Dynamic Behavior and Coexistence of Intelligent Radio Spectrum Access Systems

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Outline

1. Introduction to this project
   - Motivation, objectives, expected outcomes
2. Major results
   - Optimal CRN throughput/capacity
   - Heterogeneous CRN modeling and throughput analysis
3. Simulations
4. Conclusions

1. Introduction to this project

- Heterogeneous dynamic spectrum access (DSA) systems
  - Flexible spectrum sensing/access strategies
  - Flexible transmission parameters
  - Flexible software implementations
- Coexistence of heterogeneous intelligent users
  - Competition and cooperation → complex dynamics → impact efficiency & fairness of spectrum access

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- Objective of this project
  - Develop a framework for modeling and analyzing coexistence behavior of heterogeneous DSA systems
    - Support new DSA techniques/systems development
  - Employ thought-provoking methodologies from theoretical ecology to study coexistence of intelligent users
    - Evolution of cooperation, population dynamic models
  - Promote integration between wireless communications and theoretical ecology

- Expected outcomes
  - DSA analysis framework
    - Developing techniques integrating Markov Model Bank, evolutionary game theory, evolution of cooperation, etc
    - Modeling and analyzing dynamic interactions among different DSA strategies
  - Spectrum-usage model stimulated by similar population dynamic models in ecosystems
    - Modeling and analyzing spectrum sharing of large DSA systems
  - A framework for DSA policy modeling and analysis
    - Support policy design and optimization

2. Major Results

- What is the best a DSA/CRN can do?
  - Formulated sum-of-ratios linear fractional programming (SoR-LFP) to derive theoretically optimal CRN throughput
    - A benchmark for evaluating the optimality of practical DSA/CRN strategies
- What is the performance of practical CRN?
  - Developed Markov Model Bank (MMB) to model heterogeneous CRN and to analyze throughput
  - Developed Network decomposition techniques for feasible and efficient analysis

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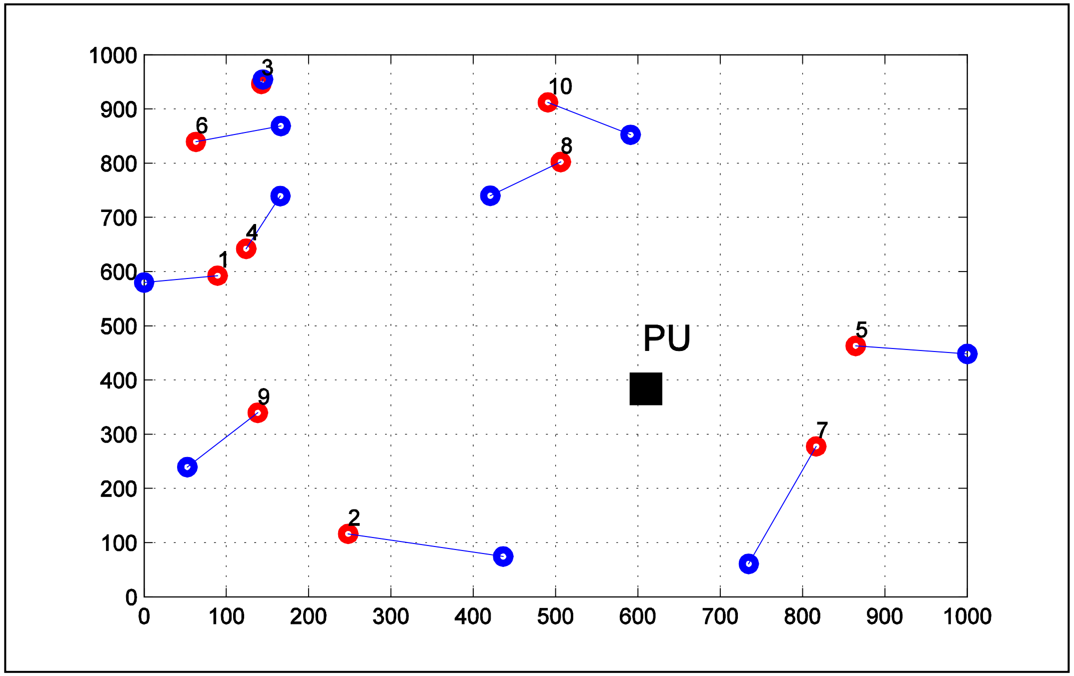
2.1 System Model

- Consider CRN with I secondary users (SU) and K channels
  - Channel available probability θ_k, SU offered load α_i

SUs optimize their channel access (via power control) for best CRN throughput.



- What is the throughput of the CRN under heterogeneous setting?
- Heterogeneous CRN performance analysis is challenging
  - Mostly done by simulation rather than analysis
  - Limited analysis results exist for simplified & homogeneous CRN, or for small CRN with a few users only
  - Optimal benchmark performance is unknown

2.2 Optimal throughput

- Optimal power control for max sum-capacity

$$\sum_{i=1}^{I}\log\left(1+\frac{P_i h_i}{\sum_{j\neq i}P_j h_j+P_N}\right)$$

  - Centralized optimization: non-convex, still a challenge
  - Distributed optimization: Iterative water-filling, various game-theoretic solutions, etc
- We explore: sum-of-ratios linear fractional programming (SoR-LFP)

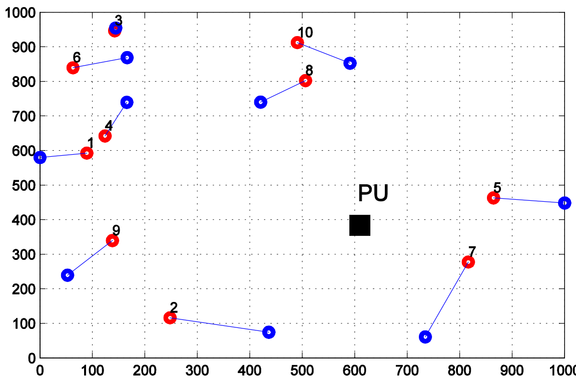
- Assume SUs allocate powers optimally among all channels under individual power constraint

$$0 \le \sum_{k=1}^{K} P_i^k \le \overline{P}_i$$

- Basic equations for SU
  - Signal, SNR, sum throughput

$$y_i^k(n) = \sqrt{P_i^k}\, h_{ii}^k s_i(n) + \sum_{j=1, j\neq i}^{I} \sqrt{P_j^k}\, h_{ji}^k s_j(n) + v_i^k(n)$$

$$\gamma_i^k = \frac{P_i^k |h_{ii}^k|^2}{\sum_{j=1, j\neq i}^{I} P_j^k |h_{ji}^k|^2 + \sigma_i^{k\,2}}, \qquad R = \sum_{i=1}^{I} R_i$$



- Formulation of centralized optimization problem

$$R = \max_{\{P_i^k\}} \sum_{i=1}^{I} \alpha_i \sum_{\ell=1}^{L_m} \log\left(1 + \frac{P_i^{k_\ell} |h_{ii}^{k_\ell}|^2}{\sum_{j=1, j\neq i}^{I} P_j^{k_\ell} |h_{ji}^{k_\ell}|^2 + \sigma_i^{k_\ell\,2}}\right)$$

$$\text{s.t.} \quad \sum_{\ell=1}^{L_m} P_i^{k_\ell} \leq \overline{P}_i, \;\; P_i^{k_\ell} \geq 0.$$

- We find this can be treated as a variation of SoR-LFP. Other variations include popular metrics like $\sum_i \gamma_i, \quad \sum_i 10\log_{10}\gamma_i, \quad \prod_i R_i$

- Sum-of-ratios linear fractional programming

$$\max_{\{x_1,\cdots,x_J\}} \sum_{i=1}^{I} \frac{a_{i0}+a_{i1}x_1+\cdots+a_{iJ}x_J}{b_{i0}+b_{i1}x_1+\cdots+b_{iJ}x_J}$$

  - A global optimization problem that has wide applications, decades of research
  - Generally non-convex. But there are some algorithms to solve it.
  - Great effort is still needed to revise/re-develop the algorithms to solve our problems.

4-ratio 4-variable example ($0 \le x_i \le 1$):

$$\gamma_1 = \frac{5.4+1.6x_1+1.7x_2+6.9x_3+2.3x_4}{8.7+1.1x_1+8.2x_2+2.6x_3+1.8x_4}$$

$$\gamma_2 = \frac{10+7.9x_1+6x_2+7.5x_3+9.1x_4}{5.8+9.6x_1+8.7x_2+8x_3+2.6x_4}$$

$$\gamma_3 = \frac{0.8+3.1x_1+2.6x_2+4.5x_3+1.5x_4}{5.5+0.8x_2+4.3x_3+1.5x_4}$$

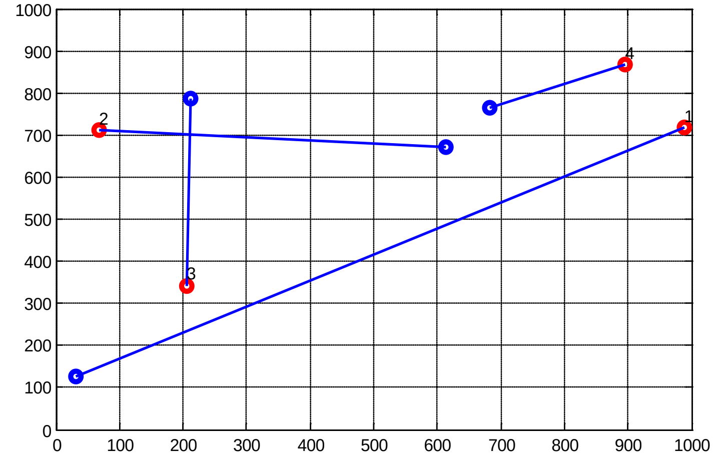
$$\gamma_4 = \frac{4.4+5.3x_1+6.5x_2+0.8x_3+8.3x_4}{1.4+7.7x_1+4x_2+9.1x_3+1.4x_4}$$

$$\max \sum_{i=1}^{4} \gamma_i = 7.8, \quad \max \sum_{i=1}^{4} 10\log_{10} \gamma_i = 3.9$$

$$\max \sum_{i=1}^{4} \log_2(1+\gamma_i) = 5.4$$

$$\max \prod_{i=1}^{4} \log_2(1+\gamma_i) = 1.4$$

Example: 4 users access 1 channel



$$\gamma_\Sigma = \underset{[x_1,x_2,x_3,x_4]}{\text{Max}} \sum_{i=1}^{4} \gamma_i$$

Optimal solution:

$[x_1, x_2, x_3, x_4] = [0,0,0,1]$

$\gamma_\Sigma = 6.75$

$$R = \underset{[x_1,x_2,x_3,x_4]}{\text{Max}} \sum_{i=1}^{4} \log_2(1+\gamma_i)$$

Optimal solution:

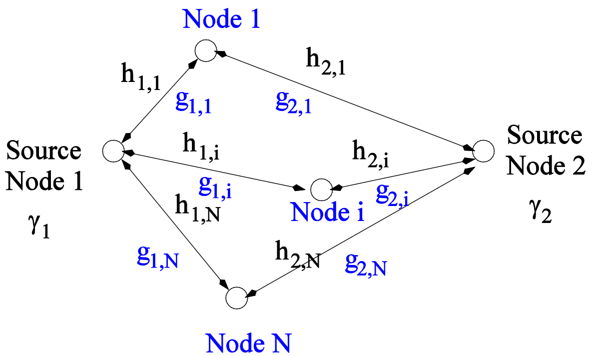
$[x_1, x_2, x_3, x_4] = [0,0,1,1]$

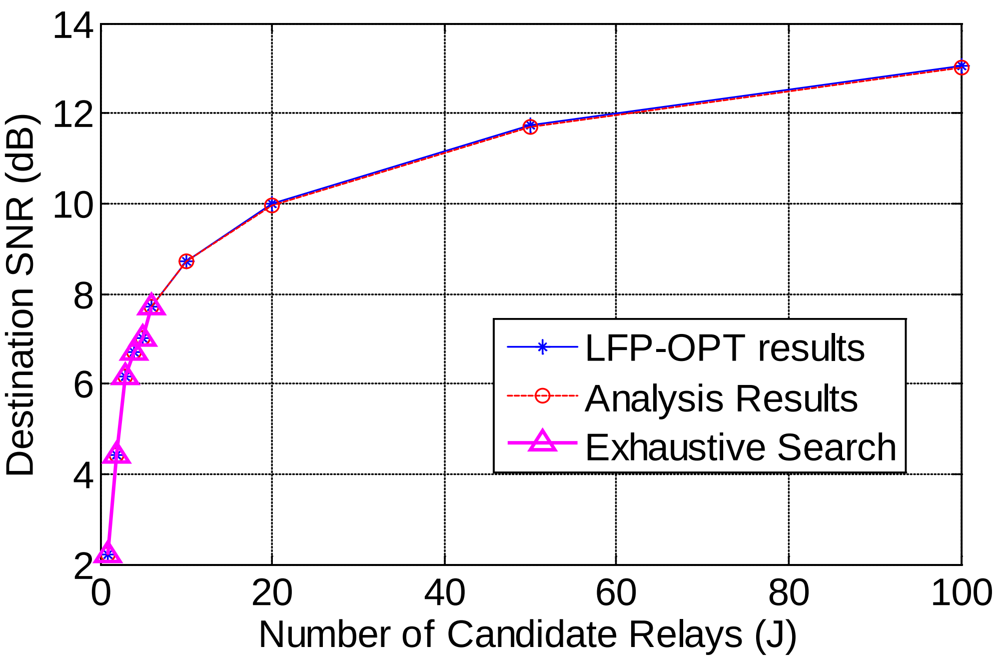
$R = 3.39$

- Our current algorithm can work with a large number of variables, but with a few ratios only
  - Need to improve convergence if there are more ratios

Simulation of two-way relaying:

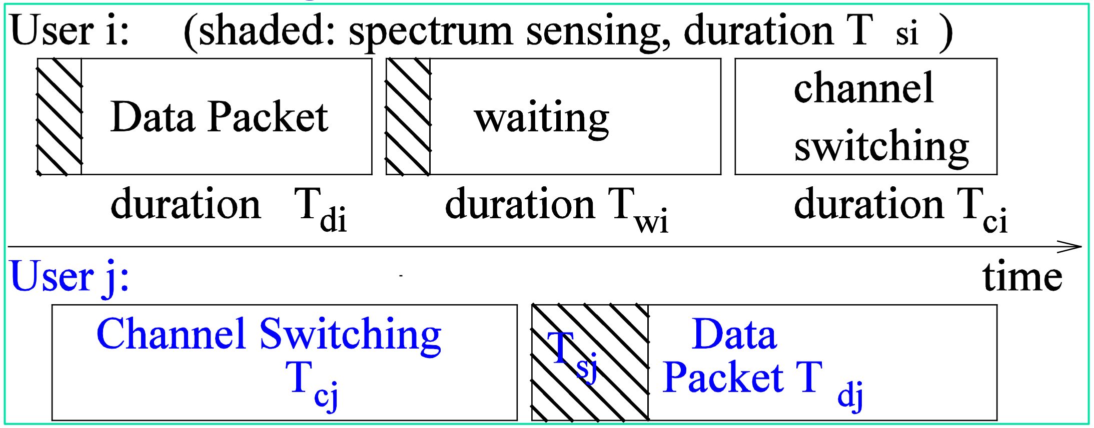
$$\max_{\{\mathbf{z}\}} \gamma_1 + \gamma_2 = \frac{(\mathbf{a} \odot \mathbf{b})^T \mathbf{z}}{1 + \mathbf{b}^T \mathbf{z}} + \frac{(\mathbf{c} \odot \mathbf{d})^T \mathbf{z}}{1 + \mathbf{d}^T \mathbf{z}},$$



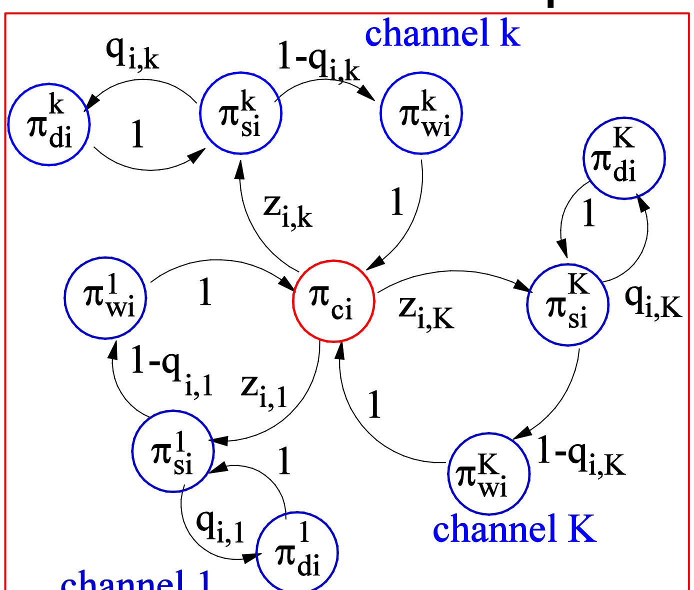


2.3 Modeling CRN and Analyzing Throughput

- Model CRN's four basic working modes
  - Spectrum sensing: duration T_{si}^{k}, SNR threshold Γ_{si}^{k}
  - Spectrum access (data packet transmission): duration T_{di}^{k}, max transmission power $\overline{P}_{i}$
  - Idling: duration T_{wi}^{k}
  - Channel switching: duration T_{ci}^{k}



- Markov model bank (MMB)
  - N Markov chains: A separate chain for each user
  - $3K+1$ states in each separated Markov chain
  - Users & chains connected implicitly by transitional probability q_{si}^{k}



q_{si}^{k} : prob. of channel sensed available

$z_{i,k}$: prob. of channel selection

π_{si}^{k} : prob. of spectrum sensing

π_{di}^{k} : prob. of data transmission

π_{wi}^{k} : prob. of ideling

π_{ci} : prob. of channel switching

- Essential idea of MMB
  - Reduce complexity of Markov chains, leave complexity to transitional probability analysis
    - Convenient for modeling heterogeneous systems
    - Feasible mutual interference analysis
    - Efficient network decomposition
- Steady-state probability

$$\begin{bmatrix} \mathbf{A}_1 & & & \mathbf{a}_1 \\ & \ddots & & \vdots \\ & & \mathbf{A}_K & \mathbf{a}_K \\ \mathbf{b} & \cdots & \mathbf{b} & -1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_K \\ \pi_{ci} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ 0 \end{bmatrix}$$

$$\mathbf{A}_k = \begin{bmatrix} -1 & 1 & 0 \\ q_{si}^k & -1 & 0 \\ 1-q_{si}^k & 0 & -1 \end{bmatrix}, \mathbf{x}_k = \begin{bmatrix} \pi_{si}^k \\ \pi_{di}^k \\ \pi_{wi}^k \end{bmatrix}$$

$$\Rightarrow$$

$$\pi_{si}^k = \frac{z_{i,k}}{\left(1+2\sum_{\ell=1}^{K}\frac{z_{i,\ell}}{1-q_{i,\ell}}\right)(1-q_{i,k})}$$

- General throughput expression

$$R_i = \sum_{k=1}^{K} U_{i,k} \sum_{\ell=1}^{2^{N-1}} \log(1+\gamma_{i,k}(\ell)) \prod_{j\in S_{i,\ell}} \beta_{j,k} \prod_{j\notin\{i\bigcup S_{i,\ell}\}} (1-\beta_{j,k})$$

$$q_{i,k} = \theta_k \alpha_i \sum_{\ell=1}^{2^{N-1}} I_{i,k}(\ell) \prod_{j\in S_{i,\ell}} \beta_{j,k} \prod_{j\notin\{i\bigcup S_{i,\ell}\}} (1-\beta_{j,k})$$

$$\beta_{j,k} = \frac{z_{j,k} q_{j,k} T_{dj,k}}{1-q_{j,k}} \frac{1}{T_{cj} + \sum_{\ell=1}^{K} \left[T_{sj,\ell} + q_{j,\ell} T_{dj,\ell} + (1-q_{j,\ell}) T_{wj,\ell} \right] \frac{z_{j,\ell}}{1-q_{j,\ell}}}$$

  - Complexity is high since all users ($i = 1 \ldots I$) and channels ($k = 1 \ldots K$) are coupled together

- Apply network decomposition for efficiency
  - Spatial decoupling: separate weak interferer from strong interferer, like CSMA
  - Channel decoupling: users in different channels become uncorrelated, via translation of $z_{i,k}$ to $x_{i,k}$

$$R_i = \sum_{k=1}^{K} c_{i,k} q_{i,k} x_{i,k}, \quad q_{i,k} = \frac{1}{a_{i,k}} \prod_{j=1, j \neq i}^{N} (1 - b_j q_{j,k} x_{j,k})$$

Removed $q_{j,\ell}$

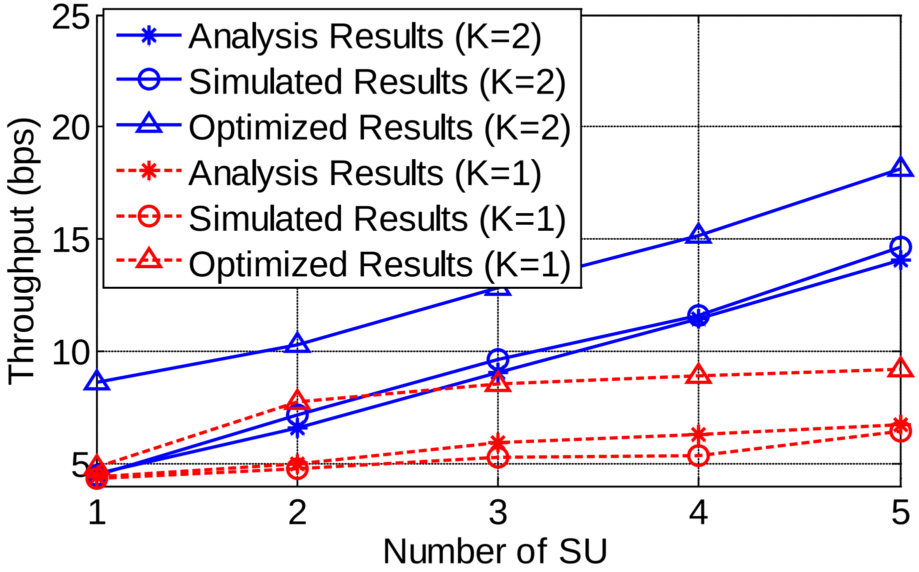
  - User decoupling: each user's throughput can be evaluated individually, via invariance property

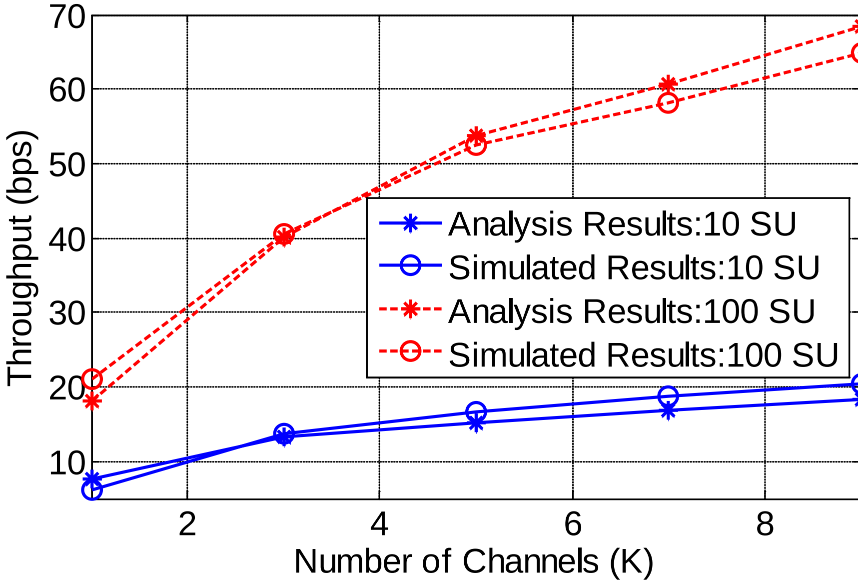
$$q_{i,k} = \frac{a_{i,k} - \sqrt{a_{i,k}^2 - 4 a_{i,k} b_i x_{i,k} D_k}}{2 a_{i,k} b_i x_{i,k}}$$

Removed $q_{j,k}$

3. Simulations

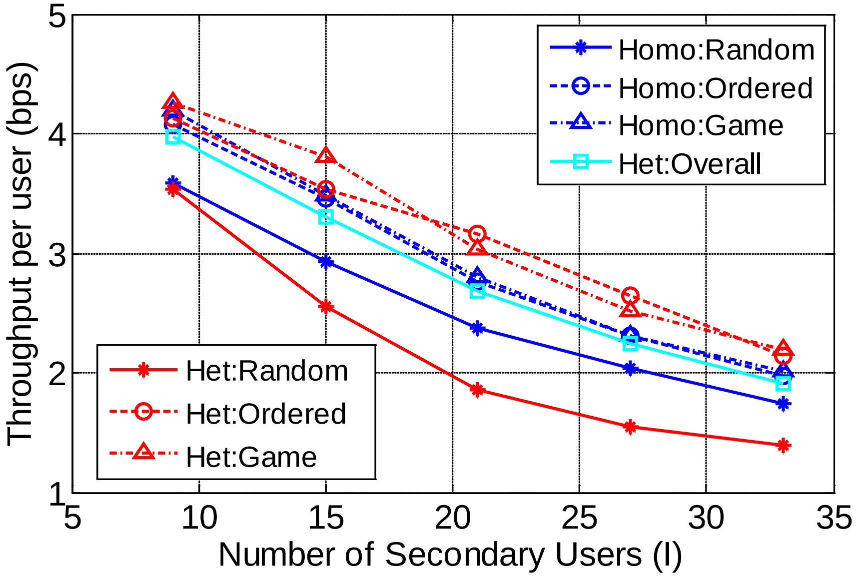
Random network with distance-based path-loss model. Random PU activity in K white-space channels.

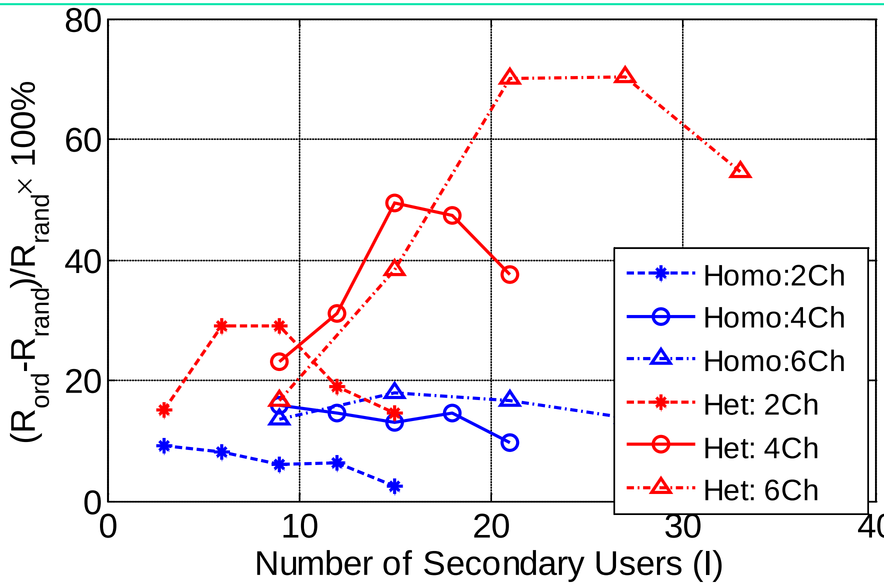




1. Analysis results are verified as accurate.
2. Gap between CRN throughput and optimized throughput.

Random network. Three access strategies: random, fixed order, potential game (minimize interference).





1. Coexistence reduces throughput of random-access strategies.
2. Unfairness is more severe for larger networks.

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4. Conclusions

- This project is to develop a framework to study the coexistence of heterogeneous DSA systems, inspired by theoretical ecology.
- We developed Markov Model Bank (MMB) to model and analyze CRN,
  - MMB allows network decomposition for efficient analysis.
- We formulated Sum-of-Ratios Linear Fractional Programming (SoR-LFP) for benchmark optimal CRN throughput.