Population Dynamic Human Behavioral Models for Smart Grid Demand Side Management

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***Abstract*—Demand side management (DSM) plays a critical role in scheduling and optimizing the energy consumption in the smart grid. Considering the critical yet complex human behavior issue, we develop new population dynamic models to describe the behavior of DSM users and use them to analyze the performance of DSM. We first introduce an accurate Markov model for the DSM population. We show that this model can be converted into a form similar to the popular SIR (susceptible-infected-recovered) model in mathematical biology. Then, we formulate a composite model that integrates the new DSM user behavioral model with a game theoretic DSM scheme. The convergence and the equilibrium of the composite model are studied both analytically and numerically. Experiments are conducted to determine the important model parameters.**

***Index Terms*—demand side management, smart grid, human behavior, Markov model, population dynamic model**

I. Introduction

Programs implemented by utility companies which aim to control energy consumption of their customers are known as demand-side management (DSM) [1]. DSM is becoming critical in today's smart grid because at the customer side there is an emergence of many heavy appliances such as plug-in hybrid vehicles (PHEVs) and large energy sources such as storage batteries and solar panels [2]. Their impact to the grid has to be mitigated by effective DSM. Fortunately, smart grid technologies such as smart meters and home energy management systems (HEMS) can greatly help the DSM deployment [3].

In conventional DSM, utility companies directly control their customer's appliances. In contrast, in smart grids autonomous DSM is more desirable since customers can determine their own demand and response. Game-theory has been an effective tool to help autonomous DSM realize promising objectives in cost minimization, peak-to-average power ratio reduction, grid reliability/efficiency improvement, etc [4]-[7].

Although DSM has been investigated extensively, a critical issue, i.e., the impact of human behavior, has not been addressed sufficiently. Since a DSM system consists of a large number of distributed users with different interests, the behavior of these users critically affects its performance [8]-[10]. In fact, applying game theory in DSM is a way to address the human behavior issue. It uses the rationality assumption to model the selfish and competitive nature of human behaviors. However, the rationality assumption also sets a severe limit to its effectiveness in DSM human behavior study [11][12].

In this paper, we develop new human behavioral models for DSM. Following [13]-[15], we set up a Markov model for DSM users and derive both linear and nonlinear population dynamic models to describe the DSM user behavior. Interestingly, the model we derived is similar to a biological model SISa (susceptible-infected-susceptible with autonomous infection) which was applied successfully in human behavior modeling and prediction [16][17]. The integration of this model with DSM schemes provides a better understanding of the practical performance of DSM.

The paper is organized as follows. In Section II, a smart grid DSM system is formulated. In Section III, new human behavioral models are developed. In Section IV, a composite DSM model is used to evaluate the DSM performance. Simulations are conducted in Section V and conclusions are presented in Section VI.

II. Smart grid DSM system

Consider a smart grid consisting of one energy source (utility company) and N energy users (customers). Each user has a smart meter with two-way communication capability and is equipped with HEMS to control household appliances. The energy source has a central controller to communicate with the users and to collect their energy usage data through the smart meters.

Let $\mathcal{N} = \{1, 2, \cdots, N\}$ be the user set. Each user $n \in \mathcal{N}$ has a number of household appliances $a \in \mathcal{A}_n$, where $\mathcal{A}_n$ denotes the set of all the appliances of the user n. Each appliance consumes energy $x_{n,a}(h)$ during time $h \in \mathcal{H}$, where $\mathcal{H} = \{1, 2, \cdots, H\}$ is the optimization time horizon. For example, one-day ahead energy consumption scheduling with hourly step-size means $\mathcal{H} = \{1, 2, \cdots, 24\}$. The energy usage (or load) of the whole system during time h is

$$L(h) = \sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} x_{n,a}(h). \tag{1}$$

Following [6], we define the cost of energy usage as $f_h(L(h))$, which is a function of the energy usage $L(h)$. Each function $f_h(\ell)$, $h \in \mathcal{H}$, is assumed convex with respect to ℓ

(the total energy usage) during the hour h. A special example is the monotonically increasing quadratic cost function

$$f_h(L(h)) = a_h L^2(h) + b_h L(h) + c_h, \tag{2}$$

where $a_h > 0$, $b_h \geq 0$ and $c_h \geq 0$ are time-dependent parameters.

The total energy usage of the whole system is

$$\sum_{h\in\mathcal{H}} L(h) = \sum_{h\in\mathcal{H}} \sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} x_{n,a}(h) \tag{3}$$

and the total cost of the system is

$$C(\mathbf{x}) = \sum_{h\in\mathcal{H}} f_h(L(h)) = \sum_{h\in\mathcal{H}} f_h \left(\sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} x_{n,a}(h) \right), \tag{4}$$

where

$$\mathbf{x} = \{\mathbf{x}_n | \forall n \in \mathcal{N}\}, \quad \mathbf{x}_n = \{x_{n,a}(h) \mid \forall a \in \mathcal{A}_n, \ h \in \mathcal{H}\}. \tag{5}$$

Besides the energy usage cost, we consider also the utility of the energy usage appliances. Each user $n \in \mathcal{N}$ has a number of different household appliances $a \in \mathcal{A}_n$. Each appliance, while consuming energy $x_{n,a}(h)$ during time h (with certain cost), brings utility which is characterized by a utility function $U_{n,a}(x_{n,a}(h))$. Utility functions for some major appliances are developed in [5], such as air conditioner, refrigerator, PHEV, clothes washer, dishwasher, lighting and entertainment. The total utility of the system is

$$U(\mathbf{x}) = \sum_{h\in\mathcal{H}} \sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} U_{n,a}(x_{n,a}(h)) \tag{6}$$

All the utility functions $U_{n,a}(x_{n,a}(h))$ are assumed concave over $x_{n,a}(h)$.

The DSM problem considered in this paper is to schedule and shift the energy usage profile $\mathbf{x}$ so as to maximize the social welfare

$$\begin{aligned} W(\mathbf{x}) \stackrel{\triangle}{=} U(\mathbf{x}) - C(\mathbf{x}) &= \sum_{h\in\mathcal{H}} \sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} U_{n,a}(x_{n,a}(h)) \\ &- \sum_{h\in\mathcal{H}} f_h \left(\sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} x_{n,a}(h) \right). \end{aligned} \tag{7}$$

Because $U(\mathbf{x})$ is concave of $x_{n,a}(h)$ and $C(\mathbf{x})$ is convex of $x_{n,a}(h)$, the social welfare $W(\mathbf{x})$ is concave with respect to $x_{n,a}(h)$.

Recall that each appliance a of the user n consumes a total energy $E_{n,a}$. We assume that it can work during time set $\mathcal{T}_{n,a} \subseteq \mathcal{H}$. During each time $h \in \mathcal{T}_{n,a}$, this appliance has an upper bound and a lower bound on its energy consumption, which are denoted as $\gamma_{n,a}^{\min}$ and $\gamma_{n,a}^{\max}$, respectively. This gives linear constraints on $\mathbf{x}$ as

$$\begin{cases} E_{n,a} = \sum_{h\in\mathcal{T}_{n,a}} x_{n,a}(h), & \\ x_{n,a}(h) = 0, & \text{if } \ h \notin \mathcal{T}_{n,a}, \\ \gamma_{n,a}^{\min} \leq x_{n,a}(h) \leq \gamma_{n,a}^{\max}, & \text{if } \ h \in \mathcal{T}_{n,a}. \end{cases} \tag{8}$$

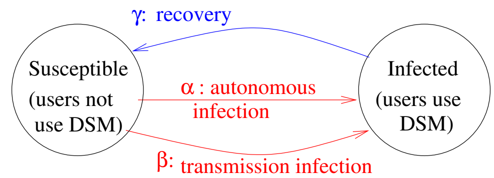

Fig. 1. A two-state Markov model for each DSM user.

Under the linear constraints (8), the social welfare $W(\mathbf{x})$ has a global maximum, which can be found in the convex optimization

$$\max_{\mathbf{x}} \ W(\mathbf{x}), \quad \text{s.t.,} \quad \mathbf{x}_n \in \mathcal{X}_n, \quad \forall\, n \in \mathcal{N}, \tag{9}$$

where $\mathcal{X}_n$ is the domain of the optimization variable $\mathbf{x}_n$ defined as

$$\mathcal{X}_n = \{\mathbf{x}_n | x_{n,a}(h) \ \text{ satisfies } \ (8) \ \text{ for } \ \forall \ \ a \in \mathcal{A}_n, h \in \mathcal{H}\}. \tag{10}$$

This problem can also be converted into decentralized optimization problems in more practical implementations [5][6].

III. Population dynamic human behavioral models for smart grid

A. An exact Markov model for DSM population

To model the complex human behavior with a tractable formulation, in this paper we adopt a population dynamic approach and consider the Markov model shown in Fig. 1 for each DSM user. We define two states: the *susceptible* state (i.e., not using the DSM scheme), and the *infected* state (i.e., using the DSM scheme). All users in the *susceptible* state form the set $\mathcal{S}$, while all the other users form the set $\mathcal{I}$. Thus, $\mathcal{S} \subseteq \mathcal{N}$, $\mathcal{I} \subseteq \mathcal{N}$, $\mathcal{S} \cap \mathcal{I} = \phi$, and $\mathcal{S} \cup \mathcal{I} = \mathcal{N}$.

From Fig. 1, a user in the set $\mathcal{S}$ can autonomously, or spontaneously, switch to the set $\mathcal{I}$, which means adopting the DSM scheme. This happens with probability α, where $0 \leq \alpha \leq 1$. For example, after a sustainability education, each user has probability α to adopt the DSM scheme. Each user in the set $\mathcal{I}$ has probability γ, where $0 \leq \gamma \leq 1$, to switch back to $\mathcal{S}$. For example, a user may find the DSM scheme inconvenient to use and thus abandon it.

Finally, the users in the set $\mathcal{I}$ may infect the users in the set $\mathcal{S}$. For example, a user may adopt the DSM scheme if some of his friends have adopted it. This happens with probability β, where $0 \leq \beta \leq 1$. It models the effect of social networking or mutual imitation among the DSM users. Note that the probability β is a networking-related parameter. In other words, a user n in the state $\mathcal{S}$ will remain in the state $\mathcal{S}$ with probability $(1-\beta)^{I_n}$, where I_n is the total number of users in the state $\mathcal{I}$ that have connection to the user n. It is also the number of neighbors of the user i. Obviously, the user n will switch to the state $\mathcal{I}$ with probability $1-(1-\beta)^{I_n}$.

To describe the state evolution, define the state of the user n as $\xi_n(t)$ at time t, where $n \in \mathcal{N}$. We let $\xi_n(t) = 0$ if the user n is in the set $\mathcal{S}$ and $\xi_n(t) = 1$ if the user n is in the set $\mathcal{I}$. The state of the overall DSM population is $\boldsymbol{\xi}(t) = (\xi_1(t), \cdots, \xi_N(t))$. There are 2^N states $\boldsymbol{\xi}(t) \in \{0,1\}^N$.

We use discrete Markov chains in this paper. The transitional probability between the states can be defined as

$$\mathbb{P}[\boldsymbol{\xi}(t+1) = \mathbf{y} | \boldsymbol{\xi}(t) = \mathbf{z}] = \prod_{n \in \mathcal{N}} \mathbb{P}[\xi_n(t+1) = y_n | \boldsymbol{\xi}(t) = \mathbf{z}], \tag{11}$$

where $\mathbf{y} = (y_1, \cdots, y_N)$ and $\mathbf{z} = (z_1, \cdots, z_N)$ are state values. Note that both y_n and z_n have values 0 or 1. Based on the transitional diagram of Fig. 1, we have

$$\begin{aligned}&\mathbb{P}[\xi_n(t+1) = y_n | \boldsymbol{\xi}(t) = \mathbf{z}] \\ &= \begin{cases} (1-\beta)^{I_n(t)}(1-\alpha), & \text{if } (z_n, y_n) = (0,0) \\ 1-(1-\beta)^{I_n(t)}(1-\alpha), & \text{if } (z_n, y_n) = (0,1) \\ \gamma, & \text{if } (z_n, y_n) = (1,0) \\ 1-\gamma, & \text{if } (z_n, y_n) = (1,1). \end{cases}\end{aligned} \tag{12}$$

Note that $I_n(t)$ is the number of neighboring users of the user n that are in the state $\mathcal{I}$ at time t.

Let $P_{\mathcal{S},n}(t)$ and $P_{\mathcal{I},n}(t)$ be the probabilities of the user n in the states $\mathcal{S}$ and $\mathcal{I}$ at time t, respectively. We have $P_{\mathcal{S},n}(t) = 1 - P_{\mathcal{I},n}(t)$ because there are two states only. The Markov state evolution is described by

$$\begin{aligned}&P_{\mathcal{S},n}(t+1) \\ &= P_{\mathcal{S},n}(t)\mathbb{P}[y_n = 0 | z_n = 0] + P_{\mathcal{I},n}(t)\mathbb{P}[y_n = 0 | z_n = 1] \\ &= (1-\beta)^{I_n(t)}(1-\alpha)P_{\mathcal{S},n}(t) + \gamma P_{\mathcal{I},n}(t),\end{aligned} \tag{13}$$

and

$$\begin{aligned}&P_{\mathcal{I},n}(t+1) \\ &= P_{\mathcal{I},n}(t)\mathbb{P}[y_n = 1 | z_n = 1] + P_{\mathcal{S},n}(t)\mathbb{P}[y_n = 1 | z_n = 0] \\ &= (1-\gamma)P_{\mathcal{I},n}(t) + \left[1 - (1-\beta)^{I_n(t)}(1-\alpha)\right] P_{\mathcal{S},n}(t).\end{aligned} \tag{14}$$

B. Nonlinear/linear models from mean-field approximation

The exact Markov model (13)(14) is computationally prohibitive for analysis. To simplify it, we consider the mean-field approximation, where the exact probabilities $P_{\mathcal{S},n}(t)$ and $P_{\mathcal{I},n}(t)$ are replaced by their mean-field approximations $\overline{P}_{\mathcal{S},n}(t)$ and $\overline{P}_{\mathcal{I},n}(t)$, respectively. In addition, $(1-\beta)^{I_n(t)}$ is approximated as $\prod_{j \in \mathcal{N}_n} [\overline{P}_{\mathcal{I},j}(t)(1-\beta) + (1 - \overline{P}_{\mathcal{I},j}(t))] = \prod_{j \in \mathcal{N}_n} (1 - \beta \overline{P}_{\mathcal{I},j}(t)) \approx 1 - \beta \sum_{j \in \mathcal{N}_n} \overline{P}_{\mathcal{I},j}(t)$, where $\mathcal{N}_n$ denotes the set of neighboring users of the user n. Then we have

$$\begin{aligned}&\overline{P}_{\mathcal{S},n}(t+1) \\ &= \left(1 - \beta \sum_{j \in \mathcal{N}_n} \overline{P}_{\mathcal{I},j}(t)\right)(1-\alpha)\overline{P}_{\mathcal{S},n}(t) + \gamma \overline{P}_{\mathcal{I},n}(t).\end{aligned} \tag{15}$$

Similarly, the mean-field approximation of (14) is

$$\begin{aligned}\overline{P}_{\mathcal{I},n}(t+1) &= (1-\gamma)\overline{P}_{\mathcal{I},n}(t) \\ &+ \left[1 - \left(1 - \beta \sum_{j \in \mathcal{N}_n} \overline{P}_{\mathcal{I},j}(t)\right)(1-\alpha)\right] \overline{P}_{\mathcal{S},n}(t).\end{aligned} \tag{16}$$

Put all the mean-field Markov state approximations into the $N \times 1$ vectors

$$\begin{cases} \overline{\mathbf{P}}_{\mathcal{S}}(t) \triangleq [\overline{P}_{\mathcal{S},1}(t), \cdots, \overline{P}_{\mathcal{S},N}(t)]^T \\ \overline{\mathbf{P}}_{\mathcal{I}}(t) \triangleq [\overline{P}_{\mathcal{I},1}(t), \cdots, \overline{P}_{\mathcal{I},N}(t)]^T \end{cases} \tag{17}$$

where $(\cdot)^T$ denotes matrix/vector transpose. Then (15)-(16) can be written in vector form as

$$\overline{\mathbf{P}}_{\mathcal{S}}(t+1) = (1-\alpha)\left(\mathbf{1} - \beta \mathbf{A} \overline{\mathbf{P}}_{\mathcal{I}}(t)\right) \odot \overline{\mathbf{P}}_{\mathcal{S}}(t) + \gamma \overline{\mathbf{P}}_{\mathcal{I}}(t), \tag{18}$$

$$\begin{aligned}\overline{\mathbf{P}}_{\mathcal{I}}(t+1) =& (1-\gamma)\overline{\mathbf{P}}_{\mathcal{I}}(t) \\ &+ \left[\mathbf{1} - (1-\alpha)\left(\mathbf{I} - \beta \mathbf{A} \overline{\mathbf{P}}_{\mathcal{I}}(t)\right)\right] \odot \overline{\mathbf{P}}_{\mathcal{S}}(t).\end{aligned} \tag{19}$$

where $\mathbf{A}$ is the $N \times N$ adjacency matrix of the DSM population, $\mathbf{1} = [1, \cdots, 1]^T$, and $\odot$ denotes direct element-wise multiplication.

To linearize the above nonlinear model, various techniques can be applied [13][14]. One of the ways is to simply discard the factor $\overline{P}_{\mathcal{S},n}(t)$ from (15), which gives

$$\widetilde{P}_{\mathcal{S},n}(t+1) = \gamma \widetilde{P}_{\mathcal{I},n}(t) + (1-\alpha)\left(1 - \beta \sum_{j \in \mathcal{N}_n} \widetilde{P}_{\mathcal{I},j}(t)\right). \tag{20}$$

Since $\overline{P}_{\mathcal{S},n}(t) \le 1$, we have $\overline{P}_{\mathcal{S},n}(t+1) < \widetilde{P}_{\mathcal{S},n}(t+1)$. Therefore, if $\widetilde{P}_{\mathcal{S},n}(t+1)$ converges, then the nonlinear model $\overline{P}_{\mathcal{S},n}(t+1)$ also converges. This in fact provides a valid way to analyze the convergence of the nonlinear model. Similarly, from (16) we can obtain

$$\begin{aligned}\widetilde{P}_{\mathcal{I},n}(t+1) &= (1-\gamma)\widetilde{P}_{\mathcal{I},n}(t) + \widetilde{P}_{\mathcal{S},n}(t) \\ &- (1-\alpha)\left(1 - \beta \sum_{j \in \mathcal{N}} \widetilde{P}_{\mathcal{I},j}(t)\right).\end{aligned} \tag{21}$$

Considering the vector $\widetilde{\mathbf{P}}_{\mathcal{S}}(t) = [\widetilde{P}_{\mathcal{S},1}(t), \cdots, \widetilde{P}_{\mathcal{S},N}(t)]^T$ and the fact that $\overline{\mathbf{P}}_{\mathcal{I}}(t) = \mathbf{1} - \overline{\mathbf{P}}_{\mathcal{S}}(t)$, from (20) we can get

$$\begin{aligned}\widetilde{\mathbf{P}}_{\mathcal{S}}(t+1) =& \left((1-\alpha)\beta \mathbf{A} - \gamma \mathbf{I}\right) \widetilde{P}_{\mathcal{S}}(t) \\ &+ \left(1 - \alpha + \gamma + (1-\alpha)\beta \mathbf{A}\right) \mathbf{1},\end{aligned} \tag{22}$$

where $\mathbf{I}$ is the $N \times N$ identity matrix. Therefore, if the magnitude of the maximum eigenvalue of the adjacency matrix $\mathbf{A}$ is less than $\frac{\gamma}{\beta(1-\alpha)}$, the linear model (20)(21) is convergent, which in turn means the nonlinear model (15) (16) is convergent as well.

C. A population dynamic model for homogeneous DSM

For further simplification, we consider a well-mixed homogeneous population where each user has equal probability to imitate any other user. Then we have $\overline{P}_{\mathcal{I},n}(t) = \overline{P}_{\mathcal{I}}(t)$, $\overline{P}_{\mathcal{S},n}(t) = \overline{P}_{\mathcal{S}}(t)$, and $\mathcal{N}_n \approx \mathcal{N}$. The nonlinear mean-field model (15) and (16) can be simplified to

$$\overline{P}_{\mathcal{S}}(t+1) = (1 - N\beta\overline{P}_{\mathcal{I}}(t))(1-\alpha)\overline{P}_{\mathcal{S}}(t) + \gamma\overline{P}_{\mathcal{I}}(t) \quad (23)$$

$$\overline{P}_{\mathcal{I}}(t+1) = (1-\gamma)\overline{P}_{\mathcal{I}}(t) + \alpha\overline{P}_{\mathcal{S}}(t) + (1-\alpha)N\beta\overline{P}_{\mathcal{I}}(t)\overline{P}_{\mathcal{S}}(t). \quad (24)$$

Since the average number of users in the sets $\mathcal{S}$ and $\mathcal{I}$ at time t are, respectively, $S(t) = N\overline{P}_{\mathcal{S}}(t)$ and $I(t) = N\overline{P}_{\mathcal{I}}(t)$, from (23) and (24) we can obtain a nonlinear time-difference equation based model

$$\begin{cases} S(t+1) - S(t) = -\beta(1-\alpha)S(t)I(t) + \gamma I(t) - \alpha S(t) \\ I(t+1) - I(t) = \beta(1-\alpha)S(t)I(t) - \gamma I(t) + \alpha S(t) \end{cases} \quad (25)$$

which describes the evolution of the DSM population. Note that $I(t) + S(t) = N$.

Interestingly, the model (25) is similar to the SISa (susceptible-infected-susceptible with autonomous) model of [16][17] in mathematical biology. The SISa model was based on the analogy from the well-known SIR (susceptible-infected-recovered) virus propagation model and was designed specifically for modeling the inter-personal propagation of human behaviors, states, ideas, emotions, etc. Its effectiveness has been validated by real data in [16] (modeling the spread of obesity) and in [17] (modeling the spread of emotions such as content).

Proposition 1. In the equilibrium of (25), the probability for a user to adopt the DSM scheme is

$$P_I = \frac{I(t)}{N} = \frac{1}{2}\left(1 - \frac{\alpha+\gamma}{(1-\alpha)\beta N} + \sqrt{\left(1 - \frac{\alpha+\gamma}{(1-\alpha)\beta N}\right)^2 + \frac{4\alpha}{(1-\alpha)\beta N}}\right). \quad (26)$$

Proof. The equilibrium is obtained when $S(t+1) - S(t) = I(t+1) - I(t) = 0$. Considering that $I(t) + S(t) = N$, at equilibrium, we have

$$(1-\alpha)\beta I(t)(N - I(t)) - \gamma I(t) + \alpha(N - I(t)) = 0. \quad (27)$$

Solving this equation for $I(t)$, we can get (26). □

As a market penetration problem, in order to get at least $I(t) \geq NP_I$ users to $\mathcal{I}$, from (26) we require

$$\gamma < \left((1-\alpha)\beta N + \frac{\alpha}{P_I}\right)(1 - P_I). \quad (28)$$

By improving DSM performance (i.e., maximizing social welfare $W(\mathbf{x})$), we can increase β and reduce γ for (28) to be satisfied.

IV. Integrating human behavior model with DSM

Let us consider the DSM optimization (9) first. Based on the SISa model (25), only the $I(t)$ users in the set $\mathcal{I}$ participate in the optimization. Therefore, the cost function (4) and utility function (6) are changed to

$$C(\mathbf{x}) = \sum_{h\in\mathcal{H}} f_h\left(\sum_{n\in\mathcal{I}}\sum_{a\in\mathcal{A}_n} x_{n,a}(h) + \sum_{m\in\mathcal{S}}\sum_{a\in\mathcal{A}_m} x_{m,a}(h)\right), \quad (29)$$

and

$$U(\mathbf{x}) = \sum_{h\in\mathcal{H}}\sum_{a\in\mathcal{A}_n}\left(\sum_{n\in\mathcal{I}} U_{n,a}(x_{n,a}(h)) + \sum_{m\in\mathcal{S}} U_{m,a}(x_{m,a}(h))\right) \quad (30)$$

If the central controller can still read the overall energy consumption data $L_m(h) = \sum_{a\in\mathcal{A}_m} x_{m,a}(h)$ of each user in $\mathcal{S}$ (those who do not participate in the DSM) through smart meters, it calculates $E_S(h) = \sum_{m\in\mathcal{S}} L_m(h)$ and apply the new cost and utility functions

$$C_1(\mathbf{x}_I) = \sum_{h\in\mathcal{H}} f_h\left(\sum_{n\in\mathcal{I}}\sum_{a\in\mathcal{A}_n} x_{n,a}(h) + E_S(h)\right), \quad (31)$$

$$U_1(\mathbf{x}_I) = \sum_{h\in\mathcal{H}}\sum_{a\in\mathcal{A}_n}\left(\sum_{n\in\mathcal{I}} U_{n,a}(x_{n,a}(h)) + \sum_{m\in\mathcal{S}} U_{m,a}(L_m(h))\right) \quad (32)$$

where $\mathbf{x}_I = \{\mathbf{x}_n | n \in \mathcal{I}\}$.

Otherwise, if the data $L_m(h)$ is not available (e.g., due to lack of smart meters), then the central controller has to skip $x_{m,a}(h)$. The cost and utility functions become

$$\tilde{C}_1(\mathbf{x}_I) = \sum_{h\in\mathcal{H}} f_h\left(\sum_{n\in\mathcal{I}}\sum_{a\in\mathcal{A}_n} x_{n,a}(h)\right), \quad (33)$$

$$\tilde{U}_1(\mathbf{x}_I) = \sum_{h\in\mathcal{H}}\sum_{n\in\mathcal{I}}\sum_{a\in\mathcal{A}_n} U_{n,a}(x_{n,a}(h)) \quad (34)$$

In both cases, we have constraint $\mathbf{x}_n \in \mathcal{X}_n, \ \forall n \in \mathcal{N}$. Now we can optimize the social welfare (9) with the new cost and utility functions, i.e.,

$$W_1(\mathbf{x}_I) = U_1(\mathbf{x}_I) - C_1(\mathbf{x}_I), \ \text{ or } \ \tilde{W}_1(\mathbf{x}_I) = \tilde{U}_1(\mathbf{x}_I) - \tilde{C}_1(\mathbf{x}_I). \quad (35)$$

We assume that the infection probability β and the recovery probability γ are functions of the social welfare $W_1(\mathbf{x}_I)$ or $\tilde{W}_1(\mathbf{x}_I)$. The rationale is that the probability for a user to adopt or abandon the DSM scheme depends on the social welfare of the scheme. The higher the social welfare, the smaller the probability γ and the higher the probability β. Some typical functions can be used, such as linear function $g(W) = \mu W + \eta$, exponential function $g(W) = \eta(1 - e^{-\mu W})$, or sigmoid/logistic function $g(W) = \eta/(1 + e^{-\mu(W - W_0)})$, with appropriate parameters.

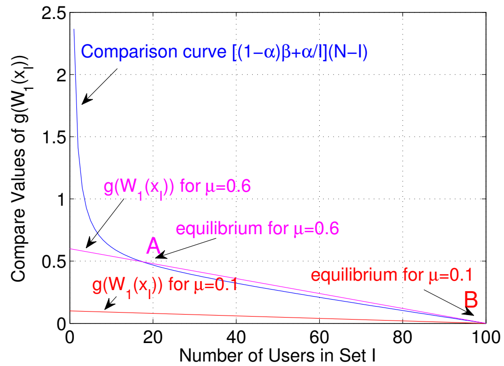

Fig. 2. Equilibrium analysis: Compare $\gamma = g(W_1(\mathbf{x}_I)) = \mu W_1(\mathbf{x}_I)$ with $[(1-\alpha)\beta + \alpha/I(t)](N - I(t))$. Note that $g(W_1(\mathbf{x}_I))$ may not be linear but is monotonically non-increasing in $I(t)$. $N = 100$.

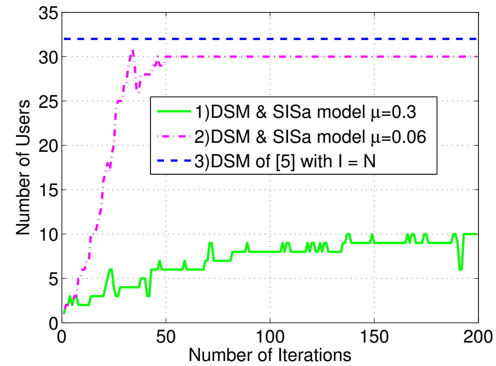

Fig. 3. Convergence in terms of number of users adopting DSM.

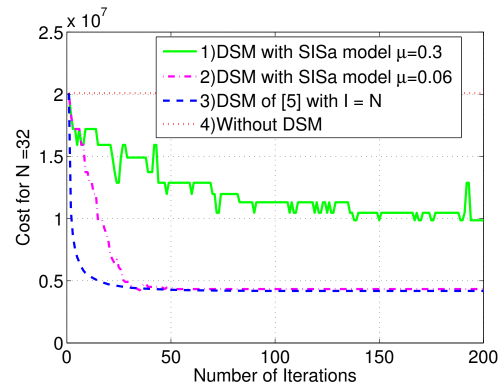

Fig. 4. Convergence in terms of energy usage cost.

Therefore, in our composite model, the new SISa model (25) affects the social welfare $W_1(\mathbf{x}_I)$ or $\tilde{W}_1(\mathbf{x}_I)$ through the user set $\mathcal{I}$, while the social welfare affects the SISa model through the parameters β and γ. Since the SISa model evolves at a much slower pace than the optimization (9), we assume that during each iteration of (25) we have time to finish a new optimization (9) and use the optimized social welfare $W_1(\mathbf{x}_I)$ or $\tilde{W}_1(\mathbf{x}_I)$ to update β and γ.

Proposition 2. Assume monotonically increasing cost functions $f_h(\ell)$. For any two subsets $\mathcal{I}_1 \subseteq \mathcal{N}$ and $\mathcal{I}_2 \subseteq \mathcal{N}$, if $\mathcal{I}_1 \subseteq \mathcal{I}_2$, then $W_1(\mathbf{x}_{I_1}) \leq W_1(\mathbf{x}_{I_2})$ and $\tilde{W}_1(\mathbf{x}_{I_1}) \leq \tilde{W}_1(\mathbf{x}_{I_2})$.

Proof. For cost minimization $C(\mathbf{x})$, since the cost functions $f_h(\ell)$, $h \in \mathcal{H}$, are monotonically increasing, the minimum values of both (31) and (33) are achieved at the same $\min_{\mathbf{x}_I} \sum_{n\in\mathcal{I}} \sum_{a\in\mathcal{A}_n} x_{n,a}(h)$. Therefore, $C_1(\mathbf{x}_I) = \tilde{C}_1(\mathbf{x}_I)$, which means the energy consumption data of the users in $\mathcal{S}$ can be safely omitted from the cost optimization. In addition, if $\mathcal{I}_1 \subseteq \mathcal{I}_2$, then

$$\sum_{m\in\mathcal{I}_2\setminus\mathcal{I}_1} \sum_{a\in\mathcal{A}_m} x_{m,a}(h) + \min_{\mathbf{x}_{I1}} \sum_{n\in\mathcal{I}_1} \sum_{a\in\mathcal{A}_n} x_{n,a}(h) \geq \min_{\mathbf{x}_{I2}} \sum_{n\in\mathcal{I}_2} \sum_{a\in\mathcal{A}_n} x_{n,a}(h) \qquad (36)$$

for all $h \in \mathcal{H}$. Hence $C_1(\mathbf{x}_{I_1}) \geq C_1(\mathbf{x}_{I_2})$. Similarly, for the utility maximization of $U(\mathbf{x})$, we can easily see that $U_1(\mathbf{x}_{I_1}) \leq U_1(\mathbf{x}_{I_2})$ and $\tilde{U}_1(\mathbf{x}_{I_1}) \leq \tilde{U}_1(\mathbf{x}_{I_2})$ because the overall utility is a summation of all the user's utilities. Combining the cost $C(\mathbf{x})$ and the utility $U(\mathbf{x})$ together, we can prove the proposition. □

The more users involved in the DSM, the better the performance. Higher market penetration is always better.

To analyze $I(t)$ of the set $\mathcal{I}$ at equilibrium, from (25)(28), the system can converge to the best case $\mathcal{I} = \mathcal{N}$ if $I(t+1) - I(t) > 0$ for any t, which means $\gamma < [(1-\alpha)\beta + \alpha/I(t)](N - I(t))$ for all possible sets $\mathcal{I} \subset \mathcal{N}$. If $\gamma > [(1-\alpha)\beta + \alpha/I(t)](N - I(t))$ for all $\mathcal{I} \subset \mathcal{N}$, then no user participates in the DSM and $\mathcal{I} = \phi$. The system may consist of a mixture of $\mathcal{I}$ and $\mathcal{S}$ users otherwise.

By comparing γ with $[(1-\alpha)\beta + \alpha/I(t)](N - I(t))$, we can analyze the convergence and equilibrium of the composite model. An example is illustrated in Fig. 2, where $\gamma = g(W_1(\mathbf{x}_I)) = \mu W_1(\mathbf{x}_I)$ with constant μ. Point B is a desirable convergent equilibrium since $\mathcal{I} = \mathcal{N}$. However, point A is also a convergent equilibrium for the case of $\mu = 0.6$. Depending on the parameters, the initial set $\mathcal{I}$ may converge to either A or B.

V. Simulations

First, we consider a simple system with 32 households. The parameters of the SISa model are $\beta = 0.005$, $\alpha = 0.019$, and $\gamma = \mu W/W_{\max}$ where $W_{\max}$ is the maximum social welfare.

To clearly illustrate the effects of human behavior on DSM, simulations are conducted for 4 different scenarios: 1) without using DSM or SISa; 2) DSM model of [5] without SISa ($\mathcal{I} = \mathcal{N}$); 3) DSM model of [5] integrated with SISa and a relatively high $\mu = 0.3$; and 4) DSM model of [5] integrated with SISa and a relatively low $\mu = 0.06$. The initial conditions of $\mathcal{I}$ for our composite models are set at $\mathcal{I} = \phi$.

Fig. 3 shows the evolution of the number of users in the set $\mathcal{I}$. Note that the equilibria fit well with the analysis results illustrated in Fig. 2. Electricity costs for all different scenarios are calculated and plotted in Fig. 4. The SISa parameters play a critical role on the convergence of the composite models and the market penetration of the DSM schemes.

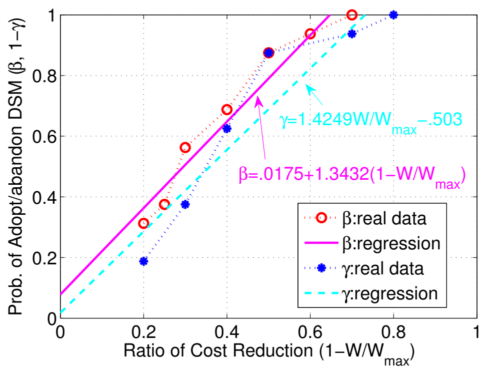

Fig. 5. Survey data and regression results give β and γ as functions of W/W_{max}.

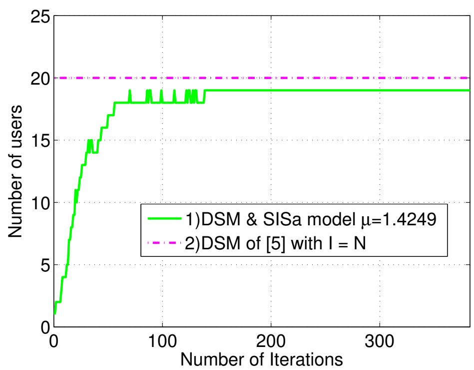

Fig. 6. Convergence in terms of number of users adopting DSM, with experiment data.

To determine the SISa model parameters in a more realistic setting, we did a survey experiment in a class of 20 students. Students were asked whether they would like to adopt the DSM, for what a cost change they would cancel the DSM, and for what a cost change they would recommend the DSM to friends. With the collected data, we determined $\alpha = 0.15$ based on the first question. The regression of the data of the second equation gave $\gamma = 1.4249W/W_{\max} - 0.503$, while the regression of the data of the third equation gave $\beta = 0.0175 + 1.3432(1 - W/W_{\max})$, see Fig. 5. We used these parameters in the simulation of a DSM system of 20 users. The simulation results are shown in Figs. 6 and 7. Similarly, we can see that the effectiveness of the DSM scheme is highly related to the human behavior.

VI. Conclusions

In this paper, we develop population dynamic human behavioral models for smart grid demand side management (DSM). A composite DSM model is presented by integrating a new SISa model with a game-theoretic DSM scheme. The convergence and the equilibria of the composite model are studied by analysis and simulation, which demonstrates the importance of addressing human behavior in DSM.

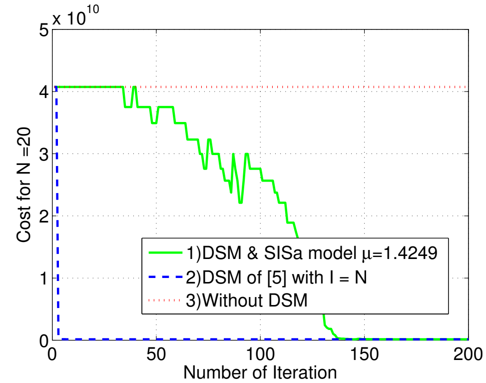

Fig. 7. Convergence in terms of energy usage cost, with experiment data.

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