Optimal relay selection for transmission rate maximisation in multi-hop wireless networks

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A fundamental problem for wireless networks is how to select relays from all available network nodes to realise the optimal multi-hop relaying between a source node and a destination node. Mutual interference among wireless nodes makes this problem challenging. A surprising result of the reported work is that interference-free multi-hop relaying can be achieved in full-duplex decode-and-forward relaying. The broadcast nature of wireless transmissions can be exploited without suffering from mutual interference. Then, an efficient relay selection algorithm is developed that finds the optimal hop count and all the relays to maximise the source-destination multi-hop transmission rate. The complexity of the algorithm is $O(N^2)$ only, where N is the number of available network nodes or network size. Interestingly, this wireless network algorithm is similar to the well-known Dijkstra's algorithm of wired networks. Simulations are conducted to demonstrate its optimality and efficiency.

Introduction: In networks, one of the basic problems is to select optimal relays to maximise the transmission rate between a source node and a destination node via multi-hop relaying. In wired networks, there are many well-known algorithms such as Dijkstra's algorithm [1] developed to solve this problem efficiently. In wireless networks, however, the problem becomes significantly more challenging because the broadcasting nature of wireless transmissions creates complex mutual interference among wireless nodes.

Theoretically, wireless network capacity and optimal multi-hop relay selection are described by exhaustive search over all possible node combinations [2, 3]. Unfortunately, exhaustive search has prohibitively high computational complexity. Specifically, the complexity is exponential in network size or the number of available network nodes. Owing to this complexity hurdle, most relay network capacity research is limited to very small networks with one or two hops [2]. Most multi-hop relaying study is limited to a few fixed relaying nodes only [4]. Optimal hop count and optimal multi-hop relay selection have been studied in [5, 6] but for a special linear network with fixed relaying nodes only. The exponentially complex relay selection problem is still an open challenge.

Noting the key issue of interference, we show in this Letter that interference-free multi-hop relaying can be realised by full-duplex decode-and-forward relays. This resolves fundamentally the mutual interference issue, and leads to efficient algorithms for optimal relay selection in arbitrarily large wireless networks.

Multi-hop relaying model: For an ad-hoc wireless network with $N+2$ nodes, we consider the problem of determining a multi-hop transmission path from a source node to a destination node. Let node 0 be the source node, node $N+1$ be the destination node, and the index set $\mathcal{N} = \{1, 2, \ldots, N\}$ denote all candidate relay nodes. We need to determine the optimal hop count (number of hops) $h+1$, where $0 \le h \le N$ and $h = 0$ means direct source-destination transmission without relaying. We also need to select a relay node r_j for each hop j, where $r_j \in \mathcal{N}$, $1 \le j \le h$, to maximise the source-destination multi-hop transmission rate. For notational simplicity, we define $r_0 \triangleq 0$ and $r_{h+1} \triangleq N+1$.

We consider causal full-duplex decode-and-forward relays. While a relay is receiving a packet, it can transmit simultaneously another packet that it has already decoded. This relay model is adopted widely in information theory research [2, 5]. Its practical implementation is also promising, as demonstrated by a number of research activities in recent years [6].

We adopt a slotted multi-hop packet forwarding scheme. During slot k, the source node r_0 encodes a packet $\boldsymbol{u}(k)$ into $\boldsymbol{u}_0(k)$ and transmits it. Each relay r_j receives (and decodes) a packet $\boldsymbol{u}(k{-}j{+}1)$ while transmitting simultaneously another packet $\boldsymbol{u}_j(k{-}j)$. The destination node r_{h+1} receives (and decodes) packet $\boldsymbol{u}(k{-}h)$. This procedure is illustrated in Fig. 1 (top). Each hop node r_j, $1 \le j \le h+1$, receives the summation of all transmitted signals

$$\boldsymbol{x}_j(k) = \sum_{i=0}^{h} \sqrt{P_{r_i} G_{r_i,r_j}}\, \mathrm{e}^{\mathbb{I}\theta_{r_i,r_j}} \boldsymbol{u}_i(k-i) + \boldsymbol{v}_j(k) \qquad (1)$$

where P_{r_i} is the transmission power of the node r_i, $\sqrt{G_{r_i,r_j}}\,\mathrm{e}^{\mathbb{I}\theta_{r_i,r_j}}$ is the instantaneous propagation channel coefficient from the transmitting node r_i to the receiving node r_j, $\mathbb{I} = \sqrt{-1}$, and $\boldsymbol{v}_j(k)$ is additive white Gaussian noise (AWGN). $\boldsymbol{x}_j(k)$, $\boldsymbol{u}_i(k)$ and $\boldsymbol{v}_j(k)$ are vectors containing all the samples in the slot k. The signals are illustrated in Fig. 1 (bottom).

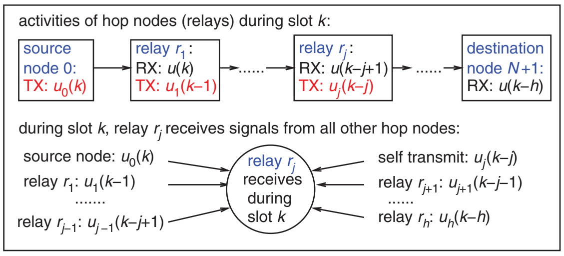

Fig. 1 *Transmission and receiving schedule of multi-hop relaying*

RX denotes receiving and decoding; TX denotes transmitting

We assume complex flat fading channels with gain $G_{i,j}$, zero-mean AWGN with power σ_j^2, and individual relay power limit $0 \le P_j \le P_j^{\max}$. All encoded packets $\boldsymbol{u}_j(t)$ have unit power.

Multi-hop transmission rate optimisation: Since the relay r_j has full knowledge of packets transmitted by itself and by relays in its following hops, it can subtract signals $\boldsymbol{u}_i(k{-}i)$, $i = j, j+1, \ldots, h$, from the mixture (1). The received signal (1) can thus be reduced to

$$\hat{\boldsymbol{x}}_j(k) = \sum_{i=0}^{j-1} \sqrt{P_{r_i} G_{r_i,r_j}}\, \mathrm{e}^{\mathbb{I}\theta_{r_i,r_j}} \boldsymbol{u}_i(k-i) + \boldsymbol{v}_j(k) \qquad (2)$$

To decode packet $\boldsymbol{u}(k{-}j{+}1)$ in slot k, it needs to detect the signal $\boldsymbol{u}_{j-1}(k{-}j{+}1)$ from (2) by treating all the other signal contents as interference. The signal-to-interference-plus-noise ratio (SINR) is

$$\gamma_j(k) = \frac{P_{r_{j-1}} G_{r_{j-1},r_j}}{\sum_{i=0}^{j-2} P_{r_i} G_{r_i,r_j} + \sigma_j^2} \qquad (3)$$

The achievable data rate is $\log_2[1+\gamma_j(k)]$. In (3) we see that there is no interference coming from r_j's following hop relays. However, there is interference coming from its preceding hop relays.

The key point for complete interference-free multi-hop relaying is to exploit the fact that the packet $\boldsymbol{u}(k{-}j{+}1)$ is not only contained in signal $\boldsymbol{u}_{j-1}(k{-}j{+}1)$. This packet has in fact been re-encoded into signals $\boldsymbol{u}_i(k{-}j{+}1)$ and transmitted in slots $k{-}j{+}1{+}i$ by the preceding relays r_i, respectively, for all $0 \le i \le j-1$. Therefore, to decode the packet $\boldsymbol{u}(k{-}j{+}1)$ in slot k, the optimal way for the relay r_j is to store and exploit all the j signals $\boldsymbol{x}_j(k{-}j{+}1{+}i)$ received in the past j slots $k{-}j{+}1{+}i$, $0 \le i \le j-1$, by a successive interference cancellation (SIC) procedure.

Specifically, before decoding the packet $\boldsymbol{u}(k{-}j{+}1)$ in slot k, the relay r_j has already decoded all packets $\boldsymbol{u}(t)$, $t \le k-j$. Subtracting signals related to these known packets, the signal received in slot $k{-}j{+}1{+}i$ is reduced to

$$\begin{aligned}\tilde{\boldsymbol{x}}_j(k-j+1+i) &= \sum_{\ell=0}^{i} \sqrt{P_{r_\ell} G_{r_\ell,r_j}}\, \mathrm{e}^{\mathbb{I}\theta_{r_\ell,r_j}} \boldsymbol{u}_\ell(k-j+1+i-\ell) \\ &\quad + \boldsymbol{v}_j(k-j+1+i), \quad 0 \le i \le j-1\end{aligned} \qquad (4)$$

For each i of (4), the relay r_j can detect the signal $\boldsymbol{u}_i(k{-}j{+}1)$, which is transmitted by the preceding relay r_i in slot $k{-}j{+}1{+}i$, with SINR

$$\gamma_j(k-j+1+i) = \frac{P_{r_i} G_{r_i,r_j}}{\sum_{\ell=0}^{i-1} P_{r_\ell} G_{r_\ell,r_j} + \sigma_j^2} \qquad (5)$$

The overall rate of r_j is thus

$$\begin{aligned}R_{r_j} &= \sum_{i=0}^{j-1} \log_2\left[1 + \gamma_j(k-j+1+i)\right] \\ &= \log_2\left(1 + \frac{\sum_{i=0}^{j-1} P_{r_i} G_{r_i,r_j}}{\sigma_j^2}\right)\end{aligned} \qquad (6)$$

After decoding the packet $\boldsymbol{u}(k{-}j{+}1)$, the relay r_j can subtract it from all its received signals (4) to prepare for the decoding of the next packet in the next slot. This SIC procedure is repeated by all relays in all slots.

The most interesting result is that there is no mutual interference left in the rate (6). In other words, multi-hop relaying becomes interference-free. What is more, each relay can collect the transmission power of all its preceding relays. This means a nice and surprising property: enjoying benefits of wireless broadcasting without suffering from interference.

The source-destination transmission rate is defined as $\min_{1\le j\le h+1} R_{r_j}$. The problem of hop count determination, relay node selection, and rate optimisation can be formulated as max-min optimisation

$$R = \max_{\substack{0\le h\le N \\ r_\ell \in \mathcal{N}, 1\le \ell \le h}} \min_{1\le j\le h+1} \log_2\left(1 + \frac{\sum_{i=0}^{j-1} P_{r_i} G_{r_i,r_j}}{\sigma_j^2}\right) \quad (7)$$

under node power constraint $0 \le P_j \le P_j^{\max}$, $0 \le j \le N$.

Efficient algorithm for multi-hop relay selection: To solve (7), rather than an exhaustive search over all possible h and relay combinations, more efficient algorithms can be developed. Since the rate R increases monotonically with relaying powers, each relay simply transmits at full power, that is, $P_{r_i} = P_{r_i}^{\max}$. A relay is not affected by the relays in its following hops. Instead, it only increases their rates. Based on these observations, we have the following efficient algorithm to solve (7).

Optimal multi-hop relay selection algorithm
for iteration j=1, 2, …, N, do
Update rates R_i for all remaining nodes $i \ne r_\ell$, $1 \le \ell \le j-1$. Select relay $r_j = \text{argmax}_{i\ne r_\ell} R_i$ for hop j. Update current multi-hop rate $R = \min_{1\le\ell\le j} R_{r_\ell}$. If r_j=N+1, then h=j−1, $R = \min\{R, R_{r_j}\}$, stop. If $R \le R_{N+1}$, then h=j, r_{h+1}=N+1, stop.

The algorithm begins with $r_0 = 0$. In each iteration j, we select, from all the remaining N–j+2 candidate nodes (include the destination node), a node with the highest rate as the relay r_j in hop j. Rates of the remaining candidate nodes are updated (calculated) based on (6) and relays selected for hops 1 to $j-1$. The algorithm stops with optimal hop count h, maximum rate R, and relay selections r_j, $1 \le j \le h$.

Proposition: The optimal multi-hop relay selection algorithm finds the optimal hop count h, selects relays r_j and maximises transmission rate R of (7) with computational complexity $O(N^2)$.

Proof: Let us consider the iteration j. The current multi-hop rate up to relay r_{j-1} is $R^{(j-1)} = \min_{1\le\ell\le j-1} R_{r_\ell} = \min_{1\le\ell\le j-1} \log_2\left[1 + \sigma_\ell^{-2} \sum_{i=0}^{\ell-1} P_{r_i} G_{r_i,r_\ell}\right]$. We need to select a relay r_j from the rest (N+1)–($j-1$) nodes. We first update their rates $R_i^{(j)}$ according to (6). If the node r_j has the maximum rate, then it should become the relay node in the hop j because $\min\{R^{(j-1)}, R_{r_j}^{(j)}\} \ge \min\{R^{(j-1)}, R_i^{(j)}\}$ for all i. This remains true for arbitrary relay selection patterns in subsequent iterations. Specifically, comparing the case of not selecting r_j as a relay in hop j to the case of selecting r_j, we can easily show that the former still has smaller rates for any relay selection pattern without r_j in subsequent hops. On the other hand, for a pattern that selects r_j being a relay in a subsequent hop $q > j$, we can get higher rates by moving r_j forward to the hop j.
If $r_j = N$+1 or $R^{(j)} \le R_{N+1}^{(j)}$, then no extra hop can further increase the multi-hop transmission rate.

As to computational complexity, in the worst case the algorithm runs N iterations. In each iteration j, it updates N–j+2 rates. Therefore it calculates a total of $\sum_{j=1}^{N} (N - j + 2) = (N^2 + 3N)/2$ rates, which has complexity $O(N^2)$ if implemented as iterative updating. □

For the simplest 3-node relay network ($N = 1$), it is easy to verify that this algorithm gives the same optimal decode-and-forward rate as the theoretical analysis result given in [2].

This wireless algorithm is essentially similar to the well-known Dijkstra's algorithm. The major difference lies in history dependence. Node rates are not fixed. Rather, they are changed by each new relay selected during each iteration.

Simulations: We simulated a wireless network whose nodes are placed randomly within a square of 1000 × 1000 m. We considered two scenarios: Rand (source and destination nodes are placed randomly) and fixed (source is in the original point and destination is in position (1000,1000)). The channel gain between two nodes with distance d_{ij} is $G_{i,j} = K d_{ij}^{-3}$. Parameters and transmission powers are normalised so that a transmission distance of 1000 m has a signal-to-noise ratio of 10 dB.

For each network size N, we generated 1000 random networks, ran our algorithm in each of them, and calculated the average multi-hop rate. We denote the result of our algorithm by 'New Alg', and compare it with the direct (no relay) transmission result ('Direct') and the brute-force exhaustive search result ('Exhaust'). Simulation results in Fig. 2 clearly show that the proposed algorithm gives the same result as the exhaustive search method. This demonstrates that our proposed algorithm is optimal. The proposed algorithm works efficiently for even extremely large networks.

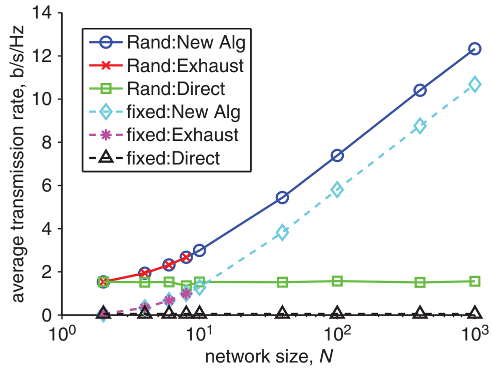

Fig. 2 *Average multi-hop transmission rate R against network size N*

Conclusion: In this Letter we first show that full-duplex decode-and-forward relaying with SIC can realise interference-free multi-hop relaying. Then we develop an efficient algorithm to find the optimal hop count and select relays to maximise the multi-hop transmission rate. The new algorithm is similar to the Dijkstra's algorithm, and is useful for exploring large wireless networks.

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One or more of the Figures in this Letter are available in colour online.

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