COMPETITIVE PERFORMANCE OF MULTIPLE SPECTRUM ACCESS STRATEGIES COEXISTING IN HETEROGENEOUS COGNITIVE RADIO NETWORKS

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ABSTRACT

This paper develops a computationally efficient method to evaluate the competitive performance of multiple spectrum access strategies when they coexist and compete with each other in a heterogeneous cognitive radio network (CRN). We use a Markov Model Bank (MMB) to model the operations of the heterogeneous CRN. Various spectrum sensing errors, spectrum sensing strategies, and spectrum access strategies can be conveniently modeled. In particular, three typical spectrum access strategies are analyzed. Throughput expressions for each CRN user can be derived even when different strategies are adopted by different users. Simulation shows the critical fairness challenge among the CRN users when different strategies compete in the heterogeneous CRN.

***Index Terms*—** cognitive radio network, Markov chain, throughput, interference, coexistence

1. INTRODUCTION

Cognitive radio network (CRN) allows the secondary user (SU) to reuse the spectrum licensed exclusively to the primary user (PU). The SUs sense the spectrum and selectively access the spectrum that is not in use by the PUs. The SUs must vacate the spectrum whenever the PUs become active.

Considering the flexible software-based implementation and the flexible regulation over the technologies that the SU may adopt, CRN will be heterogeneous in nature. Different SUs may use different transmission parameters, spectrum sensing strategies, and spectrum access strategies. For spectrum access in particular, many different strategies have been proposed, from the plain listen-before-talk (LBT) [1] to the more complex game-theoretic strategies [2] [3].

It is a challenge and mostly an open area to investigate the competitive coexistence of multiple spectrum access strategies in CRN. The large number of dissimilar SUs makes the modeling and analysis of heterogeneous CRN substantially challenging. Although CRN performance analysis has been widely reported [4]-[10], most existing work considers homogeneous CRN, or small heterogeneous CRN only, under some major simplifications. Specifically, details in spectrum sensing and spectrum access procedures were omitted in [6] and [7]. The coexistence and competition among multiple spectrum access strategies were not addressed in [8]-[10].

To deal with this challenge, we developed a framework called Markov Model Bank (MMB) and used it to model and analyze heterogeneous CRN in [11][12]. Network decomposition techniques were developed to de-correlate the complex mutual interference coupling among the SUs, based on which we showed that each SU's throughput could be analyzed independently from others by finding a special polynomial root [12]. Nevertheless, such derivations were made without considering the spectrum sensing errors. In addition, only a generic random spectrum access model was considered. It was unknown how practical spectrum access strategies behave under this framework.

In this paper, based on [11] [12], we study the competitive coexistence of multiple practical spectrum access strategies in a heterogeneous CRN. We will address the spectrum sensing errors and consider three representative spectrum access strategies, i.e., two LBT-based strategies and one game-theoretic strategy.

The organization of this paper is as follows. In Section 2, we give the system model and the MMB framework. In Section 3, we derive both the general and the simplified throughput expressions with the consideration of spectrum sensing errors. Then in Section 4, we analyze three typical spectrum access strategies. Simulations are conducted in Section 5, and conclusions are given in Section 6.

2. SYSTEM MODEL

The CRN we consider in this paper consists of N randomly distributed SUs and some PUs. Each SU is a pair of transmitting node and receiving node. For heterogeneous CRN, different SUs may have different transmission parameters. We assume there are J channels for the SUs to sense and access.

Each SU follows the three basic cognitive radio operation procedures: spectrum sensing, data transmission, and channel switching. If the spectrum sensing indicates a channel is available for secondary access, then the SU transmits a data packet. If the spectrum sensing indicates the channel is not available, then the SU conducts channel switching with some spectrum access strategy.

We denote the probability that the channel k is not used by any PU as a random variable θ_k. Due to spectrum sensing errors, the SU i has probability $\phi_{i,k}^n$ of detecting an available channel k as unavailable, and has probability $\phi_{i,k}^p$ of detecting an unavailable channel k as available. We further assume that the offered load of the SU i be α_i, i.e, with probability α_i the SU i has data packet to transmit. In addition, spectrum access strategies depend on the mutual interference (i.e., the competition) among the SUs as well.

We use a Markov Model Bank (MMB) to model the competitive spectrum access among the N SUs. The MMB consists of a separate Markov chain for each SU. The Markov chain for the SU i is shown in Fig. 1, where $\pi_{i,k}^s$ and $\pi_{i,k}^d$ are the probabilities of the SU i staying in the channel sensing state and the data packet transmis-

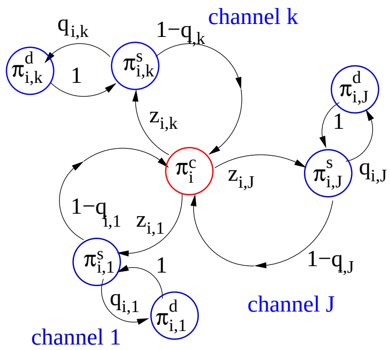

Fig. 1. Markov Model Bank (MMB) for heterogenous CRN. Only the Markov chain for the SU i is shown. Each SU is modeled by a similar Markov chain.

sion state, respectively, in the channel k. The probability π_i^c refers to the probability that the SU i staying in the channel switching state. The time durations of the spectrum sensing, data transmission, and channel switching states are $T_{i,k}^s$, $T_{i,k}^d$, and T_i^c, respectively.

The transitional probability $q_{i,k}$ denotes the probability that the channel k can be used by the SU i to transmit a data packet. It is a composite result of the PU activity, the SU's offered load, and the activity of other SUs. We define

$$q_{i,k} \triangleq \hat{\theta}_{i,k}\alpha_i \mathbb{P}[\gamma_{i,k}^s < \Gamma_{i,k}], \tag{1}$$

where $\gamma_{i,k}^s$ is the signal-to-noise ratio (SNR) of the SU i during spectrum sensing (caused by the interference from other SUs' transmission in the channel k), and $\Gamma_{i,k}$ is a pre-defined SNR threshold for detecting whether the channel k is used by other SUs. The probability $\hat{\theta}_{i,k}$ denotes the SU i's PU sensing results

$$\hat{\theta}_{i,k} = \theta_k(1-\phi_{i,k}^n) + (1-\theta_k)\phi_{i,k}^p, \tag{2}$$

with the two types of spectrum sensing errors considered. Since $\gamma_{i,k}^s$ represents the mutual interference among the SUs, different SUs' Markov chains are coupled together tightly via $q_{i,k}$.

The transitional probability $z_{i,k}$ denotes the probability for the SU i to choose the channel k during the channel switching state. It is the model of spectrum access strategies. If each SU can choose one and only one channel each time, we have

$$\sum_{k=1}^{J} z_{i,k} = 1, \quad i = 1, \cdots, N. \tag{3}$$

3. THROUGHPUT EXPRESSIONS FOR HETEROGENEOUS CRN

In order to find the throughput, we need to find the steady-state probability of the MMB states first. Thanks to the separation among different SUs' Markov chains (conditioned on $q_{i,k}$), the steady-state probability of all the $N(2J+1)$ states $\pi_{i,k}^s$, $\pi_{i,k}^d$, and π_i^c can be derived in closed-form. Specifically, based on Fig. 1, the SU i's $2J+1$ steady-state probabilities are the solutions to

$$\begin{bmatrix} \mathbf{A}_1 & & & \mathbf{a}_1 \\ & \ddots & & \vdots \\ & & \mathbf{A}_J & \mathbf{a}_J \\ \mathbf{b}_1^T & \cdots & \mathbf{b}_J^T & a \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_J \\ \pi_i^c \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ 0 \end{bmatrix}, \tag{4}$$

where $\mathbf{b}_k^T = \begin{bmatrix} 1-q_{i,k} & 0 \end{bmatrix}$, $\mathbf{a}_k = \begin{bmatrix} z_{i,k} & 0 \end{bmatrix}^T$, and

$$\mathbf{A}_k = \begin{bmatrix} -1 & 1 \\ q_{i,k} & -1 \end{bmatrix}, \quad \mathbf{x}_k = \begin{bmatrix} \pi_{i,k}^s \\ \pi_{i,k}^d \end{bmatrix}, \quad a = -\sum_{k=1}^{J} z_{i,k}. \tag{5}$$

From [11], under the constraint $\pi_i^c + \sum_{k=1}^J(\pi_{i,k}^d + \pi_{i,k}^s) = 1$, the steady state probabilities are

$$\begin{cases} \pi_{i,k}^s = \frac{z_{i,k}}{2(1-q_{i,k})\sum_{\ell=1}^J \frac{z_{i,\ell}}{1-q_{i,\ell}}}, \\ \pi_{i,k}^d = q_{i,k}\pi_{i,k}^s, \\ \pi_i^c = \frac{1}{2\sum_{\ell=1}^J \frac{z_{i,\ell}}{1-q_{i,\ell}}}. \end{cases} \tag{6}$$

To evaluate the state probabilities in (6), we need to analyze $q_{i,k}$, which is to analyze the mutual interference among all the SUs. From the definition in (1), $q_{i,k}$ means that the SU i can use the channel k to transmit a data packet when there is no PU activity sensed in this channel, the SU i has a data packet to transmit, and the SNR $\gamma_{i,k}^s$ indicates that the interference from all the other SUs in this channel is low enough.

The SNR $\gamma_{i,k}^s$ is in fact the interference-to-noise ratio in the spectrum sensing state,

$$\gamma_{i,k}^s = \frac{1}{\sigma_{i,k}^2}\sum_{j=1, j\neq i}^{N} P_{j,k}g_{j,i,k}f_{j,k} \tag{7}$$

where $\sigma_{i,k}^2$ is the SU i receiver's noise power in the channel k, $P_{j,k}$ is the SU j's transmission power, $g_{j,i,k}$ is the channel gain from the SU j's transmitter to the SU i's receiver, and $f_{j,k}$ is a binary indicator. Specifically, $f_{j,k} = 1$ means that the SU j is transmitting in the channel k, while $f_{j,k} = 0$ means the opposite.

The indicator $f_{j,k}$ can be modeled as a Bernoulli random variable with probability

$$\beta_{j,k} \triangleq \mathbb{P}[f_{j,k} = 1] = \frac{\pi_{j,k}^d T_{j,k}^d}{\pi_j^c T_j^c + \sum_{\ell=1}^J \left(\pi_{j,\ell}^s T_{j,\ell}^s + \pi_{j,\ell}^d T_{j,\ell}^d\right)}. \tag{8}$$

One of the advantages of MMB is that the SU's activities can be described simply by the steady-state probabilities (6). From (6), we can derive

$$\beta_{j,k} = \frac{z_{j,k}\frac{q_{j,k}T_{j,k}^d}{(1-q_{j,k})T_j^c}}{1 + \sum_{\ell=1}^J \frac{z_{j,\ell}}{1-q_{j,\ell}}\left(\frac{T_{j,\ell}^s}{T_j^c} + q_{j,\ell}\frac{T_{j,\ell}^d}{T_j^c}\right)}. \tag{9}$$

Based on (1), (7) and (9), the transitional probability $q_{i,k}$ is the cumulative distribution of the weighted summation of $N-1$ Bernoulli random variables $f_{j,k}$, $j = 1, \cdots, N$, and $j \neq i$. If the number of SUs N is small enough, we can search exhaustively through all the 2^{N-1} different $f_{j,k}$ combinations to calculate $q_{i,k}$, which is detailed below.

First, define the set of $N-1$ SU indices

$$\mathcal{S}_i \triangleq \{1, \cdots, i-1, i+1, \cdots, N\}. \tag{10}$$

The set $\mathcal{S}_i$ has 2^{N-1} distinct subsets $\mathcal{S}_i(\ell) \subseteq \mathcal{S}_i$, where $\ell = 1, \cdots, 2^{N-1}$. We use $\mathcal{S}_i(\ell)$ to denote the case when all the SUs listed in this subset are transmitting in the channel k, while none of the SUs not listed in this subset is transmitting in the channel k. In this case, we define

$$u_{i,k}(\ell) \triangleq \mathcal{I}_{i,k}(\ell) \prod_{j\in\mathcal{S}_i(\ell)} \beta_{j,k} \prod_{j\notin\mathcal{S}_i(\ell)} (1-\beta_{j,k}). \tag{11}$$

where the indicator function

$$\mathcal{I}_{i,k}(\ell) \triangleq \begin{cases} 1, & \text{if } \sum_{j\in\mathcal{S}_i(\ell)} P_{j,k}g_{j,i,k} < \sigma_{i,k}^2\Gamma_{i,k} \\ 0, & \text{else} \end{cases} \tag{12}$$

Then, based on the definitions in (10)-(12), the transitional probability $q_{i,k}$ in (1) can be calculated as

$$q_{i,k} = \hat{\theta}_{i,k}\alpha_i \sum_{\ell=1}^{2^{N-1}} u_{i,k}(\ell). \tag{13}$$

With the steady state probabilities (6) and $q_{i,k}$ (13), we can derive the throughput of each SU. The throughput of an SU in a channel is defined as the product between the channel capacity and the data transmission duty ratio which is the fraction of time spent in successful data transmission. Similar to (13), searching exhaustively over all the subsets $\mathcal{S}_i(\ell)$, we can find the throughput of the SU i in all the J channels as

$$R_i = \sum_{k=1}^{J} \frac{\theta_k(1-\phi_{i,k}^n)\beta_{i,k}}{\hat{\theta}_{i,k}} \frac{\sum_{\ell=1}^{2^{N-1}} \log(1+\gamma_{i,k}(\ell))u_{i,k}(\ell)}{\sum_{\ell=1}^{2^{N-1}} u_{i,k}(\ell)}. \tag{14}$$

In (14), the interference from all the other SUs is reflected in the channel capacity $\log(1+\gamma_{i,k}(\ell))$ via the SNR

$$\gamma_{i,k}(\ell) = \frac{P_{i,k}g_{i,i,k}}{\sum_{j\in\mathcal{S}_i(\ell)} P_{j,k}g_{j,i,k} + \sigma_{i,k}^2}. \tag{15}$$

We need the denominator in (14) because the fraction in (14) is the average channel capacity and many $u_{i,k}(\ell)$ are zero.

The exhaustive method of evaluating R_i (14) is general and accurate. In particular, since it considers exhaustively all possible simultaneous transmissions of the SUs, it is powerful to analyze the complex mutual coupling among the SUs. However, the computational complexity becomes prohibitively high for larger CRN. In the sequel, we will develop computationally efficient expressions.

Just as the practical MAC-layer protocols like CSMA, the majority of CRN spectrum access strategies allow only one SU to use a channel within a certain geographical area at a time. Because the objective of our method is to evaluate the coexistence of practical spectrum access strategies, we exploit this property to avoid considering many simultaneous transmissions, which is the key to reduce drastically the computational complexity.

Specifically, if the channel k is sensed as available to the SU i, then it means that any SU in the set $\mathcal{N}_{i,k} = \{j \mid P_{j,k}g_{j,i,k} \geq \sigma_{i,k}^2\Gamma_{i,k}\}$ is not transmitting in this channel. Define another small enough threshold $\Gamma'_{i,k} \leq \Gamma_{i,k}$. We further skip the interference from all the SUs in the set $\mathcal{M}_{i,k} = \{j \mid P_{j,k}g_{j,i,k} < \sigma_{i,k}^2\Gamma'_{i,k}\}$ because their impact to the SU i is negligibly small. Re-define the set $\mathcal{S}_i$ in (10) to

$$\mathcal{S}_{i,k} = \{j \mid 1 \leq j \leq N, j \neq i, j \notin \mathcal{N}_{i,k}, j \notin \mathcal{M}_{i,k}\}, \tag{16}$$

which has $L_{i,k}$ elements. We use $\mathcal{S}_{i,k}(\ell) \in \mathcal{S}_{i,k}$ to denote its $2^{L_{i,k}}$ subsets. Then, the transitional probability and the throughput expressions (13)(14) become

$$q_{i,k} = \hat{\theta}_{i,k}\alpha_i \prod_{j\in\mathcal{N}_{i,k}} (1-\beta_{j,k}) \sum_{\ell=1}^{2^{L_{i,k}}} u_{i,k}(\ell) \tag{17}$$

$$R_i = \sum_{k=1}^{J} \frac{\theta_k(1-\phi_{i,k}^n)\beta_{i,k}}{\hat{\theta}_{i,k}} \times \frac{\sum_{\ell=1}^{2^{L_{i,k}}} \log(1+\gamma_{i,k}(\ell))u_{i,k}(\ell)}{\sum_{\ell=1}^{2^{L_{i,k}}} u_{i,k}(\ell)}. \tag{18}$$

Note that we just need to replace $\mathcal{S}_i(\ell)$ with $\mathcal{S}_{i,k}(\ell)$. Since we can make $L_{i,k} \ll N$, we can reduce greatly the computational complexity. The error of R_i due to skipping $\mathcal{M}_{i,k}$ is usually negligibly small.

In particular, if the threshold $\Gamma_{i,k}$ is small enough, we can simply let $\Gamma'_{i,k} = \Gamma_{i,k}$. In this case, $\mathcal{S}_{i,k}$ is empty, $L_{i,k} = 0$, and we have extremely simplified expressions

$$q_{i,k} = \hat{\theta}_{i,k}\alpha_i \prod_{j\in\mathcal{N}_{i,k}} (1-\beta_{j,k}), \tag{19}$$

$$R_i = \sum_{k=1}^{J} \frac{\theta_k(1-\phi_{i,k}^n)\beta_{i,k}}{\hat{\theta}_{i,k}} \log\left(1+\frac{P_{i,k}g_{i,i,k}}{\sigma_{i,k}^2}\right). \tag{20}$$

There is no exhaustive search any more. The complexity is reduced from $O(2^N)$ to $O(N)$.

4. SPECTRUM ACCESS STRATEGIES

The effect of spectrum access strategies to the CRN throughput performance is reflected in the transitional probabilities $z_{i,k}$. A unique advantage of MMB is that $z_{i,k}$ can be derived for each individual SU and for each individual spectrum access strategy separately, without considering explicitly other SUs or other spectrum access strategies. The resulted $z_{i,k}$ can be applied directly to all the expressions in Section 3. A pure random spectrum access strategy was used in [11], whereas $z_{i,k}$ were optimized in [12]. However, it is unknown what is the $z_{i,k}$ for some typical and practical spectrum access strategies. In this section, we address this issue.

The first strategy is listen-before-talk (LBT) [1] with **Random** channel selection. With this strategy, each SU selects randomly one of the J channels with equal probability. Therefore, we have

$$z_{i,k} = \frac{1}{J}. \tag{21}$$

The second strategy is LBT with **Ordered** (or Round-Robin) channel selection. Each SU i examines the channels according to the ordered list $\{r(1), r(2), \cdots, r(J)\}$ where $1 \leq r(j) \leq J$. In particular, this can model the strategies when the SU intends to stay on the same channel whenever the data transmission is successful. In this case, we just need to let $r(1)$ to be the currently used channel.

Proposition 1. The transitional probabilities $z_{i,r(k)}$ for the LBT with ordered channel selection are

$$z_{i,r(k)} = \prod_{j=1}^{k-1} \alpha_i^{-1}(1-z_{i,r(j)})q_{i,r(k)}, \quad k=1,\cdots,J, \tag{22}$$

with $z_{i,r(1)} = \alpha_i^{-1}q_{i,r(1)}$ and $z_{i,r(J)} = \prod_{j=1}^{J-1}(1-z_{i,r(j)})$.

Proof. The probability for the SU i to select the first channel $r(1)$ is $z_{i,r(1)} = \hat{\theta}_{i,r(1)}\mathbb{P}[\gamma_{i,r(1)}^s < \Gamma_{i,r(1)}]$, which is $\alpha_i^{-1}q_{i,r(1)}$. The probability of selecting the second channel $r(2)$ is $z_{i,r(2)} = (1-\hat{\theta}_{i,r(1)})\mathbb{P}[\gamma_{i,r(1)}^s < \Gamma_{i,r(1)}])\hat{\theta}_{i,r(2)}\mathbb{P}[\gamma_{i,r(2)}^s < \Gamma_{i,r(2)}]$, which equals $\alpha_i^{-1}(1-z_{i,r(1)})q_{i,r(2)}$. By induction we can get (22). $\square$

The third strategy we consider in this paper is the **Game** theoretic spectrum access strategy in [2]. The idea is to select the channel to minimize both the interference to all the other SUs and the interference from all the other SUs, i.e.,

$$\max_{\{k\}} \; -\sum_{j=1,j\neq i}^{N} P_{j,k}g_{j,i,k}f_{j,k} - \sum_{j=1,j\neq i}^{N} P_{i,k}g_{i,j,k}f_{j,k}. \tag{23}$$

$$= \sum_{j\neq i}(-P_{j,k}g_{j,i,k} - P_{i,k}g_{i,j,k})f_{j,k} \triangleq \sum_{j\neq i} h_{i,j,k}f_{j,k}.$$

It was shown in [2] that (23) is a potential game, which means that some distributed implementation of (23) guarantees convergence to the Nash Equilibrium. Nevertheless, the performance analysis is non-trivial, especially when the CRN is heterogeneous.

Proposition 2. The transitional probabilities $z_{i,k}$ for the SU i using the strategy (23) are

$$z_{i,k} = \sum_{\ell=1}^{2^{N-1}} \mathcal{J}_{i,k}(\ell) \prod_{j\in\mathcal{S}_i(\ell)} \beta_{j,k} \prod_{j\notin\mathcal{S}_i(\ell)} (1-\beta_{j,k}), \tag{24}$$

where

$$J_{i,k}(\ell) = \begin{cases} 1, & \text{if } \sum_{j\in\mathcal{S}_i(\ell)} h_{i,j,k} \geq \\ & \quad \max_{m\neq k} \sum_{j\notin\{i\cup\mathcal{S}_i(\ell)\}} h_{i,j,m}\beta_{j,m} \\ 0, & \text{else} \end{cases} \tag{25}$$

Proof. According to (23), we have $z_{i,k} = \mathbb{P}[\sum_{j\neq i} h_{i,j,k} f_{j,k} \geq \sum_{j\neq i} h_{i,j,m} f_{j,m}]$ for all $m = 1, \cdots, J$ and $m \neq k$. Considering the exhaustive search over all combination of $f_{j,k}$, we can obtain (24)-(25). □

Note that reduced complexity expressions are also available, similar to the derivation of (18) or (19).

In summary, to calculate the throughput of the heterogeneous CRN, first, we calculate $\{q_{i,k}, z_{i,k}\}$ by solving a system of non-linear equations (e.g., (13) and (24), or their reduced-complexity alternatives). The full-complexity version involves at most $2NJ$ non-linear equations, whose roots can be found numerically by, e.g., the function fsolve() in MATLAB. Then, we calculate the throughput $\{R_i\}$ via (14) or the reduced-complexity alternative (18) or (19).

5. SIMULATIONS

We evaluated numerically the analysis results in Sections 3 and 4 and compared them with the Monte-Carlo simulation of the heterogeneous CRN. We simulated a CRN where the SUs' positions were randomly generated within a square of 1000×1000 meters. The path-loss were calculated as $10^8 d_{ij}^{-2.6}$ where d_{ij} is the propagation distance.

First, we calculated the sum throughput $R = \sum_{i=1}^{N} R_i$ for each of the three spectrum access strategies (**Random**, **Ordered**, **Game**), and showed that the results fit well with the simulated results, as can be seen in Fig. 2.

Next, we simulated both homogeneous CRN and heterogeneous CRN where SUs have equal probability of adopting the three strategies. We calculated the average throughput per user for SUs in each strategy category. Results in Fig. 3 indicate that the **Ordered** and the **Game** strategies out-compete the **Random** strategy. The unfairness in spectrum access due to competition can be seen even more clearly in Fig. 4, where the SUs with the **Ordered** strategy can gain much more throughput than the SUs with the **Random** strategy. The unfairness due to competition becomes more severe in larger heterogeneous CRN.

6. CONCLUSION

This paper develops MMB (Markov Model Bank) as an efficient approach to analyze the heterogeneous CRN and the competition of multiple spectrum access strategies. Throughput is derived analytically with the consideration of spectrum sensing errors and some practical spectrum access strategies. Simulation results show the severe competition unfairness among the spectrum access strategies.

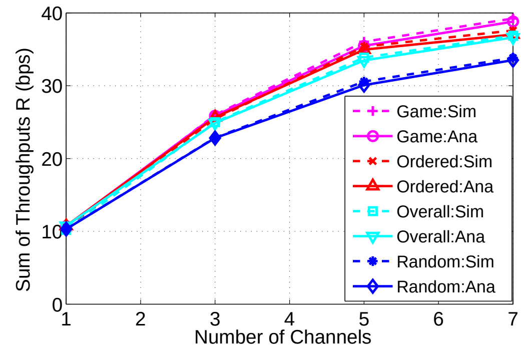

Fig. 2. CRN sum throughput for the three spectrum access strategies: analysis results fit well with simulation results.

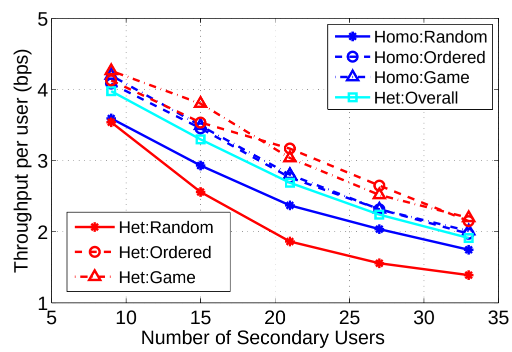

Fig. 3. Average throughput per user for homogeneous CRN and for heterogeneous CRN, with the three spectrum access strategies.

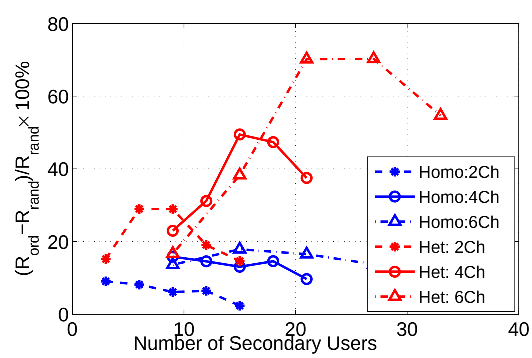

Fig. 4. SUs with **Ordered** LBT tend to have larger competitive advantage over SUs with **Random** LBT in larger CRNs with more SUs or more channels.

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