Performance Analysis of Coexisting Secondary Users in Heterogeneous Cognitive Radio Network

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***Abstract*—In this paper, we propose an analysis framework for the performance of heterogeneous cognitive radio networks with multiple different secondary users (SU) coexisting and competing for spectrum access. In order to model the operation of different SUs in a feasible manner, we use a bank of Markov models, where each SU is modeled by a separate Markov model. The individual Markov models are connected implicitly in their transitional probabilities. Expressions for the transitional probabilities are derived by analyzing the mutual interference among the SUs. The throughput of each SU and the overall CRN can thus be calculated. In addition, to set up a comparison basis for evaluating the optimality of the CRN throughput, we optimize the SU channel access by solving a sum-of-ratios linear fractional programming. Simulations are conducted to verify the proposed techniques and to compare the throughput of the CRN under random channel access over the throughput under the optimized channel access.**

I. Introduction

Cognitive radio network (CRN) and dynamic spectrum access (DSA) are new radio spectrum access technologies that can potentially ameliorate the spectrum shortage problem faced by today's rapidly increasing wireless services. It allows new wireless systems to reuse the spectrum currently occupied exclusively by primary systems. As a unique feature, CRN conducts spectrum sensing and accesses the spectrum that is not occupied by the primary users (PU). It must vacate the spectrum if finding that the PU becomes active.

There are extensive research published in CRN. However, the majority of the existing work on CRN are for homogeneous CRN, where all SUs have the same strategies and parameters, such as the performance analysis of CRN MAC protocols [1]-[5]. In reality, CRNs are highly likely heterogeneous. Different SUs may have different implementations, different spectrum sensing and spectrum access algorithms. Since the PU signals are different temporarily and geographically, the SUs may adopt more conservative or more aggressive policies toward sensing errors. Some SUs may exploit handshaking or common messaging more than others. The flexible software implementation makes CRNs subject to user modifications. There are very limited study on heterogeneous CRN and the coexistence of a large number of different SUs in heterogeneous CRN.

It has been a challenge to analyze the performance of CRN. It is especially challenging to analyze the performance of heterogeneous CRN with multiple different cognitive radio users coexisting and competing for spectrum access. The large number of different SUs makes the performance analysis substantially difficult, as evidenced in [5] even though significant simplifications were adopted for Markov modeling.

In this paper, we develop a framework to analyze the performance of heterogeneous CRN with a large number of different SUs. We use a Markov Model Bank (MMB), i.e., a bank of Markov models, to model the operations of the CRN. Each SU has its own Markov model, and the interference from all the other SUs is abstracted into its state transition probability. This makes it tractable to treat the complex coupling among a large number of SUs. We focus on analyzing the throughput under a generic random spectrum access strategy only. Consideration of more advanced strategies such as [6] will be reported elsewhere. In addition, since it is important to evaluate the optimality of the CRN performance, or how far away the CRN performance is from certain ideal performance [7], we further develop a method to optimize SUs spectrum access by solving a sum-of-ratios linear fractional programming.

This paper is organized as follows. In Section II, we give the system model. In Section III, we develop the framework of throughput analysis of the heterogeneous CRN. In Section IV, we develop the method to optimize the throughput. Simulations are then conducted in Section V to compare the throughput of the heterogeneous CRN to the optimized throughput. Conclusions are then given in Section VI.

II. System model

The CRN we consider in this paper consists of a set of I SUs and some PUs. Each SU is a transmitting-receiving pair of nodes. When transmitting to the receiving node, the transmitter node creates interference to other SUs as well. An example of the network is shown in Fig. 1. We assume there are K channels (spectrum resource) available for the I SUs to choose from.

Each SU follows the generic cognitive radio operation policy that includes four states: spectrum sensing, data transmission, waiting (idling) and channel switching, as illustrated in Fig. 2. The operation of a cognitive radio always begins with the spectrum sensing state. If the spectrum sensing indicates the channel is available for secondary access, then the cognitive radio transmits a data packet, and the model shifts into the data transmission state. If the spectrum sensing indicates the channel is not available, then the cognitive radio enters the waiting (idling) state which is followed by the channel

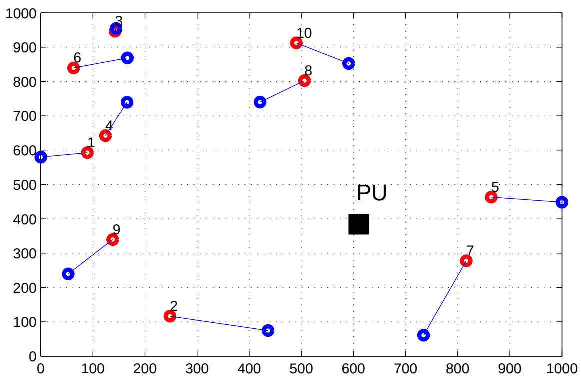

Fig. 1. An example of the CRN with $I = 10$ secondary users and one primary user.

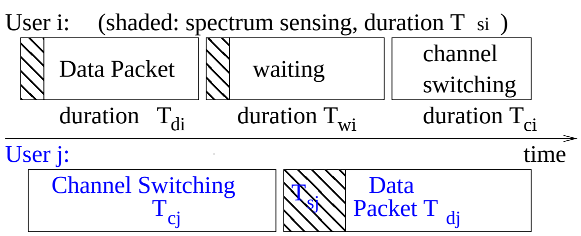

Fig. 2. Illustration of the cognitive radio operation with four basic states: spectrum/channel sensing, data packet transmission, waiting (idling) and channel switching.

switching state. In this paper we consider the generic channel selection strategy: each SU randomly selects a channel to sense and access. Each of the K channels thus has probability $\frac{1}{K}$ being selected. This also means that each SU uses just one channel each time. We assume that the operation always shifts back to the spectrum sensing state after the channel switching state or the data transmission state.

Spectrum sensing results depend on the PU activity, the interference from other SUs, and the offered load of the SU. We assume that the SU can successfully detect the activity of the PUs which is described by the probability θ_k, i.e., the channel k is not accessed by any PU. We further assume that the offered load of the SU i be α_i, i.e, with probability α_i the SU i has data packet to transmit. Such assumptions allow us to focus on analyzing the mutual interference among the SUs, which is necessary for studying their coexistence.

Let P_i^k be the transmission power of the SU i when using the channel k, where $i = 1, \cdots, I$, and $k = 1, \cdots, K$. The signal received by the SU i's receiver when using the channel k is

$$y_i^k(n) = \sqrt{P_i^k} h_{ii}^k s_i(n) + \sum_{j=1, j\neq i}^{I} f_j^k \sqrt{P_j^k} h_{ji}^k s_j(n) + v_i^k(n) \tag{1}$$

where $s_i(n)$ is the discrete-time signal transmitted by the SU i, h_{ji}^k is the complex Gaussian distributed flat fading channel from the SU j's transmitter to the SU i's receiver, and $v_i^k(n)$ is the AWGN with zero-mean and power σ_i^{k2}. The variable $f_j^k = 1$ (or 0) denotes the SU j is (not) currently using the channel k.

Without PU activity, the signal-to-noise ratio (SNR) during the SU's spectrum sensing state is

$$\gamma_{si}^k = \frac{\sum_{j=1, j\neq i}^{I} f_j^k P_j^k |h_{ji}^k|^2}{\sigma_i^{k2}} \tag{2}$$

We assume that if the SNR is larger than some detection threshold Γ_{si}^k, then the SU makes a decision that the channel is occupied by other SUs, and is thus not available for transmission.

During data transmission slot, the SNR of the SU i in the channel k is

$$\gamma_i^k = \frac{P_i^k |h_{ii}^k|^2}{\sum_{j=1, j\neq i}^{I} f_j^k P_j^k |h_{ji}^k|^2 + \sigma_i^{k2}} \tag{3}$$

Note that a channel may be shared by multiple SUs simultaneously only if their sensing SNRs γ_{si}^k are small enough.

As the performance metric, we consider the summation of the throughput R_i of all the I SUs

$$R = \sum_{i=1}^{I} R_i, \tag{4}$$

where R_i is the throughput that the SU i obtained in all the channels. The throughput depends on the fraction of time spent in data packet transmission state and the capacities of transmission channels $\log(1 + \gamma_i^k)$.

III. HETEROGENEOUS CRN THROUGHPUT ANALYSIS

We use a Markov Model Bank (MMB) to model the operation of the I SUs. As shown in Fig. 3, the MMB consists of an individual Markov model for each SU, where π_{si}^k,π_{di}^k,π_{wi}^k are the probabilities of the SU i staying in the channel sensing, data packet transmission and waiting states when accessing the channel k. The state probability π_{ci} refers to channel switching. The transitional probability q_{si}^k denotes the probability that the channel can be used for data packet transmission, which is the result of spectrum sensing. Although the different SU's Markov models look separated from each other, they are inter-related through this transitional probability.

For each SU i, let the durations of the spectrum sensing slot, data transmission slot, waiting slot, and channel switching slot be T_{si}^k, T_{di}^k, T_{wi}^k and T_{ci}, respectively. In heterogeneous CRN, different SUs may have different parameter values, as illustrated in Fig. 2.

In the sequel, to derive the CRN throughput, we first analyze the steady state of the MMB, and then derive the transitional probabilities q_{si}^k through interference analysis.

First, conditioned on q_{si}^k, we can analyze the steady state property of the MMB by considering each SU separately. This is one of the key advantages of our proposed MMB for analyzing large heterogeneous CRNs. According to the steady state property of the Markov models, we can find the state probabilities π_{si}^k, π_{di}^k, π_{wi}^k and π_{ci} of the SU i from

$$\begin{bmatrix} \mathbf{A}_1 & & & \mathbf{a}_1 \\ & \ddots & & \vdots \\ & & \mathbf{A}_K & \mathbf{a}_K \\ \mathbf{b} & \cdots & \mathbf{b} & -1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_K \\ \pi_{ci} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \vdots \\ \mathbf{0} \\ 0 \end{bmatrix}, \tag{5}$$

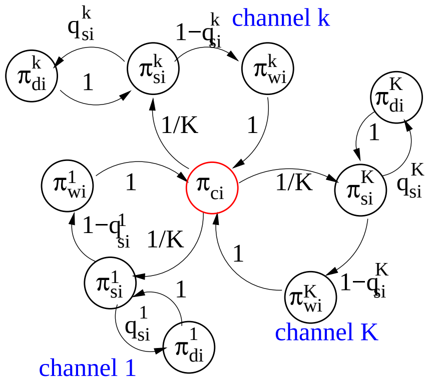

Fig. 3. Markov Model Bank (MMB) for heterogenous CRN. Only the Markov model for the SU i is shown. All other SUs have similar Markov models.

where $\mathbf{b} = [0, 0, 1]$, and

$$\mathbf{A}_k = \begin{bmatrix} -1 & 1 & 0 \\ q_{si}^k & -1 & 0 \\ 1 - q_{si}^k & 0 & -1 \end{bmatrix}, \mathbf{a}_k = \begin{bmatrix} \frac{1}{K} \\ 0 \\ 0 \end{bmatrix}, \mathbf{x}_k = \begin{bmatrix} \pi_{si}^k \\ \pi_{di}^k \\ \pi_{wi}^k \end{bmatrix}.$$

Under the constraint

$$\pi_{ci} + \sum_{k=1}^{K} (\pi_{di}^k + \pi_{si}^k + \pi_{wi}^k) = 1, \tag{6}$$

we can solve (5) to find all the states' steady state probabilities. Specifically, from (5) we can derive

$$\pi_{di}^k = q_{si}^k \pi_{si}^k, \; \pi_{wi}^k = (1 - q_{si}^k)\pi_{si}^k, \; \pi_{ci} = K(1 - q_{si}^k)\pi_{si}^k. \tag{7}$$

Substituting (7) into the constraint (6) gives

$$2 \sum_{k=1}^{K} \pi_{si}^k = 1 - \pi_{ci}. \tag{8}$$

Considering the last equation in (7), we see

$$\frac{\pi_{si}^k}{\pi_{si}^\ell} = \frac{1 - q_{si}^\ell}{1 - q_{si}^k}. \tag{9}$$

From (8) and (9) we have the steady-state probabilities

$$\begin{cases} \pi_{ci} = \frac{K}{K + 2\sum_{\ell=1}^{K} \frac{1}{1 - q_{si}^\ell}} \\ \pi_{si}^k = \frac{1}{\left(K + 2\sum_{\ell=1}^{K} \frac{1}{1 - q_{si}^\ell}\right)(1 - q_{si}^k)} \end{cases} \tag{10}$$

The other state probabilities π_{di}^k and π_{wi}^k can be obtained from (10) and (7).

Next, to specify q_{si}^k, we need to analyze the mutual interference among the SUs. In our model, the transitional probability

$$q_{si}^k = \theta_k \alpha_i P[\gamma_{si}^k < \Gamma_{si}^k]. \tag{11}$$

From (2), The sensing SNR depends on other SUs through the binary parameter f_j^k, where $f_j^k = 1$ denotes the SU j is currently transmitting in the channel k. Based on the steady state probabilities, we have

$$P[f_j^k = 1] = \delta_{si}^k \delta_{dj}^k, \tag{12}$$

where the probability that the SU i senses the channel k is

$$\delta_{si}^k = \frac{\pi_{si}^k T_{si}^k}{\pi_{ci} T_{ci} + \sum_{\ell=1}^{K} \left(\pi_{si}^\ell T_{si}^\ell + \pi_{di}^\ell T_{di}^\ell + \pi_{wi}^\ell T_{wi}^\ell\right)}, \tag{13}$$

whereas the probability that the SU j is transmitting data packet in the channel k is

$$\delta_{dj}^k = \frac{\pi_{dj}^k T_{dj}^k}{\pi_{cj} T_{cj} + \sum_{\ell=1}^{K} \left(\pi_{sj}^\ell T_{sj}^\ell + \pi_{dj}^\ell T_{dj}^\ell + \pi_{wj}^\ell T_{wj}^\ell\right)}. \tag{14}$$

Based on (10), we can readily derive

$$P[f_j^k = 1] = \frac{q_{sj}^k T_{si}^k T_{dj}^k}{Q_i^k Q_j^k} \tag{15}$$

where Q_i^k and Q_j^k are

$$Q_i^k = K(1 - q_{si}^k) T_{ci} + \sum_{\ell=1}^{K} \left[T_{si}^\ell + q_{si}^\ell T_{di}^\ell + (1 - q_{si}^\ell) T_{wi}^\ell\right] \frac{1 - q_{si}^k}{1 - q_{si}^\ell}. \tag{16}$$

The probability

$$P[\gamma_{si}^k < \Gamma_{si}^k] = P\left[\frac{1}{\sigma_i^{k2}} \sum_{j=1, j \neq i}^{I} P_j^k |h_{ji}^k|^2 f_j^k < \Gamma_{si}^k\right] \tag{17}$$

is the probability distribution of the weighted summation of $I - 1$ Bernoulli random variables f_j^k. When the number of SUs I is not too big, we can simply list all possible values of the weighted summation with their corresponding probability mass so as to find the probability (17). Otherwise, we can take some appropriate approximations, such as the Gaussian approximation, to simplify the calculation.

With the state and transitional probabilities, we can define and evaluate the average throughput of the SU i as

$$R_i = \sum_{k=1}^{K} \delta_{di}^k E[\log(1 + \gamma_i^k)] = \sum_{k=1}^{K} \frac{q_{si}^k T_{di}^k}{Q_i^k} E[\log(1 + \gamma_i^k)], \tag{18}$$

where γ_i^k is the data transmission slot SNR defined in (3). The expectation is conducted over the Bernoulli random variables f_j^k on which the SNR γ_i^k depends. Similarly as the evaluation of (17), we can list all possible cases of the f_j^k with the corresponding probabilities and channel capacities, and find the average capacity $E[\log(1 + \gamma_i^k)]$ for the channel k.

In the sequel, we consider an assumption in order to analyze heterogeneous CRN with a large number of SUs efficiently. We assume that the sensing threshold Γ_{si}^k is small when compared with the mutual interference among the SUs. In other words,

$$\Gamma_{si}^k < \min_{j=1, j \neq i}^{J} \frac{P_j^k |h_{ji}^k|^2}{\sigma_i^{k2}}. \tag{19}$$

This assumption is valid if the CRN exploits its extreme sensitivity of spectrum sensing, or if the CRN have limited spatial spread.

This assumption simplifies the transitional probability (11) into

$$\begin{aligned} q_{si}^k &= \theta_k \alpha_i P[f_j^k = 0, \forall\, j \neq i] \\ &= \theta_k \alpha_i \prod_{j=1, j\neq i}^{I} \left(1 - \frac{q_{sj}^k T_{si}^k T_{dj}^k}{Q_i^k Q_j^k}\right). \end{aligned} \quad (20)$$

Similarly, the throughput (18) is reduced to

$$R_i = \sum_{k=1}^{K} \frac{q_{si}^k T_{di}^k}{Q_i^k} \log\left(1 + \frac{P_i^k |h_{ii}^k|^2}{\sigma_i^{k2}}\right). \quad (21)$$

The throughput is now a function of the transitional probabilities q_{si}^k only. It is not explicitly relied on all other SUs' activity. In this case, the data transmission is conducted without interference from other SUs due to the perfect spectrum sensing. There is no need of searching exhaustively through all other SUs activities.

Note that for large CRN, another way of simplification is to exploit jointly the simplified expression (21) and the exhaustive expression (18) by classifying the SUs into two classes: those cause large interference (and thus in close distance) to the SU i, and those with small interference (and thus in long distance) to the SU i. This can be conducted by adjusting the spectrum sensing threshold Γ_{si}^k in practice.

Recall that to find the throughput R_i (21) we need to evaluate all transitional probabilities q_{si}^k, which can be calculated from (20). Obviously q_{si}^k is a function of all other q_{sj}^ℓ, $j = 1, \cdots, I$ and $\ell = 1, \cdots, K$. Non-linear numerical algorithms can be used to find some solutions to q_{si}^k. Obviously, there may have many different solutions. These different solutions are corresponding to the different throughputs of the heterogeneous CRN, or the different equilibriums of the heterogeneous CRN when the SUs compete for spectrum access.

IV. CRN Throughput Optimization

The derivation in the Section III is based on the distributed (uncooperative) spectrum access. Obviously, it may not achieve the best throughput obtainable under the centralized (cooperative) spectrum access optimization. In order to provide a basis for measuring the optimality of CRN operations, in this section we develop a way to optimize the SUs' spectrum access and to evaluate the optimized throughput.

From the system model in Section II, P_i^k is the transmission power of the SU i spent in the channel k, where $i = 1, \cdots, I$, and $k = 1, \cdots, K$. Differently from the previous sections, we assume that each SU can use multiple available channels simultaneously under an overall transmission power limit $\bar{P}_i$.

Consider an arbitrary set of L_m channels

$$C_m = \{k_1, \cdots, k_{L_m}\},\ 1 \leq k_\ell \leq K,\ 0 \leq L_m \leq K. \quad (22)$$

Note that there are altogether 2^K such sets, i.e., $m = 1, \cdots, 2^K$. The probability that only the channels in C_m are not occupied by the PUs is

$$P[C_m] = \prod_{\ell=1}^{L_m} \theta_{k_\ell} \prod_{k=1, k\notin C_m}^{K} (1 - \theta_k). \quad (23)$$

For each set C_m, we can determine the optimal transmission power P_i^k, where $1 \leq i \leq I$ and $k \in C_m$, to maximize the sum of the throughputs of all the I SUs, i.e.,

$$\begin{aligned} R = & \\ \max_{\{P_i^k\}} \quad & \sum_{i=1}^{I} \alpha_i \sum_{\ell=1}^{L_m} \log\left(1 + \frac{P_i^{k_\ell} |h_{ii}^{k_\ell}|^2}{\sum_{j=1, j\neq i}^{I} P_j^{k_\ell} |h_{ji}^{k_\ell}|^2 + \sigma_i^{k_\ell 2}}\right) \\ \text{s.t.} \quad & \sum_{\ell=1}^{L_m} P_i^{k_\ell} \leq \bar{P}_i,\ \ P_i^{k_\ell} \geq 0. \end{aligned} \quad (24)$$

Note that $P_i^k = 0$ for all $k \notin C_m$.

Define the $IL_m \times 1$ dimensional normalized transmission power vector

$$\mathbf{z}_m = \left[\frac{P_1^{k_1}}{\bar{P}_1}, \cdots, \frac{P_1^{k_{L_m}}}{\bar{P}_1}, \cdots, \frac{P_I^{k_1}}{\bar{P}_I}, \cdots, \frac{P_I^{k_{L_m}}}{\bar{P}_I}\right]^T. \quad (25)$$

We can rewrite (24) into

$$\begin{aligned} R = \quad \max_{\mathbf{z}_m} \quad & \sum_{i=1}^{I} \alpha_i \sum_{\ell=1}^{L_m} \log\left(1 + \frac{\mathbf{a}_{i\ell}^T \mathbf{z}_m}{\mathbf{b}_{i\ell}^T \mathbf{z}_m + 1}\right), \\ \text{s.t.} \quad & \mathbf{B}\mathbf{z}_m \leq \mathbf{1},\ \ \mathbf{z}_m \geq \mathbf{0}. \end{aligned} \quad (26)$$

where the $IL_m \times 1$ dimensional vector

$$\mathbf{a}_{i\ell} = \left[0, \cdots, 0, \frac{\bar{P}_i |h_{ii}^{k_\ell}|^2}{\sigma_i^{k_\ell 2}}, 0, \cdots, 0\right]^T, \quad (27)$$

has only one none-zero element in the entry $L_m(i-1) + \ell$. The vector

$$\mathbf{b}_{i\ell} = \left[0, \cdots, 0, \frac{\bar{P}_1 |h_{1i}^{k_\ell}|^2}{\sigma_i^{k_\ell 2}}, 0, \cdots, 0, \frac{\bar{P}_I |h_{Ii}^{k_\ell}|^2}{\sigma_i^{k_\ell 2}}, 0, \cdots, 0\right]^T \quad (28)$$

has $I-1$ non-zero elements, which are $\bar{P}_j |h_{ji}^{k_\ell}|^2 / \sigma_i^{k_\ell 2}$ in the entry $L_m(j-1) + \ell$ for all $j = 1, \cdots, I$, and $j \neq i$. The $I \times IL_m$ matrix $\mathbf{B}$ is block-diagonal

$$\mathbf{B} = \begin{bmatrix} \mathbf{1}^T & & \\ & \ddots & \\ & & \mathbf{1}^T \end{bmatrix}. \quad (29)$$

where $\mathbf{1}$ is the $L_m \times 1$ dimensional vector with all entries as 1.

After obtaining the optimal transmission powers by solving (26), the CRN throughput is thus maximized. Specifically, for each available channel set C_m, the SU i has optimal channel rate

$$R_i(C_m) = \sum_{\ell=1}^{L_m} \log\left(1 + \frac{P_i^{k_\ell} |h_{ii}^{k_\ell}|^2}{\sum_{j=1, j\neq i}^{I} P_j^{k_\ell} |h_{ji}^{k_\ell}|^2 + \sigma_i^{k_\ell 2}}\right). \quad (30)$$

The optimal throughput of the SU i is thus

$$R_i = \alpha_i \sum_{m=1}^{2^K} R_i(C_m) P[C_m], \quad (31)$$

and the overall CRN throughput is $R = \sum_{i=1}^{I} R_i$.

Note that (26) can be solved by the sum-of-ratios linear fractional programming (LFP) [8]-[11]. Even though it is not in the standard linear fractional ratio form, we can still simply

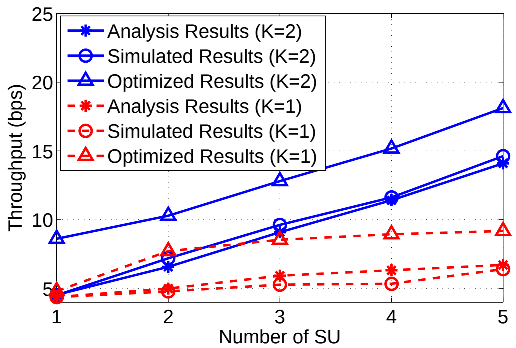

Fig. 4. CRN throughput when $I = 1, \cdots, 5$ SUs compete for $K = 1$ channel or $K = 2$ channels.

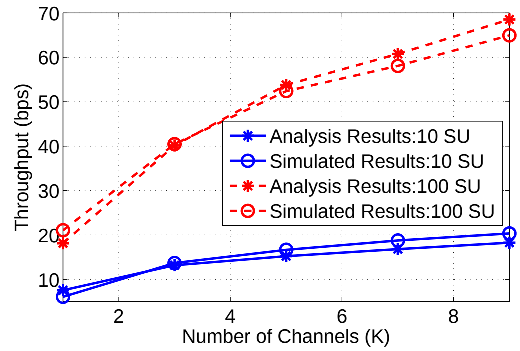

Fig. 5. CRN throughput as function of the number of channels, under two situations: $I = 10$ SUs or $I = 100$ SUs.

modify our optimization framework and algorithm developed in [12] to optimize the sum of the logarithms of linear fractions (26). Because of the sparsity of the optimization parameters (each fraction relies on a few optimization variables only), we may need to modify the nominators of (26) to $(\mathbf{a}_{i\ell} + \epsilon)^T \mathbf{z}_m$, where $\epsilon \longrightarrow 0$ is an arbitrarily small positive constant.

V. Simulations

In this section, we report our simulation of the numerical evaluation of the CRN throughput as well as the optimized CRN throughput. We simulated a random CRN where the nodes' positions were randomly generated within a square of 1000×1000 meters. The edge SNRs were calculated as $10^8 d_{ij}^{-2.6}$ where d_{ij} is the propagation distance. The SUs' offered loads and the PUs' activities were uniformly distributed from 0 to 1 unless otherwise stated. We use the expressions in Section III to calculate the throughput of all CRN SUs ("Analysis Results") and use the expressions in Section IV to optimize the throughput of all the CRN SUs ("Optimized Results"). We use Monte-Carlo simulations to find the "Simulated Results" for comparison.

First, we compared the "Analysis results" to the "Simulation results" as well as the "Optimized Results" in Fig. 4. Due to the slow convergence of the optimization of large networks, we focused on relatively small networks with $K = 1$ or 2 channels and up to $I = 5$ SUs. Each SU had a constant 0.9 offered load, so we could focus on the optimal channel rate. It can be clearly seen that the "Analysis results" fit well with the simulated results. In addition, the CRN throughput has a large gap below the optimal throughput. Therefore, there are still much room to improve the performance of the CRN, where better spectrum access strategies can play an important role.

Next, for relatively large CRN with up to $K = 9$ channels and up to $I = 100$ users, we compared the "Analysis results" to the "Simulated results" in Fig. 5. The SU's offered load was randomly distributed in this case. CRN througput increases with more channels and more SUs. Again, the analysis fits well to the simulated results.

VI. Conclusion

In this paper, we develop an MMB (Markov Model Bank) based framework for the performance analysis of heterogeneous CRN. MMB uses an individual Markov chain to model each SU's operations, and embeds the complex mutual interference among the SUs in the transitional probabilities. MMB provides a feasible approach to analyze large heterogeneous CRNs. In addition, to set up a comparison basis to evaluate the optimality of CRN, we develop a method to optimize CRN spectrum access and throughput. This task is full-filled by the sum-of-ratios linear fractional programming. CRN throughputs are derived in both cases, and simulations are then conducted to show their validity.

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