OFDM Transmission Scheme for Asynchronous Two-Way Multi-Relay Cooperative Networks with Analog Network Coding

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***Abstract*—For two-way relaying assisted by analog network coding, most investigation so far is based on perfect synchronization assumption. In contrast, in this paper we consider the more practical asynchronism assumption, and develop a new OFDM transmission scheme that is robust to the lack of synchronization in both timing and carrier frequency. In our scheme, the relays' signals are constructed by fusing several OFDM symbols received from the source nodes transmissions. The source node receivers can successfully demodulate the received OFDM signals after mitigating effectively multiple carrier frequency offsets and multiple timing phase offsets. Simulations are conducted to demonstrate its superior performance. This scheme has the same bandwidth efficiency as the conventional OFDM transmission, and can achieve the same relaying gain as the existing multiple relay transmissions. By relieving the stringent synchronization requirement, this scheme leads to simplified relay design, which makes it more practical to exploit multiple relays in two-way relaying networks.**

Keywords*—*Two-way relaying, analog network coding, OFDM, synchronization, carrier frequency offset

I. Introduction

Cooperative relaying is one of the important technologies for the LTE and the future cellular systems. Compared with the one-way cooperative relaying that has been used in the 4G systems currently, two-way cooperative relaying is much more promising because it is able to exploit analog (physical-layer) network coding (ANC) to drastically boost transmission bandwidth efficiency. In two-way relaying, all source nodes broadcast their transmitted signals simultaneously to the relays. These signals are combined in the air before reaching the relaying nodes, which is called analog network coding [1]. The relaying nodes then broadcast the received signals back to the source nodes. With the knowledge of their own transmitted signals, the source nodes decode the ANC-coded signals and detect the signals transmitted from other sources [2][3]. Since the relays may not be able to decode the ANC coded signals, amplify-and-forward (AF) relaying is usually adopted, where each relay node simply retransmits its received signals with appropriate amplification.

In the future dense cellular systems, there is great opportunity for multiple relays to work together. Multiple relays can enhance the relaying performance much more greatly than a single relay node. In [4] [5] , closed-form solutions were developed to select optimally multiple relays for maximum destination SNR. With some efficient distributed algorithms, relays can determine whether to participate in relaying by themselves. Some necessary system information, such as the knowledge of the channels, can be obtained via feedback or reciprocity [6][7].

Unfortunately, there is a major hurdle to the practical use of multi-relay two-way relaying: the difficulty of synchronizing timing and carrier frequency among the distributed nodes. Although two-way relaying has been a focused research area for many years, most of such research are based on the ideal assumption of perfect synchronization. In reality, the distributed nature makes it almost impossible to guarantee perfect synchronization. It was shown in [8] that OFDM could be applied to tolerate certain timing mismatch, but carrier frequency mismatch was still a challenge. On the other hand, we developed asynchronous transmission techniques in [9][10], but not for two-way relaying with ANC.

In this paper, we address the issue of ANC-coded two-way relaying in frequency-selective fading channels without perfect synchronization. The "synchronization" in this paper refers specifically to the synchronization of the timing and carrier frequency of all the distributed transmitters. Signals of distributed transmitters should have the same symbol timing when arriving at a receiver, otherwise there is timing phase offset (TPO). Similarly, there is carrier frequency offset (CFO) if the signals arriving at a receiver have slightly different carrier frequencies. Many practical phenomena cause TPO and CFO, such as clock drifting and oscillator parameter drifting. Nevertheless, fundamentally, in distributed two-way relaying networks, different propagation distances among the transmitters and receivers make it almost impossible to avoid TPO. Similarly, different moving speeds/directions of the distributed nodes make it virtually impossible to avoid CFO.

In frequency selective fading environment, orthogonal frequency division multiplexing (OFDM) is a popular choice due to its high performance and low implementation cost. Unfortunately, OFDM is especially sensitive to CFO [11]. As a

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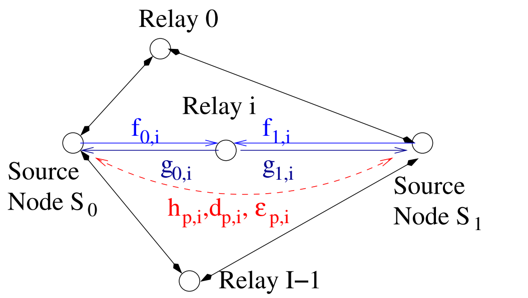

Fig. 1. Two-way relaying network with two source nodes and I relay nodes, where $f_{p,i}(n)$, $g_{p,i}(n)$ are forward and backward channel responses, whereas $h_{p,i}(n)$, $d_{p,i}$ and $\epsilon_{p,i}$ are the composite channel response, timing phase offset (TPO), and carrier frequency offset (CFO) between the two source nodes via the relay i, respectively.

matter of fact, it has been mostly an open problem for OFDM to mitigate multiple CFOs.

We propose a new OFDM transmission scheme for distributed two-way ANC-coded relaying, considering a pair of source nodes and multiple relaying nodes with frequency selective fading channels. We exploit the special structure of the new OFDM signals to tolerate TPOs and to guarantee complete mitigation of CFOs. As a result, no accurate synchronization in timing and carrier is needed. The relays just need to fuse and filter the received baseband signals. They do not need to conduct OFDM demodulation. Therefore, effectively they are still AF relays. The filter coefficients can be optimized for enhancing signal-to-noise ratio (SNR) based on [5] or approaching beamforming performance based on [4].

The organization of this paper is as follows. In Section II, we give the system model. In Section III, we develop the new OFDM transmission scheme. Simulations are conducted in Section IV and conclusions are given in Section V.

II. System Model

Consider a wireless two-way relaying network with two source nodes (S_0 and S_1), and I relay nodes, as illustrated in Fig. 1. The forward channel from the source node S_p, $p = 0, 1$, to the relay node i, $i = 0, \cdots, I-1$, is denoted as $f_{p,i}(n)$. The backward channel from the relay node i to the source node p is $g_{p,i}(n)$. The source-to-source composite channels, TPOs and CFOs $h_{p,i}(n)$, $d_{p,i}$ and $\epsilon_{p,i}$ will be defined in the sequel. The channels are all multi-tap FIR filter channels considering frequency selective fading and sampling timing mismatch. Due to the space limit, we assume all the nodes have the knowledge of these parameters in this paper.

We assume that the network uses a dual-phase relaying scheme with analog network coding. During the first phase, the source nodes broadcast their signals to all the relays. Each of the relays receives the mixture of the two source signals, fuses and filters it appropriately. During the second phase, each relay broadcasts the resulted signal back to the two source nodes. Each of the source nodes then detects the other source node's signal. Note that each relay's transmitted signal still consists of ANC-encoded two source signals. The essential point of ANC decoding is that each source node knows its own transmitted signal, and can thus subtract it from the received mixture.

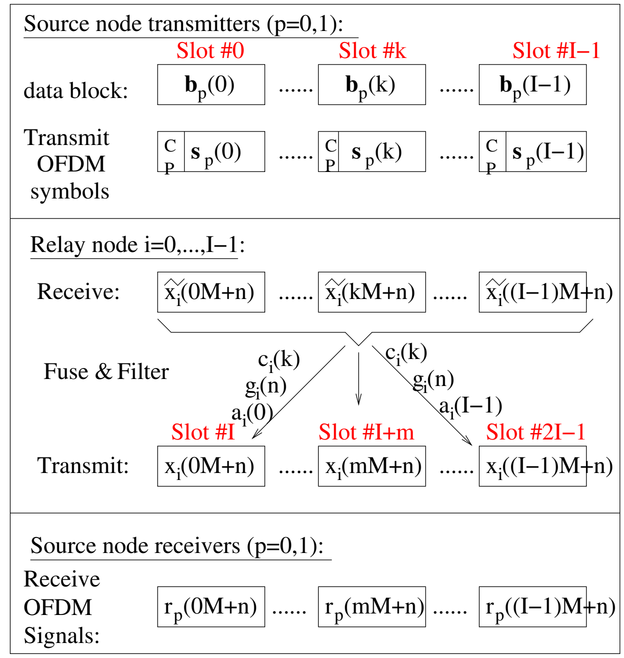

Fig. 2. A transmission session of the dual phase two-way relaying scheme. During the first I transmission slots, each of the source nodes transmits I OFDM symbols. During each of the next I transmission slots, each relay transmits a fused and filtered OFDM symbol. All the signals still keep the OFDM signal structure.

We consider a transmission session consisting of $2I$ transmission slots. Each of the source nodes uses the first I transmission slots to transmit I OFDM symbols to the relays, one OFDM symbol per slot. Each OFDM symbol consists of a block of N data symbols. Then, during the next I transmission slots, each of the relays transmits its signal back to the source nodes. This scheme is outlined in Fig. 2.

During the kth transmission slot in the first phase, where $0 \leq k \leq I-1$, each source node p has a block of data symbols $\mathbf{b}_p(k) = [b_p(kN), \cdots, b_p(kN+N-1)]^T$ to transmit. The block length N is the length of the OFDM symbol or FFT. To generate OFDM signals, the source node first conducts IFFT

$$\mathbf{s}_p(k) = \mathbf{F}^{-1}\mathbf{b}_p(k). \tag{1}$$

where $\mathbf{F}$ is the $N \times N$ FFT matrix. The vector $\mathbf{s}_p(k) = [s_p(kN), \cdots, s_p(kN+N-1)]^T$. After adding cyclic prefix (CP), the source node transmits the sequence

$$\tilde{s}_p(kM+n) = \begin{cases} s_p(kN+n), & \text{if } \ 0 \leq n \leq N-1 \\ s_p(kN+N-n), & \text{if } \ -N_c \leq n \leq -1 \\ 0, & \text{else} \end{cases} \tag{2}$$

where $M = N + N_c$, and the parameter N_c is the length of the CP, which should be sufficiently long to match the channel orders plus delays. We will specify the N_c requirements in the next section.

With the sufficiently long CP length N_c, each OFDM symbol's transmission and reception are independent from others. Therefore, without loss of generality, we consider the kth transmission slot only. The signal received by the relay node i is

$$\tilde{x}_i(kM+n) = \sum_{p=0}^{1} f_{p,i}(n) * \tilde{s}_p(kM+n-\tau_{p,i}^f) \times e^{j[\omega_{p,i}^f(kM+n)+\theta_{p,i}^f]} + v_i(kM+n). \quad (3)$$

where $*$ denotes discrete convolution, $\tau_{p,i}^f$ includes the propagation delay and timing mismatch from the source node p to the relay i (we consider explicitly the integer delays only, since the fractional parts of the delays are modeled into the channel impulse responses), $[\omega_{p,i}^f(kM+n)+\theta_{p,i}^f]$ denotes the carrier frequency/phase mismatch, and $v_i(kM+n)$ is the AWGN of the relay receiver. The two source nodes can hardly synchronize their signals to all the I relay nodes simultaneously considering the different distances and moving speeds/directions. Note that in each slot we have exactly $M = N + N_c$ samples, i.e., $-N_c \le n \le N-1$ in (3).

With all the signals received during the first I transmission slots, each relay node i constructs a new OFDM signal by fusing, filtering and scrambling,

$$x_i(mM+n) = a_i(m) q_i(n) * \sum_{k=0}^{I-1} c_i(k) \tilde{x}_i(kM+n) = a_i(m) \sum_{k=0}^{I-1} c_i(k) q_i(n) * \tilde{x}_i(kM+n), \quad (4)$$

where $m = 0, \cdots, I-1$ means that the new OFDM signal consists of I OFDM symbols (blocks), each symbol will be transmitted in a slot during the second phase. $c_i(k)$ is the fusing parameter used to combine all the I OFDM packets received during the first phase. $a_i(m)$ is the scrambling parameter which makes the new OFDM signal different for different m. $q_i(n)$ is the linear filter applied by the relay i to adjust the transmission power (in each OFDM sub-channel). Note that all the parameters $c_i(k)$, $a_i(m)$, and $q_i(n)$ can be periodic or aperiodic. For example, we can use a code $\{a_i(m) : \forall\, m\}$ that is periodic with period I.

The newly formed signal $x_i(mM+n)$ is transmitted during the second phase (i.e., the last I transmission slots). The relay i transmits $x_i(mM+n)$ during the slot $I+m$.

In our scheme, the relays do not need to demodulate the OFDM signals. They just down-convert the received signals into baseband, fuse the signals sample by sample, filter the results, up-convert them to passband before transmission. There is no accurate synchronization needed. As a matter of fact, it is allowable if the relays introduce extra TPOs and CFOs. This greatly simplifies the relay design, which is important when exploiting multiple relays.

During the last I transmission slots, the source nodes receive signal

$$r_p(mM+n) = \sum_{i=0}^{I-1} g_{p,i}(n) * x_i(mM+n-\tau_{p,i}^b) \times e^{j[\omega_{p,i}^b(mM+n)+\theta_{p,i}^b]} + \tilde{v}_p(mM+n), \quad (5)$$

where $m = 0, \cdots, I-1$ denotes the slot index (OFDM symbol index), $\tau_{p,i}^b$ is the timing mismatch of the backward transmission, $[\omega_{p,i}^b(mM+n)+\theta_{p,i}^b]$ is the carrier mismatch, and $\tilde{v}_p(mM+n)$ is the AWGN of the source node receiver.

In summary, this transmission scheme uses $2I$ transmission slots to transmit $2I$ OFDM symbols in each transmission session, and thus has the same bandwidth efficiency as the conventional OFDM which transmits one OFDM symbol in each slot. The gain of exploiting I relays is achieved in terms of diversity, which can be optimized by designing the filters $q_i(n)$, similarly as existing two-way relaying studies such as [4][5].

Based on (5), the source node p needs to detect the other source node's signal. However, there are up to I different CFOs and up to I TPOs in (5). It is a challenge to both mitigate the CFOs/TPOs while conducting OFDM demodulation. Next, we will develop efficient algorithms for each source node to successfully demodulate the received signals.

III. CFO/TPO Mitigation and Symbol Detection

A. OFDM structure of the source nodes' received signals

From the signal models (3)-(5), it is easy to see that within the signal $r_p(mM+n)$ received by the source node p, there is its own transmitted signal $\tilde{s}_p(kM+n)$, which is included by the form $\sum_{i=0}^{I-1} a_i(m) \sum_{k=0}^{I-1} c_i(k) g_{p,i}(n) * q_i(n-\tau_{p,i}^b) * f_{p,i}(n-\tau_{p,i}^b) * \tilde{s}_p(kM+n-\tau_{p,i}^b-\tau_{p,i}^f) e^{j[\omega_{p,i}^b(mM+n)+\theta_{p,i}^b+\omega_{p,i}^f(kM+n)+\theta_{p,i}^f]}$. With the knowledge about $\tilde{s}_p(kM+n)$, the source node p can estimate the necessary parameters such as the channels, and subtract $\tilde{s}_p(kM+n)$ from the mixture. This is ANC decoding.

What is remaining in the received signal includes both the unknown signal of the other source node $1-p$ and the noise. First, the noise includes both the relay amplified noise and the source node receiver's noise, which can be written as

$$u_p(mM+n) = \sum_{i=0}^{I-1} a_i(m) \sum_{k=0}^{I-1} c_i(k) g_{p,i}(n) * q_i(n-\tau_{p,i}^b) * v_i(kM+n-\tau_{p,i}^b) + \tilde{v}_p(mM+n). \quad (6)$$

With zero-mean AWGN $v_i(kM+n)$ and $\tilde{v}_p(mM+n)$, the noise (6) is zero-mean AWGN with power

$$\sigma_{u_p}^2 = \Big(\sum_{i=0}^{I-1} E[|a_i(m)|^2] \sum_{k=0}^{I-1} E[|c_i(k)|^2] E[\|g_{p,i}(n)\|^2] E[\|q_i(n)\|^2] \Big) \sigma_{v_i}^2 + \sigma_{\tilde{v}_p}^2. \quad (7)$$

If the coefficients $a_i(m)$ and $c_i(k)$ have unit magnitude and the maximum transmission power of each AF relay filter is $P_i = E[\|q_i(n)\|^2]$, then the noise power becomes

$$\sigma_{u_p}^2 = \left(I \sum_{i=0}^{I-1} P_i E[\|g_{p,i}(n)\|^2] \right) \sigma_{v_i}^2 + \sigma_{\tilde{v}_p}^2. \quad (8)$$

Next, the other source node's signal is included by the form $\sum_{i=0}^{I-1} a_i(m) \sum_{k=0}^{I-1} c_i(k) g_{p,i}(n) * q_i(n-\tau_{p,i}^b) * f_{1-p,i}(n-\tau_{p,i}^b) * \tilde{s}_{1-p}(kM+n-\tau_{p,i}^b-\tau_{1-p,i}^f) e^{j[\omega_{p,i}^b(mM+n)+\theta_{p,i}^b+\omega_{1-p,i}^f(kM+n)+\theta_{1-p,i}^f]}$.

After ANC decoding, the received signal (5) is changed to

$$y_p(mM+n) = \sum_{i=0}^{I-1} a_i(m) h_{p,i}(n) * z_i(n-d_{p,i}) \times e^{j(\epsilon_{p,i} n + \phi_{p,i})} + u_p(mM+n), \quad (9)$$

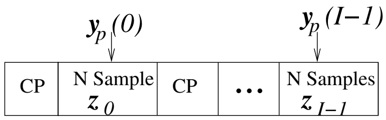

Fig. 3. Source node received signal during the second phase can be looked as the OFDM transmissions of I OFDM symbols.

where

$$z_i(n) = \sum_{k=0}^{I-1} c_i(k) e^{jkM\omega^f_{1-p,i}}\ \tilde{s}_{1-p}(kM+n), \tag{10}$$

includes all the source signal components, and

$$h_{p,i}(n) = g_{p,i}(n) * q_i(n-\tau^b_{p,i}) * f_{1-p,i}(n-\tau^b_{p,i}) \tag{11}$$

denotes the composite channel response from the source node $(1-p)$'s transmitter to the source node p's receiver via the relay node i. Similarly, $d_{p,i} = \tau^f_{1-p,i} + \tau^b_{p,i}$ is the composite TPO, $\epsilon_{p,i} = \omega^f_{1-p,i} + \omega^b_{p,i}$ is the composite CFO, and $\phi_{p,i} = mM\omega^b_{p,i} + \theta^f_{1-p,i} + \theta^b_{p,i}$ is the composite phase.

Let the length of composite channel response $h_{p,i}(n)$ be $L_{p,i}$. If the OFDM CP length satisfies

$$N_c > \max_{p=0,1;i=0,\cdots,I-1} (L_{p,i} + d_{p,i}), \tag{12}$$

then the transmission in each of the $2I$ transmission slots is not interfered by other transmissions. In other words, there is no inter-block-interference (IBI). Therefore, the signal in each of the transmission block preserves the OFDM structure. In particular, the signal $y_p(\cdot)$ can be looked as generated from transmitting I OFDM symbols, as illustrated in Fig. 3.

B. TPO/CFO resistant receiver

In the sequel, to simplify expressions, we use the maximum channel length

$$L = \max_{p=0,1;i=0,\cdots,I-1} L_{p,i}. \tag{13}$$

So each channel $h_{p,i}(n)$ is an FIR filter channel with coefficients $[h_{p,i}(0), \cdots, h_{p,i}(L)]$.

Without loss of generality, let us consider the ANC-decoded signal $y_p(mM+n)$ in the $(I+m)$th transmission slot, where $m = 0, \cdots, I-1$ and $n = -N_c, \cdots, N-1$. Because each of the I received sample blocks does not interference with the other blocks (thanks to the CP), as in conventional OFDM demodulator, we remove the first N_c CP samples and put the rest N samples into an $N \times 1$ vector $\mathbf{y}_p(m) = [y_p(mM), \cdots, y_p(mM+N-1)]^T$. Then the signal (9) can be written in the vector form as

$$\mathbf{y}_p(m) = \sum_{i=0}^{I-1} \mathbf{E}_i(m)\tilde{\mathbf{H}}_i\tilde{\mathbf{z}}_i + \mathbf{u}_p(m), \tag{14}$$

where the $N \times N$ diagonal matrix

$$\mathbf{E}_i(m) = a_i(m)e^{j\phi_{p,i}} \begin{bmatrix} e^{j0\epsilon_{p,i}} & & \\ & \ddots & \\ & & e^{j(N-1)\epsilon_{p,i}} \end{bmatrix} \tag{15}$$

is the CFO matrix (since it includes all the CFOs), and the $N \times (N+L)$ matrix

$$\tilde{\mathbf{H}}_i = \begin{bmatrix} h_{p,i}(L) & \cdots & h_{p,i}(0) & & \\ & \ddots & & \ddots & \\ & & h_{p,i}(L) & \cdots & h_{p,i}(0) \end{bmatrix}, \tag{16}$$

is the channel matrix. The source signal vector and the noise vector are, respectively,

$$\tilde{\mathbf{z}}_i = \begin{bmatrix} z_i(-d_{p,i}-L) \\ \vdots \\ z_i(N-1-d_{p,i}) \end{bmatrix}, \mathbf{u}_p(m) = \begin{bmatrix} u_p(mM) \\ \vdots \\ u_p(mM+N-1) \end{bmatrix}. \tag{17}$$

From (2) and (10), we know that both $\tilde{s}_{1-p}(kM+n)$ and $z_i(n)$, for $-N_c \le n \le N-1$, have OFDM signal structure. More specifically, we have

$$z_i(n) = z_i(N+n), \quad \text{for} \quad -N_c \le n \le -1. \tag{18}$$

Therefore, based on (14) and following similarly the procedure of [9][10], we remove the L repeated samples from $\tilde{\mathbf{z}}_i$, and restructure the $(N+L) \times 1$ vector $\tilde{\mathbf{z}}_i$ into the new $N \times 1$ vector

$$\mathbf{z}_i = \begin{bmatrix} z_i(0) \\ \vdots \\ z_i(N-1) \end{bmatrix}. \tag{19}$$

Correspondingly, the channel matrix $\tilde{\mathbf{H}}_i$ in (14) and (16) should be changed to an $N \times N$ circulant matrix

$$\mathbf{H}_i(d_{p,i}) = \begin{bmatrix} \mathbf{0}_{N-d_{p,i}-L} & h_{p,i}(L) & \cdots & h_{p,i}(0) & \mathbf{0}_{d_{p,i}-1} \\ \vdots & & & & \vdots \\ \mathbf{0}_{N-d_{p,i}-L-1} & h_{p,i}(L) & \cdots & h_{p,i}(0) & \mathbf{0}_{d_{p,i}} \end{bmatrix}, \tag{20}$$

where $\mathbf{0}_d$ denotes the $1 \times d$ zero vector. Note that the rows of (20) are the right cyclic shift of its first row.

After the above changes, the equation (14) has the new form

$$\mathbf{y}_p(m) = \sum_{i=0}^{I-1} \mathbf{E}_i(m)\mathbf{H}_i(d_{p,i})\mathbf{z}_i + \mathbf{u}_p(m). \tag{21}$$

In ideal OFDM signals without CFOs, the sample vector (21) becomes $\mathbf{y}_p(m) = \sum_{i=0}^{I-1} a_i(m)\mathbf{H}_i(d_{p,i})\mathbf{z}_i + \mathbf{u}_p(m)$. According to the conventional OFDM demodulation, we just need to take FFT of $\mathbf{y}_p(m)$, which diagonalizes $\mathbf{H}_i(d_{p,i})$. Then the signal $\mathbf{z}_i$ can be estimated. However, the multiple CFOs $\epsilon_{p,i}$ in (21) make this way of demodulation invalid. Specifically, the presence of $\mathbf{E}_i(m)$ prevents the diagonalization of $\mathbf{H}_i(d_{p,i})$ by conducting FFT on $\mathbf{y}_p(m)$. Therefore, we need to look for new ways to cancel the CFO matrices $\mathbf{E}_i(m)$ first.

An important feature of the signal formulation (21) is that the delay d_i is contained in the channel matrix $\mathbf{H}_i(d_{p,i})$ only, whereas the CFO $\epsilon_{p,i}$ is contained in the diagonal CFO matrix $\mathbf{E}_i(m)$ only. Because the channel matrix $\mathbf{H}_i(d_{p,i})$ is independent of CFO, CFO can be mitigated before OFDM demodulation. Once CFO is mitigated, $d_{p,i}$ just introduces phase shifts to the frequency domain channels during OFDM demodulation.

Notice from (21) that the sample vectors $\mathbf{y}_p(m)$, $0 \le m \le I-1$, are different from each other in the CFO matrices $\mathbf{E}_i(m)$ only. They share the same $\mathbf{H}_i(d_{p,i})$ and $\mathbf{z}_i$. This observation serves as our ground for removing CFO. As per (21) and Fig. 3, stacking together all the I sample vectors, we have

$$\begin{bmatrix} \mathbf{y}_p(0) \\ \vdots \\ \mathbf{y}_p(I-1) \end{bmatrix} = \sum_{i=0}^{I-1} \begin{bmatrix} \mathbf{E}_i(0) \\ \vdots \\ \mathbf{E}_i(I-1) \end{bmatrix} \mathbf{H}_i(d_{p,i})\mathbf{z}_i + \begin{bmatrix} \mathbf{u}_p(0) \\ \vdots \\ \mathbf{u}_p(I-1) \end{bmatrix} \quad (22)$$

which for notational simplicity can be defined as

$$\mathbf{Y}_p = \sum_{i=0}^{I-1} \mathbf{Q}_i \mathbf{H}_i(d_{p,i})\mathbf{z}_i + \mathbf{U}. \quad (23)$$

Note that $\mathbf{Y}_p$ has dimension $IN \times 1$, and $\mathbf{Q}_i$ has dimension $IN \times N$.

Our basic idea of removing CFO is thus to design $N \times IN$ CFO-cancellation matrix $\mathbf{X}_i$ for each i, such that

$$\mathbf{X}_i \mathbf{Q}_k = \delta_{i,k}\mathbf{I}_N = \begin{cases} \mathbf{I}_N, & \text{if } i = k \\ \mathbf{0}_{N\times N}, & \text{if } i \neq k \end{cases} \quad (24)$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix.

A direct solution to (24) is

$$\mathbf{X}_i = \left[\mathbf{0}_{N\times(i-1)N},\ \mathbf{I}_N,\ \mathbf{0}_{N\times(I-i)N}\right] \left[\mathbf{Q}_0, \cdots, \mathbf{Q}_{I-1}\right]^+, \quad (25)$$

where $(\cdot)^+$ denotes pseudo-inverse. After obtaining the matrix $\mathbf{X}_i$, we apply it on $\mathbf{Y}_p$ to get

$$\mathbf{X}_i \mathbf{Y}_p = \mathbf{H}_i(d_{p,i})\mathbf{z}_i + \mathbf{X}_i\mathbf{U}, \quad (26)$$

where $i = 0, \cdots, I-1$. This procedure removes the CFO matrix completely.

After the CFO matrix is removed, because $\mathbf{H}_i(d_{p,i})$ is $N \times N$ circulant, FFT can be conducted on (26)

$$\mathbf{F}\mathbf{X}_i\mathbf{Y}_p = \mathbf{D}_i\mathbf{F}\mathbf{z}_i + \mathbf{F}\mathbf{X}_i\mathbf{U}, \quad (27)$$

where $\mathbf{D}_i = \mathbf{F}\mathbf{H}_i(d_{p,i})\mathbf{F}^{-1}$ is $N \times N$ diagonal matrix. The diagonal entries are the channel gains in the frequency domain.

Remember that the final objective is to detect the source symbols $\mathbf{b}_{1-p}(k)$ which are contained in $\mathbf{z}_i$. For this, we need to study the results of $\mathbf{F}\mathbf{z}_i$ in (27). From (19), (10) and (2), we know that

$$\begin{aligned} \mathbf{z}_i &= \sum_{k=0}^{I-1} c_i(k) e^{jkM\omega^f_{1-p,i}} \mathbf{s}_{1-p}(k) \\ &= \sum_{k=0}^{I-1} c_i(k) e^{jkM\omega^f_{1-p,i}} \mathbf{F}^{-1}\mathbf{b}_{1-p}(k) \end{aligned} \quad (28)$$

Therefore, applying FFT on $\mathbf{z}_i$, we have

$$\begin{aligned} \mathbf{F}\mathbf{z}_i &= \sum_{k=0}^{I-1} c_i(k) e^{jkM\omega^f_{1-p,i}} \mathbf{b}_{1-p}(k) \\ &= \mathbf{B}_{1-p}\mathbf{E}_i\mathbf{c}_i, \end{aligned} \quad (29)$$

where the $N \times I$ matrix

$$\mathbf{B}_{1-p} = [\ \mathbf{b}_{1-p}(0) \quad \cdots \quad \mathbf{b}_{1-p}(I-1)\ ] \quad (30)$$

includes all the data symbols transmitted by the source node $1-p$, which are what the source node p needs to detect. The $I \times I$ diagonal matrix

$$\mathbf{E}_i = \text{diag}\{e^{j0M\omega^f_{1-p,i}}, \cdots, e^{j(I-1)M\omega^f_{1-p,i}}\} \quad (31)$$

is another CFO matrix due to the forward channel CFO. The $I \times 1$ vector $\mathbf{c}_i = [c_i(0), \cdots, c_i(I-1)]^T$ consists of all the fusion parameters used by the relay i to fuse the I received OFDM packets (during the first I transmission slots).

Combining (27) and (29), we have

$$\mathbf{F}\mathbf{X}_i\mathbf{Y}_p = \mathbf{D}_i\mathbf{B}_{1-p}\mathbf{E}_i\mathbf{c}_i + \mathbf{F}\mathbf{X}_i\mathbf{U}. \quad (32)$$

In order to estimate the unknown data in $\mathbf{B}_{1-p}$, we need to exploit the signals (32) for all $i = 0, \cdots, I-1$. Define

$$\begin{aligned} \mathbf{A} &= \left[\mathbf{D}_0^+\mathbf{F}\mathbf{X}_0\mathbf{Y}_p, \cdots, \mathbf{D}_{I-1}^+\mathbf{F}\mathbf{X}_{I-1}\mathbf{Y}_p\right], \\ \mathbf{C} &= \left[\mathbf{c}_0, \cdots, \mathbf{c}_{I-1}\right], \\ \mathbf{W} &= \left[\mathbf{D}_0^+\mathbf{F}\mathbf{X}_0\mathbf{U}, \cdots, \mathbf{D}_{I-1}^+\mathbf{F}\mathbf{X}_{I-1}\mathbf{U}\right]. \end{aligned} \quad (33)$$

We have

$$\mathbf{A} = \mathbf{B}_{1-p}\mathbf{E}_i\mathbf{C} + \mathbf{W}. \quad (34)$$

The source node $1-p$ transmitted data can be detected from

$$\hat{\mathbf{B}}_{1-p} = \mathbf{A}\mathbf{C}^{-1}\mathbf{E}_i^{-1} + \mathbf{W}\mathbf{C}^{-1}\mathbf{E}_i^{-1}. \quad (35)$$

C. Efficient algorithm and computational complexity

In the detection of the fused data (35), the matrix $\mathbf{E}_i$ is $I \times I$ diagonal, so its inverse is trivial. The matrix $\mathbf{C}$ is the $I \times I$ fusion coefficient matrix, which can be designed offline. Therefore, the inverse $\mathbf{C}^{-1}$ is available *a priori*. The computational complexity of (35) is $O(NI^2)$ to detect NI symbols.

The OFDM demodulation, specifically, the FFT operation (27) is almost identical to the conventional OFDM demodulation. The use of FFT makes its computation very efficient. Note that the conventional OFDM needs $O(NI \log N)$ for transmitting NI symbols.

Perhaps the majority of the computation comes from (25)-(26), i.e., the cancellation of the CFOs. Following similarly to [10], we can see that (25) always has solutions. By analyzing the special structure of the CFO matrices, we can find that the ℓ^{th} row of the matrix $\mathbf{X}_i$ has only I non-zero entries. Put these I entries into the vector $\mathbf{w}_i(\ell)$. We have

$$\mathbf{w}_i(\ell) = \mathbf{T}^{-1}\mathbf{e}_i e^{-j[\epsilon_{p,i}(N+\ell)+\phi_{p,i}]} \quad (36)$$

where $\mathbf{e}_i$ is the $I \times 1$ unit vector (has value 1 in the i^{th} entry and zero elsewhere), and the $I \times I$ matrix

$$\mathbf{T} = \begin{bmatrix} a_0(0) & \cdots & a_0(I-1)e^{j\epsilon_{p,0}M(I-1)} \\ \vdots & & \vdots \\ a_{I-1}(0) & \cdots & a_{I-1}(I-1)e^{j\epsilon_{p,I-1}M(I-1)} \end{bmatrix} \quad (37)$$

consists of both the CFO values and the scrambling parameters.

By (36) we can obtain the I non-zero entries of the ℓ^{th} row of $\mathbf{X}_i$. Doing this for all the N rows $0 \le \ell \le N-1$,

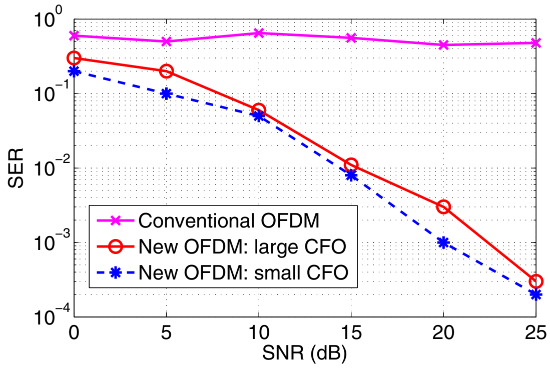

Fig. 4. SER vs. SNR under various CFOs. Our proposed scheme has CFO-resistance performance, while conventional method fails.

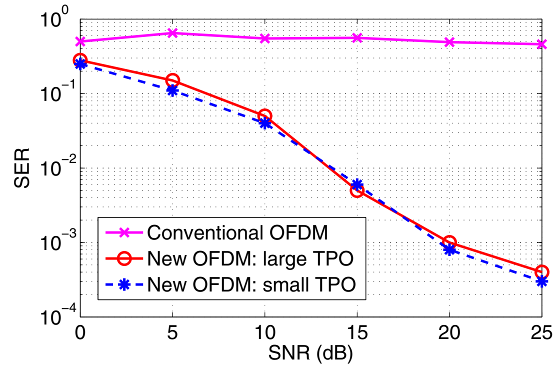

Fig. 5. SER vs. SNR under various TPOs. Our proposed scheme has TPO-independent performance. Conventional method fails because of CFO.

we obtain the CFO mitigation matrix $\mathbf{X}_i$ for the relay i. The complexity is in the order of $O(NI^3)$.

Therefore, this new OFDM transmission scheme is computationally efficient. First, it can exploit all the nice features of the FFT-based efficient processing of the conventional OFDM. Second, although we need to calculate the inverse of large matrix for the CFO-cancellation matrix $\mathbf{X}_i$, the computation is effectively reduced to the inverse of the much smaller $I \times I$ matrix $\mathbf{T}$ in (37). Note that I, the number of relays, is usually much smaller than the OFDM symbol length N. In addition, this just needs to be calculated once for all the transmission sessions if the CFOs are constant and the scrambling codes are periodic.

IV. Simulations

In order to evaluate the performance of our proposed transmission scheme, we simulated a system with four relays. The parameters we used were $N = 32$, QPSK, $I = 4$. Randomly generated channels with order $L = 4$ were used. 10000 runs of the program were conducted to derive the average symbol error rate (SER) under various CFOs and TPOs.

To simulate the performance of our scheme in mitigating various CFOs and TPOs, we set the maximum possible CFOs between any two transmitters as 0.5 of the OFDM bin width for large CFO cases or 0.01 of the OFDM bin width for small CFO cases. Similarly, for TPOs, the maximum available values were 10 and 1 for large and small cases, respectively. The actual CFOs and TPOs were randomly generated according to uniform distribution between 0 and the maximum possible values.

The simulation results shown in Fig. 4 indicate that under various CFOs, our scheme can cancel all CFOs and have reliable SER performance, while the conventional OFDM transmission fails. Similarly, the results in Fig. 5 show that our scheme does not lose performance under TPOs. Note that the conventional OFDM fails because there are small CFOs introduced in this case.

V. Conclusions

We propose a new OFDM transmission scheme to support two-way relaying with multiple amplify-and-forward relays and analog network coding. This scheme tolerates TPO and guarantees complete CFO cancellation. In this scheme, the relays fuse multiple received OFDM signals together and scramble them into multiple OFDM symbols. The source receivers remove CFOs by an efficient algorithm. Simulations show that this scheme has superior performance in asynchronous environment.

References

[1] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: analog networking coding," in *Proc. ACM Special Interest Group Data Commun. (SIGCOMM)*, Kyoto, Japan, Aug. 2007.

[2] M. Zaeri-Amirani, S. Shahbazpanahi, T. Mirfakhraie, and K. Ozdemir, "Performance tradeoffs in amplify-and-forward bidirectional network beamforming," *IEEE Trans. Signal Processing*, vol. 60, no. 8, pp. 4196-4209, Aug. 2011.

[3] Y. Jing and S. ShahbazPanahi, "Max-Min optimal joint power control and distributed beamforming for two-way relay networks under per-node power constraints," *IEEE Trans. Signal Processing*, vol. 60, no. 12, pp. 6576-6589, Dec. 2012.

[4] M. Zeng, R. Zhang, and S. Cui, "On design of collaborative beamforming for two-way relay networks," *IEEE Trans. Signal Processing*, vol. 59, no. 5, pp. 2284-2295, May 2011.

[5] C. Xiong and X. Li, "Optimal relay selections in two-way amplify-and-forward networks," *the 47th Annual Conference on Information Sciences and Systems* (CISS'2013), Johns Hopkins University, Mar. 20-23, 2013.

[6] G. S. Smith, "A direct derivation of a single-antenna reciprocity relation for the time domain," *IEEE Trans. on Antennas and Propagation*, vol. 52, no. 6, pp. 1568-1577, June 2004.

[7] M. Guillaud, D. T. Slock and R. Knopp, "A practical method for wireless channel reciprocity exploitation through relative calibration," in *Proc. Intl. Symp. on Signal Processing and its Applications* (ISSPA'05), Sydney, Australia, Aug. 2005.

[8] Z. Li, X. G. Xia and B. Li, "Achieving full diversity and fast ML decoding via simple analog network coding for asynchronous two-way relay networks," *IEEE Trans. Commun.*, vol. 57, no. 12, pp. 3672-3681, Dec. 2009.

[9] X. Li, F. Ng, and T. Han, "Carrier frequency offset mitigation in asynchronous cooperative OFDM transmissions," *IEEE Trans. Signal Processing*, vol. 56, no. 2, pp. 675-685, Feb. 2008.

[10] X. Li, J. Feldman and W. Cadeau, "An OFDM-based distributed transmission scheme for uncoordinated transmitters without carrier frequency and timing synchronization," *IEEE Military Communications Conference* (MILCOM'2012), Orlando, FL, Oct. 2012.

[11] S.-H. Tsai, Y.-P. Lin and C.-C. J. Kuo, "An approximately MAI-free multi-access OFDM system in carrier frequency offset environment," *IEEE Trans. Signal Processing*, vol. 53, no. 11, pp. 4339-4353, Nov. 2005.