Interference Immune Multi-hop Relaying and Efficient Relay Selection Algorithm for Arbitrarily Large Half-Duplex Gaussian Wireless Networks

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***Abstract*—It has been a challenge to develop efficient algorithms for relay selection and relay power optimization in arbitrarily large multi-hop wireless networks. In this paper, considering Gaussian networks with half-duplex decode-and-forward relays, we develop a practical network-wide signal processing procedure with which the relay's data rate is not degraded by mutual interference. Multi-hop relaying is thus immune to mutual interference. Then, we develop an algorithm to find approximately the optimal hop count and the optimal relays for source-destination transmission rate maximization. With a quadratic complexity $O(N^2)$, where N is the network size, this algorithm is efficient for arbitrarily large wireless networks. More interestingly, this algorithm is similar to the well-known Dijkstra's algorithm of wired networks.**

Keywords—successive interference cancellation, multi-hop relay, wireless networks, signal to noise and interference ratio, algorithm

I. INTRODUCTION

For wireless ad-hoc networks consisting of an arbitrary number of wireless nodes, a basic problem, i.e., selecting relays to construct multi-hop relaying path to maximize source-destination transmission rate, is a long-standing open problem. In wired networks, sophisticated algorithms such as Dijkstra's algorithm can be used to solve this basic problem [1]. In wireless networks, however, this basic multi-hop relay selection problem is difficult because broadcasting nature of wireless transmissions creates complex mutual interference among wireless nodes.

Theoretically, multi-hop relay selection can be formulated as an exhaustive search optimization over all possible node combinations [2]-[4]. Unfortunately, exhaustive search has prohibitively high computational complexity. Due to this complexity hurdle, most research of relay selection and relay power optimization, conducted in the fields of cooperative communications and relay networks, is limited to small networks with one or two hops, or with a few nodes only [5]. Similarly, most multi-hop relay research is based on fixed multi-hop relay structures without the flexibility of selecting relays from all available network nodes [6]-[8].

Multi-hop relaying is important for many conventional applications such as wireless ad hoc networks and for many emerging applications such as vehicular networks and Google's balloon networks (where one of the major hurdles is the capacity of the backbone multi-hop relaying). Existing works show that multi-hop capacity reduces rapidly with the number of hops due to mutual interference [9]. Fortunately, fundamental results in network information theory indicate that the rate of multi-hop relaying with decode-and-forward relays is not affected by the mutual interference among the relays [10][3]. Nevertheless, such results were derived under some ideal rather than practical assumptions such as full-duplex relaying, asymptotic equipartition property (AEP) and infinitely long random codes. Moreover, no efficient algorithm is developed for relay selection to take this benefit.

The major contribution of this paper is thus to develop practical signal processing algorithms to realize interference-immune multi-hop relaying, with half-duplex relays and practical coding techniques. In addition, based on the interference-immune phenomenon, we develop efficient algorithms for optimal relay selection in arbitrarily large wireless networks.

The organization of this paper is as follows. In Section II, we give the multi-hop wireless network model with half-duplex decode-and-forward relays. In Section III, we develop techniques for interference immune multi-hop relaying and develop an efficient algorithm for multi-hop relay selection. Simulations are conducted in Section IV. Conclusion is then given in Section V.

II. MULTI-HOP WIRELESS NETWORK MODEL

In a wireless network with $N + 2$ nodes, we need to construct a multi-hop transmission path to forward data packets from a source node to a destination node. Without loss of generality, denote the source node as node 0, and the destination node as node $N + 1$. All the other N nodes are candidates for relay selection. The multi-hop relay selection problem considered in this paper is to determine the optimal number of hops (hop count), to select optimally a relay for each hop, and to determine the relay's transmission power so as to maximize the source-destination transmission rate.

Let the index set $\mathcal{N} = \{1, 2, \cdots, N\}$ denote all the N candidate relay nodes. Let the hop count be $h+1$, where $0 \leq h \leq N$ and $h = 0$ means direct source-destination transmission without relaying. As shown in Fig. 1, we define the relay node in hop i as r_i, where $r_i \in \mathcal{N}$ for all $1 \leq i \leq h$. For notational simplicity, we define $r_0 \triangleq 0$ and $r_{h+1} \triangleq N + 1$.

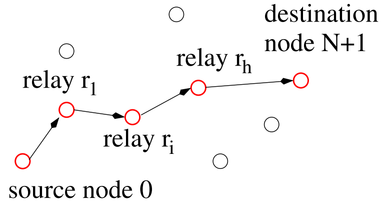
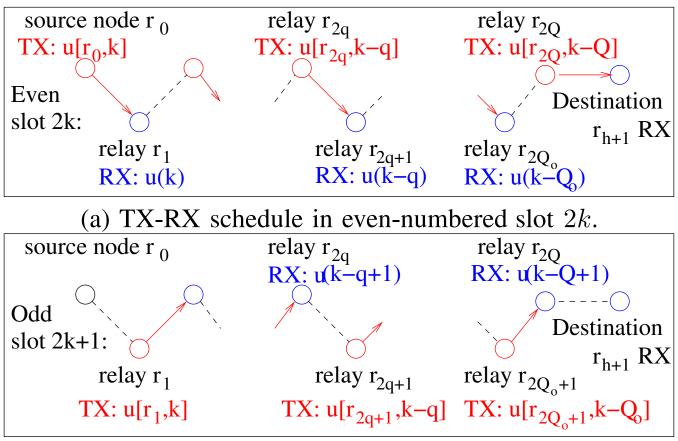


Fig. 1. A multi-hop wireless network with $h+1$ hops from the source node 0 to the destination node $N+1$. A node $r_i \in \mathcal{N}$ is selected from all the N network nodes as relay in hop i, $1 \leq i \leq h$.



(a) TX-RX schedule in even-numbered slot $2k$.

(b) TX-RX schedule in odd-numbered slot $2k+1$.

Fig. 2. Transmission and receiving schedule of half-duplex multi-hop relaying. Only even hop count h is shown. Multi-hop with odd hop count h is similar.

We consider causal half-duplex decode-and-forward relays in this paper. A relay works either in the receiving (RX) state or the transmitting (TX) state. In the RX state, the relay receives and decodes packets, during which it can not transmit. In the TX states, the relay can transmit only those packets that it has already decoded during previous RX states. The data transmission/receiving rate of each node, or of the overall multi-hop path, is called decode-and-forward rate since other relaying strategies, such as amplify-and-forward [5], are not considered in this paper.

We adopt a slotted half-duplex multi-hop packet forwarding scheme. In general, we let even-numbered relays transmit in even-numbered slots, and odd-numbered relays transmit in odd-numbered slots. Define $Q = \lfloor \frac{h}{2} \rfloor$, where $\lfloor x \rfloor$ is the maximum integer not larger than x. As shown in Fig. 2(a), during an even-numbered slot $2k$, $k = 0, 1, \cdots$, each even-numbered relay r_{2q}, $q = 0, 1, \cdots, Q$, re-encodes a packet $\mathbf{u}(k-q)$ into a signal $\mathbf{u}[r_{2q}, k-q]$ and transmits this signal. For example, the source node r_0 re-encodes the packet $\mathbf{u}(k)$ into $\mathbf{u}[r_0, k]$ and transmits the signal $\mathbf{u}[r_0, k]$ in the slot $2k$. During this slot, each odd-numbered relay r_{2q+1} receives (and decodes) a packet $\mathbf{u}(k-q)$, where $q = 0, 1, \cdots, Q_o$, and

$$Q_o = \begin{cases} Q, & \text{if } h \text{ is } odd; \\ Q-1, & \text{if } h \text{ is } even. \end{cases} \tag{1}$$

Next, as shown in Fig. 2(b), during the odd-numbered slot $2k+1$, each odd-numbered relay r_{2q+1}, $0 \leq q \leq Q_o$, re-encodes a packet $\mathbf{u}(k-q)$ into signal $\mathbf{u}[r_{2q+1}, k-q]$ and transmits this signal. Each even-numbered relay r_{2q}, $0 \leq q \leq Q$, receives and decodes a packet $\mathbf{u}(k-q+1)$. We assume that the source node r_0 does not transmit in this slot.

We assume that the destination node r_{h+1} can receive signals in both even-numbered and odd-numbered slots, and use them to decode packets. For example, it receives and decodes the packet $\mathbf{u}(k-Q)$ during two slots $2k-1$ and $2k$ in Fig. 2. Note that it can use its signals received in other slots as well to help decode this packet.

Due to the broadcasting nature of wireless transmissions, each hop node receives the summation of the signals transmitted from all the transmitting relays. Consider the even-numbered slot $2k$ first, where all even-numbered relays r_{2q} conduct transmission, while all odd-numbered relays r_{2q+1} conduct reception. The signal received by the relay r_{2q+1} is

$$\mathbf{x}[r_{2q+1}, 2k] = \sum_{i=0}^{Q} \sqrt{P(r_{2i}) G(r_{2i}, r_{2q+1})} e^{j\theta(r_{2i}, r_{2q+1})} \times \mathbf{u}[r_{2i}, k-i] + \mathbf{v}[r_{2q+1}, 2k], \tag{2}$$

where $P(r_{2i})$ is the transmission power of the node r_{2i}, $\sqrt{G(r_{2i}, r_{2q+1})} e^{j\theta(r_{2i}, r_{2q+1})}$ is the instantaneous propagation channel coefficient from the transmitting node r_{2i} to the receiving node r_{2q+1}, $j = \sqrt{-1}$, and $\mathbf{v}[r_{2q+1}, 2k]$ is additive white Gaussian noise (AWGN). Note that $\mathbf{x}[r_{2q+1}, 2k]$, $\mathbf{u}[r_{2i}, k]$ and $\mathbf{v}[r_{2q+1}, 2k]$ are vectors containing all the samples in the slot $2k$.

Similarly, during the odd-numbered slot $2k+1$, all odd-numbered relays r_{2q+1} conduct transmission, while all even-numbered relays r_{2q} conduct reception. Specifically, the relay r_{2q}, $0 \leq q \leq Q$, receives signal

$$\mathbf{x}[r_{2q}, 2k+1] = \sum_{i=0}^{Q_o} \sqrt{P(r_{2i+1}) G(r_{2i+1}, r_{2q})} e^{j\theta(r_{2i+1}, r_{2q})} \times \mathbf{u}[r_{2i+1}, k-i] + \mathbf{v}[r_{2q}, 2k+1]. \tag{3}$$

Based on (2) and (3), the destination node r_{h+1} receives signals $\mathbf{x}[r_{h+1}, 2k]$ and $\mathbf{x}[r_{h+1}, 2k+1]$ during slots $2k$ and $2k+1$, respectively.

Define the receiving/transmission data rate of each relay r_i as $R(r_i)$, and the source-destination transmission rate as

$$R = \min_{1 \leq i \leq h+1} R(r_i). \tag{4}$$

The multi-hop relay selection problem considered in this paper is to find the optimal h and r_i, $1 \leq i \leq h$, so as to maximize R.

We assume complex flat fading channels with gain $G(i,j)$ from node i to node j, zero-mean AWGN with power $\sigma^2(i)$ for node i in all slots, and individual relay power limit $0 \leq P(i) \leq P^{\max}(i)$. All re-encoded signals $\mathbf{u}[i,t]$ have unit power. We also assume that all channel coefficients and re-encoding rules are public knowledge.

III. Interference Immune Multi-hop Relaying and Relay Selection

A. *Interference immune phenomenon in multi-hop relaying*

Let us derive the rate expression for the odd-numbered relay r_{2q+1} first. This relay node receives signal $\mathbf{x}[r_{2q+1}, 2k]$ in even-numbered slot $2k$, as shown in (2) and Fig. 2(a). Because the relay r_{2q+1} has full knowledge of packets transmitted by relays in its subsequent hops, it can subtract signals $\mathbf{u}[r_{2i}, k-i]$ for all $i = q+1, q+2, \cdots, Q$ from the mixture (2). The received signal (2) can thus be reduced to

$$\hat{\mathbf{x}}[r_{2q+1}, 2k] = \sum_{i=0}^{q} \sqrt{P(r_{2i})G(r_{2i}, r_{2q+1})} e^{j\theta(r_{2i}, r_{2q+1})} \times \mathbf{u}[r_{2i}, k-i] + \mathbf{v}[r_{2q+1}, 2k]. \quad (5)$$

In this slot, the relay r_{2q+1} needs to decode the packet $\mathbf{u}(k-q)$ to prepare for the transmission of it in the next slot. This means that it needs to detect the signal $\mathbf{u}[r_{2q}, k-q]$ from (5). Treating all the other signal contents as interference, the signal-to-interference-plus-noise ratio (SINR) for the relay r_{2q+1} to detect signal $\mathbf{u}[r_{2q}, k-q]$ with (5) is

$$\Gamma(r_{2q+1}, r_{2q}) = \frac{P(r_{2q})G(r_{2q}, r_{2q+1})}{\sum_{i=0}^{q-1} P(r_{2i})G(r_{2i}, r_{2q+1}) + \sigma^2(r_{2q+1})}. \quad (6)$$

The achievable data rate is $0.5 \log_2(1 + \Gamma(r_{2q+1}, r_{2q}))$, where the factor 0.5 is due to the half duplexity.

In (6), we see that there is no mutual interference caused by the relays in the subsequent hops. But there is still mutual interference coming from the relays in the preceding hops. Fortunately, such mutual interference can be compensated for if we exploit the characteristics of multi-hop relaying: a packet is transmitted repeatedly by multiple nodes in multiple slots.

Specifically, the packet $\mathbf{u}(k-q)$ is not only transmitted by the one-hop ahead relay r_{2q} (as signal $\mathbf{u}[r_{2q}, k-q]$ in slot $2k$). This packet has in fact been re-encoded by all preceding even-numbered relays r_{2i} into signals $\mathbf{u}[r_{2i}, k-q]$ and transmitted in slots $2(k-q+i)$, $0 \le i \le q$, respectively. Therefore, to decode the packet $\mathbf{u}(k-q)$ in slot $2k$, the optimal way for the relay r_{2q+1} is to store and exploit all these q signals $\mathbf{x}[r_{2q+1}, 2(k-q+i)]$ that have been received in the past q even-numbered slots $2(k-q+i)$, $0 \le i \le q$. We call it a network-wide signal processing procedure, where successive interference cancellation (SIC) is used to process signals from multiple transmitting nodes during multiple time slots.

Proposition 1. With network-wide signal processing, the relay r_{2q+1}, $0 \le q \le Q_o$, can achieve the optimal transmission rate

$$R(r_{2q+1}) = \frac{1}{2} \log_2 \left(1 + \frac{\sum_{i=0}^{q} P(r_{2i})G(r_{2i}, r_{2q+1})}{\sigma^2(r_{2q+1})}\right), \quad (7)$$

which is independent and thus free of mutual interference.

Proof. (7) can be proved information-theoretically following [10]. In this paper we propose a more practical signal processing approach instead. Before decoding the packet $\mathbf{u}(k-q)$ in slot $2k$, the relay r_{2q+1} has already decoded and transmitted all packets $\mathbf{u}(t)$, $t \le k-q$. Subtracting signals related to these known packets, the signal received in slot $2(k-q+i)$, $0 \le i \le q$, is reduced to

$$\tilde{\mathbf{x}}[r_{2q+1}, 2(k-q+i)] = \sum_{\ell=0}^{i} \sqrt{P(r_{2\ell})G(r_{2\ell}, r_{2q+1})} \times e^{j\theta(r_{2\ell}, r_{2q+1})} \mathbf{u}[r_{2\ell}, k-q+i-\ell] + \mathbf{v}[r_{2q+1}, 2(k-q+i)]. \quad (8)$$

Based on (8), for each i, the relay r_{2q+1} can detect a signal $\mathbf{u}[r_{2i}, k-q]$, which is the signal transmitted from the preceding relay r_{2i} in slot $2(k-q+i)$. The SINR for this signal detection is

$$\Gamma(r_{2q+1}, r_{2i}) = \frac{P(r_{2i})G(r_{2i}, r_{2q+1})}{\sum_{\ell=0}^{i-1} P(r_{2\ell})G(r_{2\ell}, r_{2q+1}) + \sigma^2(r_{2q+1})}. \quad (9)$$

Note that (6) is a special case of (9) with $i = q$.

To combine these signals, the optimal way is to exploit the re-encoding procedure. Specifically, each relay r_{2i} re-encodes $\mathbf{u}(k-q)$ into $\mathbf{u}[r_{2i}, k-q]$ appropriately so that the relay r_{2q+1} decodes a different portion of this packet from each signal $\tilde{\mathbf{x}}[r_{2q+1}, 2(k-q+i)]$. This realizes the optimal rate for the relay r_{2q+1} as

$$R(r_{2q+1}) = \frac{1}{2} \sum_{i=0}^{q} \log_2 \left(1 + \Gamma(r_{2q+1}, r_{2i})\right). \quad (10)$$

This rate equals (7) because the denominator (mutual interference) items in the SINR $\Gamma(r_{2q+1}, r_{2i})$ expressions are cancelled nicely by each other.

After decoding the packet $\mathbf{u}(k-q)$, the relay r_{2q+1} can subtract it from all its received signals (8) to prepare for the decoding of the next packet in the next slot. This network-wide signal processing procedure is repeated by all the odd-numbered relays in all the even-numbered slots.

A remaining problem is how the even-numbered relays r_{2i}, $0 \le i \le q$, re-encodes the packet $\mathbf{u}(k-q)$ so as to satisfy (10) and (7) for all odd-numbered relays r_{2q+1}, $0 \le q \le Q_o$. Note that each even-numbered relay r_{2i} needs to transmit a portion of its data to the odd-numbered relay r_{2q+1}. Denote the rate of such portion of data as $R(r_{2i}, r_{2q+1})$, which is within the overall rate $R(r_{2i})$. This portion of rate is constrained by the channel SINR, i.e.,

$$R(r_{2i}, r_{2q+1}) \le 0.5 \log_2(1 + \Gamma(r_{2q+1}, r_{2i})). \quad (11)$$

Then, the rate of the relay r_{2q+1} is $\sum_{i=0}^{q} R(r_{2i}, r_{2q+1}) = R(r_{2q+1})$.

For all the odd-numbered relays, their rates can be described by the triangular matrix equation

$$\begin{bmatrix} R(r_0, r_1) & & \\ \vdots & \ddots & \\ R(r_0, r_{2Q_o+1}) & \cdots & R(r_{2Q_o}, r_{2Q_o+1}) \end{bmatrix} \mathbf{1} = \mathbf{R}_o, \quad (12)$$

where $\mathbf{R}_o = [R(r_1), R(r_3), \cdots, R(r_{2Q_o+1})]^T$ is the $(Q_o+1) \times 1$ dimensional rate vector of all the odd-numbered relays, and $\mathbf{1}$ is an $(Q_o+1) \times 1$ vector with all elements being 1. By solving (12), we can determine the value of each rate portion $R(r_{2i}, r_{2q+1})$.

Based on $R(r_{2i}, r_{2q+1})$, one of the ways of conducting re-encoding is random re-encoding with superposition codes. Let $\mathbf{b} = [b_1, \cdots, b_M]$ denote the symbols of a packet. The relay r_{2i} re-encodes $\mathbf{b}$ into $[c_1^{2i}, \cdots, c_M^{2i}] = \mathbf{b}\mathbf{E}_{2i}$, where $\mathbf{E}_{2i}$ is an $M \times M$ full-rank re-encoding matrix for the relay r_{2i}. Then the first ℓ symbols $\mathbf{c}_\ell^{2i} = [c_1^{2i}, \cdots, c_\ell^{2i}]$, where $\ell/M \ge R(r_{2i}, r_{2q+1})/R(r_{2i})$, are assigned with appropriate transmission power so that the SINR $\Gamma(r_{2i}, r_{2q+1})$ can be satisfied. In this way, the relay r_{2q+1} can receive successfully $\mathbf{c}_\ell^{2i}$. With all such symbols received from all the even-numbered relays, the relay r_{2q+1} can decode the packet $\mathbf{b}$ by solving the equation

$$[\mathbf{c}_\ell^0, \cdots, \mathbf{c}_\ell^{2q}] = \mathbf{b}\mathbf{D}_{2q+1}, \tag{13}$$

where $\mathbf{D}_{2q+1}$ consists of the corresponding columns of all the re-encoding matrices $\mathbf{E}_{2i}$ and has full row rank with probability 1. □

The most interesting observation is that there is no mutual interference left in the relay rate (7). In other words, multi-hop relaying becomes immune to mutual interference. What's more, each relay can collect the transmission power of all the transmitting relays in its preceding hops. This means a nice and surprising property: Enjoy benefits of wireless broadcasting without suffering from interference.

Similarly, we can analyze the SINRs and rates of the even-numbered relays.

Proposition 2. With network-wide signal processing, the relay r_{2q}, $0 \le q \le Q$, can achieve the optimal transmission rate

$$R(r_{2q}) = \frac{1}{2}\log_2\left(1 + \frac{\sum_{i=0}^{q-1} P(r_{2i+1})G(r_{2i+1}, r_{2q})}{\sigma^2(r_{2q})}\right), \tag{14}$$

which is independent and thus free of mutual interference.

Proof. The derivation of (14) is very similar to the derivation of (7). For detecting signals from the relay r_{2i+1}, the relay r_{2q} has SINR

$$\Gamma(r_{2q}, r_{2i+1}) = \frac{P(r_{2i+1})G(r_{2i+1}, r_{2q})}{\sum\limits_{\ell=0}^{i-1} P(r_{2\ell+1})G(r_{2\ell+1}, r_{2q}) + \sigma^2(r_{2q})}. \tag{15}$$

The overall rate of the relay r_{2q} is $R(r_{2q}) = 0.5\sum_{i=0}^{q-1}\log_2(1 + \Gamma(r_{2q}, r_{2i+1}))$ which can be shown equal to (14).

For the re-encoding problem, denote $R(r_{2i+1}, r_{2q})$ as the rate transmitting from the relay r_{2i+1} to the relay r_{2q}, where the constraint is

$$R(r_{2i+1}, r_{2q}) \le 0.5\log_2(1 + \Gamma(r_{2q}, r_{2i+1})). \tag{16}$$

The rates $R(r_{2i+1}, r_{2q})$ can be found by solving the following triangular matrix equation

$$\begin{bmatrix} R(r_1, r_2) & & & \\ R(r_1, r_4) & R(r_3, r_4) & & \\ \vdots & & \ddots & \\ R(r_1, r_{2Q}) & \cdots & \cdots & R(r_{2Q-1}, r_{2Q}) \end{bmatrix} \mathbf{1} = \mathbf{R}_e, \tag{17}$$

where $\mathbf{R}_e = [R(r_2), R(r_4), \cdots, R(r_{2Q})]^T$ is the $Q \times 1$ dimensional rate vector of all the even-numbered relays. Iterative random re-encoding can be conducted similarly as the odd-numbered relay case. □

Finally, for the destination node $r_{h+1} = N+1$, since it can receive signals in both even-numbered and odd-numbered slots, its optimal rate should be the summation of (7) and (14), i.e.,

$$R(r_{h+1}) = \frac{1}{2}\log_2\left(1 + \frac{\sum_{i=0}^{Q} P(r_{2i})G(r_{2i}, r_{h+1})}{\sigma^2(r_{h+1})}\right) + \frac{1}{2}\log_2\left(1 + \frac{\sum_{i=0}^{Q_o} P(r_{2i+1})G(r_{2i+1}, r_{h+1})}{\sigma^2(r_{h+1})}\right). \tag{18}$$

There is no mutual interference in (18). Therefore, the destination node is also immune to mutual interference.

B. *Efficient algorithm for multi-hop relay selection*

The problem of hop count determination, relay node selection, and multi-hop rate (4) optimization can be formulated as max-min optimization

$$R = \max_{\substack{0 \le h \le N \\ r_\ell \in \mathcal{N}, 1 \le \ell \le h}} \min_{1 \le i \le h+1} R(r_i) \tag{19}$$

under node power constraint $0 \le P(i) \le P^{\max}(i), 0 \le i \le N$.

To solve (19), rather than exhaustive search over all possible h and relay combinations, more efficient algorithms can be developed. First, because the rate R increases monotonically with relaying powers, each relay should simply transmit at full power, i.e., $P(r_i) = P^{\max}(r_i)$. This resolves the challenging power control issue. Second, a relay is not affected by the relays in its subsequent hops. Based on this fact, we can start from determining the first hops sequentially. Finally, a relay only increases the rates of the relays in its subsequent hops. With this result, we can try a greedy procedure to select all possible relays with large enough decode-and-forward rates.

We can use the following efficient algorithm to solve (19) approximately.

Algorithm 1: Half-duplex Multi-hop Relay Selection
initialize: $r_0 = 0$, $\mathcal{N} = \{1, \cdots, N+1\}$
for iteration $j = 1, 2, \cdots, N$, do
Update rates R_i for all remaining nodes $i \in \mathcal{N}$.
Select relay $r_j = \arg\max_{i \in \mathcal{N}} R_i$ for hop j.
Update node set $\mathcal{N} := \mathcal{N} \setminus \{r_j\}$.
Update current multi-hop rate $R = \min_{1 \le \ell \le j} R_{r_\ell}$.
If $r_j = N+1$, then $h = j-1$, $R = \min\{R, R_{r_j}\}$, stop.
If $R \le R_{N+1}$, then $h = j$, $r_{h+1} = N+1$, stop.
output: h, R, r_j, $j = 1, \cdots, h$.

The algorithm begins with $r_0 = 0$. In each iteration j, we select, from all the remaining $N - j + 2$ candidate nodes (include the destination node), a node with the highest rate as the relay r_j in hop j. Rates of the remaining candidate nodes are updated (calculated) based on (7), (14) and (18) for odd j, even j, and destination node, respectively. Relays selected in previous iterations 1 to $j-1$ are used to update the node rates. The rate updating procedure can be implemented iteratively, where each node keeps two rates (for even and odd j) updated and stored. The algorithm stops with hop count h and relay selections r_j, $1 \le j \le h$.

As to computational complexity, in the worst case the algorithm runs N iterations. In each iteration j, it updates $N - j + 2$ rates. Therefore, it calculates a total of $\sum_{j=1}^{N}(N -$

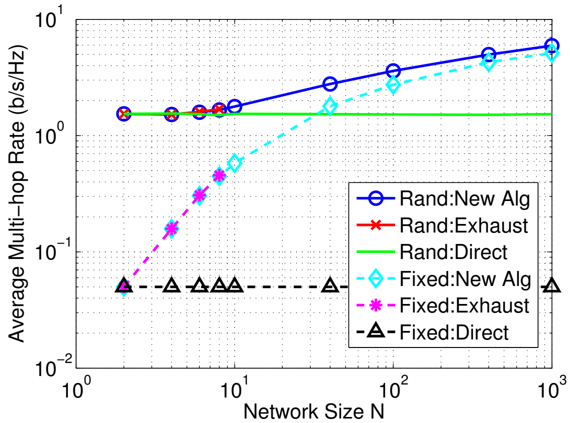


Fig. 3. Average multi-hop transmission rate R of random wireless networks.

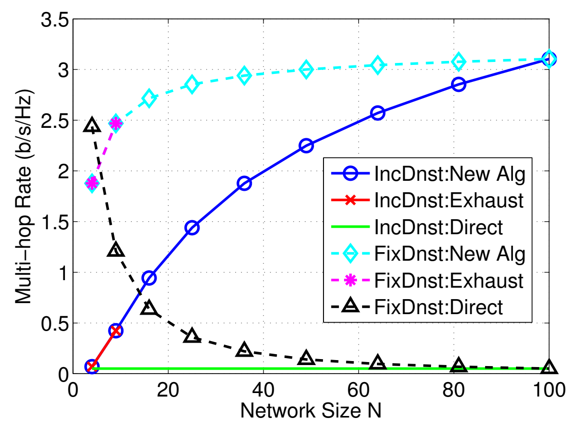


Fig. 4. Multi-hop transmission rate R of fixed grid networks of N nodes.

$j+2) = (N^2+3N)/2$ rates, which has complexity $O(N^2)$ if implemented as iterative updating.

This wireless algorithm is essentially similar to the well-known Dijkstra's algorithm. The major difference lies in history dependence. Node rates are not fixed. But rather, they are changed by each new relay selected during each iteration.

IV. Simulations

In the first simulation setting, we simulate a wireless network whose nodes are placed randomly within a square of 1000×1000 meters. We consider two scenarios: **Rand** (source and destination nodes are placed randomly) and **Fixed** (source is in the original point and destination is in position (1000,1000)). The channel gain between two nodes with distance d_{ij} is $G_{i,j} = K d_{ij}^{-3}$. Parameters and transmission powers are normalized so that a transmission distance of 1000 meters has signal-to-noise ratio (SNR) 10 dB.

For each network size N, we generate 1000 random networks, run our algorithm in each of them, and calculate average multi-hop rate. We denote the result of our algorithm by "**New Alg**", and compare it with the direct (no relay) transmission result ("**Direct**") and the brute-force exhaustive search result ("**Exhaust**"). The exhaustive search method works for small network size only due to its exponential complexity. Simulation results are shown in Fig. 3. Note that the **Rand** cases have higher rates than the **Fixed** cases because the latter have larger source-destination distance which usually leads to longer hop distance when N is small.

In the second simulation setting, we consider a fixed grid network, where wireless nodes are placed evenly on a $\sqrt{N} \times \sqrt{N}$ square grid. The grid distance either shrinks with N to keep constant network area and increased density (**IncDnst**), or remains constant for fixed node density (**FixDnst**). Obviously, the former case will have high multi-hop rates than the latter. Simulation results are shown in Fig. 4.

Simulation results clearly show that the proposed algorithm gives almost the same result as the exhaustive search method. This demonstrates that our proposed algorithm is near optimal. The proposed algorithm works efficiently for even extremely large networks. Although Algorithm 1 is an approximate algorithm only, simulations indicate that it can achieve the optimal solution of (19) in majority of cases. In addition, it can achieve average transmission rates that are very close, within 2%, to the optimal average transmission rates.

V. Conclusion

In this paper we first develop a network-wide signal processing procedure for half-duplex decode-and-forward relays to realize interference immune multi-hop relaying. Then we develop an efficient multi-hop relay selection algorithm to find approximately the optimal hop count and select relays to maximize multi-hop transmission rate. The new algorithm is similar to Dijkstra's algorithm, and is efficient for exploring large wireless networks. Simulations are conducted to verify its efficiency and near optimal performance.

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