

[46] A. Hyvärinen, J. Karhunen, and E. Oja, *Independent Component Analysis*. New York: Wiley, 2001.

[47] J.-F. Cardoso and A. Souloumiac, "Blind beamforming for non-gaussian signals," *IEE Proceedings-F*, vol. 140, no. 6, pp. 362–370, Dec. 1993.

[48] N. D. Sidiropoulos, "A short list of sound sources," http://www.telecom.tuc.gr/∼nikos/BSS_Nikos.html.