Markov Model Bank for Heterogenous Cognitive Radio Networks with Multiple Dissimilar Users and Channels

Xiaohua Li and Chengyu Xiong
Department of Electrical and Computer Engineering
State University of New York at Binghamton
Binghamton, NY 13902
Email: {xli,cxiong1}@binghamton.edu

***Abstract*—In this paper, we develop a Markov Model Bank (MMB) to analyze the heterogeneous cognitive radio networks (CRN) with a large number of dissimilar secondary users (SU) coexisting and competing for multiple dissimilar channels. The MMB consists of a separate Markov chain for each SU. The Markov chains are connected implicitly by a few state transitional probabilities that can be derived by analyzing the mutual interference among the SUs. We first develop the expressions of the transitional probabilities and the throughput in the general heterogeneous users setting. Then, by exploiting some special advantages of the MMB in spatial, channel and user de-correlation, we reduce the complexity of evaluating such expressions to a great extent so as to make it feasible to analyze the throughput of large heterogeneous CRNs under various channel access strategies. Simulations are conducted to verify the proposed approaches.**

Keywords*—*cognitive radio network, Markov chain, throughput, interference, dynamic spectrum access

I. INTRODUCTION

Cognitive radio network (CRN) and dynamic spectrum access (DSA) allow new wireless systems to reuse the spectrum currently occupied exclusively by primary systems. As a unique feature, CRN conducts spectrum sensing and allows the secondary users (SU) accesses the spectrum that is not occupied by the primary users (PU). The SUs must vacate the spectrum whenever the PUs become active.

Due to the inherent flexibility of cognitive radios, large CRN may be heterogeneous in nature. Different SUs may have different implementations, different spectrum sensing and spectrum access algorithms. For example, the PU's signals are different temporarily and geographically, which may lead the SUs to adopt either more conservative or more aggressive polices toward sensing errors. Some SUs may exploit handshaking or control channels more than others. The flexible software implementations are subject to modification.

Unfortunately, it has been a challenge and a mostly open area to analyze the heterogeneous CRN with multiple dissimilar users coexisting and competing for multiple dissimilar channels. Although there have been extensive research published in CRN, including spectrum sensing, Physical-layer modulation, MAC/Network layer protocols, hardware/testbed development, etc, very limited study has been conducted on the coexistence of a large number of users in heterogeneous CRNs. There is a class of work on the CRN transmission power/capacity analysis based on the physical-layer interference analysis, but they are conducted under certain simplified communication models [1]. There is another class of work on CRN MAC protocol design and analysis, such as the performance analysis of the CSMA-type protocols [2]-[4]. Nevertheless, the majority of these works are for homogeneous CRN, where all SUs have the same strategies and parameters.

The large number of dissimilar users makes the modeling and analysis of heterogeneous CRN substantially challenging. Most of the existing work on CRN analysis that consider more or less the heterogeneous nature of CRN, such as [5]-[12], have to make substantial simplifications. As a typical example, SU's competition for spectrum access is skipped in [5] and [6] since simultaneous transmissions are considered only, where spectrum access becomes just the constant arrival/departing rate. In addition, most of such work focus on the simple scenario of two or three users only for easy Markov chain modeling. The general case becomes prohibitively complex due to the rapidly increased number of states and transitional probabilities in the Markov chain.

In this paper, we develop an innovative Markov Model Bank (MMB) for modeling and analyzing the heterogeneous CRN with multiple dissimilar SUs and multiple dissimilar channels. We focus on evaluating the SU throughput under certain spectrum access strategies. Our basic idea is to decompose the complex Markov model into a bank of relatively separated Markov chains, each for a SU. This makes the Markov chains easy to analyze. The major complexity is now involved in the state transitional probabilities which include the mutual interference among all the SUs. Fortunately, MMB allows us to exploit spatial, channel and user de-correlations to make interference analysis feasible. As a result, the MMB approach makes it tractable to analyze large heterogeneous CRNs.

The organization of this paper is as follows. In Section II, we give the system model. In Section III, we develop the MMB and give throughput expressions in the general setting. Then in Section IV, we exploit the de-correlation properties to develop more efficient expressions for evaluating the throughput. Simulations are conducted in Section V, and conclusions are given in Section VI.

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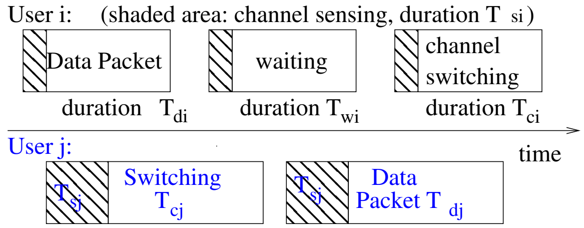

Fig. 1. A snapshot of cognitive radio transmissions. For each SU i or j, there are four basic operation states: channel sensing, data packet transmission, waiting, and channel switching.

II. System Model

The CRN we consider in this paper consists of a set of N randomly distributed SUs and some PUs. Each SU is a transmitting-receiving pair of nodes, where the transmitting node transmits to the receiving node. While transmitting to the receiving node, the transmitter creates interference to other users. We assume there are K channels available for the N users to choose from.

For each SU, we consider a cognitive radio transmission model that includes four states: spectrum sensing, data transmission, waiting and channel switching. A snapshot of the CRN operation is shown in Fig. 1. The working sequence of a cognitive radio always begins with the spectrum sensing. If the spectrum sensing indicates the channel is available for secondary access, then the cognitive radio transmits a data packet, and the model shifts into the data transmission state. If the spectrum sensing indicates the channel is not available (due to the PU activity, interference from other SUs activity, or the lack of offered load from this SU), then the cognitive radio enters the waiting state, which will be followed by the channel switching state. We assume that the cognitive radio always shifts back to the spectrum sensing state after either the channel switching state or the data transmission state.

A SU's spectrum sensing results actually depend on the PUs' activity, the other SUs' activity, as well as this SU's offered load. we assume that the SU can successfully detect the activity of the PUs. Therefore, we can denote the activity of the PUs in each channel k by a random variable θ_k, which denotes the probability that the channel k is not used by any PU. We further assume that the offered load of the SU i be α_i, i.e, with probability α_i the SU i have data packet to transmit. Under these simplifications, we can then focus on analyzing the coexistence of SUs via their mutual interference. Specifically, in the spectrum sensing state, if the signal to noise ratio (SNR) $\gamma_{si,k}$ in the channel k detected by the SU i is larger than the detection threshold $\Gamma_{i,k}$, then the SU i makes a decision that the channel is not available for transmission. It turns to the waiting state.

Let $P_{i,k}$ be the transmission power that the SU i spends in the channel k, where $i = 1, \cdots, N$, and $k = 1, \cdots, K$. In the data transmission state, the signal received by the SU i's receiver in the channel k is

$$y_{i,k}(n) = \sqrt{P_{i,k}} h_{ii,k} s_i(n) + \sum_{j=1, j\neq i}^{N} f_{j,k} \sqrt{P_{j,k}} h_{ji,k} s_j(n) + v_{i,k}(n) \tag{1}$$

where $s_i(n)$ is the signal transmitted by the SU i, $h_{ji,k}$ is the flat fading channel from the SU j's transmitter to the SU i's receiver, and $v_{i,k}(n)$ is the AWGN with zero-mean and power $\sigma^2_{i,k}$. The variable $f_{j,k} = 1$ or 0 denotes whether the SU j is currently using the channel k. The instantaneous SNR of the SU i in the channel k during data transmission is thus

$$\gamma_{i,k} = \frac{P_{i,k}|h_{ii,k}|^2}{\sum_{j=1,j\neq i}^{N} f_{j,k} P_{j,k} |h_{ji,k}|^2 + \sigma^2_{i,k}} \tag{2}$$

Similarly, in the spectrum sensing state, the instantaneous SNR of the SU i in the channel k is

$$\gamma_{si,k} = \frac{\sum_{j=1,j\neq i}^{N} f_{j,k} P_{j,k} |h_{ji,k}|^2}{\sigma^2_{i,k}} \tag{3}$$

In this paper, we focus on analyzing the throughput of the CRN under certain spectrum access strategies. Each SU i has a throughput R_i, which is the sum throughput of the SU obtained in all the K channels. The R_i can be calculated from the probability of the data transmission state, and the transmission capacities of each channel, i.e., $\log(1 + \gamma_{i,k})$. Only the data transmission state is counted toward the throughput.

Based on the SUs' throughput, there are multiple performance metrics that can be used to evaluate the CRN performance, such as the proportional fair metric or the sum throughput metric, i.e.,

$$R = \prod_{i=1}^{N} R_i, \quad \text{or} \quad R = \sum_{i=1}^{N} R_i. \tag{4}$$

III. Heterogeneous CRN Throughput in the General Setting

We use a Markov Model Bank (MMB) to model the competitive spectrum access among the N CRN users. The MMB consists of a Markov chain for each SU, as shown in Fig. 2, where π^k_{si},π^k_{di},π^k_{wi} are the probabilities of the SU i staying in the channel sensing, data packet transmission and waiting state in the channel k. The probability π_{ci} refers to the probability that the SU i staying in the channel switching state. The transitional probability $q_{i,k}$ denotes the probability that the channel k can be used by the SU i to transmit a data packet. It is a composite result of PU detection, SU carrier sensing, as well as the offered load. The transitional probability $z_{i,k}$ denotes the probability for the SU i to choose the channel k to sense and access. It is determined by the channel access strategy, such as the random selection ($z_{i,k} = 1/K$), the channel selection based on PU information ($z_{i,k} \sim \theta_k$), or the more advanced game-theoretic strategies. In this paper, we study the CRN throughput under the given $z_{i,k}$. We assume $0 \leq z_{i,k} \leq 1$ and

$$\sum_{k=1}^{K} z_{i,k} = 1, \quad i = 1, \cdots, N, \tag{5}$$

which means that a SU just uses one channel for data transmission at a time.

Note that although the different SUs' Markov chains look separated from each other, their transitional probabilities and state probabilities are inter-related via $q_{i,k}$. As a matter of fact,

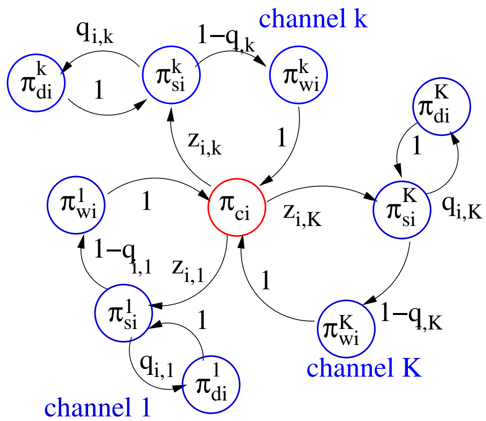

Fig. 2. Markov Model Bank (MMB) for multi-user heterogenous CRN, where only the Markov chain for the SU i is shown. Each of the SUs has a similar Markov chain. Each Markov chain has $3K+1$ states: 3 states in each channel k, plus a single channel switching state for all the channels. The transitional probabilities $q_{i,k}$ and $z_{i,k}$ are determined by the spectrum sensing and the spectrum access strategies.

the MMB approach simplifies the Markov model, but increases the complexity of $q_{i,k}$. Fortunately, we will show that MMB can bring some useful de-correlation properties to reduce the complexity.

For each SU i, the durations of the spectrum sensing, data transmission, waiting, and channel switching are $T_{si,k}$, $T_{di,k}$, $T_{wi,k}$ and T_{ci}, respectively.

As the first advantage, conditioned on $q_{i,k}$ and $z_{i,k}$, MMB allows us to analyze the steady state property of each SU independently. Based on Fig. 2, we can calculate the probabilities of all the states π_{si}^k, π_{di}^k, π_{wi}^k and π_{ci} of the SU i by solving the following equation

$$\begin{bmatrix} \mathbf{A}_1 & & & \mathbf{a}_1 \\ & \ddots & & \vdots \\ & & \mathbf{A}_K & \mathbf{a}_K \\ \mathbf{b} & \cdots & \mathbf{b} & -1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_K \\ \pi_{ci} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ 0 \end{bmatrix}, \tag{6}$$

where $\mathbf{b} = [0,0,1]$, $\mathbf{a}_k = [z_{i,k}, 0, 0]^T$, and

$$\mathbf{A}_k = \begin{bmatrix} -1 & 1 & 0 \\ q_{i,k} & -1 & 0 \\ 1-q_{i,k} & 0 & -1 \end{bmatrix}, \quad \mathbf{x}_k = \begin{bmatrix} \pi_{si}^k \\ \pi_{di}^k \\ \pi_{wi}^k \end{bmatrix}. \tag{7}$$

Proposition 1. Under the constraint $\pi_{ci} + \sum_{k=1}^K (\pi_{di}^k + \pi_{si}^k + \pi_{wi}^k) = 1$, the steady state probabilities are

$$\pi_{si}^k = \frac{z_{i,k}}{\left(1 + 2\sum_{\ell=1}^K \frac{z_{i,\ell}}{1-q_{i,\ell}}\right)(1-q_{i,k})} \tag{8}$$

and

$$\pi_{di}^k = q_{i,k}\pi_{si}^k, \ \pi_{wi}^k = (1-q_{i,k})\pi_{si}^k, \ \pi_{ci} = \frac{1-q_{i,k}}{z_{i,k}}\pi_{si}^k. \tag{9}$$

Proof. The equation (9) can be easily obtained from (6)-(7). The constraint in the proposition gives $2\sum_{k=1}^K \pi_{si}^k = 1 - \pi_{ci}$. Considering also the fact that

$$\frac{\pi_{si}^k}{\pi_{si}^\ell} = \frac{z_{i,k}(1-q_{i,\ell})}{z_{i,\ell}(1-q_{i,k})}, \tag{10}$$

we can readily prove (8).

The most complex task is perhaps to analyze $q_{i,k}$ which requires us to analyze the mutual interference among all the SUs. Based on our MMB model, we define

$$q_{i,k} = \theta_k \alpha_i \mathcal{P}[\gamma_{si,k} < \Gamma_{i,k}], \tag{11}$$

which means that the SU i can use the channel k to transmit a data packet only if there is no PU activity in the channel k, the SU i has a data packet to transmit, and the sensing SNR $\gamma_{si,k}$ (3) indicates that the interference from all the other SUs using the channel k simultaneously is low enough.

The probability that another SU j is also using the channel k can be modeled as

$$\begin{aligned} \beta_{j,k} &= \mathcal{P}[f_{j,k} = 1] \qquad (12) \\ &= \frac{\pi_{dj}^k T_{dj,k}}{\pi_{cj}T_{cj} + \sum_{\ell=1}^K \left(\pi_{sj}^\ell T_{sj,\ell} + \pi_{dj}^\ell T_{dj,\ell} + \pi_{wj}^\ell T_{wj,\ell}\right)}. \end{aligned}$$

According to the Proposition 1, we can derive

$$\begin{aligned} \beta_{j,k} = & \frac{z_{j,k} q_{j,k} T_{dj,k}}{1-q_{j,k}} \\ & \times \frac{1}{T_{cj} + \sum_{\ell=1}^K [T_{sj,\ell} + q_{j,\ell}T_{dj,\ell} + (1-q_{j,\ell})T_{wj,\ell}] \frac{z_{j,\ell}}{1-q_{j,\ell}}}. \end{aligned} \tag{13}$$

In other words, $f_{j,k}$ is a Bernoulli random variable with probability $\beta_{j,k}$.

Therefore, the probability of having no significant interference from other SUs

$$\mathcal{P}[\gamma_{si,k} < \Gamma_{i,k}] = \mathcal{P}\left[\frac{1}{\sigma_{i,k}^2} \sum_{j=1, j\neq i}^N P_{j,k}|h_{ji,k}|^2 f_{j,k} < \Gamma_{i,k}\right] \tag{14}$$

can be calculated from the cumulative distribution of the weighted summation of $N-1$ Bernoulli random variables $f_{j,k}$, $j = 1, \cdots, N$, $j \neq i$. Since $\gamma_{si,k}$ is a discrete random variable with at most 2^{N-1} different values, one of the ways to evaluate (14) is to list all possible values of the weighted summation and their corresponding probability mass, and then to add together all those values less than $\Gamma_{i,k}$. This is convenient when the number of SUs N is not large. This is also convenient when the fading is severe or the SUs are separated by large distances, which makes the number of summation items small.

For the most general case, define the set of SU indices

$$\mathcal{S}_i = \{1, \cdots, i-1, i+1, \cdots, N\}. \tag{15}$$

Define $\mathcal{S}_{i,\ell} \subseteq \mathcal{S}_i$ as the subset of $\mathcal{S}_i$. Obviously, there are altogether 2^{N-1} different subsets, i.e., $\ell = 1, \cdots, 2^{N-1}$. Then

$$q_{i,k} = \theta_k \alpha_i \sum_{\ell=1}^{2^{N-1}} \mathcal{I}_{i,k}(\ell) \prod_{j\in\mathcal{S}_{i,\ell}} \beta_{j,k} \prod_{j\notin\{i\bigcup\mathcal{S}_{i,\ell}\}} (1-\beta_{j,k}), \tag{16}$$

where the indicator function

$$\mathcal{I}_{i,k}(\ell) \triangleq \begin{cases} 1, & \text{if } \sum_{j\in\mathcal{S}_{i,\ell}} P_{j,k}|h_{ji,k}|^2 < \sigma_{i,k}^2\Gamma_{i,k} \\ 0, & \text{else} \end{cases} \tag{17}$$

means whether the interference from set of transmitting SUs $\mathcal{S}_{i,\ell}$ is small enough to the ith SU.

Based on the state transitional probabilities $q_{i,k}$ derived from (16) and the state probabilities derived from Proposition 1, we can then evaluate the throughput R_i or R. First, with the state probabilities, the fraction of time used by the SU i for successful data transmission in the channel k is

$$U_{i,k} = \frac{\pi_{di}^k T_{di}^k}{\pi_{ci} T_{ci} + \sum_{\ell=1}^K \left(\pi_{si}^\ell T_{si}^\ell + \pi_{di}^\ell T_{di}^\ell + \pi_{wi}^\ell T_{wi}^\ell \right)}, \quad (18)$$

which can be deducted to $U_{i,k} = \beta_{i,k}$ similarly to (13). Then we can define the average throughput of the SU i as

$$R_i = \sum_{k=1}^K U_{i,k} \sum_{\ell=1}^{2^{N-1}} \log(1+\gamma_{i,k}(\ell)) \times \prod_{j\in\mathcal{S}_{i,\ell}} \beta_{j,k} \prod_{j\notin\{i \bigcup \mathcal{S}_{i,\ell}\}} (1-\beta_{j,k}), \quad (19)$$

where $\gamma_{i,k}(\ell)$ is the SNR during the data packet transmission

$$\gamma_{i,k}(\ell) = \frac{P_{i,k}|h_{ii,k}|^2}{\sum_{j\in\mathcal{S}_{i,\ell}} P_{j,k}|h_{ji,k}|^2 + \sigma_{i,k}^2}. \quad (20)$$

Therefore, in the most general setting, we can outline as follows a way to evaluate the throughput of the heterogeneous CRN under certain spectrum access strategies. First, for a channel accessing strategy described by the probabilities $\{z_{i,k}\}$, we calculate $\{q_{i,k}\}$ by solving a system of non-linear equations (16) under constraints $0 \le q_{i,k} \le 1$. With the $\{q_{i,k}\}$, we can then calculate the throughputs $\{R_i\}$ via (19), as well as the performance metric R defined in (4).

The equation system (16) consists of NK nonlinear equations. We can find the roots by existing root searching algorithms. Note that there may have many solutions to $q_{i,k}$, each of which represents a special working mode of the CRN, where the SUs may have different throughputs. It is an interesting future objective to study the complex behavior of heterogeneous CRN through the solutions to (16).

On the other hand, the computational complexity of this way of performance evaluation in the general setting is prohibitively high for large CRN. Fortunately, as we will shown in the next section, we can exploit some useful de-correlation properties of the MMB to develop more efficient expressions. These expressions can provide more incisive information about the heterogeneous CRN performance. They can also help to provide intuitive initial conditions for solving (16) numerically.

IV. Throughput of CRN with Co-located SUs

For complexity reduction, first we need to exploit the fact that most Medium Access Control (MAC)-layer protocols such as Carrier Sense Multiple Access (CSMA) avoid simultaneous transmission among multiple SUs. It was reported that simultaneous transmission among multiple SUs usually leads to inefficient performance [6]. Therefore, we can consider very low detection SNR thresholds $\Gamma_{i,k}$, and assume that *i*) $q_{i,k} = 0$ whenever there is a SU j with $P_{j,k}|h_{ji,k}|^2/\sigma_{i,k}^2 \ge \Gamma_{i,k}$, and that *ii*) $q_{i,k}$ is independent from all the SUs j with $P_{j,k}|h_{ji,k}|^2/\sigma_{i,k}^2 < \Gamma_{i,k}$. Under this assumption, we effectively subdivide the CRN into different cells or clusters, where the SUs within a cluster can not transmit at the same time, while the SUs from different clusters do not affect each other. We call the SUs inside a cluster as co-located SUs. This means the de-correlation of the SUs in the spatial domain.

Focusing on one cluster, we assume all the N SUs are co-located without loss of generality. In this case, a channel is available to the SU i only if no other SUs or PUs are using it. Obviously, the spatial de-correlation helps us avoid the exhaustive search over the 2^{N-1} sets in (16) and (19), and thus substantially reduce the computational complexity.

To further simplify the expressions, we assume that each SU uses the same slot length parameters in all the channels, i.e., $T_{si,k} = T_{si}$, $T_{di,k} = T_{di}$, $T_{wi,k} = T_{wi}$, for all $k = 1, \cdots, K$. We also assume that $T_{di} = T_{wi}$, i.e., the SUs wait for just one data packet transmission duration when the channel is not available, see Fig. 2. Note that both assumptions are practical enough.

Under such assumptions, we can simplify the expressions for $q_{i,k}$ and R_i in (16) and (19) into

$$\begin{aligned} R_i &= \sum_{k=1}^K \beta_{i,k} \log\left(1 + \frac{P_{i,k}|h_{ii,k}|^2}{\sigma_{i,k}^2}\right), \\ q_{i,k} &= \theta_k \alpha_i \prod_{j=1, j\neq i}^N (1-\beta_{j,k}), \\ \beta_{i,k} &= \frac{z_{i,k} q_{i,k} T_{di}}{(1-q_{i,k})(T_{si}+T_{di}) \sum_{\ell=1}^K \frac{z_{i,\ell}}{1-q_{i,\ell}}}. \end{aligned} \quad (21)$$

In (21), each $q_{i,k}$ still depends explicitly on all the SUs and all the channels. Fortunately, MMB allows us to take channel de-correlation to remove its explicit dependence on other channels $\ell \neq k$. To do this, define

$$x_{i,k} = \frac{z_{i,k}}{(1-q_{i,k}) \sum_{\ell=1}^K \frac{z_{i,\ell}}{1-q_{i,\ell}}}, \quad (22)$$

From $0 \le q_{i,k} \le 1$ and $\sum_{k=1}^K z_{i,k} = 1$, we can easily see that

$$0 \le x_{i,k} \le 1, \quad \sum_{k=1}^K x_{i,k} = 1. \quad (23)$$

Proposition 2. $z_{i,k}$ is uniquely determined by $x_{i,k}$ via

$$z_{i,k} = \frac{x_{i,k}(1-q_{i,k})}{\sum_{\ell=1}^K x_{i,\ell}(1-q_{i,\ell})}. \quad (24)$$

Proof. (24) can be readily proved from (22)-(23). Details skipped due to space limit.

With $x_{i,k}$, we can then change (21) into

$$R_i = \sum_{k=1}^K c_{i,k} q_{i,k} x_{i,k}, \quad (25)$$

$$q_{i,k} = \frac{1}{a_{i,k}} \prod_{j=1, j\neq i}^N (1 - b_j q_{j,k} x_{j,k}), \quad (26)$$

where $c_{i,k} = \log(1 + P_{i,k}|h_{ii,k}|^2/\sigma_{i,k}^2) T_{di}/(T_{si}+T_{di})$, $a_{i,k} = 1/(\theta_k \alpha_i)$, and $b_j = T_{dj}/(T_{sj}+T_{dj})$.

Due to the channel de-correlation in (26), conditioned on $x_{i,k}$ which is actually a new way to describe the same channel access strategy, we can solve each set of N probabilities $q_{i,k}$, $i = 1, \cdots, N$, from N nonlinear equations, rather than the NK probabilities altogether.

From (26), we find that

$$a_{i,k}q_{i,k}(1 - b_i q_{i,k}x_{i,k}) = a_{j,k}q_{j,k}(1 - b_j q_{j,k}x_{j,k}). \quad (27)$$

Therefore, we can further take user de-correlation by defining a special parameter D_k such that

$$a_{i,k}q_{i,k}(1 - b_i q_{i,k}x_{i,k}) = D_k, \quad (28)$$

from which $q_{i,k}$ can be specified in the closed-form expression

$$q_{i,k} = \frac{a_{i,k} \pm \sqrt{a_{i,k}^2 - 4a_{i,k}b_i x_{i,k} D_k}}{2a_{i,k}b_i x_{i,k}}. \quad (29)$$

Note that we usually just need the "minus"-based solution because $x_{i,k}$ is small. If $x_{i,k}$ is small enough, then we have approximately $q_{i,k} \approx D_k / a_{i,k}$.

The value of D_k can be obtained from the constraint

$$D_k = \prod_{j=1}^{N}(1 - b_j q_{j,k}x_{j,k}) = \left(\prod_{j=1}^{N} a_{j,k}q_{j,k}\right)^{K-1}, \quad (30)$$

by substituting $q_{i,k}$ with (29).

Therefore, for CRN performance analysis, we first find the roots to (30), and then use the values D_k to calculate $q_{i,k}$ as well as the throughput R_i. The fact that all the SUs have the same D_k makes the performance analysis and comparison feasible. Computationally, another advantage is that it is much easier to evaluate numerically the roots of a single equation (30) than a system of N nonlinear equations. There are even closed-form solutions available under appropriate approximations. Due to the severe space limit in this paper, details will be reported elsewhere.

V. Simulations

In this section, we report our simulation of the numerical evaluation of the CRN throughput based on both the general expressions in Section III and the more efficient expressions in Section IV. We simulated a random CRN of $N = 4$ secondary users and $K = 2$ channel. The nodes' positions were randomly generated within a square of 1000×1000 meters. The SNRs were calculated as $10^8 d_{ij}^{-2.6}$ where d_{ij} is the propagation distance. We evaluate the sum throughput of all the CRN SUs under various (normalized) SU transmission powers $P_{i,k}$. Simulation results were shown in Fig. 3. It can be clearly seen that the analysis results derived from the expressions in Sections III and IV fit well with each other. This is not a surprise because the simulated CRN is a co-located network. In addition, the analysis results fit well with the Monte-Carlo simulated results, which verifies that our MMB approach is valid.

VI. Conclusion

We developed a MMB (Markov Model Bank) approach to evaluate the throughput of heterogeneous CRN with a large number of different users. By using a separate Markov chain for each SU, the MMB provides spatial, channel and user de-correlations to simplify the mutual interference analysis. This makes the MMB feasible to deal with large heterogeneous CRNs.

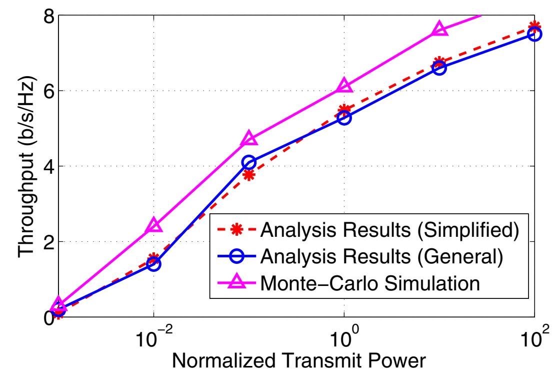

Fig. 3. CRN sum throughput under various (normalized) transmission powers (identical for all the SUs).

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