Chapter 1: Making Economic Decisions

1-1

A survey of students answering this question indicated that they thought that about 40% of their decisions were conscious decisions.

1-2

(a) Yes. The choice of an engine has important money consequences, so it would be suitable for engineering economic analysis.

(b) Yes. Important economic and social consequences. Some might argue that the social consequences are more important than the economics.

(c) ? Probably there are a variety of considerations much more important than the economics.

(d) No. Picking a career on an economic basis sounds terrible.

(e) No. Picking a wife on an economic basis sounds even worse.

1-3

Of the three alternatives, the $150,000 investment problem is <u>most</u> suitable for economic analysis. There is not enough data to figure out how to proceed, but if the "desirable interest rate" were 9%, then foregoing it for one week would mean an immediate loss of:

1/52 (0.09) = 0.0017= 0.17%

It would take over a year at 0.15% more to equal the 0.17% foregone now.

The candy bar problem is suitable for economic analysis. Compared to the investment problem it is, of course, trivial.

Joe's problem is a real problem with serious economic consequences. The difficulty may be in figuring out what one gains if he pays for the fender damage, instead of having the insurance company pay for it.

1-4

Gambling, the stock market, drilling for oil, hunting for buried treasure—there are sure to be a lot of interesting answers. Note that if you could double your money every day, then:

2^x (\$300) = \$1,000,000

and *x* is less than 12 days.

1-5

Maybe their stock market "systems" don't work!

1-6

It may look simple to the owner because he is not the one losing a job. For the three machinists it represents a major event with major consequences.

1-7

For most high school seniors there probably are only a limited number of colleges and universities that are feasible alternatives. Nevertheless, it is still a complex problem.

1-8

It really is not an economic problem solely — it is a complex problem.

1-9

Since it takes time and effort to go to the bookstore, the minimum number of pads might be related to the smallest saving worth bothering about. The maximum number of pads might be the quantity needed over a reasonable period of time, like the rest of the academic year.

1-10

While there might be a lot of disagreement on the "correct" answer, only automobile insurance represents a substantial amount of money and a situation where money might be the primary basis for choosing between alternatives.

1-11

The overall problems are all complex. The student will have a hard time coming up with examples that are truly simple or intermediate until he/she breaks them into smaller and smaller subproblems.

1-12

These questions will create disagreement. None of the situations represents rational decision making.

Choosing the same career as a friend might be OK, but it doesn't seem too rational.

Jill didn't consider all the alternatives.

Don thought he was minimizing cost, but it didn't work. Maybe rational decision making says one should buy better tools that will last.

1-13

Possible objectives for NASA can be stated in general terms of space exploration or the generation of knowledge or they can be stated in very concrete terms. President Kennedy used the latter approach with a year for landing a man on the moon to inspire employees. Thus the following objectives as examples are concrete. No year is specified here, because unlike President Kennedy we do not know what dates may be achievable.

Land a man safely on Mars and return him to earth by———.
Establish a colony on the moon by———.
Establish, a permanent space station by———.
Support private sector tourism in space by———.
Maximize fundamental knowledge about science through *x* probes per year or for *$y* per year.
Maximize applied knowledge about supporting man's activities in space through *x* probes per year or for *$y* per year.

Choosing among these objectives involves technical decisions (some objectives may be prerequisites for others), political decisions (balance between science and applied knowledge for man's activities), and economic decisions (how many dollars per year can be allocated to NASA).

However, our favorite is a colony on the moon, because a colony is intended to be permanent and it would represent a new frontier for human ingenuity and opportunity. Evaluation of alternatives would focus on costs, uncertainties, and schedules. Estimates of these would rely on NASA's historical experience, expert judgment, and some of the estimating tools discussed in Chapter 2.

1-14

This is a challenging question. One approach might be:

(a) Find out what percentage of the population is left-handed.

(b) What is the population of the selected hometown?

(c) Next, market research might be required. With some specific scissors (quality and price) in mind, ask a random sample of people if they would purchase the scissors. Study the responses of both left-handed and right-handed people.

(d) With only two hours available, this is probably all the information one could collect. From the data, make an estimate.

A different approach might be to assume that the people interested in left-handed scissors in the future will be about the same as the number who bought them in the past.

(a) Telephone several sewing and department stores in the area. Ask two questions:
(i) How many pairs of scissors have you sold in one year (or six months)?
(ii) What is the ratio of sales of left-handed scissors to regular scissors?

(b) From the data in (a), estimate the future demand for left-handed scissors.

Two items might be worth noting:
1. Lots of scissors are universal and are equally useful for left- and right-handed people.
2. Many left-handed people probably never have heard of left-handed scissors.

1-15

Possible alternatives might include:

1. Live at home.
2. Live in a room in a private home in return for work in the garden, etc.
3. Become a Resident Assistant in a university dormitory.
4. Live in a camper — or tent — in a nearby rural area.
5. Live in a trailer on a construction site in return for "keeping an eye on the place."

1-16

A common situation is looking for a car where the car is purchased from either the first dealer or the most promising alternative from the newspaper's classified section. This may lead to an acceptable or even a good choice, but it is highly unlikely to lead to the best choice. A better search would begin with *Consumer Reports* or some other source that summarizes many models of vehicles. While reading about models, the car buyer can be identifying alternatives and clarifying which features are important. With this in mind, several car lots can be visited to see many of the choices. Then either a dealer or the classifieds can be used to select the best alternative.

1-17

Choose the better of the undesirable alternatives.

1-18

(a) Maximize the difference between output and input.
(b) Minimize input.
(c) Maximize the difference between output and input.
(d) Minimize input.

1-19

(a) Maximize the difference between output and input.
(b) Maximize the difference between output and input.
(c) Minimize input.
(d) Minimize input.

1-20

Some possible answers:

1. There are benefits to those who gain from the decision, but no one is harmed (Pareto optimum).
2. Benefits flow to those who need them most (Welfare criterion).
3. Minimize air pollution or other specific item.
4. Maximize total employment on the project.
5. Maximize pay and benefits for some group (e.g., union members).
6. Most aesthetically pleasing result.
7. Fit into normal workweek to avoid overtime.
8. Maximize the use of the people already within the company.

1-21

Surely planners would like to use criterion (a). Unfortunately, people who are relocated often feel harmed, no matter how much money, etc., they are given. Thus planners consider criterion (a) unworkable and use criterion (b) instead.

1-22

Major benefits typically focus on better serving future demand for travel measured in vehicles per day (extra market), lower traffic accident rates (extra market), time lost due to congestion (extra market), happy drivers (intangible), and urban renewal of decayed residential or blighted industrial areas (intangible).

Major costs include the money spent on the project (market), the time lost to travelers due to construction caused congestion (extra market), unhappy drivers (intangible), and the lost residences and businesses of those displaced (intangible).

1-23

The extra direct costs would be two nights stay at the hotel and two days of meals or (2) ($100 + $40) = $280. The savings on the airplane ticket would be $800 – $200 = $600. Thus, staying the extra two days saves $600 – $280 = $320. The intangibles will probably associated with your personal life, e.g., a dinner and bridge party with friends missed on Friday evening, your daughter's soccer game missed on Saturday morning, the lawn not mowed on Saturday afternoon, a Church service missed on Sunday morning, etc. These may be missed without drastic consequences. However, you may have a golf/business game with a client scheduled on Saturday afternoon that could have consequences related to your job and perhaps worth the $320 extra expense.

1-24

The remaining costs for the year are:

(a) and (b) Alternatives and their costs:

1. To stay in the dormitory the rest of the year
   Food: 8 months at $300/month = $2400

2. To stay in the dormitory the balance of the first semester; apartment for second semester (consider person paying $700 for second semester)
   Housing: 4½ months × $200 apartment − $700 dorm = $200
   Food: 3½ months × $300 + 4½ × $250 = $1,950
   Total = $2,150

3. Move into an apartment now
   Housing: 8 months × $200 apartment – 8 × $100 dorm = $800
   Food: 8 months × $250 = $2000
   Total = $2,800

(c) He should stay in the dormitory for the rest of this semester and then move into an apartment. This alternative (#2) is the lowest cost.

1-25

"In decision making the model is mathematical."

1-26

The situation is an example of the failure of a low-cost item that may have major consequences in a production situation. While there are alternatives available, one appears so obvious that that foreman discarded the rest and asks to proceed with the replacement.

One could argue that the foreman, or the plant manager, or both are making decisions. There is no single "right" answer to this problem.

1-27

While everyone might not agree, the key decision seems to be in providing Bill's dad an opportunity to judge between purposely limited alternatives. Although suggested by the clerk, it was Bill's decision.

(One of my students observed that his father would not fall for such a simple deception, and surely would insist on the weird shirt as a subtle form of punishment.)

1-28

Plan A: Profit = Income − Cost = $800 – $600 = $200/acre
Plan B: Profit = Income − Cost = $1,900 – $1,500 = $400/acre
Plan C: Profit = Income − Cost = $2,250 – $1,800 = $450/acre
Plan D: Profit = Income − Cost = $2,500 – $2,100 = $400/acre

To maximize profit, choose Plan C.

1-29

Each student's answer will be unique, but there are likely to be common threads. Alternatives to their current major are likely to focus on other fields of engineering and science, but answers are likely to be distributed over most fields offered by the university. Outcomes include degree switches, courses taken, changing dates for expected graduation, and probable future job opportunities.

At best criteria will focus on joy in the subject matter and a good match for the working environment that pleases that particular student. Often economic criteria will be mentioned, but these are more telling when comparing engineering with the liberal arts than when comparing engineering fields. Other criteria may revolve around an inspirational teacher or an influential friend or family member. In some cases, simple availability is a driver. What degree programs are available at a campus or which programs will admit a student with a 2.xx GPA in first-year engineering?

At best, the process will follow the steps outlined in this chapter. At the other extreme, a student's major may have been selected by the parent and may be completely mismatched to the student's interests and abilities.

Students shouldn't lightly abandon a major, as changing majors represents real costs in time, money, and effort and real risks that the new choice will be no better a fit. Nevertheless, it is a large mistake to not change majors when a student now realizes the major is not for them.

1-30

The most common large problem faced by undergraduate engineering students is where to look for a job and which offer to accept. This problem seems ideal for listing student ideas on the board or overhead transparencies. It is also a good opportunity for the instructor to add more experienced comments.

1-31

1. Recognize problem – I'm going to graduate in one more semester and I need to decide what I'm going to do.
2. Define the goal or objective – I do not want to move back in with my parents. I would much rather be independent, live on my own and do something that I enjoy.
3. Assemble relevant data – How much money do I need to live on my own? Where would it be best for me to live so I can continue with my favorite activities? How important is it that I be close to family? Are jobs available that allow me to do what I enjoy? What types of teaching assistantship are available?
4. Identify feasible alternatives – Find a job near my hometown or at least in my home state. Apply for graduate assistantships at several universities.
5. Select the criteria to determine the best alternative – Will I enjoy what I will be doing? Will it provide me with enough money to live on? Will I be able to continue with my favorite activities?
6. Construct a model – List possible job activities and study topics and assign each a number from 1 to 10 based on personal preference. Make a range of acceptable remuneration and assign a 1 for below range, 2 for within range, and
3 for above range. List favorite activities and assign each a number between 1 and 3 depending on how much you like to do it.
7. Predict each alternative's outcomes or consequences – For this scenario there will be two steps here. First, use the model and decision criteria to decide to which jobs and graduate schools to apply. Second, when you receive offers, use the model again.
8. Choose the best alternative – Choose the job or graduate school offer having the largest number.
9. Audit the result – In six months reflect on your decision. Are you happy? Have you earned enough money to live on? Are you doing activities that you enjoy?

1-32

Test marketing and pilot plant operation are situations where it is hoped that solving the subproblems gives a solution to the large overall problem. On the other hand, Example 3-1 (shipping department buying printing) is a situation where the sub-problem does not lead to a proper complex problem solution.

1-33

The criterion will be to maximize net after-tax income considering risk, social and environmental factors, and ethicality.

1-34

Ethics consists of standards of behavior, conduct, and moral judgment.

1-35

The criteria would be legality, balance (equity and fairness), harmfulness to others, ability to live with yourself.

1-36

(a) The IEEE Code of Ethics emphasizes: (1) responsibility in decision making consistent with safety, health and welfare of the public, and avoiding endangerment of the public and environment, (2) avoiding conflicts of interest, (3) being honest when using data, (4) rejecting bribery, (5) improving understanding of technology, (6) maintaining and improving technical competence, (7) honest criticizing of technical work and proper crediting the contributions of others, (8) treating all people fairly, (9) avoiding injury to others by false or malicious action, and (10) assisting others in their professional development.

(b) The NSPE Code of Ethics is much more detailed than IEEE's code. All of the items listed above are covered in one way or another in the NSPE code. The NSPE code includes much more detail about the conduct of an engineer in his employment, his disclosure of his work, his interactions with other firms, and his interactions with the public.

1-37

Student answers will vary depending on their experience or what they find. In Table 1-1, the author offers some excellent examples of ethical lapses than can occur at the various steps of the design process. It would be hoped that some positive ethical occurrences (i.e., opposites of lapses) will be included in their analyses.

1-38

(a) Ethical issues that might arise include: (1) excessive road improvements in areas where assembly members live or own property, (2) acquiring land for building a new school in areas where school board members live or own property, (3) approving building improvements that favors the hiring of relatives or using a company owned by one of the school board members, (4) firing a person for personal reasons not related to their job performance, (5) promoting a personal agenda not in step with sound teaching practices or at odds with the vast majority of the scientific community.

(b) Many large cities have City Ethics Commissions to administer and enforce the laws related to government ethics, campaign finance, and lobbyist activities. They may engage in mandated programs, introduce ethic reforms, conduct investigations, audit campaigns, summarize disclosure reports, provide advice about the law, prepare statements of incompatible activities for various departments, boards and commissions, and meet with community groups.

(c) Student answers will vary depending on what they find.

1-39

(a) Ethical issues that might arise include: (1) the road improvement may be intended to benefit a new large business or mall at the expense of existing small businesses who loose business during the construction and/or who suffer parking loss after the construction, (2) local businesses may lose business because commuters can travel through the area much faster, (3) road improvements usually mean widening so local residences and businesses may loose property to the improvement, (4) the road improvement may divert money away from other more cost effective projects, (5) the improvement that mostly aids commuters may, in fact, be paid for by a bond issue that is ultimately paid off by local property and sales taxes.

(b) Student answers will vary depending on what they find.

(c) Many cities have Ethics Boards that can address these issues. Many states allow such boards to be more restrictive than the minimum standards set by state laws.

1-40

Student answers to this question will be highly variable. What follows below is only a sample of what you may expect.

(a) It would seem that the mostly likely ethical question to arise here is the use of eminent domain to shift the ownership of property from one private party to another. It is well established in the U.S. Constitution that the Federal Government (5th amendment) and State Governments (14th amendment) can take private property for "public use" provided there is "just compensation." However, to shift ownership to another private party for indirect benefits like increased taxes is not as clear cut and would seem to require an ethical analysis perhaps using a utilitarian principle; i.e., do the benefits outweigh the disbenefits for all parties concerned?

(b) Student answers will vary depending on what they find. If you need to point to an example, try the recent New Trumbell, Connecticut, case.

(c) In Indiana, eminent domain procedure is established in law by the State Legislature. The most recent change, effected in 2006, redefined the term "public use" to specifically exclude "the public benefit of economic development including an increase in tax base, tax revenues, employment, or general economic health." It would seem that the ethical problem suggested in part (a) no longer exists here.

1-41

Student answers to this question will be highly variable depending on what they find.

(a) The most obvious ethical issue would be a conflict of interest where a certain project is promoted that, if funded, would help the company for which the engineer works or has ties to through family, friends or, in the worst case, ownership. Along these lines of favoring a particular company, other conflicts could be relaxation of environmental regulations, special tax considerations, changing fee structures by regulated utilities, etc.

(b) The Indiana State Ethics Commission consists of five Commissioners. They are appointed by the Governor and serve for four-year staggered terms. The Commission holds monthly public meetings during which it issues Advisory Opinions and receives complaints filed by the Inspector General. Members may not be elected officials, state employees, or lobbyists. No more than three may be from the same political party. The committee has a published Code of Ethics. In addition, there is an Indiana Utility Regulatory Commission (IURC) which among its duties is to set utility rates. No one can serve on the commission if they have a demonstrable conflict of interest. The IURC has a published set of ethical considerations. Other states will have similar ethics commissions.

(c) Student answers will vary depending on what they find. An example here may be difficult to locate.

1-42

Student answers to this question will be highly variable depending on what they find.

(a) Possible ethical conflicts that may arise are: (1) working in a governmental regulatory capacity and having a financial interest in a private concern that the regulations cover, (2) using previous governmental contacts to influence favorable legislation for a private industry, (3) using secret or classified information learned in governmental work to make financial investments after becoming a private citizen, (4) using your influence as a private person on a public works project to promote a favorite but, perhaps, unsafe design, (5) taking a job involving public contracts in which you participated as a public employee.

(b) Most states have an Ethics Commission at least minimally charged with educating public and former public employees about ethical rules, which, when violated, could lead to civil and criminal penalties.

(c) Student answers will vary depending on what they find. An example here may be difficult to locate.

1-43

Student answers to this question will be highly variable depending on what they find.

(a) Possible ethical and legal conflicts that may arise are: (1) exploitation of workers can be effected by placing them on salary with no extra pay for overtime, (2) workers may "fake" work in order to receive overtime pay, (3) the existence of overtime pay may be used by employers to "force" employees to work longer hours, i.e., "don't complain, you're getting paid for it," (4) an employer may make you work 70 hours one week and only 10 the next but only pay you for a normal 80 hours every two weeks (probably illegal), (5) your employer may fire you for challenging questionable overtime practices (probably illegal).

(b) The federal government regulates overtime law with the Fair Labor Standards Act (FLSA) of 1938, however, almost 42% of laborers are exempt for one reason or another from this act. The Wage and Hour Division of the U. S. Labor Department is charged with enforcing the FLSA. States may pass additional overtime laws as is the case in California.

(c) Student answers will vary depending on what they find.

1-44

Student answers to this question will be highly variable depending on what they find.

(a) Possible ethical and legal conflicts that may arise are: (1) legislators may pass laws favorable to large campaign donors, (2) lobbyists may present unfounded "facts" when arguing for favorable action, (3) lobbyists may provide favors (airplane travel, vacations, campaign money, etc.) to obtain desired legislation, (4) advocacy organizations may prepare documents that are one-sided and ignore or distort relevant scientific data, (5) legislators may expend taxpayer funds for unapproved purposes.

(b) Student answers will vary depending on what they find.

(c) The Office of Government Ethics exercises leadership in the executive branch to "prevent conflicts of interest on the part of government employees, and to resolve those conflicts when they occur." The U.S. Senate passed legislation in the Spring of 2006 restricting lobbyist gift-giving and making lobbying activities more open. Many people feel that much more work needs to done in this area.

1-45

Student answers to this question will be highly variable. What follows below is only a sample of what you may expect.

(a) Projects may be funded that benefit small numbers of people compared to the proportion of funding required, that benefit a company with ties to the congressman's family or friends or in which the congressman's "blind trust" owns stock, that benefit industries that are major polluters, that benefit special interest groups that have helped elect the congressman, or that lead to expressways or bridges named after the congressman himself!

(b) $1.25M has been earmarked in the 2007 Senate Transportation, Housing and Urban Development Appropriation Bill for the Fort Wayne Clinton Street bridge replacement ($1M) and an update to signage for the Fort Wayne International Airport ($0.25M). The bridge replacement should "improve transportation access to the downtown area and spur economic development." The airport signage should help the airport to "continue to serve the people and businesses of Northeast Indiana." A non-federal match of 20% in required. Both Senators from Indiana supported this legislation (a Democrat and a Republican).

Comment: This example is used to illustrate the fact that not all pork barrel earmarking is blatantly unethical, nor obviously unethical, nor simply unethical. Perhaps one can argue that the signage is frivolous (the airport is very easy to find, even in the dark!); however, the bridge replacement is anything but frivolous considering the state of infrastructure in the U.S. today. Ethical questions are rarely ever clear cut.

(c) The U.S. Senate Select Committee on Ethics is charged with dealing with matters related to senatorial ethics. Senate rules require the Ethics Committee to be evenly divided between the Democrats and the Republicans, no matter who controls the Senate.

1-46

Student answers to this question will be highly variable depending on what they find.

(a) Possible ethical conflicts that may arise are: (1) moving an industry to a third-world country to take advantage of lax environmental laws, (2) exporting garbage or toxic waste to underdeveloped countries, (3) selling insecticides to third-world countries that are banned in the west, (4) exploiting third-world countries for their oil, timber, and minerals.

(b) Student answers will vary depending on what they find. An example here may be difficult to locate.

(c) Many binding international agreements concerning international environmental law are in existence. They cover such topics as atmospheric and water pollution through wildlife and biodiversity protection.

1-47

Student answers to this question will be highly variable depending on what they find.

(a) Possible ethical conflicts that may arise are: (1) moving an industry to a third-world country to take advantage of lax health and safety laws, (2) moving an industry to a third-world country to take advantage of nonexistent child labor laws, (3) agreeing to build a dangerous chemical plant in a foreign country that insists on plant staffing with little educated but supposedly "trained" local workers.

(b) Student answers will vary depending on what they find. An example here may be difficult to locate.

(c) Many organizations exist throughout the world that are making an attempt through education and exposure to attack this intractable problem. Some examples are: (1) Office of Health, Safety and Security in the USA through international studies, (2) National Institute for Occupational Safety and Health Hazards in the UK through publication of a magazine, (3) International Chemical Workers Union Council Center for Worker Health and Safety Education with many consortium members through training and publications, and (4) Clean Clothes Campaign through a code of conduct, publications and international campaigns.

1-48

Student answers to this question will be highly variable depending on what they find.

(a) Possible ethical conflicts that may arise are: (1) a project that disrupts the environment more than intended, say a dam or road, (2) a project that causes disruption of social mores, say mechanized farm machinery where beasts of burden have been used for millennia, (3) a project with too high of operating costs that are not sustainable by the indigenous population, say a sewer system, (4) a project that over stresses the environment, say too much logging or too many tourists.

(b) Student answers will vary depending on what they find. An example here may be difficult to locate.

(c) Many national and international organizations exist that offer education and advice related to sustainable development. Some are (no favoritism intended) U. S. Government's Sustainable Development Partnerships (SDP), U. K. Sustainable Development Commission, World Business Council for Sustainable Development, European Sustainable Development Network, International Institute for Sustainable Development, etc.

1-49

Student answers to this question will be highly variable. What follows below is only a sample of what you may expect.

(a) Bribery can cause people to make purchases that do not reward the most efficient producer, can result in substandard or even dangerous products being sold to an unsuspecting public, can degrade the respect one has towards fellow human beings, and can produce cynicism and distrust of institutions.

(b) Student answers to this question will be highly variable depending on what they find. The literature is replete with examples so one should not be hard to locate. Most will involve the wanton disregard of any ethical principles in the pursuit of monetary gain.

(c) The Organization for Economic Cooperation and Development (OECD) Convention on Combating Bribery of Foreign Public Officials in International Business Transactions (Anti-bribery Convention) has been ratified by all 35 signatories as of 2004. The main attempt of the OECD is to promote the putting in place and then enforcement of anti-bribery laws under the convention in each of the signing countries. Also, the Foreign Corrupt Practices Act allows for Federal prosecution in the United States for cases of bribery of foreign officials.

1-50

(a) Ford certainly did NOT hold paramount the safety, health, and welfare of the public. Their public statements at the time certainly were NOT objective and truthful as they continued to claim that the Pinto was a safe automobile to drive. They did NOT avoid deceptive acts. Finally, they did NOT conduct themselves honorably, responsibly, and ethically (although perhaps they were lawful). One would like to believe that the Pinto disaster was only due to management decisions and that engineers were not involved in the deception.

(b) It would seem that the "greatest good" was limited to the company itself. The "greatest number" were the owners of the Pinto.

(c) This is the "ultimate question" that you as an engineer dread! The best first thing to do is to reveal the problem through all available channels within the company. Leave no legitimate way untried. The second, assuming you are being thwarted by a mid-level manager, is to go as close to the top as you can without being anonymous. A last resort, assuming the defect can result in injury to humans, is to whistle blow outside the company but beware of the potential negative consequences to your future.

1-51

Itemized expenses: $0.223 × 18,000 miles + $2,000 = $6,014
Based on Standard Mileage Rate: $0.32 × $18,000 = $5,760

Itemizing produces a larger reimbursement.

Breakeven: Let x = mileage at which both methods yield the same amount.
x = $2,000/($0.32 – $0.223) = <u>20,619 miles</u>

1-52

The fundamental concept here is that we will trade an hour of study in one subject for an hour of study in another subject so long as we are improving the total results. The stated criterion is to “get as high an average grade as possible in the combined classes.” (This is the same as saying “get the highest combined total score.”)

Since the data in the problem indicate that additional study always increases the grade, the question is how to apportion the available 15 hours of study among the courses. One might begin, for example, assuming five hours of study on each course. The combined total score would be 190.

Decreasing the study of mathematics one hour reduces the math grade by 8 points (from 52 to 44). This hour could be used to increase the physics grade by 9 points (from 59 to 68). The result would be:

Math	4 hours	44
Physics	6 hours	68
Engr. Econ.	5 hours	79
Total	15 hours	191

Further study would show that the best use of the time is:

Math	4 hours	44
Physics	7 hours	77
Engr. Econ.	4 hours	71
Total	15 hours	192

1-53

Saving = 2 [$185.00 + (2 × 90 miles) ($0.60/mile)] = $586.00/week

1-54

Area A: Preparation Cost = $2 \times 10^6 \times \$2.35$ = \$4,700,000

Area B: Difference in Haul
0.60 × 5 miles = 3.0 miles
0.20 × –2 miles = –0.4 miles
0.20 × 0 = 0 miles
Total = 2.6 miles average additional haul

Cost of additional haul/load = 2.6 mi/15 mph × \$35/hr = \$6.07

Since truck capacity is 20 m^3:
Additional cost/cubic yard = \$6.07/20 m^3 = \$0.303/m^3

For 14 million cubic meters:
Total Cost = $14 \times 10^6 \times \$0.303$ = \$4,240,000

<u>Area B with its lower total cost is preferred.</u>

1-55

3,000 gallon capacity = 3,000 gallons/7.48 gal/cf = 401 cubic ft. capacity

Let: L = tank length in feet
d = tank diameter in feet

The volume of a cylindrical tank equals the end area × length:
Volume = $(\Pi/4)\ d^2L = 401$ cf
$L = (401 \times 4)/(\Pi\ d^2)$

The total surface area is the two end areas + the cylinder surface area:
$S = 2\ (\Pi/4)\ d^2 + \Pi\ dL$

Substitute in the equation for L:
$S = (\Pi/2)\ d^2 + \Pi d\ [(401 \times 4)/(\Pi d^2)]$
$= (\Pi/2)d^2 + 1{,}604d^{-1}$

Take the first derivative and set it equal to zero:
$dS/dd = \Pi d - 1{,}604d^{-2} = 0$

$\Pi d = 1{,}604/d^2$

$d^3 = 1{,}604/\Pi = 510$

d = 8'

Subsitute back to find L:
$L = (401 \times 4)/(\Pi d^2) = 1{,}604/(\Pi 8^2) = 8'$

<u>Tank diameter = 8'</u>
<u>Tank length = 8'</u>

1-56

Quantity Sold per Week	Selling Price	Income	Cost	Profit
300 packages	$0.60	$180	$104	$75
600	$0.45	$270	$210	$60
1,200	$0.40	$480	$336	$144
1,700	$0.33	$561	$425*	$136
			$400**	$161
2,500	$0.26	$598	$460	$138

* buy 1,700 packages at $0.25 each
** buy 2,000 packages at $0.20 each

Conclusion: <u>Buy 2.000 packages at $0.20 each. Sell at $0.33 each.</u>

1-57

Time Period	Daily Sales in Time Period	Cost of Groceries	Hourly Cost	Hourly Profit
0600–0700	$20	$14	$10	–$4
0700–0800	$40	$28	$10	+$2
0800–0900	$60	$42	$10	+$8
0900–1200	$200	$140	$30	+$30
1200–1500	$180	$126	$30	+$24
1500–1800	$300	$210	$30	+$60
1800–2100	$400	$280	$30	+$90
2100–2200	$100	$70	$10	+$20
2200–2300	$30	$21	$10	–$1
2300–2400	$60	$42	$10	+$8
2400–0100	$20	$14	$10	–$4

The first profitable operation is in 0700–0800 time period. In the evening the 2200–2300 time period is unprofitable, but next hour's profit more than makes up for it.

Conclusion: <u>Open at 0700, close at 2400.</u>

1-58

			Outcome		
Alternative	Price	Net Income per Room	Rate	No. Room	Net Income
1	$36	$24	100%	50	$1,200
2	$42	$30	94%	47	$1,410
3	$48	$36	80%	40	$1,440
4	$54	$42	66%	33	$1,386
5	$48	$36	70%	35	$1,260
6	$54	$42	68%	34	$1,428
7	$62	$50	66%	33	$1,650
8	$68	$56	56%	28	$1,568

To maximize net income, Joy should not advertise and charge $62 per night.

1-59

Profit = Income – Cost
= PQ – C where $PQ = 35Q - 0.02Q^2$
$C = 4Q + 8,000$

$d(\text{Profit})/dQ = 31 - 0.04Q = 0$

Solve for Q:
$Q = 31/0.04 = 775$ units/year

$d^2 (\text{Profit})/dQ^2 = -0.04$

The negative sign indicates that profit is maximum at Q equals 775 units/year.
Answer: Q = 775 units/year

1-60

Basis: 1,000 pieces

Individual Assembly: $22.00 × 2.6 hours × 1,000 = $57,200 $57.20/unit
Team Assembly: 4 × $13.00 × 1.0 hours × 1,000 = $52,00 $52.00/unit

<u>Team Assembly is less expensive.</u>

1-61

Let t = time from the present (in weeks)
Volume of apples at any time = (1,000 + 120t − 20t)
Price at any time = \$3.00 − \$0.15t

Total Cash Return (TCR) = (1,000 + 120t − 20t) (\$3.00 – \$0.15t)
= \$3,000 + \$150t − \$15t^2

This is a minima–maxima problem.
Set the first derivative equal to zero and solve for t.

dTCR/dt = \$150 – \$30t = 0

t = \$150/\$30 = 5 weeks

$d^2TCR/dt^2 = -10$

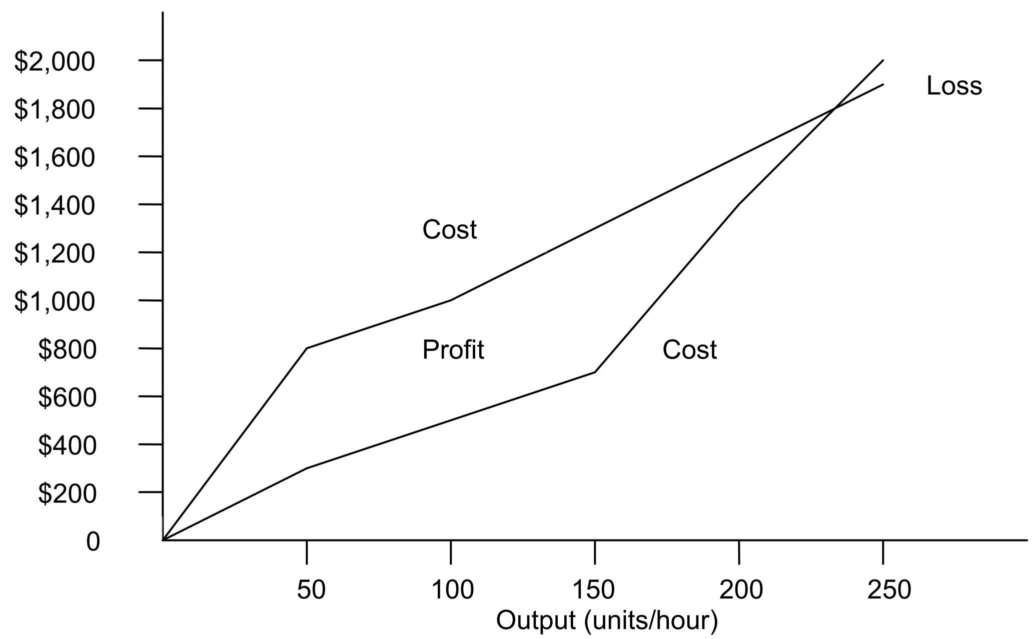
(The negative sign indicates the function is a maximum for the critical value.)

At t = 5 weeks:
<u>Total Cash Return (TCR) = \$3,000 + \$150 (5) – \$15 (25) = \$3,375</u>

1-62

(a) The suitable criterion is to maximize the difference between output and input. Or simply, maximize net profit. The data from the graphs may be tabulated as follows:

Output Units/Hour	Total Cost	Total Income	Net Profit
50	$300	$800	$500
100	$500	$1,000	$500
150	$700	$1,350	$650 ←
200	$1,400	$1,600	$200
250	$2,000	$1,750	–$250



(b) Minimum input is, of course, zero, and maximum output is 250 units/hr (based on the graph). Since one cannot achieve maximum output with minimum input, the statement makes no sense.

Chapter 2: Engineering Costs and Cost Estimating

2-1

(a) 500 parts
Average cost = $13
Marginal cost = $13

(b) 1500 parts
Average cost = ((1000)($13) + (500)($12)) / 1500 = $ 12.67
Marginal cost = $12

(c) 2500 parts
Average cost = ((1000)($13) + (1500)($12)) / 2500 = $12.40
Marginal cost = $12

(d) 3500 parts
Average cost = ((1000)($13) + (2000)($12) + 500($11)) / 3500 = $12.14
Marginal cost = $11

2-2

(a) 75 hours
Average cost = 0
Marginal cost = 0

(b) 125 hours
Average cost = (25)($75) / 125 = $15
Marginal cost = $75

(c) 250 hours
Average cost = (150)($75) / 250 = $45
Marginal cost = $75

2-3

Unit Manufacturing Cost

(a) Daytime Shift = ($2,000,000 + $9,109,000)/23,000
= $483/unit

(b) Two Shifts = [($2,400,000 + (1 + 1.25) ($9,109,000)]/46,000
= $497.72/unit

<u>Second shift increases unit cost.</u>

2-4

(a) Monthly Bill:

50 × 30	= 1,500 kw-hr @ $0.086	= $129.00
	= 1,300 kw-hr @ $0.066	= $85.80
Total	= 2,800 kw-hr	= $214.80

Average Cost = $214.80/2,800 = $129.00
Marginal Cost (cost for the next kw-hr) = $0.066 because the 2,801st kw-hr is in the 2nd bracket of the cost structure.

($0.066 for 1,501–3,000 kw-hr)

(b) Incremental cost of an additional 1,200 kw-hr/month:

200 kw-hr × $0.066	= $13.20
1,000 kw-hr × $0.040	= $40.00
1,200 kw-hr	$53.20

(c) New equipment:
Assuming the basic conditions are 30 HP and 2,800 kw-hr/month
Monthly bill with new equipment installed:

50 × 40 =	2,000 kw-hr at $0.086	= $172.00
	900 kw-hr at $0.066	= $59.40
	2,900 kw-hr	= $231.40

Incremental cost of energy = $231.40 – $214.80 = $16.60
Incremental unit cost = $16.60/100 = $0.1660/kw-hr

2-5

x = no. of maps dispensed per year

(a) Fixed Cost (I) = \$1,000
(b) Fixed Cost (II) = \$5,000
(c) Variable Costs (I) = 0.900
(d) Variable Costs (II) = 0.100
(e) Set Total Cost (I) = Total Cost (II)
\$1,000 + 0.90 x = \$5,000 + 0.10 x
thus x = 5,000 maps dispensed per year.
The student can visually verify this from the figure.
(f) System I is recommended if the annual need for maps is <5,000
(g) System II is recommended if the annual need for maps is >5,000
(h) Average Cost @ 3,000 maps:
TC(I) = (0.9) (3.0) + 1.0 = 3.7/3.0 = \$1.23 per map
TC(II) = (0.1) (3.0) + 5.0 = 5.3/3.0 = \$1.77 per map
Marginal Cost is the variable cost for each alternative, thus:
Marginal Cost (I) = \$0.90 per map
Marginal Cost (II) = \$0.10 per map

2-6

x = number of campers

(a) Total Cost = Fixed Cost + Variable Cost
= \$48,000 + \$80 (12) x
Total Revenue = \$120 (12) x

(b) Breakeven when Total Cost = Total Revenue
\$48,000 + \$960 x = \$1,440 x
\$4,800 = \$480 x
x = 100 campers to breakeven

(c) Capacity is 200 campers
80% of capacity is 160 campers
@ 160 campers x = 160

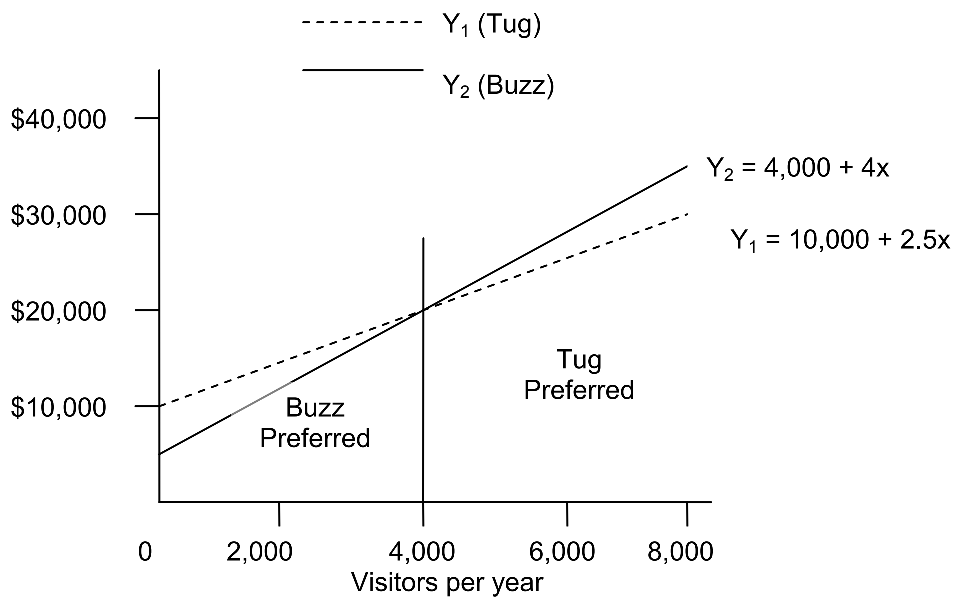
Total Cost = \$48,000 + \$80 (12) (160) = \$201,600
Total Revenue = \$120 (12) (160) = \$230,400
Profit = Revenue – Cost = \$230,400 − \$201,600 = \$28,800

2-7

(a) x = number of visitors per year
Breakeven when: Total Costs (Tugger) = Total Costs (Buzzer)
$10,000 + $2.5 x = $4,000 + $4.00 x
x = 4000 visitors is the breakeven quantity

(b) See the figure below that plots costs as a function of the number of visitors:

X	Y1 (Tug)	Y2 (Buzz)
0	10,000	4,000
4,000	20,000	20,000
8,000	30,000	36,000



2-8

x = annual production

(a) Total Revenue = ($200,000/1,000) x = $200 x

(b) Total Cost = $100,000 + ($100,000/1,000)x = $100,000 + $100 x

(c) Set Total Cost = Total Revenue
$200 x = $100,000 + $100 x
$100 x = $100,000
x = $100,000/$100 = 1,000 units per year

The student can visually verify this from the figure.

(d) Total Revenue = \$200 (1,500) = \$300,000
Total Cost = \$100,000 + \$100 (150) = \$250,000
Profit = \$300,000 − \$250,000 = \$50,000

(e) Given that the slope of the cost equation is \$100, the cost for one more unit is:
Marginal Cost = \$100
At 1500 units, the Total Cost = \$100,000 + \$100(1500) = \$250,000
Average Cost = \$250,000/1500 units = \$166.67/unit

2-9

x = annual production

Let's look at the graphical solution first, where the cost equations are:
Total Cost (A) = \$20 x + \$100,000
Total Cost (B) = \$5 x + \$200,000
Total Cost (C) = \$7.5 x + \$150,000

[See graph below]

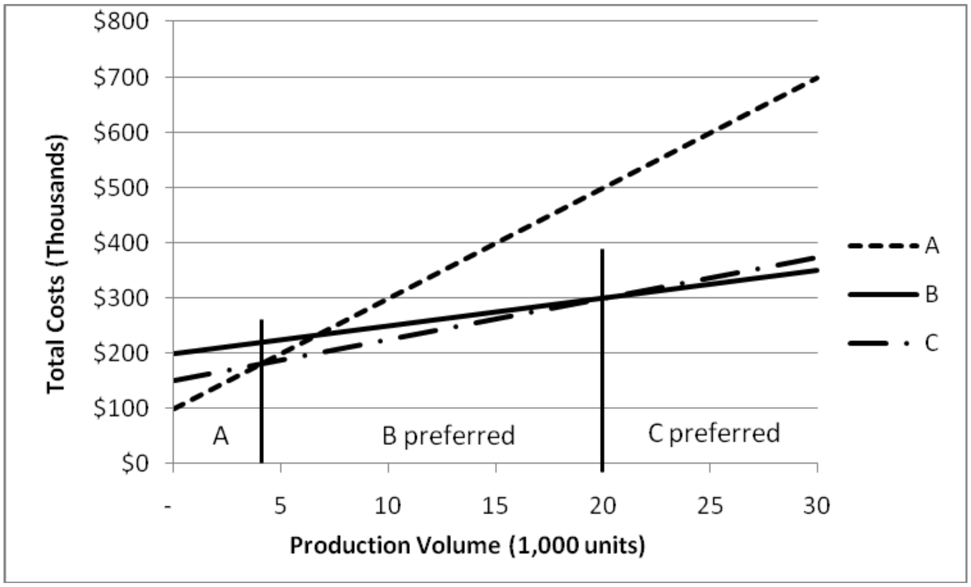
Quatro Hermanas wants to minimize costs over all ranges of x. From the graph we see that there are three breakeven points: A & B, B & C, and A & C. Only A & C and B & C are necessary to determine the minimum cost alternative over x. Mathematically the breakeven points are:

A & C: \$20 x + \$100,000 = \$7.5 x + \$150,000 at x = 4,000
B & C: \$5 x + \$200,000 = \$7.5 x + \$150,000 at x = 20,000

Thus our recommendation is, if:

$0 \le x < 4{,}000$ choose Alternative A
$4{,}000 \le x \le 20{,}000$ choose Alternative C
$20{,}000 \le x \le 30{,}000$ choose Alternative B

X	A	B	C
0	100	200	150
4	180	220	180
10	300	250	225
20	500	300	300
30	700	350	375



2-10

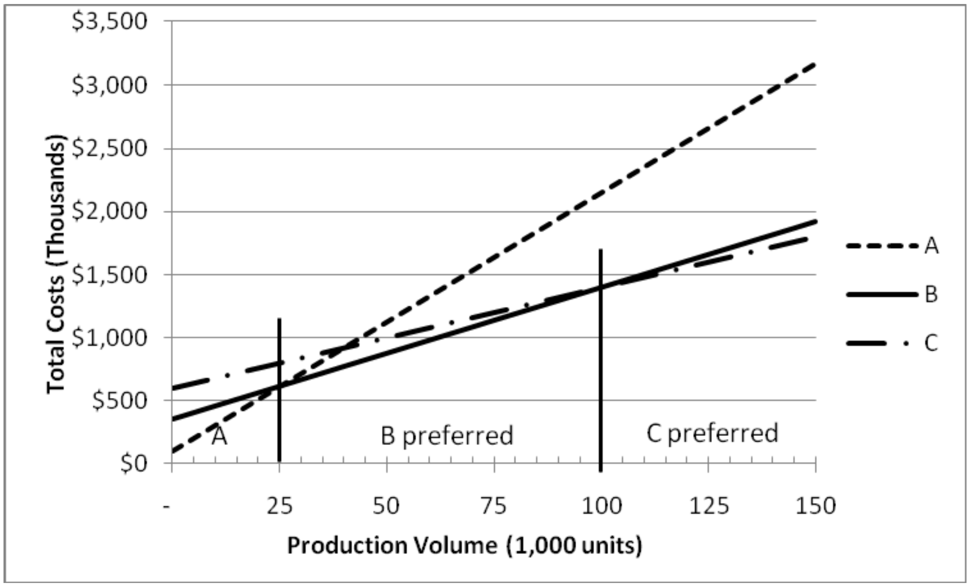
x = annual production rate

(a) There are three breakeven points for total costs for the three alternatives
A & B: \$20.5 x + \$100,000 = \$10.5 x + \$350,000 at x = 25,000
B & C: \$10.5 x + \$350,000 = \$8 x + \$600,000 at x = 100,000
A & C: \$20.5 x + \$100,000 = \$8 x + \$600,000 at x = 40,000

We want to minimize costs over the range of x, thus the A & C breakeven point is not of interest. Looking at figure below we see that if:
0 < x ≤ 25,000 choose A
25,000 ≤ x ≤ 100,000 choose B
100,000 ≤ x ≤ 150,000 choose C

(b) See graph below for Solution:

X	A	B	C
0	100	350	600
25	612.5	612.5	800
50	1,125	875	1,000
100	2,150	1,400	1,400
150	3,175	1,925	1,800



2-11

x = units/year
By hand = Painting Machine
$1.40 x = $15,000/4 + $0.20
x = $5,000/1.20 = <u>$4,167 units</u>

2-12

x = annual production units
Total Cost to Company A = Total Cost to Company B
$15,000 + $0.002 x = $5,000 + $0.05 x
x = $10,000/$0.048 = <u>208,330 units</u>

2-13

$C = \$3,000,000 - \$18,000Q + \$75Q^2$
where C = Total cost per year
Q = Number of units produced per year
Set the first derivative equal to zero and solve for Q.
$dC/dQ = -\$18,000 + \$150Q = 0$
$Q = \$18,000/\$150 = 120$

Therefore total cost is a minimum at Q equal to 120. This indicates that production below 120 units per year is most undesirable, as it costs more to produce 110 units than to produce 120 units.

Check the sign of the second derivative:
$d^2C/dQ^2 = +\$150$

The + indicates the curve is concave upward, ensuring that Q = 120 is the point of a minimum.

Average unit cost at Q = 120/year:
$= [\$3,000,000 – \$18,000 (120) + \$75 (120)^2]/120 = \$16,000$
Average unit cost at Q = 110/year:
$= [\$3,000,000 – \$18,000 (110) + \$75 (120)^2]/110 = \$17,523$

One must note, of course, that 120 units per year is <u>not</u> necessarily the optimal level of production. Economists would remind us that the optimum point is where Marginal Cost = Marginal Revenue, and Marginal Cost is increasing. Since we do not know the Selling Price, we cannot know Marginal Revenue, and hence we cannot compute the optimum level of output.

We can say, however, that if the firm is profitable at the 110 units/year level, then it will be much more profitable at levels <u>greater than</u> 120 units.

2-14

x = annual production volume (demand) = D

(a) Total Cost = \$10,875 + \$20 x
Total Revenue = (price per unit) (number sold)
= (\$0.25 D + \$250) D and if D = x
= −\$0.25 x^2 + \$250 x

(b) Set Total Cost = Total Revenue
\$10,875 + \$20 x = –\$0.25 x^2 + \$250 x
–\$0.25 x^2 + \$230 x – \$10,875 = 0

This polynomial of degree 2 can be solved using the quadratic formula:
There will be two solutions:
$x = (-b \pm (b^2 - 4ac)^{1/2})/2a = (-\$230 \pm \$205)/-0.50$
Thus x = 870 and x = 50. There are two levels of x where TC = TR.

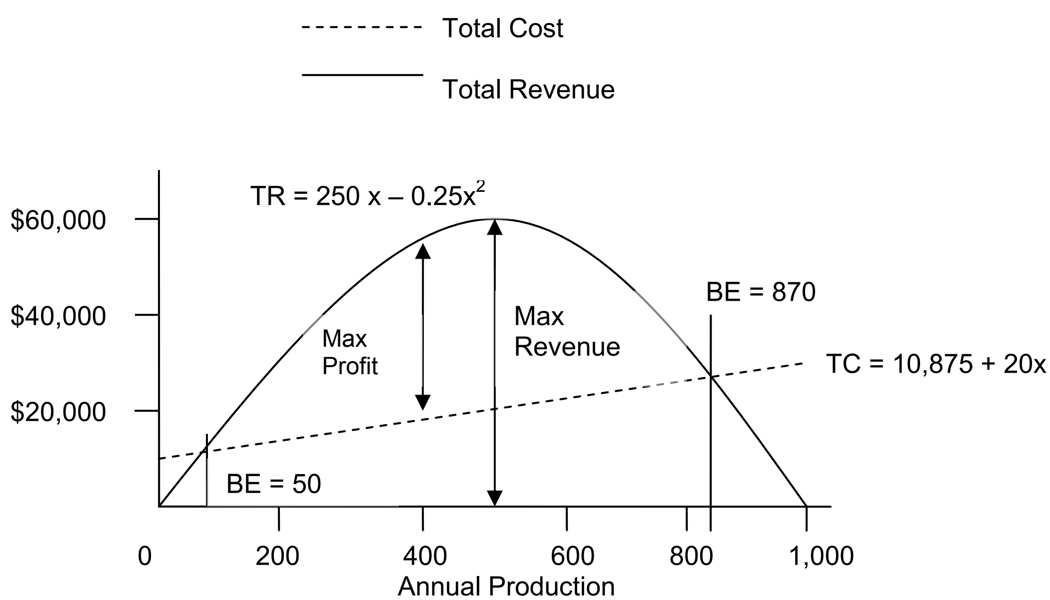
(c) To maximize Total Revenue we will take the first derivative of the Total Revenue equation, set it equal to zero, and solve for x:
TR = −\$0.25 x^2 + \$250 x
dTR/dx = −\$0.50 x + \$250 = 0
x = 500 is where we realize maximum revenue

(d) Profit is revenue – cost, thus let's find the profit equation and do the same process as in part (c).

Total Profit = (−\$0.25 x^2 + \$250 x) − (\$10,875 + \$20 x)
= −\$0.25 x^2 + \$230 x − \$10,875
dTP/dx = −\$0.50 x + \$230 = 0
x = 460 is where we realize our maximum profit

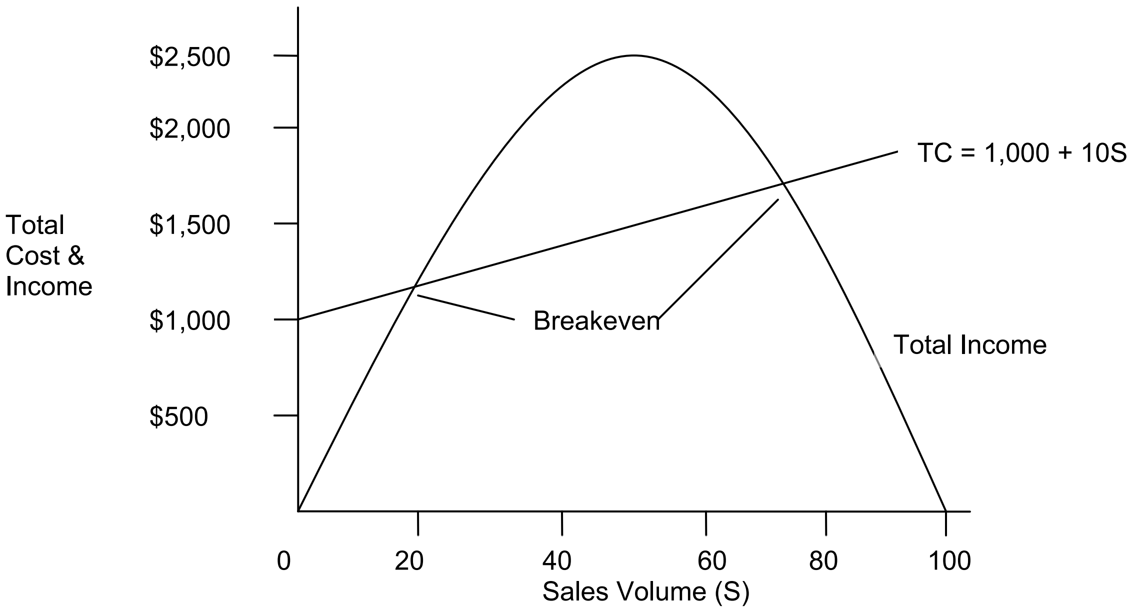
(e) See the figure below. Your answers to (a) – (d) should make sense now.

X	Total Cost	Total Revenue
0	$10,875	$0
250	$15,875	$46,875
500	$20,875	$62,500
750	$25,875	$46,875
1,000	$30,875	$0



2-15

(a)



(b) For breakeven, set Profit = 0
$-S^2 + \$90S - \$1{,}000 = \$0$

$S = (-b \pm (b^2 - 4ac)^{½})/2a = (-\$90 \pm (\$90^2 - (4)(-1)(-1{,}000))^{½})/-2$
$= 12.98, 77.02$

(c) For maximum profit
$dP/dS = -\$2S + \$90 = \$0$
$S = 45$ units

Answers: Breakeven at 14 and 77 units. Maximum profit at 45 units.

Alternative Solution: Trial & Error

Prige	Sales Volume	Total Income	Total Cost	Profit
$20	80	$1,600	$1,800	−$200
$23	77	$1,771	$1,770	$0 (Breakeven)
$30	70	$2,100	$1,700	$400
$50	50	$2,500	$1,500	$1,000
$55	45	$2,475	$1,450	$1,025
$60	40	$2,400	$1,400	$1,000
$80	20	$1,600	$1,200	$400
$87	13	$1,131	$1,130	$0 (Breakeven)
$90	10	$900	$1,100	−$200

2-16

(a) $7000 The book cost results strictly from depreciation and can be more or less than the market value.

(b) $4000 The pump could be sold for this amount. If the pump is used instead, then it would be viewed as an opportunity foregone.

(c) $6000 – 4000 – 500 = $1500 cheaper than buying the brass pump.

2-17

If sell in December: Cost per use = (500 – 200) / 52 = $5.77.

If sell in May: Cost per use = (500 – 100) / 72 = $5.56.

The cost to move it is roughly equal to 4 to 5 uses.

The longer you keep it and use it, its cost per use will continue to drop. Since there is no convenient gym at the new location and if you value the exercise that you get using the exercise equipment, then you should keep it. The moving cost is only 5% of its original cost and, presumably, its replacement cost, so moving it is cheap. Sell it in December only if you absolutely need the $200 and you feel the exercise is not bettering your health.

2-18

This is an example of a “sunk cost.” The $4,000 is a past cost and should not be allowed to alter a subsequent decision unless there is some real or perceived effect. Since either home is really an individual plan selected by the homeowner, each should be judged in terms of value to the homeowner vs. the cost. On this basis the stock plan house appears to be the preferred alternative.

2-19

In this situation the owners would have both recurring costs (repeating costs per some time period) as well as nonrecurring costs (one time costs). Below is a list of possible recurring and nonrecurring costs. Students may develop others.

Recurring Costs	Nonrecurring costs
- Annual inspection costs	- Initial construction costs
- Annual costs of permits	- Legal costs to establish rental
- Carpet replacement costs	- Drafting of rental contracts
- Internal/external paint costs	- Demolition costs
- Monthly trash removal costs	
- Monthly utilities costs	
- Annual costs for accounting/legal	
- Appliance replacements	
- Alarms, detectors, etc., costs	
- Remodeling costs (bath, bedroom)	
- Durable goods replacements (furnace, air-conditioner, etc.)	

2-20

Recurring Costs	Nonrecurring Costs
- Tuition	- Vehicle repair
- Room and board	- Doctor's fee
- Books	- Admission to out-of-town athletic event
- Gas for commuting	- Flash drive
- Automobile oil change	- Tattoo
- Morning coffee	- Flowers for girlfriend

2-21

A cash cost is a cost in which there is a cash flow exchange between or among parties. This term derives from "cash" being given from one entity to another (persons, banks, divisions, etc.). With today's electronic banking capabilities, cash costs may or may not involve "cash." "Book costs" are costs that do not involve an exchange of "cash", rather, they are only represented on the accounting books of the firm. Book costs are not represented as before-tax cash flows.

Engineering economic analyses can involve both cash and book costs. Cash costs are the before-tax cash flows usually estimated for a project (such as initial costs, annual costs, and retirement costs) as well as costs due to financing (payments on principal and interest debt) and taxes. Cash costs are important in such cases. For the engineering economist the primary book cost that is of concern is equipment depreciation, which is accounted for in after-tax analyses.

2-22

Here the student may develop several different thoughts as it relates to life cycle costs. By life cycle costs the authors are referring to any cost associated with a product, good, or service from the time it is conceived, designed, constructed, implemented, delivered, supported and retired. Firms should be aware of and account for all activities and liabilities associated with a product through its entire life cycle. These costs and liabilities represent real cash flows for the firm — either at the time or some time in the future.

2-23

Costs incurred:
- $600 purchase of refurbished notebook computer.
- $60 replace CD-ROM after two years.
- $30 purchase of wireless mouse.

Total cost of ownership: $690

Estimate benefits of ownership (over 4 years):
- $360 saved by playing games on weekends with friends instead of going to the movies.
- $200 saved by emailing instead of sending letters and making phone calls.
- $100 saved downloading music over the internet.
- $80 saved by doing business (like banking) over the internet instead of buying gas for the car.
- $30 saved by not buying paper and pens for note taking.

Total estimated benefits: $770

Yes, ownership has been worth it.

2-24

The following is a list of car ownership items that would incur costs. Each could be estimated reasonably accurately with a little effort.

(1) License, registration, title, and insurance.
(2) Gas, oil changes, and tire replacement and/or rotation.
(3) Brake pad, brake rotor, belt, wiper, battery replacement.
(4) Exhaust system repair.
(5) Major system repair.
(6) Cooling system (Radiator leak, head gasket leak, hose leak, thermostat malfunction).
(7) Drive train (Transmission repair, clutch replacement, differential replacement).
(8) Brake system (Master and/or slave cylinder replacement, brake line leak).
(9) Fuel system (Fuel pump replacement, gas tank leak, fuel line leak, fuel filter replacement).

The following is a partial list of car ownership benefits. Each would be difficult to quantify in terms of dollars. Accuracy would be low.

(1) Commuting to school and to part-time job.
(2) Running errands, shopping, and dining out with friends.
(3) Visiting parents and relatives.
(4) Traveling to other cities for entertainment, getting to an airport, attending a professional meeting, going to a job interview.
(5) Going to an out-of-town athletic event.
(6) Lugging equipment to a remote site for research.
(7) Going to the post office to mail items.

2-25

Figure 2-4 illustrates the difference between “dollars spent” and “dollars committed” over the life cycle of a project. The key point being that most costs are committed early in the life cycle, although they are not realized until later in the project. The implication of this effect is that if the firm wants to maximize value-per-dollar spent, the time to make important design decisions (and to account for all life cycle effects) is early in the life cycle. Figure 2-5 demonstrates “ease of making design changes” and “cost of design changes” over a project’s life cycle. The point of this comparison is that the early stages of the design cycle are the easiest and least costly periods to make changes. Both figures represent important effects for firms.

In summary, firms benefit from spending time, money and effort early in the life cycle. Effects resulting from early decisions impact the overall life cycle cost (and quality) of the product, good, or service. An integrated, cross-functional, enterprise-wide approach to product design serve the modern firm well.

2-26

In this chapter, the authors list the following three factors as creating difficulties in making cost estimates: One-of-a-Kind Estimates, Time and Effort Available, and Estimator Expertise. Each of these factors could influence the estimate, or the estimating process, in different scenarios in different firms. One-of-a-kind estimating is a particularly challenging aspect for firms with little corporate-knowledge or suitable experience in an industry. Estimates, bids and budgets could potentially vary greatly in such circumstances. This is perhaps the most difficult of the factors to overcome. Time and effort can be influenced, as can estimator expertise. One-of-a-kind estimates pose perhaps the greatest challenge.

2-27

(a) Gas Cost: (500 miles) (1 gal/20 mi) ($1/gal) = $25
Wear and Tear: (500 miles) ($0.08/mi) = $40
Total Cost = $25 + $40 = $65

(b) (75 years) (365 days/year) (24 hours/day) = 657,000 hrs

(c) Miles around Equator = 2 Π (4,000/2) = 12,566 mi

(d) Area of Kansas = (390) (200) = 78,000 mi^2
Area of USA = (78,000) (50 states) = 3,900,000 mi^2

2-28

Total Cost = Phone Unit Cost + Line Cost + One-Time Cost
= ($100/2) 125 + $7,500 (100) + $10,000
= $766,250

Cost to State = $766,250 (1.35) = $1,034,438

2-29

Cost (total) = Cost (paint) + Cost (labor) + Cost (fixed)

Number of Cans Needed = (6,000 ft^2/300 ft^2/can) (2 coats) = 40 cans

Cost (paint)= (10 cans) \$15 = \$150.00
= (15 cans) \$10 = \$150.00
= (15 cans) \$7.50 = \$112.50
Total Cost = \$412.50

Cost (labor)= (5 painters) (10 hrs/day) (4.5 days/job) (\$8.75/hr*painter)
= \$1,968.75

Cost (total) = \$412.50 + \$1,968.75 + \$200 = \$2,581.25

2-30

(a) Unit Cost = \$150,000/2,000 = \$75/ft^2
(b) i. If all items change proportionately, then:
Total Cost = (\$75/ft^2) (4,000 ft^2) = \$300,000
(b) ii. For items that change proportionately to the size increase we multiply by:
4,000/2,000 = 2.0 all the others stay the same.
[See table below]

Cost Item	2,000 ft^2 House Cost	Increase	4,000 ft^2 House Cost
1	(\$150,000) (0.08) = \$12,000	× 1	\$12,000
2	(\$150,000) (0.15) = \$22,500	× 1	\$22,500
3	(\$150,000) (0.13) = \$19,500	× 2	\$39,000
4	(\$150,000) (0.12) = \$18,000	× 2	\$36,000
5	(\$150,000) (0.13) = \$19,500	× 2	\$39,000
6	(\$150,000) (0.20) = \$30,000	× 2	\$60,000
7	(\$150,000) (0.12) = \$18,000	× 2	\$36,000
8	(\$150,000) (0.17) = \$25,500	× 2	\$51,000
		Total Cost	= \$295,500

2-31

(a) Unit Profit = \$410 (0.30)= \$123 or
= Unit Sales Price − Unit Cost
= \$410 (1.3) − \$410 = \$533 − \$410 = \$123

(b) Overall Batch Cost = \$410 (10,000) = \$4,100,000

(c) Of the 10,000 batch:
1. (10,000) (0.01) = 100 are scrapped in mfg.
2. (10,000 − 100) (0.03) = 297 of finished product go unsold
3. (9,900 − 297) (0.02) = 192 of sold product are not returned
Total = 589 of original batch are not sold for profit

Overall Batch Profit = (10,000 − 589) \$123 = \$1,157,553

(d) Unit Cost = 112 (\$0.50) + \$85 + \$213 = \$354
Batch Cost with Contract = 10,000 (\$354) = \$3,540,000
Difference in Batch Cost:
= BC without contract- BC with contract = \$4,100,000 − \$3,540,000
= \$560,000

SungSam can afford to pay up to \$560,000 for the contract.

2-32

$C_A/C_B = I_A/I_B$
$C_{50\ YEARS\ AGO}/C_{TODAY} = AFCI_{50\ YEARS\ AGO}/AFCI_{TODAY}$
C_{TODAY} = (\$2,050/112) (55) = \$1,007

2-33

I_{TODAY} = (72/12) (100) = 600
$C_{LAST\ YEAR}$= (525/600) (72) = \$63

2-34

From Table 2-1 the power sizing exponent is 1.13.
Cost = \$23M $(1.6/1.0)^{1.13}$ = (\$23M) (1.701) = \$3.91 M

2-35

Equipment	Cost of New Equipment minus	Trade-In Value	= Net Cost
Varnish Bath	$(75/50)^{0.80}$ (3,500) = \$4,841	\$3,500 (0.15)	= \$4,316
Power Scraper	$(1.5/0.75)^{0.22}$ (250) = \$291	\$250 (0.15)	= \$254
Paint Booth	$(12/3)^{0.6}$ (3,000) = \$6,892	\$3,000 (0.15)	= \$6,442
		Total	\$11,012

2-36

Equipment	Cost of New Equipment minus	Trade-In Value	= Net Cost
Varnish Bath	4,841 (171/154) = \$5,375	\$3,500 (0.15)	= \$4,850
Power Scraper	291 (900/780) = \$336	\$250 (0.15)	= \$298
Paint Booth	6892 (76/49) = \$10,690	\$3,000 (0.15)	= \$10,240
		Total	\$15,338

2-37

Scaling up cost:
Cost of 4,500 g/hr centrifuge = $(4,500/1,500)^{0.75}$ (40,000) = \$91,180

Updating the cost:
Cost of 4,500 model= \$91,180 (300/120) = \$227,950

2-38

Cost of VMIC − 50 today = 45,000 (214/151) = \$63,775
Using Power Sizing Model:

$(63,775/100,000) = (50/100)^x$

log (0.63775) = x log (0.50)

x = 0.65

2-39

$T(7) = T(1) \times 7^b$
$60 = (200) \times 7^b$
$0.30 = 7^b$
$\log 0.30 = b \log (7)$
$b = \log (0.30)/\log (7) = -0.62$

b is defined as log (learning curve rate)/ log 20
$b = [\log (\text{learning curve rate})/\log 2.0] = -0.62$

$\log (\text{learning curve rate}) = -0.187$
learning curve rate $= 10^{(-0.187)} = .650 = 65\%$

2-40

Time for the first pillar is
$T(10) = T(1) \times 10^{\log (0.75)/\log (2.0)}$
$T(1) = 676$ person hours

Time for the 20th pillar is
$T(20) = 676 (20^{\log (0.75)/\log (2.0)})$
$= 195$ person hours

2-41

80% learning curve in use of SPC will reduce costs after 12 months to
Cost in 12 months $= (x)\ 12^{\log (0.80)/\log (2.0)} = 0.45\ x$
Thus costs have been reduced:
$[(x - 0.45)/x]$ times $100\% = 55\%$

2-42

$T(25) = 0.60 (25^{\log (0.75)/\log (2.0)})$	= 0.16 hours/unit
Labor Cost = ($20/hr) (0.16 hr/unit)	= $3.20/unit
Material Cost = ($43.75/25 units)	= $1.75/unit
Overhead Cost = (0.50) ($3.20/units)	= $1.60/unit
Total Mfg. Cost	= $6.55/unit
Profit = (0.20) ($6.55/unit)	= $1.31/unit
Unit Selling Price	= $7.86/unit

2-43

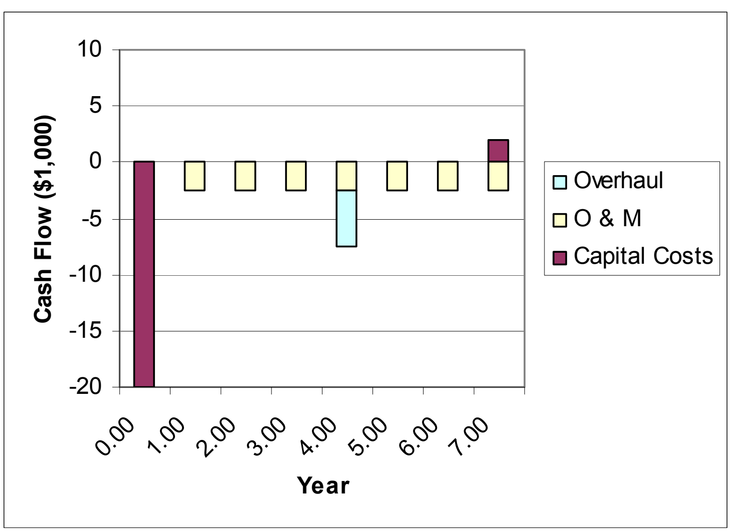
The concepts, models, effects, and difficulties associated with "cost estimating" described in this chapter all have a direct (or near direct) translation for "estimating benefits." Differences between cost and benefit estimation include: (1) benefits tend to be over-estimated, whereas costs tend to be underestimated, and (2) most costs tend to occur during the beginning stages of the project, whereas benefits tend to accumulate later in the project life comparatively.

2-44

Time	Purchase Price	Maintenance	Market Value	Total
0	−$5,000	$0	$0	−$5,000
1	−$6,000	−$1,000	$0	−$7,000
2	−$6,000	−$2,000	$0	−$8,000
3	−$6,000	−$2,000	$0	−$8,000
4	$0	−$2,000	$7,000	+$5,000

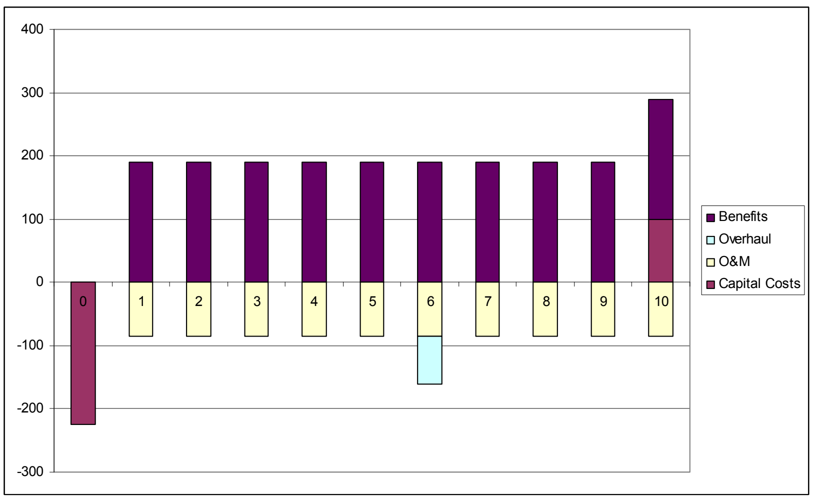
2-45

Year	Capital Costs	O & M	Overhaul
0.00	−20	0	0
1.00	0	−2.5	0
2.00	0	−2.5	0
3.00	0	−2.5	0
4.00	0	−2.5	−5
5.00	0	−2.5	0
6.00	0	−2.5	0
7.00	2	−2.5	0



2-46

Year	Capital Costs	O&M	Overhaul	Benefits
0	−225			
1		−85		190
2		−85		190
3		−85		190
4		−85		190
5		−85		190
6		−85	−75	190
7		−85		190
8		−85		190
9		−85		190
10	100	−85		190



2-47

Each student's answers will be different depending on their particular school and life situation.

As an example:

First Costs: tuition costs, fees, books, supplies, board (if paid ahead)
O & M Costs: monthly living expenses, rent (if applicable)
Salvage Value: selling books back to student union, etc.
Revenues: wages & tips, etc.
Overhauls: periodic (random or planned) mid-term expenses

The cash flow diagram is left to the student.

Chapter 3: Interest and Equivalence

3-1

Time Value of Money means simply that "money has value over time." Money has value, of course, because of what it can purchase. However, the time value of money means that ownership of money is valuable, and it is valuable because of the interest dollars that can be earned/gained due to its ownership. Understanding interest and its impact is important in many life circumstances. Examples could include some of the following:

- Selecting the best loans for homes, boats, jewelry, automobiles, etc.
- Many aspects involved with businesses ownership (payroll, taxes, etc.)
- Using the best strategies for paying off personal loans, credit cards, debt
- Making investments for life goals (purchases, retirement, college, weddings, etc.)
- Etc.

3-2

Under most circumstances $20,000 received now would be more valuable. The present worth factors for n = 4 , i.e. (P/A, i, 4), are all less than 4 for interest rates greater than 0. Receiving $5,000 per year for four years would only potentially be more valuable for certain conditions of deflation and/or transitioning into a lower incremental tax bracket.

3-3

For simple interest, the interest earned each year is a fixed amount based upon the original principal. For compound interest, you earn interest on previous year's interest as well as on the principal. Compound interest is more common.

3-4

It is entirely possible that different decision makers will make a different choice in this situation. The reason this is possible (that there is not a RIGHT answer) is that Magdalen, Miriam and Mary June all could be using a different *discounting rate* (interest rate or investment rate) as they consider the choice of $500 today versus $1,000 three years from today.

We find the interest rate at which the two cash flows are equivalent by

P = $500, F = $1000, n = 3 years, I = unknown
So, F = P (1+ i%)^n and, i% = {(F/P) ^ (1/n)} −1
Thus, i% = {(1000/500)^(1/3)} – 1 = 26%

In terms of an explanation, Magdalen wants the $500 today because she knows that she can invest it at a rate above 26% and thus have more than $1,000 three years from today. Miriam, on the other hand, could know that she does not have any investment options that would come close to earning 26% and thus would be happy to pass up on the $500 today to accept the $1,000 three years from today. Mary June, on the other hand, could be indifferent because she has another investment option that earns exactly 26%, the same rate the $500 would grow at if not accepted now. Thus, as a decision maker she would be indifferent.

Another aspect that may explain Magdalen's choice might have nothing to do with interest rates at all. Perhaps she simply has need for $500 right now to make a purchase or pay off a debt. Or, perhaps she is a pessimist and isn't convinced the $1,000 will be there in three years (a bird in the hand idea).

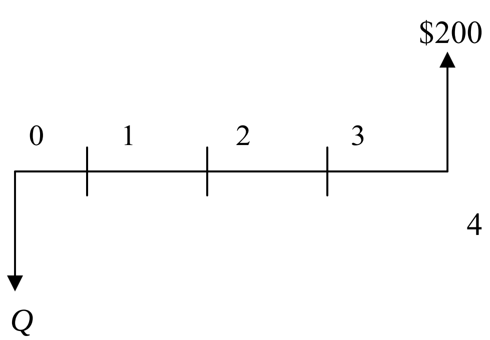
3-5

$2,000 + $2,000 (0.10 × 3) = $2,600

3-6

($5,350 – $5,000)/$_{(0.08 \times \$5,000)}$ = $350/$_{\$400}$ = 0.875 years = 10.5 months

3-7



Q = $200 (P/F, 10%, 4)
= $200 (0.683)
= $136.60

3-8

P = \$750, n = 3 years, i = 8%, F = ?
F = P $(1 + i)^n$ = \$750 $(1.08)^3$ = \$750 (1.260)
= \$945

Using interest tables:
F = \$750 (F/P, 8%, 3) = \$750 (1.360)
= \$945

3-9

$F = P (1 + i)^n$
Solve for P: $P = F/(1 + i)^n$
$P = F (1 + i)^{-n}$
P = \$150,000 $(1 + 0.10)^{-5}$ = \$150,000 (0.6209) = \$93,135

3-10

Use F = P (F/P, i, n) = P $(1 + i)^n$ = 2000 $(1 + 0.06)^n$.
(a) n = 5, F = \$2,676
(b) n = 10, F = \$3,582
(c) n = 20, F = \$6,414
(d) n = 50, F = \$36,840
(e) n = 100, F = \$678,604

3-11

Use P = F (P/F, i, n) = F $(1 + i)^{-n}$ = 20,000 $(1 + 0.07)^{-n}$.
(a) n = 5, P = \$14,260
(b) n = 10, P = \$10,167
(c) n = 20, P = \$5,168
(d) n = 50, P = \$679

3-12

Use 6000 = 5000 (F/P, i, n) = 5000 $(1 + i)^n$.

(a) n = 2, $i = \sqrt{1.2} - 1 = 0.0954$ or 9.54%

(b) n = 3, $i = \sqrt[3]{1.2} - 1 = 0.627$ or 6.27%

(c) n = 5, $i = \sqrt[5]{1.2} - 1 = 0.0371$ or 3.71%

(d) n = 10, $i = \sqrt[10]{1.2} - 1 = 0.0184$ or 1.84%

3-13

Double money at 4% simple interest:
2P = P (1 + 0.04n)
2 = (1 + 0.04n)
n = (2–1)/0.04 = 1/0.04 = 25 years

Double money at 4% compound interest:
$2P = P (1 + 0.04)^n$
log 2 = n log(1.04)

$$n = \frac{\log 2}{\log 1.04} = 17.7 \text{ years}$$

3-14

Use F = P (F/P, 8%, n) = 1000 $(1 + 0.08)^n$.

(a) F = 1360, $n = \frac{\log(1.36)}{\log(1.08)} = 4 \text{ years}$

(b) F = 2720, $n = \frac{\log(2.72)}{\log(1.08)} = 13 \text{ years}$

(c) F = 4316, $n = \frac{\log(4.316)}{\log(1.08)} = 19 \text{ years}$

(d) F = 6848, $n = \frac{\log(6.848)}{\log(1.08)} = 25 \text{ years}$

3-15

n = 63 years
i = 7.9%
F = \$175,000
P = F $(1 + i)^{-n}$
= \$175,000 $(1.079)^{-63}$
= \$1,454

3-16

(a) Interest Rates
i. Interest rate for the past year = (\$100 – \$90)/\$90 = \$10/\$90
= 0.111 or 11.1%
ii. Interest rate for the next year = (\$110 – \$100)/\$1
= 0.10 or 10%

(b) \$90 (F/P, i%, 2) = \$110
(F/P, i%, 2) = \$110/\$90= 1.222

So, $(1 + i)^2$ = 1.222
i = 1.1054 − 1 = 0.1054 = 10.54%

3-17

P = \$600
F = \$29,152,000
n = 92 years

F = P $(1 + i)^n$

\$29,152,000/\$600 = $(1 + i)^{92}$ = \$45,587

(1 + i) = $(\$48,587)^{(1/92)}$ = \$48,587

i^* = 0.124 = 12.4%

3-18

P = \$1,400 (P/A, 10%, 5) – \$80 (P/G, 10%, 5)
= \$1,400 (3.791) – \$80 (6.862)
= \$4,758.44

Using single payment factors:
P = \$1400 (P/F, 10%, 1) + \$1,320 (P/F, 10%, 2) + \$1,240 (P/F, 10%, 3) + \$1,160 (P/F, 10%, 4) + \$1,080 (P/F, 10%, 5)
= \$1,272.74 + \$1,090.85 + \$931.61 + \$792.28 + \$670.57
= \$4,758.05

3-19

F =\$8,250
n = 4 semi-annual periods
i = 4%
Find P.

P = F $(1+ i)^{-n}$ = \$8,250 $(1.04)^{-4}$ = \$8,250 (0.8548)
= \$7,052.10

Using interest tables:
P = F (P/F, 4%, 4) = \$8,250 (0.8548)
= \$7,052.10

3-20

P = \$1, n = unknown number of semiannual periods, i = 2%, F = 2

$F = P (1 + i)^n$
$2 = 1 (1.02)^n$
$2 = 1.02^n$
n = log (2) / log (1.02)
= 35

Therefore, the money will double in 17.5 years.

3-21

Calculator Solution
1% per month F = $1,000 $(1 + 0.01)^{12}$ = $1,126.83
12% per year F = $1,000 $(1 + 0.12)^{1}$ = $1,120.00
Savings in interest = $6.83

Compound interest table solution
1% per month F = $1,000 (1.127) = $1,127.00
12% per year F = $1,000 (1.120) = $1,120.00
Savings in interest = $7.00

3-22

Year	BOY Loan Bal	Interest Paid	Loan Payment	EOY Loan Bal	Cash Flow
0	$12,000	$0	$2,000	$10,000	$10,000
1	$10,000	$1,500	$2,500	$7,500	-$4,000
2	$7,500	$1,125	$2,500	$5,000	-$3,625
3	$5,000	$750	$2,500	$2,500	-$3,250
4	$2,500	$375	$2,500	$0	-$2,875
Total		$3,750	$12,000		-$3,750

3-23

Local Bank
F = $3,000 (F/P, 5%, 2) = $3,000 (1.102)
= $3,306

Out of Town Bank
F = $3,000 (F/P, 1.25%, 8) = $3,000 (1.104)
= $3,312

Additional Interest = $6

3-24

Given two cash flows that are equivalent if the interest rate is i. Which one is more valuable if the interest rate is 2i?

For rate i: $P_1 = F_1(P/F,i,2) = F_2(P/F,i,3) = P_2$

$$F_2 = F_1\frac{(P/F,i,2)}{(P/F,i,3)} = F_1\frac{(1+i)^{-2}}{(1+i)^{-3}} = F_1(1+i)$$

For rate 2i: $P_1' = F_1(1+2i)^{-2}$ *and* $P_2' = F_2(1+2i)^{-3} = F_1(1+i)(1+2i)^{-3}$

$$\frac{P_2'}{P_1'} = \frac{F_1(1+i)(1+2i)^{-3}}{F_1(1+2i)^{-2}} = (1+i)\frac{(1+2i)^2}{(1+2i)^3} = \frac{(1+i)}{(1+2i)}$$

$$P_2' = \frac{1+i}{1+2i}P_1' \text{ and since } 1+2i > 1+i \text{ have } \frac{1+i}{1+2i} < 1 \text{ so } P_2' < P_1'$$

Thus, the cash flow in diagram i is more valuable than the cash flow in diagram ii.

Example: Let F_1 = 1000 and i = 10% then $F_2 = (1000)(1 + 0.1)^1 = 1100$.
At i = 2i = 20% we have $P'_1 = 1000\ (1 + 0.2)^{-2} = 694.4$
$P'_2 = 1100\ (1 + 0.2)^{-3} = 636.6$

3-25

(P/F, i, 150) $= (1 + i)^{-150} = (1 + i)^{-(50 + 100)} = (1 + i)^{-50} (1 + i)^{-100}$
= (P/F, i, 50) (P/F, i, 100)

It would be nice, however, to preserve 3 significant figures for accuracy and (P/F, i, 100) only has one significant figure! The solution is to break up (P/F, i, 100) into two terms each of which have at least 3 significant figures, say, (P/F, i, 50) (P/F, i, 50), thus, (P/F, i, 150) = $(P/F, i, 50)^3$.

Example: Let i = 10% so
(P/F, 10%, 150) = $(0.00852)^3 = 6.185 \times 10^{-7} = (1 + 0.10)^{-150} = 6.182 \times 10^{-7}$, which is close!

Clearly, (P/F, i, 200) = $(P/F, i, 50)^4$.

3-26

(a) Future Worth
$71 million = $165,000 (F/P, i%, 61)
(F/P, i%, 61) = $71,000,000/$165,000
= 430.3

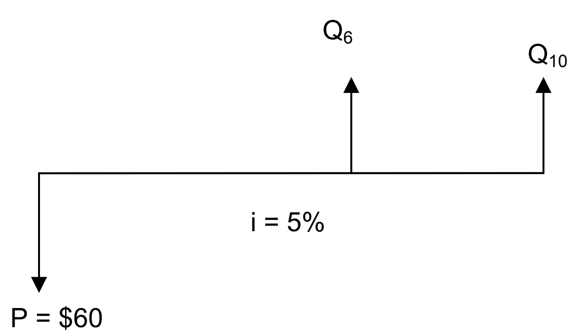
From interest tables:

(*P/A, i*%, 61)	*I*
341.7	10%
1,034.5	12%

Performing linear interpolation:
i = 10% + (2%) ((430.3 − 341.7)/(1034.5 − 341.7))
= <u>10.3%</u>

(b) In 1929, the Consumer Price Index was 17 compared to about 126 in 1990. So $165,000 in 1929 dollars is roughly equivalent to $165,000 (126/17) = $1,223,000 in 1990 dollars. The real rate of return is closer to 6.9%.

3-27



Either:
$Q_{10} = Q_6$ (F/P, 5%, 4) (1)
Q_{10} = P (F/P, 5%, 10) (2)

Since P is between and Q_6 is not, solve Equation (2),
Q_{10} = $60 (1.629)
= <u>$97.74</u>

3-28

Repayment at 4½% = \$1 billion (F/P, 4 ½%, 30)
= \$1 billion (3.745)
= \$3.745 billion

Repayment at 5 ¼% = \$1 billion $(1 + 0.0525)^{30}$
= \$4.62 billion

Saving to foreign country = \$897 million

3-29

Lump Sum Payment = \$350 (F/P, 1.5%, 8)
= \$350 (1.126)
= \$394.10

Alternate Payment = \$350 (F/P, 10%, 1)
= \$350 (1.100)
= \$385.00

Choose the alternate payment plan.

3-30

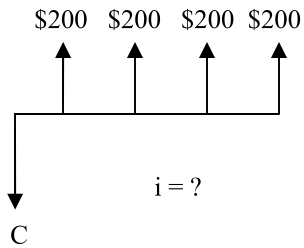
The garbage company sends out bills only six times a year. Each time they collect one month's bills one month early.

100,000 customers × \$6.00 × 1% per month × 6 times/yr = \$36,000

Chapter 4: More Interest Formulas

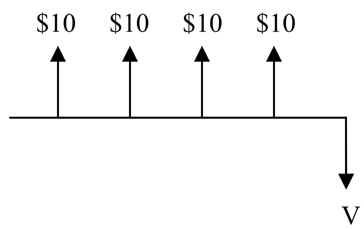
4-1

(a)



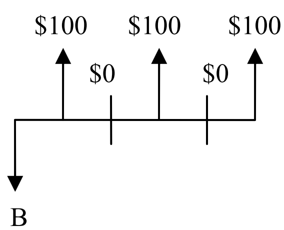
C = \$200 (P/A, 10%, 4)
C = \$200 (3.170) = \$634

(b)



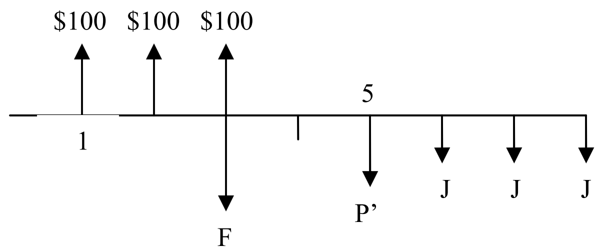
V = \$10 (*F/A*, 10%, 5) – \$10
= \$10 (6.105) – \$10
= <u>\$51.05</u>

(c)



B = \$100 (*P/F*, 10%, 1) + \$100 (*P/F*, 10%, 3) + \$100 (*P/F*, 10%, 5)
= \$100 (0.9091 + 0.7513 + 0.6209)
= \$228.13

4-2



F = \$100 (F/A, 10%, 3) = \$100 (3.310) = \$331
P' = \$331 (F/P, 10%, 2) = \$331 (1.210) = \$400.51
J = \$400.51 (A/P, 10%, 3) = \$400.51 (0.4021) = <u>\$161.05</u>

Alternate Solution: Given that we have three \$100 cash flows and three *J* cash flow, one may observe that a single cash flow, *J,* is equivalent to the future worth of a single cash flow of \$100 after five interest periods, or:
J = \$100 (F/P, 10%, 5) = \$100 (1.611) = <u>\$161.10</u>

4-3

P = A (P/A, 3.5%, n)
\$1,000 = \$50 (P/A, 3.5%, n)
(P/A, 3.5%, n) = 20

From the 3.5% interest table: n = <u>35.</u>

4-4

F = A (F/A, 10%, n)
\$35.95 = 1 (F/A, 10%, n)
(F/A, 10%, n) = 35.95

From the 10% interest table, n = <u>16.</u>

4-5

A = \$300, *i* = 5.25%, *n* = 10 years, *P* = ?
P = A (P/A, 5.25%, 10)
= $A [(1 + i)^n - 1]/[i1 + i)^n]$
= $\$300 [(1.0525)^{10} - 1]/[0.0525 (1.0525)^{10}]$
= \$300 (7.62884)
= <u>\$2,289</u>

4-6

P = \$3,000, i = 1% /month, n = 30 months, A = ?
A = P (A/P, i%, n)
A = \$3,000 (A/P, 1%, 30)
= \$3,000 (0.0387)
= <u>\$116.10</u>

4-7

P = \$3,000 + \$280 (P/A, 1%, 60) = \$3,000 + \$280 (44.955)
= <u>\$15,587</u>

4-8

F = \$2,000 (F/A, 8%, 10) (F/P, 8%, 5)
= \$2,000 (14.487) (1.469)
= <u>\$42,560</u>

4-9

Let X = toll per vehicle.
Then:
A = 20,000,000 X   *i* = 10%
F = $25,000,000   *n* = 3

20,000,000 X (F/A, 10%, 3) = $25,000,000
20,000,000 X (3.31) = $25,000,000

X = $0.38 per vehicle

4-10

P = $10,000   *i* = 12%
F = $30,000   *n* = 4

$10,000 (F/P, 12%, 4) + A (F/A, 12%, 4) = $30,000
$10,000 (1.574) + A (4.779) = $30,000

A = $2,984

4-11

From compound interest tables, using linear interpolation:

(P/A, i%, 10)	i
7.360	6%
7.024	7%

(P/A, 6.5%, 10) = ½(7.360 − 7.024) + 7.024
= 7.192

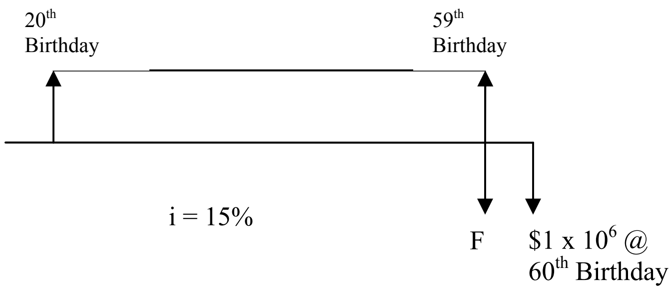
Exact computed value:
(P/A, 6.5%, 10) = 7.189
Why do the values differ? Since the compound interest factor is nonlinear, linear interpolation will not produce an exact solution.

4-12

P = A (P/A, 1.5%, n)
\$525 = \$15 (P/A, 1.5%, n)
(P/A, 1.5%, n) = 35

From the 1.5% interest table, n = 50 months.

4-13



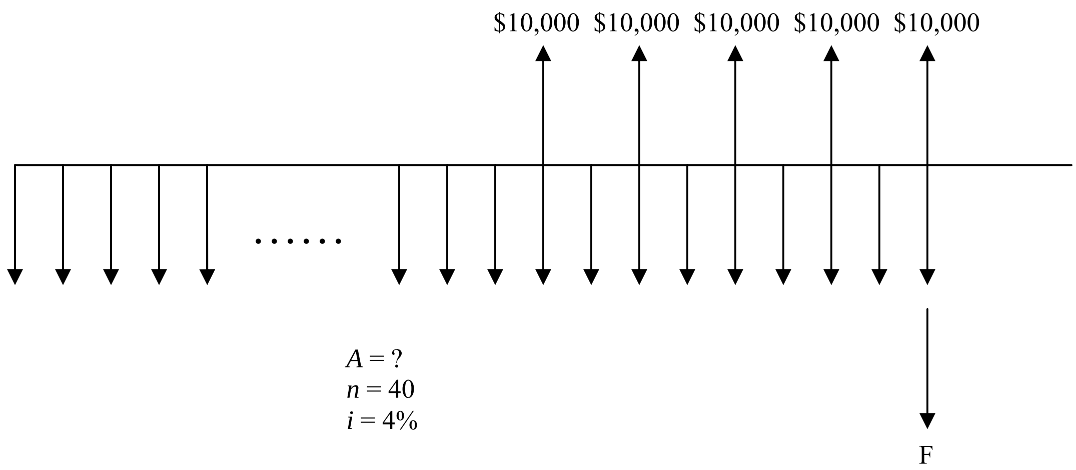
Number of yearly investments = (59 – 20 + 1) = 40
The diagram indicates that the problem is not in the form of the uniform series compound amount factor. Thus, find F that is equivalent to \$1,000,000 one year hence:

F = \$1,000,000 (P/F, 15%, 1) = \$1,000,000 (0.8696)
= \$869,600

A = \$869,600 (A/F, 15%, 40) = \$869,600 (0.00056)
= \$486.98

This result is very sensitive to the sinking fund factor. (A/F, 15%, 40) is actually 0.00056208, which makes A = \$488.78.

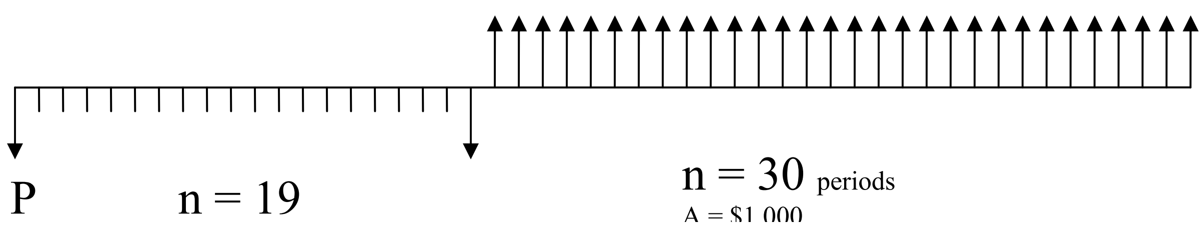
4-14



Compute F equivalent to the five $10,000 withdrawals:
F = $10,000 [(F/P, 4%, 8) + (F/P, 4%, 6) + (F/P, 4%, 4) + (F/P, 4%, 2) + 1]
= $10,000 [1.369 + 1.265 + 1.170 + 1.082 + 1]
= $58,850

Required series of 40 deposits:
A = F (A/F, 4%, 40) = $58,850 (0.0105) = <u>$618</u>

4-15

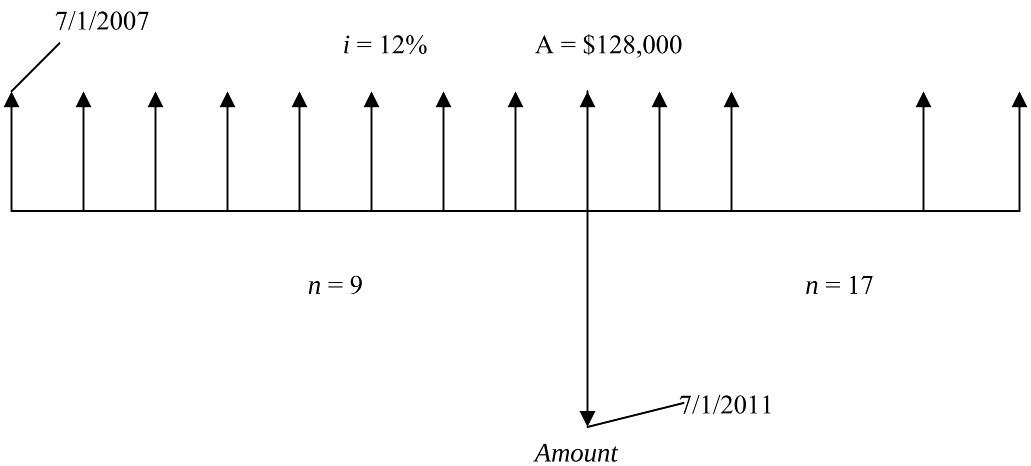


Note: There are 19 interest periods between P(40th birthday) and P' (6 months prior to 50th birthday)

P' = $1,000 (P/A, 2%, 30) = $1,000 (22.396)
= $22,396

P = P' (P/F, 2%, 19) = $22,396 (0.6864)
= <u>$15,373</u> [Cost of Annuity]

4-16



$$\text{Amount}_{7/1/2011} = \$128{,}000\ (F/A, 6\%, 9) + \$128{,}000\ (P/A, 6\%, 17)$$
$$= \$128{,}000\ [(11.491) + (10.477)]$$
$$= \underline{\$2{,}811{,}904}$$

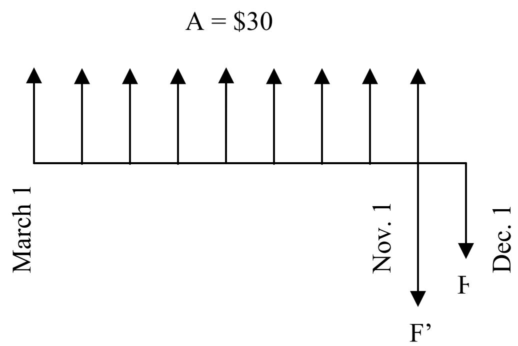
4-17

(a) Bill's monthly payment = 2/3 ($4,200) (A/P, 0.75%, 36)
= $2,800 (0.0318)
= <u>$89.04</u>

(b) Bill owed the October 1 payment plus the present worth of the 27 additional payments.

Balance = $89.04 + $89.04 (P/A, 0.75%, 27)
= $89.04 (1 + 24.360)
= <u>$2,258.05</u>

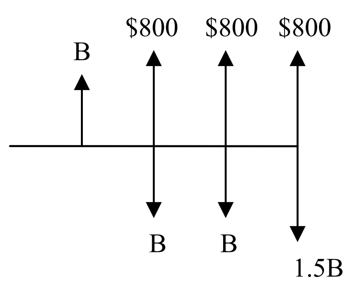
4-18



Amount on Nov 1:
F’ = $30 (F/A, ½%, 9) = $30 (9.812) = $275.46

Amount on Dec 1:
F = $275.46 (F/P, ½%, 1) = $275.46 (1.005) = 276.84

4-19

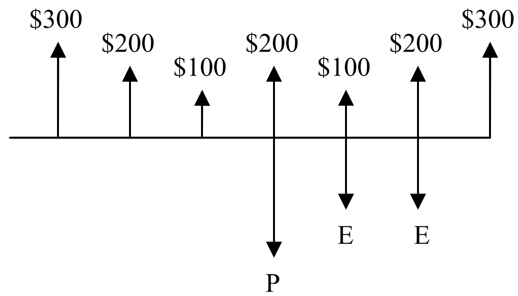


Receipts (upward) at time 0**:**
PW = B + $800 (P/A, 12%, 3) = B + $1,921.6

Expenditures (downward) at time 0**:**
PW = B (P/A, 12%, 2) + 1.5B (P/F, 12%, 3) = 2.758B

Equating:
B + $1,921.6 = 2.758B
1.758B = $1,921.6
B= $1,921.6/1.758
= $1,093.06

4-20



P = \$200 + \$100 (P/A, 10%, 3) + \$100 (P/G, 10%, 3) + \$300 (F/P, 10%, 3) + \$200 (F/P, 10%, 2) + \$100 (F/P, 10%, 1)
= \$200 + \$100 (2.487) + \$100 (2.329) + \$300 (1.331) + \$200 (1.210) + \$100 (1.100)
= \$1,432.90

E = \$1,432.90 (A/P, 10%, 2) = \$1,432.90 (0.5762) = <u>\$825.64</u>

4-21

A = \$500 (*A/P*, 1%, 16) = \$500 (0.0679)
= <u>\$33.95</u>

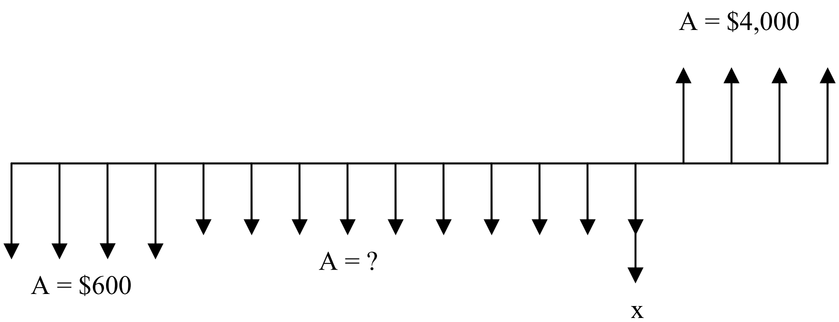
4-22

(a) *P* = \$500,000 − \$100,000 = \$400,000
n = 360
i = r/m = 0.09/12
A = ?
A = \$400,000 (A/P, 0.75%, 360) = \$400,000 (0.00805)
= <u>\$3,220</u>

(b) P = A (P/A, 0.75%, 240) = \$3,220 (111.145)
= <u>\$357,887</u>

(c) A = \$400,000 $[(e^{(0.06/12)(360)})(e^{(0.06/12)} - 1)/(e^{(0.06/12)(360)} - 1)]$
= \$400,000 [(6.05)(0.005)/(5.05)]
= <u>\$2,396</u>

4-23



To have sufficient money to pay the four $4,000 disbursements,
x = $4,000 (P/A, 5%, 4) = $4,000 (3.546)
= $14,184
This $14,184 must be accumulated by the two series of deposits.

The four $600 deposits will accumulate by x (17th birthday):
F = $600 (F/A, 5%, 4) (F/P, 5%, 10)
= $600 (4.310) (1.629)
= $4,212.59
Thus, the annual deposits between 8 and 17 must accumulate a future sum:
= $14,184 − $4,212.59
= $9,971.41

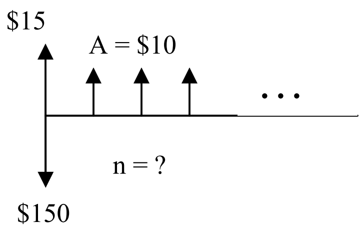
The series of ten deposits must be:
A = $9,971.11 (A/F, 5%, 10) = $9,971.11 (0.0795)
= <u>$792.73</u>

4-24

This problem may be solved in several ways. Below are two of them:
Alternative 1:
$5000 = $1,000 (P/A, 8%, 4) + *x* (P/F, 8%, 5)
= $1,000 (3.312) + *x* (0.6806)
= $3,312 + *x* (0.6806)
x = ($5,000 − $3,312)/0.6806
= <u>$2,480.16</u>

Alternative 2:
P = $1,000 (P/A, 8%, 4)
= $1,000 (3.312)
= $3,312
($5,000 − $3,312) (F/P, 8%, 5) = <u>$2,479.67</u>

4-25

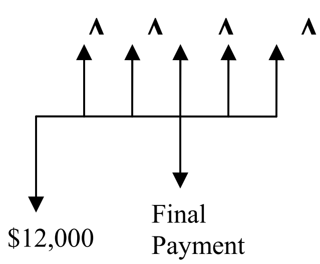


(\$150 – \$15) = \$10 (P/A, 1.5%, n)
(P/A, 1.5%, n) = \$135/\$10 = 13.5
From the 1.5% interest table we see that *n* is between 15 and 16. This indicates that there will be 15 payments of \$10 plus a last payment of a sum less than \$10.

Compute how much of the purchase price will be paid by the fifteen \$10 payments:
P= \$10 (P/A, 1.5%, 15) = \$10 (13.343)
= \$133.43

Remaining unpaid portion of the purchase price:
= \$150 – \$15 – \$133.43 = \$1.57
16th payment = \$1.57 (F/P, 1.5%, 16)
= <u>\$1.99</u>

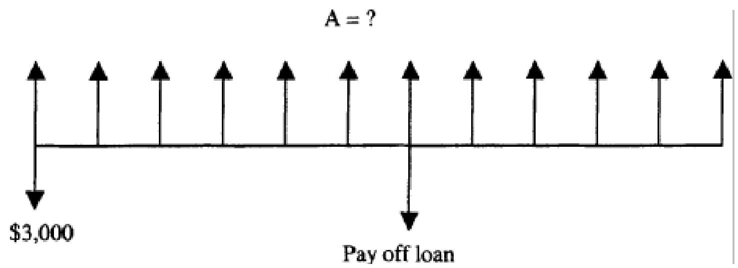
4-26



A = \$12,000 (A/P, 4%, 5)
= \$12,000 (0.2246)
= \$2,695.20
The final payment is the present worth of the three unpaid payments.

Final Payment = \$2,695.20 + \$2,695.20 (P/A, 4%, 2)
= \$2,695.20 + \$2,695.20 (1.886)
= <u>\$7,778.35</u>

4-27



Compute monthly payment:

$3,000 = A + A (P/A, 1%, 11)
= A + A (10.368)
= 11.368 A

A= $3,000/11.368
= $263.90

Car will cost new buyer:
= $1,000 + 263.90 + 263.90 (P/A, 1%, 5)
= $1263.90 + 263.90 (4.853)
= <u>$2,544.61</u>

4-28

(a) i = 8%, P = $120,000, n = 15 years, A = ?
P = $150,000 – $30,000 = $120,000
A = P (A/P, i%, n)
= $120,000 (A/P, 8%, 15)
= $120,000 (0.11683)
= <u>$14,019.55</u>
R_Y = Remaining Balance in any year, Y
R_Y = A (P/A, i%, $n - Y$)
R_7 = $14,019.55 (P/A, 8%, 8)
= $14,019.55 (5.747)
= <u>$80,570.35</u>

(b) The quantities in Table 4-38 below are computed as follows:
Column 1 shows the number of interest periods.
Column 2 shows the equal annual amount as computed in part (a) above.

The amount $14,019.55 is the total payment which includes the principal and interest portions for each of the 15 years. To compute the interest portion for year one, we must first multiply the interest rate in decimal by the remaining balance:

Interest Portion = (0.08) ($120,000) = $9,600

TABLE 4-28: SEPARATION OF INTEREST AND PRINCIPAL

YEAR	ANNUAL PAYMENT	INTEREST PORTION	PRINCIPAL PORTION	REMAINING BALANCE
0				$120,000.00
1	$14,019.55	$9,600	$4,419.55	$115,580.45
2	$14,019.55	$9,246.44	$4,773.11	$110,807.34
3	$14,019.55	$8,864.59	$5,154.96	$105,652.38
4	$14,019.55	$8,452.19	$5,567.36	$100,085.02
5	$14,019.55	$8,006.80	$6,012.75	$94,072.27
6	$14,019.55	$7,525.78	$6,493.77	$87,578.50
7*	$14,019.55	$7,006.28	$7,013.27	$80,565.23
8	$14,019.55	$6,445.22	$7,574.33	$72,990.90
9	$14,019.55	$5,839.27	$8,180.28	$64,810.62
10	$14,019.55	$5,184.85	$8,834.70	$55,975.92
11	$14,019.55	$4,478.07	$9,541.48	$46,434.44
12	$14,019.55	$3,714.76	$10,304.79	$36,129.65
13	$14,019.55	$2,890.37	$11,129.18	$25,000.47
14	$14,019.55	$2,000.04	$12,019.51	$12,981.00
15	$14,019.55	$1,038.48	$12,981.00	0

Subtracting the interest portion of $9,600 from the total payment of $14,019.55 gives the principal portion to be $4,419.55, and subtracting it from the principal balance of the loan at the end of the previous year (y) results in the remaining balance after the first payment is made in year 1 (y_1), of $115,580.45. This completes the year 1 row. The other row quantities are computed in the same fashion. The interest portion for row two, year 2 is

(0.08) ($115,580.45) = $9,246.44

*NOTE: Interest is computed on the remaining balance at the end of the preceding year and not on the original principal of the loan amount. The rest of the calculations proceed as before. Also, note that in year 7, the remaining balance as shown on Table 4-38 is approximately equal to the value calculated in (a) using a formula except for round off error.

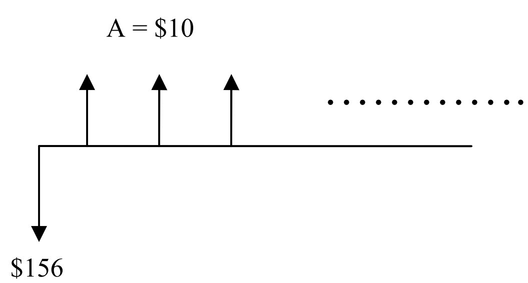
4-29

Determine the required present worth of the escrow account on January 1, 2008:
A = \$8,000, i = 5.75%, PW = ?, n = 3 years
PW = A (P/A, i%, n)
= \$8,000 + \$8,000 (P/A, 5.75%, 3)
= \$8,000 + \$8,000 $[(1 + i)^n - 1]/[i(1 + i)^n]$
= \$8,000 + \$8,000 $[(1.0575)^3 - 1]/[0.0575(1.0575)^3]$
= <u>\$29,483.00</u>

It is necessary to have \$29,483 at the end of 2007 in order to provide \$8,000 at the end of 2008, 2009, 2010, and 2011. It is now necessary to determine what yearly deposits should have been over the period 1991–2007 to build a fund of \$29,483.

A = ?, i = 5.75%, F = \$29,483, n = 18 years
A = F (A/F, i%, n) = \$29,483 (A/F, 5.75%, 18)
= \$29,483 $(i)/[(1 + i)^n - 1]$
= \$29,483 $(0.0575)/[(1.0575)^{18} - 1]$
= \$29,483 (0.03313)
= <u>\$977</u>

4-30



P = \$156 n = ?
i = 1.5% A = \$10

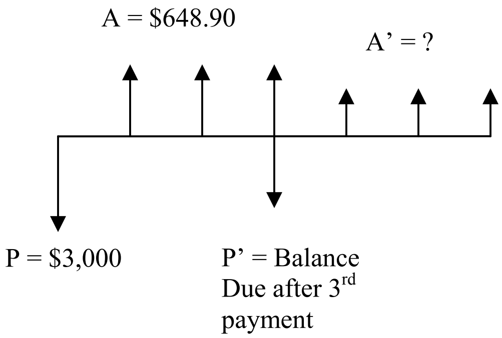
\$156 = \$10 (P/A, 1.5%, n)
(P/A, 1.5%, n) = \$156/\$10
= 15.6

From the 1.5% interest table, n is between 17 and 18. Therefore, it takes <u>18 months</u> to repay the loan.

4-31

A = P (A/P, 8%, 6)
= $3,000 (0.2163)
= $648.90

The first three payments were $648.90 each.



Balance Due after 3rd payment equals the Present Worth of the originally planned last three payments of $648.90.

P' = $648.90 (P/A, 8%, 3) = $648.90 (2.577)
= $1,672.22

Last three payments:
A' = $1,672.22 (A/P, 7%, 3) = $1,672.22 (0.3811)
= <u>$637.28</u>

4-32

P = $25,000 *n* = 60 months
i = 18% per year
= 1.5% per month

(a) A = $25,000 (A/P, 1.5%, 60)
= $635

(b) P = \$25,000 (0.98) = \$24,500
\$24,500 = \$635 (P/A, *i*%, 60)
(P/A, *i*%, 60) = \$24,500/\$635 = 38.5827

Performing interpolation using interest tables:

(P/A, i%, 60)	i
39.380	1.50%
36.964	1.75%

i% = 0.015 + (0.0025) [(39.380 – 38.5827)/(39.380 – 36.964)]
= 0.015 + 0.000825
= 0.015825
= 1.5825% per month

$i_a = (1 + 0.015825)^{12} – 1$
= 0.2073
= <u>20.72%</u>

4-33

A = P (A/P, i%, 24)
(A/P, i%, 24) = A/P = 499/10,000 = <u>0.499</u>
From the compound interest tables we see that the interest rate per month is exactly 1.5%.

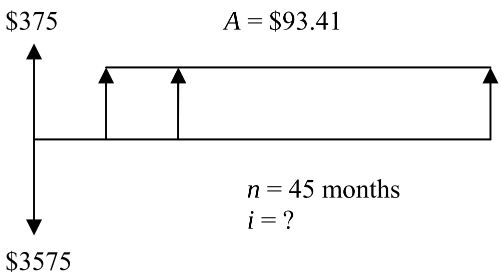
4-34

FW = FW
\$1000 (F/A, *i*%, 10) (F/P, *i*%, 4) = \$28,000

By trial and error:
Try *i* = 12% \$1,000 (17.549) (1.574) = \$27,622 *i* too low
i = 15% \$1,000 (20.304) (1.749) = \$35,512 *i* too high

Using Interpolation:
i = 12% + 3% ((\$28,000 − \$27,622)/(\$35,512 − \$27,622))
= <u>12.14%</u>

4-35



$3,575 = $375 + $93.41 (P/A, *i*%, 45)
(*P/A*, *i*%, 45) = ($3,575 − $375)/$93.41
= 34.258

From compound interest tables, *i* = <u>1.25%</u> per month. For an $800 down payment, unpaid balance is $2775.

P = $2,775, n = 45 months, *i* = 1.25%, *A* = ?
A = $2,775 (A/P, 1.25%, 45)*
= $2,775 (0.0292)
= <u>$81.03</u>

Effective interest rate = $(1 + i)^{12} - 1 = (1.0125)^{12} - 1$
= 0.161 = 16.1% per year

* Note that no interpolation is required as (*A/P,* 1.25%, 45)
= 1/(*P/A*, *i*%, 45)
= 1/34.258 = 0.0292

4-36

Given (P/F, i, n) = 0.1408 and (A/F, i, n) = 0.0408 and using the equations for each $\left[\frac{i(i+1)^n}{(1+i)^n-1}\right]=0.1408$ and $\left[\frac{i}{(1+i)^n-1}\right]=0.0408$, we have from the latter $(1+i)^n-1=24.5098i$.

This can be substituted into the first equation giving
$\left[\frac{(1+i)^n}{24.5098}\right]=0.1408$ yielding $(1+i)^n=3.45098$.

Plugging back into the first equation, we have $\left[\frac{i(3.45098)}{3.45098-1}\right]=0.1408$.

Solving for i yields i = 0.10 or 10%.

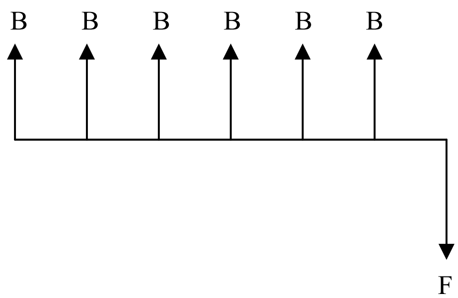
From $(1+0.01)^n=3.45098$, solving for n yields n = 13.

A simpler approach would be to just look through the tables!

4-37

Since (*A/P*, *i*%, *n*) = (A/F, *i*%, *n*) + *i*   (Equation 4-14 on page 110)
0.1728 = 0.0378 + *i*
I = 0.1728 – 0.0378 = 0.1350
i = <u>13.5%</u>

4-38



The solution may follow the general approach of the end-of-year derivation in the book.
(1) $F = B(1 + i)^n + \ldots + B(1 + i)^1$

Divide equation (1) by (1 + i):
(2) $F(1 + i)^{-1} = B(1 + i)^{n-1} + B(1 + i)^{n-2} + \ldots + B$

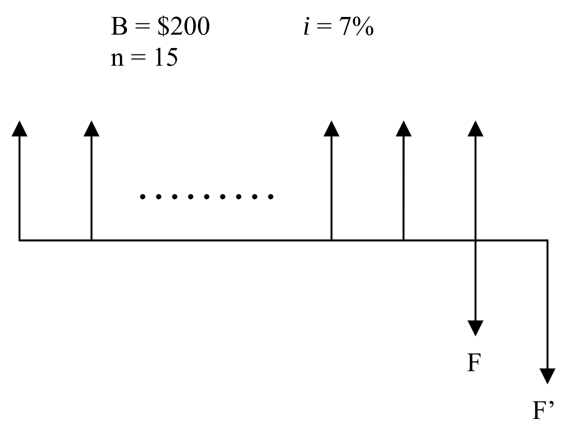
Subtract equation (2) from equation (1):
(1) − (2) $F - F(1 + i)^{-1} = B[(1 + i)^n - 1]$

Multiply both sides by (1 + i):
$F(1 + i) - F = B[(1 + i)^{n+1} - (1 + i)]$

So the equation is:
$F = B[(1 + i)^{n+1} - (1 + i)]/i$

Applied to the numerical values:
$F = 100/0.08\ [(1 + 0.08)^7 - (1.08)]$
= <u>$792.28</u>

4-39

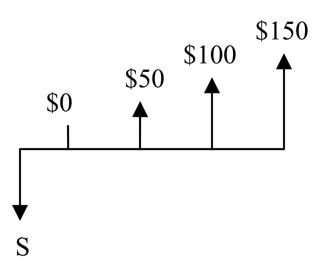


F = \$200 (F/A, *i*%, *n*) = \$200 (F/A, 7%, 15) = \$200 (25.129)
= \$5,025.80

F' = F (F/P, *i*%, *n*) = \$5,025.80 (F/P, 7%, 1) = \$5,025.80 (1.07)
= \$5,377.61

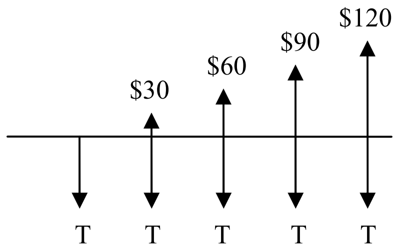
4-40

(a)



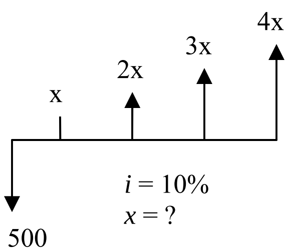
S = \$50 (*P/G*, 10%, 4) = \$50 (4.378)
= \$218.90

(b)



T = \$30 (*A/G*, 10%, 5) = \$30 (1.810)
= <u>\$54.30</u>

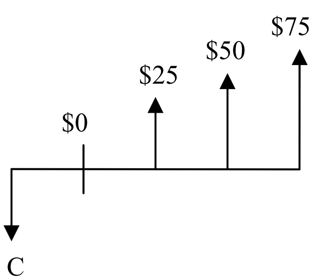
(c)



\$500 = X (*P/A*, 10%, 4) + X (*P/G*, 10%, 4)
\$500 = 7.548X
X = \$500/7.548 = \$66.24

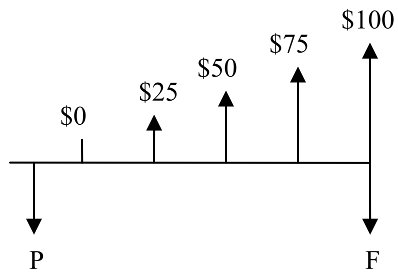
4-41

(a)



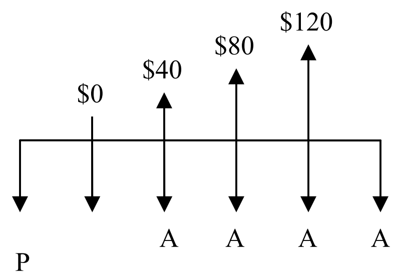
C = \$25 (*P/G*, 10%, 4)
= \$25 (4.378)
= <u>\$109.45</u>

(b)



F = \$25 (*P/G*, 10%, 5) (*F/P*, 10%, 5)
= \$25 (6.862) (1.611)
= <u>\$276.37</u>

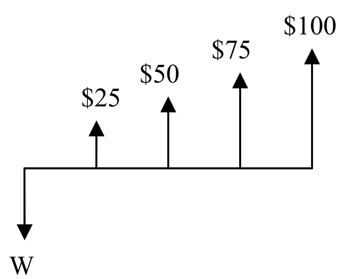
(c)



A = \$40 (*P/G*, 10%, 4) (*F/P*, 10%, 1) (*A/P*, 10%, 4)
= \$40 (4.378) (1.10) (0.3155)
= <u>\$60.78</u>

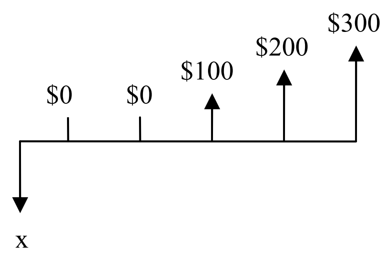
4-42

(a)



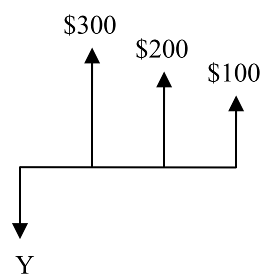
W = \$25 (*P/A*, 10%, 4) + \$25 (*P/G*, 10%, 4)
= \$25 (3.170 + 4.378)
= <u>\$188.70</u>

(b)



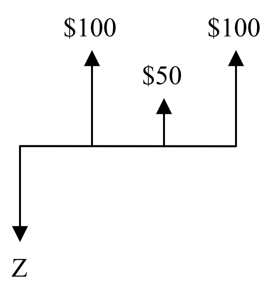
X = \$100 (*P/G*, 10%, 4) (*P/F*, 10%, 1)
= \$100 (4.378) (0.9091)
= <u>\$398.00</u>

(c)



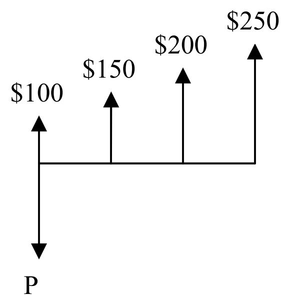
Y = \$300 (*P/A*, 10%, 3) – \$100 (*P/G*, 10%, 3)
= \$300 (2.487) – \$100 (2.329)
= <u>\$513.20</u>

(d)



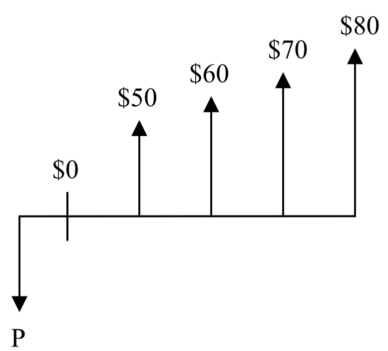
Z = \$100 (*P/A*, 10%, 3) – \$50 (*P/F*, 10%, 2)
= \$100 (2.487) – \$50 (0.8264)
= <u>\$207.38</u>

4-43



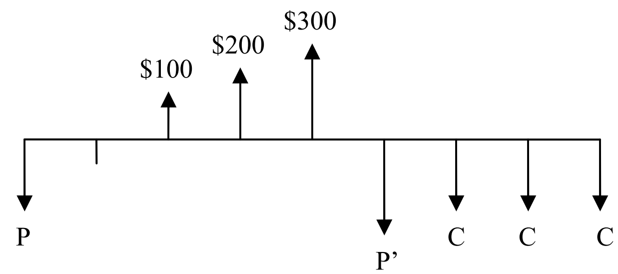
P = $100 + $150 (*P/A,* 10%, 3) + $50 (*P/G,* 10%, 3)
= $100 + $150 (2.487) + $50 (2.329)
= $589.50

4-44



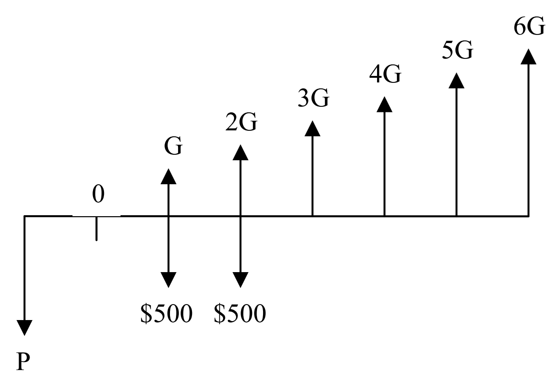
P = $10 (P/G, 15%, 5) + $40 (P/A, 15%, 4)(P/F, 15%, 1)
= $10 (5.775) + $40 (2.855) (0.8696)
= $157.06

4-45



P = \$100 (P/G, 10%, 4) = \$100 (4.378) = \$437.80
P' = \$437.80 (F/P, 10%, 5) = \$437.80 (1.611) = \$705.30
C = \$705.30 (A/P, 10%, 3) = \$705.30 (0.4021) = \$283.60

4-46

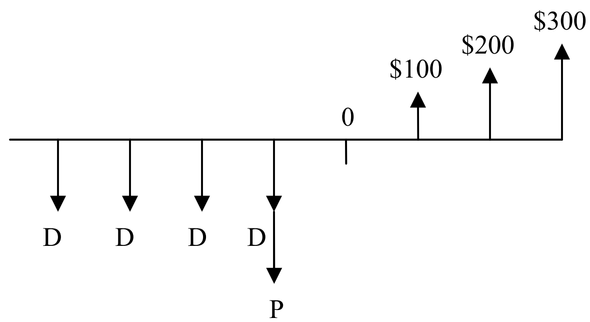


Present Worth P of the two \$500 amounts:
P = \$500 (P/F, 12%, 2) + \$500 (P/F, 12%, 3)
= \$500 (0.7972) + \$500 (0.7118)
= \$754.50

Also:
P = G (P/G, 12%, 7)
\$754.50 = G (P/G, 12%, 7)
\$754.50 = G (11.644)

G = \$754.50/11.644
= \$64.80

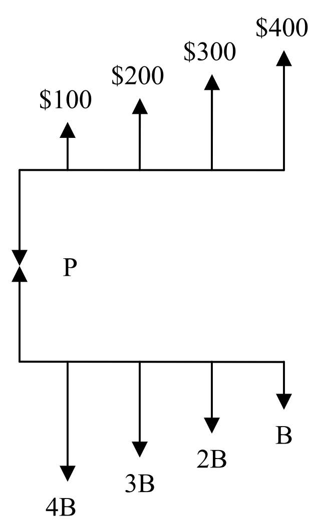
4-47



Present Worth of gradient series:

P = \$100 (P/G, 10%, 4)	= \$100 (4.378)	= \$437.80
D = \$437.80 (A/F, 10%, 4)	= \$437.80 (0.2155)	= <u>\$94.35</u>

4-48



P= \$100 (P/A, 10%, 4) + \$100 (P/G, 10%, 4)
= \$100 (3.170 + 4.378)
= \$754.80

Also:
P = 4B (P/A, 10%, 4) − B (P/G, 10%, 4)
Thus, we can equate the two cash flows to get:
4B (3.170) − B (4.378) = \$754.80
8.302B = \$754.80
B = \$754.80/8.302 = <u>\$90.92</u>

4-49

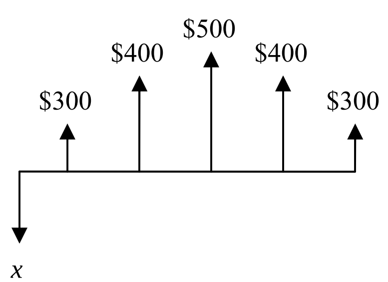
Cash flow number 1:
P_0^1 = A (P/A, 12%, 4)

Cash flow number 2:
P_0^2 = \$150 (P/A, 12%, 5) + \$150 (P/G, 12%, 5)
Since $P_0^1 = P_0^2$,
A (3.037) = \$150 (3.605) + \$150 (6.397)
A= (540.75 + 959.55)/3.037
= <u>\$494</u>

4-50

P= \$1,250 (P/A, 10%, 8) – \$250 (P/G, 10%, 8) + \$3,000 – \$250 (P/F, 10%, 8)
= \$1,250 (5.335) – \$250 (16.029) + \$3,000 – \$250 (0.4665)
= <u>\$5,545</u>

4-51

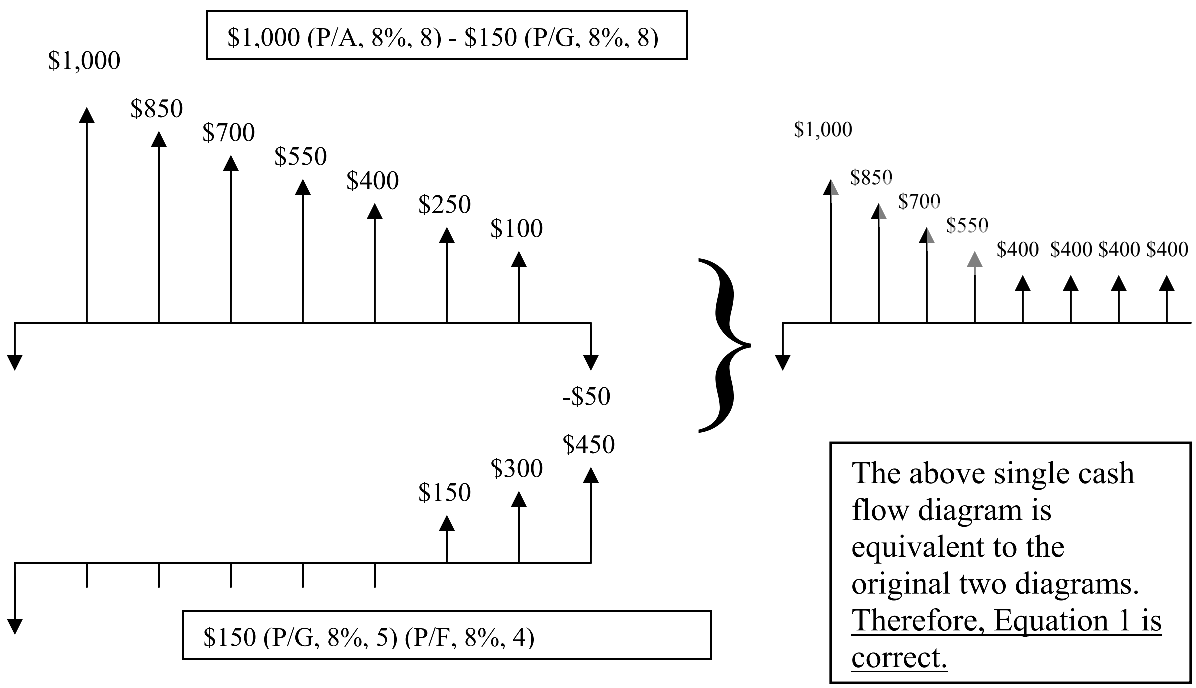


x = \$300 (*P/A,* 10%, 5) + \$100 (*P/G,* 10%, 3) + \$100 (*P/F,* 10%, 4)
= \$300 (3.791) + \$100 (2.329) + \$100 (0.6830)
= <u>\$1,438.50</u>

4-52

Correct equation is (b).
(\$50 (P/A, i%, 5) + \$10 (P/G, i%, 5) + \$50 (P/F, i%, 5))/100 = 1

4-53



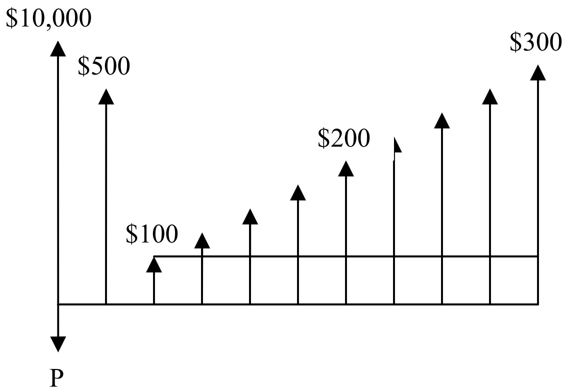
4-54

P = \$40 (*P/A,* 5%, 7) + \$10 (*P/G,* 5%, 7)
= \$40 (5.786) + \$10 (16.232)
= \$231.44 + \$162.32
= \$393.76

4-55

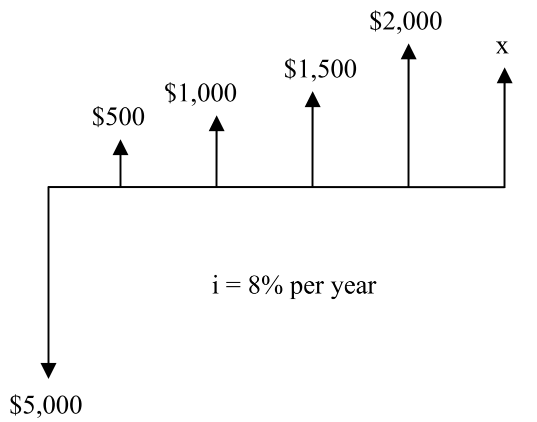
This problem has a declining gradient.
P= \$85,000 (P/A, 4%, 5) − \$10,000 (P/G, 4%, 5)
= \$85,000 (4.452) − \$10,000 (8.555)
= <u>\$292,870</u>

4-56



P = \$10,000 + \$500 (P/F, 6%, 1) + \$100 (P/A, 6%, 9) (P/F, 6%, 1) +
\$25 (P/G, 6%, 9) (P/F, 6%, 1)
= \$10,000 + \$500 (0.9434) + \$100 (6.802) (0.9434) + \$25 (24.577) (0.9434)
= <u>\$11,693.05</u>

4-57

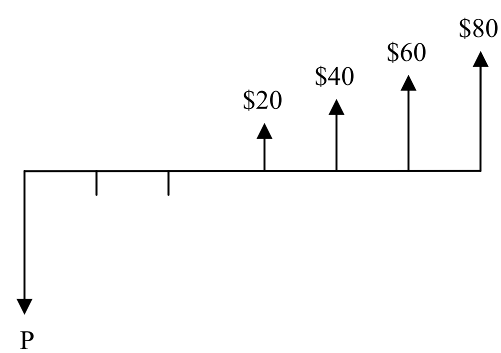


The first four payments will repay a present sum:
P= $500 (P/A, 8%, 4) + $500 (P/G, 8%, 4)
= $500 (3.312) + $500 (4.650)
= $3,981

The unpaid portion of the $5,000 is
$5,000 – $3,981 = $1,019

Thus:
x = $1,019 (F/P, 8%, 5)
= $1,019 (1.469)
= $1,496.91

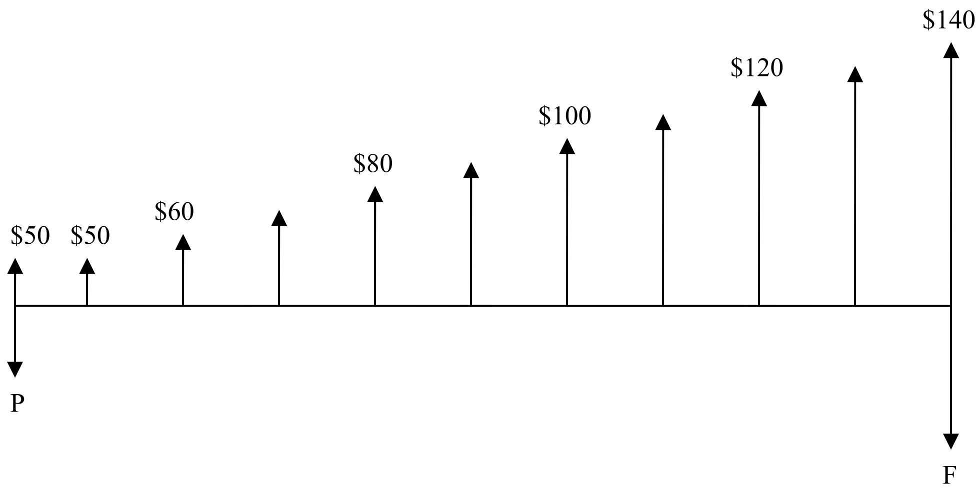
4-58



P= $20 (P/G, 8%, 5) (P/F, 8%, 1)
= $20 (7.372) (0.9529)
= $136.51

4-59

The series of deposits are beginning-of-period deposits rather than end-of-period. The simplest solution is to draw a diagram of the situation and then proceed to solve the problem presented by the diagram.

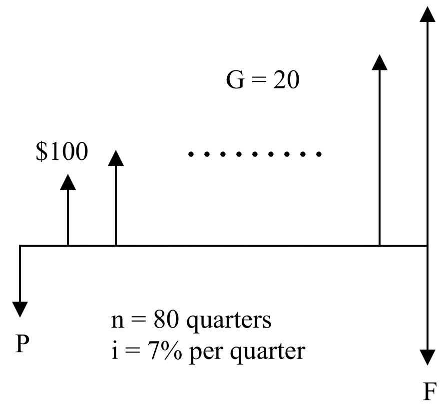


The diagram illustrates a problem that can be solved directly.

P= $50 + $50 (P/A, 3%, 10) + $10 (P/G, 3%, 10)
= $50 + $50 (8.530) + $10 (36.309)
= $839.59

F = P (F/P, 3%, 10)
= $839.59 (F/P, 3%, 10)
= $839.59 (1.344)
= <u>$1,128.41</u>

4-60



P = $100 (P/A, 7%, 80) + $20 (P/G, 7%, 80) = <u>$5,383.70</u>
F = $5,383.70 (F/P, 7%, 80) = <u>$1,207,200.00</u>

Alternate Solution:
F = [$100 + $20 (A/G, 7%, 80)] (F/A, 7%, 80)
= [$100 + $20 (13.927)] (3189.1)
= <u>$1,207,200.00</u>

4-61

We have at interest rate i: P_1 = A(P/A, i, 4) = 3B(P/A, i, 3) – B(P/G, i, 3) = P_2 . The question is what happens to the present worth when the interest rate doubles? This problem is much too complicated to solve in closed form so just try an example, e. g. let A = 100 and i = 10%, then
100(3.170) = 3B(2.487) – B(2.329) and solving for B have B = 61.77.

Next, let i = (2)(10%) = 20%, with A = 100 and B = 61.77 so
P_1' = 100 (P/A, 20%, 4) = 100(2.589) = 258.9 and
P_2' = 3(61.77) (P/A, 20%, 3) – 61.77 (P/G, 20%, 3) = 3(61.77)(2.106) – (61.77)(1.852) = 275.9

Since $P_2' > P_1'$, the (ii) cash flow is more valuable. In general, at higher interest rates, the present worth of identical cash flows is less. In this problem in sequence (ii), the equivalent cash flow is concentrated in earlier years so as interest rates go up it would have the larger present worth.

4-62

We have at interest rate i: P_1 = A(P/A, i, 4) = B(P/A, i, 4) + B(P/G, i, 4) = P_2 .
As in Problem 4-61 try an example. Let A = 100 and i = 10% so
100 (3.170) = B (3.170) + B (4.378) = (7.548) B . Thus, B = 42.00 .
Next, i = (2)(10%) = 20% so
P_1' = 100 (2.589) = 258.9
P_2' = 42 (2.589) + 42 (3.299) = 108.74 + 138.56 = 247.3.
Since $P_1' > P_2'$, the (i) cash flow is more valuable. This result should be anticipated since the

(i) cash flow has its money more concentrated at earlier years than the (ii) cash flow. As interest rates go up the (i) cash flow will be more valuable.

4-63

(a) P = 20000 (P/A, 8%, 10) + 2000 (P/G, 8%, 10)
= (20000) (6.710) + (2000) (25.977)
= $186,154

(b) P = 20000 (P/A, 10%, 8%,10)

$$= 20000 \left[\frac{1-(1+0.10)^{10}(1+0.08)^{-10}}{0.08-0.10}\right]$$

= $201,405

4-64

(a) P = 50,000 (P/A, 7%, 15) + 5,000 (P/G, 7%, 15)
= 50,000 (9.108) + (5,000) (52.446)
= $717,630

(b) P = 50,000 (P/A, 10%, 7%, 15)

$$= 50{,}000 \left[\frac{1-(1+0.10)^{15}(1+0.07)^{-15}}{0.07-0.10}\right]$$

= $856,712

4-65

(a) P = 20,000 (P/A, 10%, 10) + 2,000 (P/G, 10%, 10)
= (20,000) (6.145) + (2,000) (22.891)
= $168,682

(b) P = A_1 (P/A, i, i, n) = A_1 $[n(1+ i)^{-1}]$
= 20,000 (10) $(1+ 0.10)^{-1}$
= $181,818

4-66

(a) P = 20,000 (P/A, 8%, 10) – 2,000 (P/G, 8%, 10)
= 20,000 (6.710) – 2,000 (25.977)
= $82,246

(b) Here we have a geometric *decreasing* gradient. By inspection of the derivation of Equation 4-22, one can see that you can simply replace g with –g, so

$$P = A_1\left[\frac{1-(1-g)^n(1+i)^{-n}}{i+g}\right] = 20000\left[\frac{1-(1-0.10)^{10}(1+0.08)^{-10}}{0.08+0.10}\right]$$

= $93,166

4-67

$$\text{P}= 60{,}000\ (\text{P/A}, 8\%, 10\%, 15) = 20{,}000\left[\frac{1-(1+0.08)^{15}(1+0.10)^{-15}}{0.10-0.08}\right]$$

= $721,824

4-68

$$\text{P}= 400\ (\text{P/A}, 6\%, 10\%, 5) = 400\left[\frac{1-(1+0.06)^{5}(1+0.10)^{-5}}{0.10-0.06}\right]$$

= $1,691

4-69

$$\text{P}= 400\ (\text{P/A}, 15\%, 10\%, 5) = 400\left[\frac{1-(1+0.15)^{5}(1+0.10)^{-5}}{0.10-0.15}\right]$$

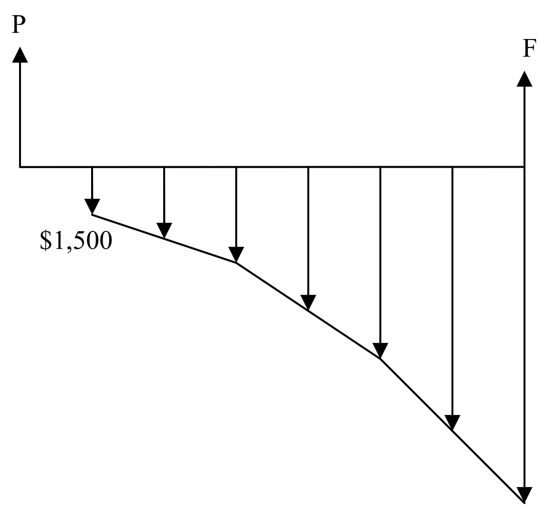
= $1,991

4-70

There are two choices:
(1) P_1 = 75,000 (P/A, 9%, 5) = $291,750
(2) P_2 = 65,000 (P/A, 5%, 9%, 5) = 65000 $\left[\frac{1-(1+0.05)^5(1+0.09)^{-5}}{0.09-0.05}\right]$ = $277,070

Suzanne should take the first choice.

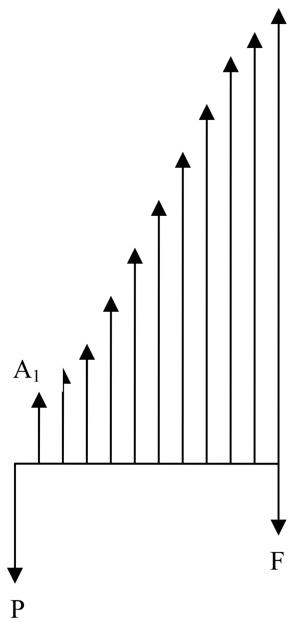
4-71



(a) Since the book only gives a geometric gradient to present worth factor, we must first solve for P and then F.
P = ? *n* = 6 *i* = 10% *g* = 8%
P = A_1 (P/A, g%, i%, n)
(P/A, g%, i%, n) = $[(1 - (1 + g)^n (1 + i)^{-n})/(i - g)]$
= $[(1 - (1.08)^6 (1.10)^{-6})/(0.10 - 0.08)]$
= 5.212
P = $1,500 (5.212) = $7,818
F = P (F/P, i%, n) = $7,818 (F/P, 10%, 6) = <u>$13,853</u>
As a check, solve with single payment factors:
$1,500.00 (F/P, 10%, 5) = $1500.00 (1.611) = $2,413.50
$1,620.00 (F/P, 10%, 4) = $1,620.00 (1.464) = $2,371.68
$1,749.60 (F/P, 10%, 3) = $1,749.60 (1.331) = $2,328.72
$1,889.57 (F/P, 10%, 2) = $1,898.57 (1.210) = $2,286.38
$2,040.73 (F/P, 10%, 1) = $2,040.73 (1.100) = $2,244.80
$2,203.99 (F/P, 10%, 0) = $2,203.99 (1.000) = <u>$2,203.99</u>
Total Amount = <u>$13,852.07</u>

(b) Here, i% = g%, hence the geometric gradient to present worth equation is
$P = A_1\, n\, (1 + i)^{-1} = \$1{,}500\ (6)\ (1.08)^{-1} = \$8{,}333$
$F = P\ (F/P, 8\%, 6) = \$8{,}333\ (1.587) =$ <u>$13,224</u>

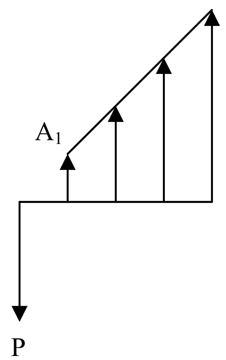
4-72



$n = 20,\ \ i = g = 8\%,\ \ F = ?$
$A = 5\%\ (\$52{,}000)$
$\quad = \$2{,}600$
$P = A_1\, n\, (1 + i)^{-1}$
$\quad = \$2{,}600\ (20)\ (1 + 0.08)^{-1}$
$\quad = \$48{,}148$

$F = P\ (F/P, i\%, n)$
$\quad = \$48{,}148\ (1 + 0.08)^{20}$
$\quad =$ <u>$224,416</u>

4-73



P = ?, i = 12%, g = 8%, n = 4

A_1 = 2nd-year salary
= 1.08 ($225,000)
= $243,000

$P = A_1 [(1-(1+g)^n (1+i)^{-n})/(i-g)]$
$= \$243{,}000[(1-(1.08)^4 (1.12)^{-4})/0.04]$
$= \$243{,}000\ [0.135385/0.04]$
= <u>$822,462</u>

4-74

Since there are annual deposits, but quarterly compounding, we must first compute the effective interest rate per year.
Effective interest rate = $(1 + i)^m - 1 = (1.02)^4 - 1 = 0.0824 = 8.24\%$

Since F = $1,000,000, we can find the equivalent P for i = 8.24% and n = 40.
P= F (P/F, 8.24%, 40)
$= \$1{,}000{,}000\ (1 + 0.0824)^{-40}$
= $42,120

Now we can insert these values in the geometric gradient to present worth equation:
$P = A_1 [(1-(1+g)^n (1+i)^{-n})/(i-g)]$
$\$42{,}120 = A_1 [(1-(1.07)^{40} (1.0824)^{-40})/(0.0824-0.0700)]$
$= A_1 (29.78)$

The first IRA deposit A_1 = $42,120/29.78 = <u>$1,414</u>

4-75

i = 1%/month
Effective Interest Rate = $(1 + i)^m - 1 = (1.01)^{12} - 1$
$= 0.127 = \underline{12.7\%}$

4-76

Effective Interest Rate = $(1 + 0.0175)^{12} - 1 = 0.2314 = \underline{23.14\%}$

4-77

Effective Interest Rate = $(1 + i)^m - 1 = (1.03)^4 - 1 = 0.1255 = \underline{12.55\%}$

4-78

Nominal Interest Rate = 12 (1.5%) = <u>18%</u>
Effective Interest Rate = $(1 + 0.015)^{12} = 0.1956 = \underline{19.56\%}$

4-79

Effective Interest Rate = $(1 + i)^m - 1$
$0.0931 = (1 + i)^4 - 1$
$1.0931 = (1 + i)^4$
$1.0931^{0.25} = (1 + i)$
$1.0225 = (1 + i)$
$i = 0.0225$
= 2.25% per quarter
= <u>9% per year</u>

4-80

Effective Interest Rate $= (1 + i)^m - 1 = (1 + (0.07/365))^{365} - 1$
$= 0.0725 = \underline{7.25\%}$

4-81

Effective interest rate = $(1 + i)^m - 1$ =
$1.61 = (1 + i)^{12}$
$(1 + i) = 1.61^{0.0833} = 1.0125$
i = .0125 = 1.25%

4-82

(a) $r = i \times m$
= (1.25%) (12)
= 15%

(b) $i_a = (1 + 0.0125)^{12} - 1$
= 16.08%

(c) A = \$10,000 (A/P, 1.25%, 48)
= \$10,000 (0.0278)
= \$278

4-83

(a) P = \$1,000 A = \$90.30 i = ? m = 12 months
\$1,000 = \$90.30 (P/A, i%, 12)
(P/A, i%, 12) = \$1,000/\$90.30 = 11.074
i = 1.25%

(b) r = (1.25%) (12)
= 15%

(c) $i_a = (1 + 0.0125)^{12} - 1$
= 16.08%

4-84

$F = P(1 + i)^n$
$\$85 = \$75(1 + i)^1$
$(1 + i)$ = \$85/\$75 = 1.133
i = 0.133 = 13.3%
Nominal Interest Rate = 13.3% (2) = 26.6%
Effective Interest Rate = $(1 + 0.133)^2 - 1$ = 0.284 = 28.4%

4-85

(a) Effective Interest Rate = $(1 + i)^m - 1 = (1 + 0.025)^4 - 1 = 0.1038$
= <u>10.38%</u>

(b) Since the effective interest rate is 10.38%, we can look backwards to compute and equivalent i for 1/252 of a year.
$(1 + i)^{252} - 1 = 0.1038$
$(1 + i)^{252} = 1.1038$
$(1 + i) = 1.1038^{1/252} = 1.000392$
Equivalent i = 0.0392% per 1/252 of a year

(c) Subscriber's Cost per Copy:
$A = P\ (A/P, i\%, n) = P\ [(i(1 + i)^n)/((1 + i)^n - 1)]$
$A = \$206\ [(0.000392\ (1 + 0.000392)^{504})/(1 + 0.000392)^{504} - 1)]$
$= \$206\ (0.002187)$
$= \$0.45$ = 45 cents per copy

To check:
Ignoring interest, the cost per copy = \$206/(2(252)) = 40.8 cents per copy
Therefore, the answer of 45 cents per copy looks reasonable.

4-86

P = \$9,500, F = \$10,000, i = ?, n = 1 six-month interest period
$F = P\ (1 + i)$
$(1 + i) = F/P = \$10,000/\$9,500 = 1.0526$
$i = .0526 = 5.26\%$
Nominal Interest Rate = 5.26% (2) = <u>10.52%</u>
Effective Interest Rate = $(1 + .0526)^2 - 1 = 0.10797$ = <u>10.80%</u>

4-87

(a) Effective Interest Rate
$i_a = (1 + r/m)^m - 1$
$= (1 + 0.06/2)^2 - 1$
$= 0.0609$
= <u>6.09%</u>
Continuous Effective Interest Rate
$i_a = e^r - 1$
$= e^{0.06} - 1$
$= 0.0618$
= <u>6.18%</u>

(b) The future value of the loan, one period (6 months) before the first repayment:
= \$2,000 (F/P, 3%, 5)
= \$2,000 (1.159)
= \$2318
The uniform payment:
= \$2,318 (A/P, 3%, 4)
= \$2,318 (0.2690)
= \$623.54 every 6 months

(c) Total interest paid:
= 4 (\$623.54) – \$2,000
= \$494.16

4-88

Common Stock Investment
P = \$1,000, n = 20 quarters, *i* = ?, *F* = \$1,307
F = P (F/P, i%, n)
\$1,307 = \$1,000 (F/P, i%, 20)
(F/P, i%, 20) = \$1,307/\$1,000
= 1.307

Performing linear interpolation using interest tables:

(P/A, i%, 20)	I
1.282	1.25%
1.347	1.50%

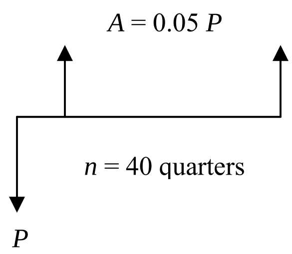
i = 1.25% + 0.25% ((1.307 − 1.282)/(1.347 − 1.282))
= 1.25% + 0.10%
= 1.35%

Nominal Interest Rate = 4 quarters / year (1.35% / quarter)
= 5.40%/year
Effective Interest Rate = $(1 + i)^m - 1 = (1.0135)^4 - 1$
= 5.51%/year

4-89

$F = P(1 + i)^n = 0.98F(1 + i)^1$
i = (1.00/0.98) − 1
= 0.0204 = 2.04%
$i_{eff} = (1 + i)^m - 1 = (1.0204)^{365/20} - 1$
= 0.4456 = 44.6%

4-90



$P = 0.05\ P\ (P/A, i\%, 40)$
$(P/A, i\%, 40) = 1/0.05 = 20$

From interest tables:

(*P/A*, *i*%, 40)	*i*
21.355	3.5%
19.793	4.0%

Performing linear interpolation:
$i = 3.5\% + 0.5\%\ ((21.355 - 20)/(21.355 - 19.793))$
$= 3.5\% + 0.5\%\ (1.355/1.562)$
$=$ <u>3.93% per quarter year</u>

Effective rate of interest $= (1 + i)^m - 1 = (1.0393)^4 - 1$
$= 0.1667 =$ <u>16.67% per year</u>

4-91

(a) $F_{16} = \$10{,}000\ (1 + 0.055/4)^{16}$
$= \$12{,}442.11$
$F_{10} = \$12{,}442.11\ (1 + 0.065/4)^{24}$
$=$ <u>$18,319.24</u>

(b) $\$18{,}319.24 = (1 + i)^{10}\ (\$10{,}000)$
$(1 + i)^{10} = \$18{,}319.24/\$10{,}000 = 1.8319$
$10 \ln (1 + i) = \ln (1.8319)$
$\ln (1 + i) = (\ln (1.8319))/10$
$= 0.0605$
$(1 + i) = 1.0624$
$i = 0.0624 =$ <u>6.24%</u>

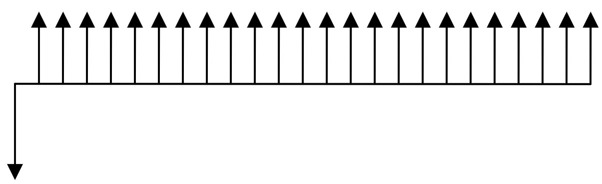
Alternative Solution
$18,319.24 = $10,000 (F/P, *i,* 10)
(F/P, *i,* 10) = 1.832

Performing interpolation:

(F/P, i%, 10)	i
1.791	6%
1.967	7%

i = 6% + [(1.832 – 1.791)/(1.967 – 1.791)] = 6.24%

4-92



P = $1,000, *n* = 24 months, *i* = ?, *A* = $47.50
P = A (P/A, i%, *n*)
$1,000 = $47.50 (P/A, i%, n)
(P/A, i%, 24) = $1,000/$47.50 = 21.053

Performing linear interpolation using interest tables:

(P/A, i%, 24)	I
21.243	1%
20.624	1.25%

i = 1% + 0.25% ((21.243 − 21.053)/(21.243 − 20.624))
= 1.077%/mo

Nominal Interest Rate = 12 months/year (1.077%/month)
= 12.92%/year

4-93

P = \$2,000, n = 50 months, i = ?, A = \$51.00
$A = P$ (A/P, i%, n)
\$51.00 = \$2,000 (A/P, i%, 50)
(A/P, i%, 50) = \$51.00/\$2,000
= 0.0255

From interest tables:
i = <u>1%/month</u>

Nominal Interest Rate = 12 months/year (1%/month)
= <u>12%/year</u>

Effective Interest Rate = $(1 + i)^m - 1 = (1.01)^{12} - 1$
= <u>12.7%/year</u>

4-94

A = 3(\$100) = \$300, i = 1.5% per quarter year, F = ?
n = 12 quarterly periods (in 3 years)

$F = A$ (F/A, i%, n) = <u>\$3,912.30</u> = \$300 (F/A, 1.5%, 12) = \$300 (13.041)

Note that this is no different from Ann's depositing \$300 at the end of each quarter, as her monthly deposits do not earn any interest until the subsequent quarter.

4-95

Monthly Payment = \$10,000 (A/P, 0.75%, 12) = \$10,000 (0.0875)
= \$875.00
Total Interest Per Year = \$875.00 × 12 – \$10,000 = \$500.00

<u>Rule of 78s</u>
With early repayment:
Interest Charge = ((12 + 11 + 10) / 78) (\$500) = \$211.54
Additional Sum (in addition to the 3rd \$875.00 payment)
Additional Sum = \$10,000 + \$211.54 interest – 3 (\$875.00) = <u>\$7,586.54</u>

<u>Exact Method</u>
Additional Sum equals present worth of the nine future payments that would have been made:
Additional Sum = \$875.00 (P/A, 0.75%, 9) = \$875.00 (8.672) = <u>\$7,588.00</u>

4-96

(a) 11.98% compounded continuously
$F = \$10{,}000\ e^{(0.1198)(4)}$
$= \$16{,}147.82$

(b) 12% compounded daily
$F = \$10{,}000\ (1 + 0.12/365)^{365\times 4}$
$= \$16{,}159.47$

(c) 12.01% compounded monthly
$F = \$10{,}000\ (1 + 0.1201/12)^{12\times 4}$
$= \$16{,}128.65$

(d) 12.02% compounded quarterly
$F = \$10{,}000\ (1 + 0.1202/4)^{4\times 4}$
$= \$16{,}059.53$

(e) 12.03% compounded yearly
$F = \$10{,}000\ (1 + 0.1203)^{4}$
$= \$15{,}752.06$

<u>Decision: Choose Alternative (b)</u>

4-97

P = 2000 cars/day, n = 2, i = 5%, F_2 = ? cars/day
$F_2 = P\ e^{in} = 2000\ e^{(0.05)(2)}$ = <u>2,210 cars/day</u>

4-98

(a) Effective Interest Rate $= (1 + i)^{m} - 1 = (1.025)^{4} - 1 = 0.1038$
= <u>10.38%</u>

(b) Effective Interest Rate $= (1 + i)^{m} - 1 = (1 + (0.10/365))^{365} - 1$
= 0.10516= <u>10.52%</u>

(c) Effective Interest Rate $= e^{r} - 1 = e^{0.10} - 1 = 0.10517$
= <u>10.52%</u>

4-99

Nominal Interest Rate = (1.75%) 12 = 21%
Effective Interest Rate = $e^m - 1 = e^{(0.21\times1)} - 1$ = 0.2337 = 23.37%

4-100

$P = Fe^{-m}$ = \$6,000 $e^{-(0.12)(2.5)}$ = \$6,000 (0.7408) = <u>\$4,444.80</u>

4-101

<u>West Bank</u>
F = P $(1 + i)^n$ = \$10,000 $(1 + (0.065/365))^{365}$ = <u>\$10,671.53</u>

<u>East Bank</u>
F = P e^m = \$10,000 $e^{(.065\times1)}$ = <u>\$10,671.59</u>
Difference = <u>\$0.06</u>

4-102

(a) $P = Fe^{-m}$
= \$8,000 $e^{-(0.08)(4.5)}$
= <u>\$5,581.41</u>

(b) $F = Pe^m$
$F/P = e^m$
ln(F/P) = m

r = (1/n) ln (F/P)
= (1/4.5) ln(\$8,000/\$5000)
= <u>10.44%</u>

4-103

(a) Continuous cash flow – continuous compounding (one period)

$F = P\hat{} \; [(e^r - 1)\,(e^m)/re^r]$

$= \$1 \times 10^9\,[(e^{0.005} - 1)\,(e^{(0.005)(1)})/(0.005\,e^{0.005})]$

$= \$1 \times 10^9\,[(e^{0.005} - 1)/0.005]$

$= \$1 \times 10^9\,(0.00501252/0.005)$

$= \$1{,}002{,}504{,}000$

Thus, the interest is <u>$2,504,000.</u>

(b) Deposits of $A = \$250 \times 10^6$ occur four times a month

Continuous compounding

r = nominal interest rate per ¼ month

= 0.005/4 = 0.00125 = 0.125%

$F = A\,[(e^m - 1)/(e^r - 1)]$

$= \$250{,}000{,}000\,[(e^{(0.00125)(4)} - 1)/(e^{(0.00125)} - 1)]$

$= \$250{,}000{,}000\,[0.00501252/0.00125078]$

$= \$1{,}001{,}879{,}000$

Here, the interest is <u>$1,879,000.</u>

So it pays <u>$625,000</u> a month to move quickly!

4-104

P = $29,000, *n* = 3 years, *F* = ?

(a) $i_a = 0.13$

$F = P\,(1 + i)^n = \$29{,}000\,(1.13)^3 = \underline{\$41{,}844}$

(b) $r = 0.1275$

$F = P\,e^{rn} = \$29{,}000\,e^{(0.1275)(3)} = \$29{,}000\,(1.4659) = \underline{\$42{,}511}$

We can see that although the interest rate was less with the continuous compounding, the future amount is greater because of the increased compounding periods (an infinite number of compounding periods). <u>Thus, the correct choice for the company is to choose the 13% interest rate and discrete compounding.</u>

4-105

A = \$1,200, *r* = 0.14/12, *n* = 7 × 12
= 0.01167 = 84 compounding periods

$F = A\,[(e^{rn} - 1)/(e^{r} - 1)]$
$= \$1,200\,[(e^{(0.01167)(84)} - 1)/(e^{0.01167} - 1)]$
$= \$1,200\,[1.66520/0.011738]$
= <u>\$170,237</u>

4-106

First Bank — Continous Compounding
Effective interest rate $i_a = e^{r} - 1 = e^{0.045} - 1 = 0.04603$
= 4.603%

Second Bank — Monthly Compounding
Effective interest rate $i_a = (1 + r/m)^{m} - 1 = (1 + 0.046/12)^{12} - 1$
= 0.04698 = 4.698%
<u>No, Barry should have selected the Second Bank.</u>

4-107

P = \$10,000, *F* = \$30,000, *i* = 5%, *n* = ?
$F = Pe^{rn}$
$\$30,000 = \$10,000\ e^{(0.05)n}$
0.05n = ln(\$30,000/\$10,000) = 1.0986
n = 1.0986/0.05
= <u>21.97 years</u>

4-108

Compute effective interest rate for each alternative
(a) <u>4.375%</u>

(b) $(1 + 0.0425/4)^{4} - 1 = (1.0106)^{4} - 1 = 0.0431$ = <u>4.31%</u>

(c) $e^{rn} - 1 = e^{0.04125} - 1 = 0.0421$ = <u>4.21%</u>

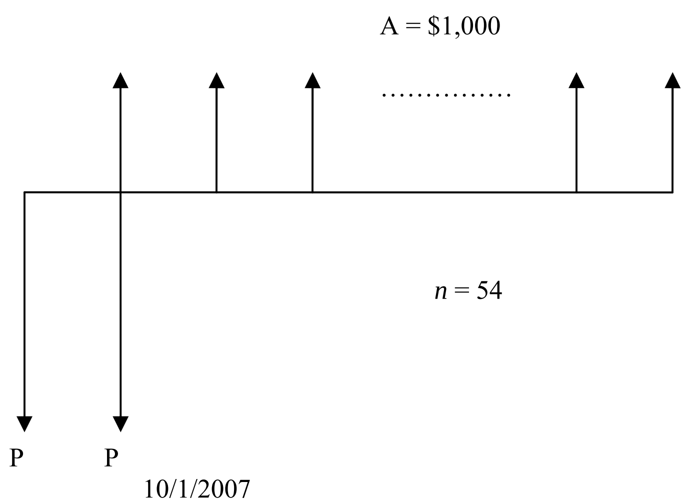
The 4 3/8% interest (a) has the highest effective interest rate.

4-109

(a) Interest Rate per 6 months = \$20,000/\$500,000 = 0.0400 = 4%
Effective Interest Rate per year. = $(1 + 0.04)^2 - 1$ = 0.0816
= 8.16%

(b) For continuous compounding:
$F = Pe^m$
\$520,000 = \$500,000 $e^{r(1)}$
r = ln(\$520,000/\$500,000) = 0.0392
= 3.92% per 6 months
Nominal Interest Rate (per year) = 3.92% (2) = 7.84% per year

4-110



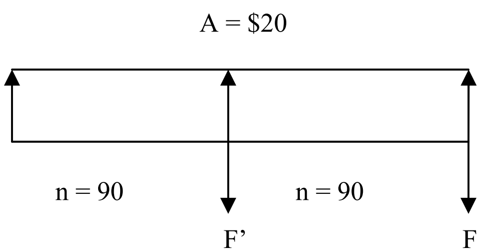
Continuous compounding
Effective interest rate/ quarter year = $e^{(0.13/4)} - 1$ = 0.03303
= 3.303%

Solution One
$P_{10/1/2007}$ = \$1,000 + \$1,000 (P/A, 3.303%, 53)
= \$1,000 + \$1,000 $[((1.03303)^{53} - 1))/(0.03303(1.03303)^{53})]$
= \$25,866

Solution Two
$P_{10/1/2007}$ = \$1,000 (P/A, 3.303%, 54) (F/P, 3.303%, 1)
= \$1,000 $[((1.03303)^{54} - 1)/(0.03303\ (1.03303)^{54})]\ (1.03303)$
= \$25,866

4-111

F = ?, n = 180 months, i = 0.50%/month, A = \$20.00
$F = A$ (*F/A,* 0.50%, 180)
Since the ½% interest table does not contain n = 180, the problem must be split into workable components. One way would be:



F = \$20 (*F/A,* ½%, 90) + \$20 (*F/A,* ½%, 90)(*F/P,* ½%, 90)
= <u>\$5,817</u>

Alternate Solution
Perform linear interpolation between n = 120 and n = 240:
F = \$20 ((*F/A,* ½%, 120) − (*F/A,* ½%, 240))/2
= <u>\$6,259</u>
Note the inaccuracy of this solution.

4-112

Compute the effective interest rate per quarterly payment period:
$i_{qtr} = (1 + 0.10/12)^3 - 1 = 0.0252 = 2.52\%$

Compute the present worth of the 32 quarterly payments:
P = A (P/A, 2.52%, 32)
$= \$3,000\ [(1.0252)^{12} - 1]/[0.0252(1.0252)^{12}]$
= \$3,000 (21.7878)
= <u>\$65,363</u>

4-113

i = 14%
n = 19 semiannual periods
i_{qtr} = 0.14/4 = 0.035
$i_{semiannual} = (1 + 0.035)^2 - 1 = 0.071225$
Can either solve for P or F first. Let's solve for F first:

$F_{1/2015}$ = A (F/A, i%, n)
= $1,000 $[(1 + 0.071225)^{19} - 1]/0.071225$
= $37,852.04

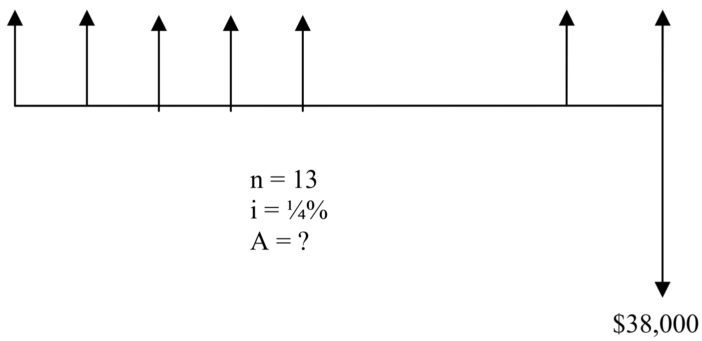
Now, we have the Future Worth at January 1, 2015. We need the Present Worth at April 1, 2008. We can use either interest rate, the quarterly or the semiannual. Let's use the quarterly with n = 27.

$P = F (1 + i)^{-n}$
= $37,852.04 $(1.035)^{-27}$
= $14,952

This particular example illustrates the concept of these problems being similar to putting a puzzle together. There was no simple formula, or even a complicated formula, to arrive at the solution. While the actual calculations were not difficult, there were several steps required to arrive at the correct solution.

4-114

i = interest rate/interest period = 0.13/52 = 0.0025 = 0.25%
Paco's Account: 63 deposits of $38,000 each, equivalent weekly deposit



A= F (A/F, *i*%, *n*)
= $38,000 (A/F, 0.25%, 13)
= $38,000 (0.0758)
= $2,880.40

For 63 deposits:
F = $2,880.40 (F/A, 0.25%, 63×13)
= $2,880.40 $[((1.0025)^{819} - 1)/0.0025]$
= $2,880.40 (2691.49)
= $7,752,570 at 4/1/2022

Amount at 1/1/2017 = $7,742,570 (P/F, 0.25%, 273)
= $7,742,570 (0.50578)
= <u>$3,921,000</u>

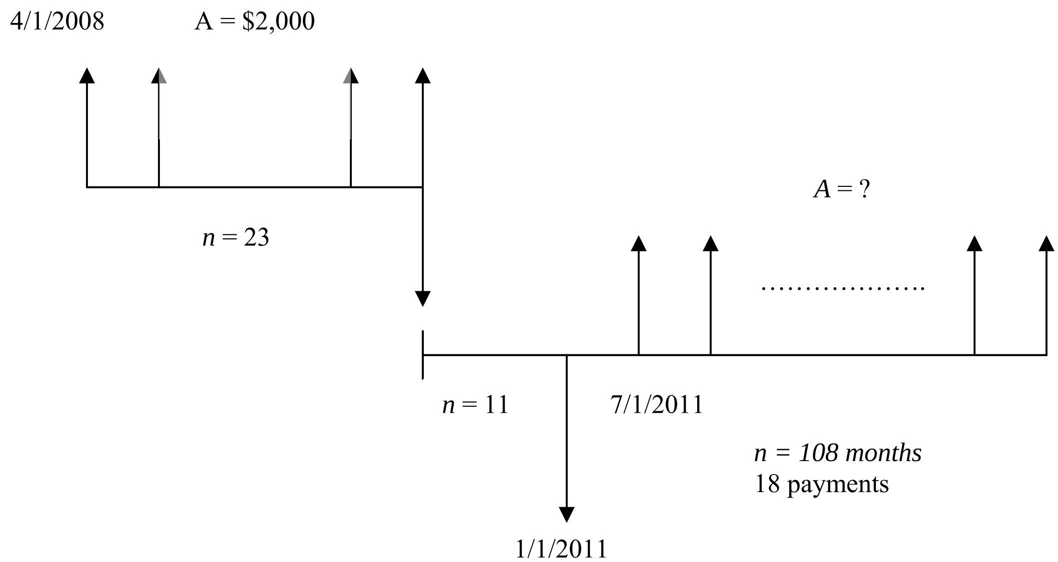
Tisha's Account: 18 deposits of $18,000 each
Equivalent weekly deposit:
A= $18,000 (A/F, 0.25%, 26)
= $18,000 (0.0373)
= $671.40

Present Worth $P_{1/1/2016}$ = $671.40 (P/A, 0.25%, 18×26)
= $671.40 $[((1.0025)^{468} - 1)/(0.0025(1.0025)^{468})]$
= $185,086

Amount at 1/1/2017 = $185,086 (F/P, 0.25%, 52)
= $185,086 (1.139)
= $211,000

Sum of both accounts at 1/1/2017 = $3,921,000 + $211,000 = <u>$4,132,000</u>

4-115



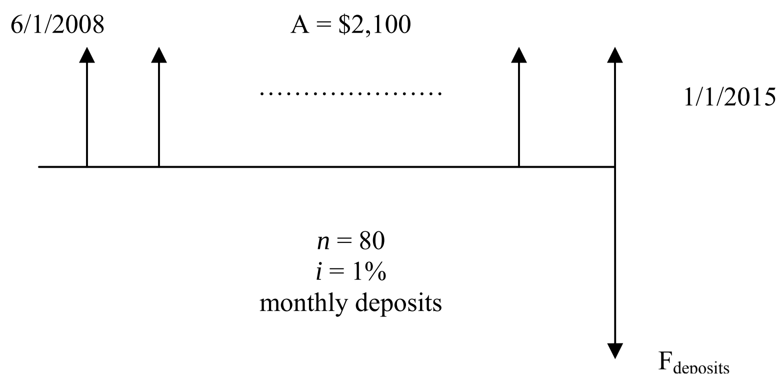
Monthly cash flows:
$F_{2/1/2010}$ = \$2,000 (F/A, 1%, 23) = \$2,000 (25.716) = \$51,432
$F_{2/1/2010}$ = \$51,432 (F/P, 1%, 11) = \$51,432 (1.116) = \$57,398

Equivalent A from 7/1/2011 through 1/1/2020 where n = 108 and i = 1%
A_{equiv} = \$57,398 (A/P, 1%, 108) = \$57,398 (0.01518)
= \$871.30

Equivalent semiannual payments required from 7/1/2011 through 1/1/2020:
$A_{semiann}$ = \$871.30 (F/A, 1%, 6) = \$871.30 (6.152)
= <u>\$5,360</u>

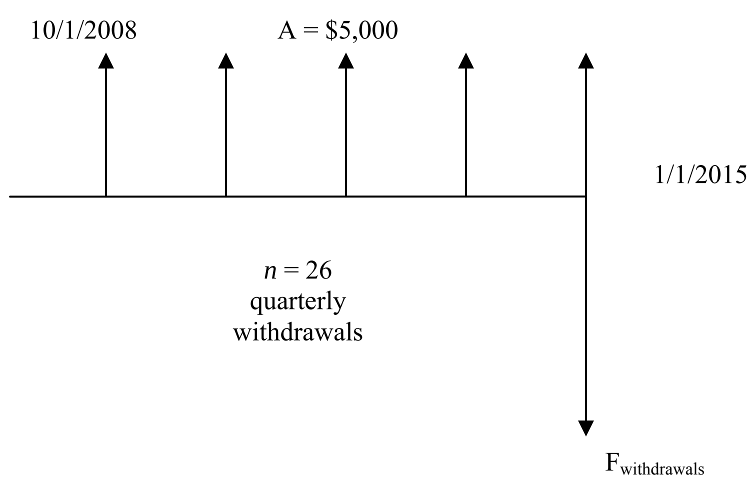
4-116

Deposits



$F_{deposits}$= $2,100 (F/A, 1%, 80)
= $255, 509

Withdrawals



Equivalent quarterly interest $i_{quarterly} = (1.01)^3 - 1$
$= 0.0303 = 3.03\%$

$F_{withdrawals}$ = $5,000 (F/A, 3.03%, 26)
= $\$5{,}000\ [((1.0303)^{26} - 1)/0.0303]$
= $193,561

Amount remaining in the account on January 1, 2015:
= $255,509 – $193,561
= <u>$61,948</u>

4-117

Amortization schedule for a $4,500 loan at 6%
Paid monthly for 24 months
P = $4,500, i = 6%/12 mo = 1/2% per month

Pmt. #	Amt. Owed BOP	Int. Owed (this pmt.)	Total Owed (EOP)	Principal (This pmt)	Monthly Pmt.
1	4,500.00	22.50	4,522.50	176.94	199.44
2	4,323.06	21.62	4,344.68	177.82	199.44
3	4,145.24	20.73	4,165.97	178.71	199.44
4	3,966.52	19.83	3,986.35	179.61	199.44
5	3,786.91	18.93	3,805.84	180.51	199.44
6	3,606.41	18.03	3,624.44	181.41	199.44
7	3,425.00	17.13	3,442.13	182.32	199.44
8	3,242.69	16.21	3,258.90	183.23	199.44
9	3,059.46	15.30	3,074.76	184.14	199.44
10	2,875.32	14.38	2,889.69	185.06	199.44
11	2,690.25	13.45	2,703.70	185.99	199.44
12	2,504.26	12.52	2,516.79	186.92	199.44
13	2,317.35	11.59	2,328.93	187.85	199.44
14	2,129.49	10.65	2,140.14	188.79	199.44
15	1,940.70	9.70	1,950.40	189.74	199.44
16	1,750.96	8.75	1,759.72	190.69	199.44
17	1,560.28	7.80	1,568.08	191.64	199.44
18	1,368.64	6.84	1,375.48	192.60	199.44
19	1,176.04	5.88	1,181.92	193.56	199.44
20	982.48	4.91	987.40	194.53	199.44
21	787.96	3.94	791.90	195.50	199.44
22	592.46	2.96	595.42	196.48	199.44
23	395.98	1.98	397.96	197.46	199.44
24	198.52	0.99	199.51	198.45	199.44
TOTALS		286.63		4499.93	

B12 = $4,500.00 (principal amount)
B13 = B12 – E12 (amount owed BOP – principal in this payment)
Column C= amount owed BOP * 0.005
Column D= Column B + Column C (principal + interest)
Column E = Column F – Column C (payment – interest owed)
Column F = Uniform Monthly Payment (from formula for A/P)
Payment 24 is the final payment. Payment amount = $199.51

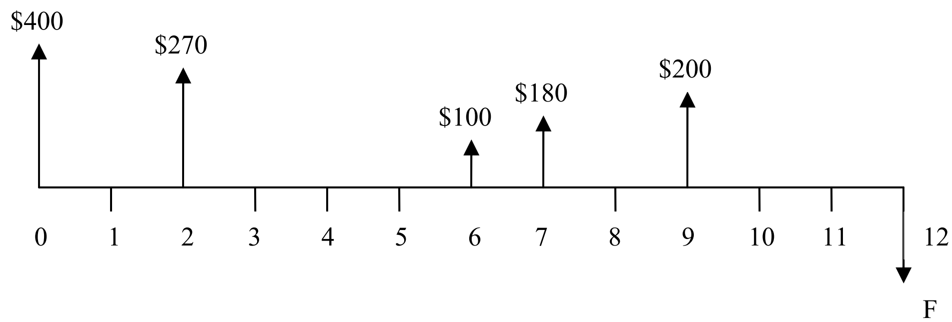
4-118

Amortization schedule for a $4,500 loan at 6% Paid monthly for 24 months
P = $4,500, i = 6%/12 mo = 1/2% per month

Pmt. #	Amt. Owed BOP	Int. Owed (this pmt.)	Total Owed (EOP)	Principal (This pmt)	Monthly Pmt.
1	4,500.00	22.50	4,522.50	176.94	199.44
2	4,323.06	21.62	4,344.68	177.82	199.44
3	4,145.24	20.73	4,165.97	178.71	199.44
4	3,966.52	19.83	3,986.35	179.61	199.44
5	3,786.91	18.93	3,805.84	180.51	199.44
6	3,606.41	18.03	3,624.44	181.41	199.44
7	3,425.00	17.13	3,442.13	182.32	199.44
8	3,242.69	16.21	3,258.90	483.79	500.00
9	2,758.90	13.79	2,772.69	185.65	199.44
10	2,573.25	12.87	2,586.12	267.13	280.00
11	2,306.12	11.53	2,317.65	187.91	199.44
12	2,118.21	10.59	2,128.80	188.85	199.44
13	1,929.36	9.65	1,939.01	189.79	199.44
14	1,739.57	8.70	1,748.27	190.74	199.44
15	1,548.83	7.74	1,556.57	191.70	199.44
16	1,357.13	6.79	1,363.92	192.65	199.44
17	1,164.48	5.82	1,170.30	193.62	199.44
18	970.86	4.85	975.71	194.59	199.44
19	776.27	3.88	780.15	195.56	199.44
20	580.71	2.90	583.61	196.54	199.44
21	384.18	1.92	386.10	197.52	199.44
22	186.66	0.93	187.59	186.66	187.59
23	0.00	0.00	0.00	0.00	0.00
24	0.00	0.00	0.00	0.00	0.00
TOTALS		256.95		4500.00	

B12 = $4,500.00 (principal amount)
B13 = B12 – E12 (amount owed BOP – principal in this payment)
Column C= amount owed BOP* 0.005
Column D= Column B + Column C (principal + interest)
Column E = Column F – Column C (payment – interest owed)
Column F = Uniform Monthly Payment (from formula for A/P)
Payment 22 is the final payment. Payment amount = $187.59

4-119



i = NIR/m = 9%/12 = 0.75%/mo

F_{12} = \$400 (F/P, 0.75%, 12) + \$270 (F/P, 0.75%, 10) + \$100 (F/P, 0.75%, 6) + \$180 (F/P, 0.75%, 5) + \$200 (F/P, 0.75%, 3)

= \$400 (1.094) + \$270 (1.078) + \$100 (1.046) + \$180 (1.038) + \$200 (1.023)

= <u>\$1,224.70</u> (same as above)

4-120

Find present worth for 10 year sequence.			
20000.00	First year amount		
7%	Increase per year		
9%	Interest rate		
Year	Increase 7%	Cash Flow	Present Worth
0			
1	0.00	20,000.00	18,348.62
2	1,400.00	21,400.00	18,011.95
3	1,498.00	22,898.00	17,681.46
4	1,602.86	24,500.86	17,357.03
5	1,715.06	26,215.92	17,038.55
6	1,835.11	28,051.03	16,725.92
7	1,963.57	30,014.61	16,419.02
8	2,101.02	32,115.63	16,117.75
9	2,248.09	34,363.72	15,822.01
10	2,405.46	36,769.18	15,531.70
		Total =	169,054.01

Increase = 0.07 * (Previous year's Cash Flow)
Cash Flow = (Previous year's Cash Flow) + (Current year's Increase)
Present Worth = Cash Flow * (1 + 0.09) ^ (–1 * Year)
Total = Sum of all Present Worth

4-121

Find present worth for 10-year sequence.			
50,000.00	First year amount		
12%	Decrease per year		
8%	Interest rate		
Year	Decrease 12%	Cash Flow	Present Worth
0			
1	0.00	50,000.00	46,296.30
2	6,000.00	44,000.00	37,722.91
3	5,280.00	38,720.00	30,737.18
4	4,646.40	34,073.60	25,045.11
5	4,088.83	29,984.77	20,407.13
6	3,598.17	26,386.60	16,628.03
7	3,166.39	23,220.20	13,548.77
8	2,786.42	20,433.78	11,039.74
9	2,452.05	17,981.73	8,995.34
10	2,157.81	15,823.92	7,329.54
		Total =	217,750.04

Decrease = 0.12 * (Previous year's Cash Flow)
Cash Flow = (Previous year's Cash Flow) – (Current year's Decrease)
Present Worth = Cash Flow * (1 + 0.08) ^ (–1 * Year)
Total = Sum of all Present Worth

4-122

<u>PW = $6.297m</u>

Year	Cash Flows ($K) − 15%	PW Factor 10%	PW ($K)
1	$2,000	0.9091	$1,818
2	$1,700	0.9264	$1,405
3	$1,445	0.7513	$1,086
4	$1,228	0.6830	$839
5	$1,044	0.6209	$648
6	$887	0.5645	$501
		Total PW	= $6,297

4-123

Year	Cash Flows ($K) − 8%	PW Factor 6%	PW ($K)
1	$10,000	0.9434	$9,434
2	$10,800	0.8900	$9,612
3	$11,664	0.8396	$9,793
4	$12,597	0.7921	$9,978
		Total PW	= $38,817

4-124

Year	Cash Flows ($K) − 15%	PW Factor 10%	PW ($K)
1	$30,000	0.9091	$27,273
2	$25,500	0.9264	$21,074
3	$21,675	0.7513	$16,285
4	$18,424	0.6830	$12,584
5	$15,660	0.6209	$9,724
6	$13,311	0.5645	$7,514
		Total PW	= $94,453

4-125

1542.5547	Yearly Payment		
9.00%	Interest Rate		
$6,000.00	Amount of Loan		
Year	9% Interest	Principal	Balance Due
0			$6,000.00
1	$540.00	$1,002.55	4,997.45
2	449.77	1,092.78	3,904.66
3	351.42	1,191.14	2,713.53
4	244.22	1,298.34	1,415.19
5	127.37	1,415.19	0.00
Note: Yearly Payment = $6000 x (A/P,9%,5)			

Interest = 0.09*(Previous year's Balance Due)
Principal = (Yearly Payment) – (Current year's Interest)
Balance Due = (Previous year's Balance Due) – (Current year's Principal)

4-126

$77.46	Monthly Payment		
0.50%	Monthly Interest Rate (6%/12)		
$900.00	Amount of Loan		
Year	0.5% Interest	Principal	Balance Due
0			$900.00
1	$4.50	$72.96	827.04
2	4.14	73.32	753.72
3	3.77	73.69	680.02
4	3.40	74.06	605.96
5	3.03	74.43	531.53
6	2.66	74.80	456.73
7	2.28	75.18	381.56
8	1.91	75.55	306.00
9	1.53	75.93	230.07
10	1.15	76.31	153.77
11	0.77	76.69	77.07
12	0.39	77.07	0.00
Note: Monthly Payment = $900 x (A/P,0.5%,12)			

Interest = 0.005*(Previous month's Balance Due)
Principal = (Monthly Payment) – (Current month's Interest)
Balance Due = (Previous month's Balance Due) – (Current month's Principal)

4-127

Payment = 11K (A/P, 1%, 36) = 11K (0.0332) = $365.2
($365.357 for exact calculations)

Month	1% Interest	$365.36 Principal	Balance Due
0			$11,000.00
1	$110.00	$255.36	10,744.64
2	107.45	257.91	10,486.73
3	104.87	260.49	10,226.24
4	102.26	263.09	9,963.15
5	99.63	265.73	9697.41
6	96.97	268.38	9429.04
7	64.29	271.07	9157.97
8	91.58	273.78	8884.19
9	88.84	276.52	8607.68
10	86.08	279.28	8328.40
11	83.28	282.07	8046.32
12	80.46	284.89	7761.43
13	77.61	287.74	7473.69
14	74.74	290.62	7183.07
15	71.83	293.53	6889.54
16	68.90	296.46	6593.08
17	65.93	299.43	6293.65
18	62.94	302.42	5991.23
19	59.91	305.45	5685.74
20	56.86	308.50	5377.28
21	53.77	311.58	5065.70
22	50.66	314.70	4751.00
23	47.51	317.85	4433.15
24	44.33	321.03	4113.13
25	41.12	324.24	3787.89
26	37.88	327.48	3460.41
27	34.60	330.75	3129.66
28	31.30	334.06	3795.60
29	27.96	337.40	3458.20
30	24.58	340.78	2117.42
31	21.17	344.18	1773.24
32	17.73	347.63	1425.61
33	14.26	351.10	1074.51
34	10.75	354.61	719.90
35	7.20	358.16	361.74
36	3.62	361.74	0.00

4-128

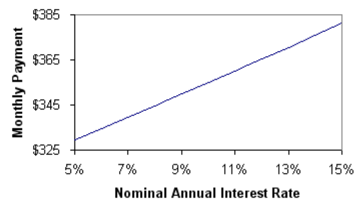
Payment = 17K (A/P, 0.75%, 60) = 17K (0.0208) = $353.60
($352.892 for exact calculations)

Month	0.75% Interest	$352.89 Principal	Balance Due	Month	0.75% Interest	$358.89 Principal	Balance Due
0			$17,000.00	30			$9,448.71
1	$127.50	$225.39	$16,774.61	31	$70.87	$282.03	9,166.68
2	125.81	227.08	16,547.53	32	68.75	284.14	8,882.54
3	124.11	228.79	16,318.74	33	66.62	286.27	8,596.27
4	122.39	230.50	16,088.24	34	64.47	288.42	8,307.85
5	120.66	232.23	15,856.01	35	62.31	290.58	8,017.27
6	118.92	233.97	15,622.04	36	60.13	292.76	7,724.51
7	117.17	235.73	15,386.31	37	57.93	294.96	7,429.55
8	115.40	237.49	15,148.81	38	55.72	297.17	7,132.38
6	113.62	239.28	14,909.54	39	53.49	299.40	6,832.98
10	111.82	241.07	14,668.48	40	51.25	301.64	6,531.33
11	110.01	242.88	14,425.59	41	48.98	303.91	6,227.43
12	108.19	244.70	14,180.89	42	46.71	306.19	5,921.24
13	106.36	246.54	13,934.35	43	44.41	308.48	5,612.76
14	104.51	278.38	13,685.97	44	42.10	310.80	5,301.96
15	102.64	250.25	13,435.72	45	39.76	313.13	4,988.83
16	100.77	252.12	13,183.60	46	37.42	315.48	4,673.36
17	98.88	254.02	12,929.58	47	35.05	317.84	4,355.52
18	96.97	255.92	12,673.66	48	32.67	320.23	4,035.29
19	95.05	257.84	12,415.82	49	30.26	322.63	3,712.66
20	93.12	259.77	12,156.05	50	27.84	325.05	3,387.62
21	91.17	261.72	11,894.33	51	25.41	327.48	6,030.13
22	89.21	263.68	11,630.64	52	22.95	329.94	2,730.19
23	87.23	265.66	11,364.98	53	20.48	332.45	2,397.77
24	85.24	237.65	11,097.33	54	17.98	334.91	2,062.86
25	83.23	269.66	10,827.67	55	15.47	337.42	1,725.44
26	81.21	271.68	10,555.98	56	12.94	339.95	1,385.49
27	79.17	273.72	10,282.26	57	10.39	342.50	1,042.99
28	77.12	275.78	10,006.48	58	7.82	345.07	697.92
29	75.05	277.84	9,728.64	59	5.23	347.66	350.27
30	72.96	279.93	9,448.71	60	2.63	350.27	0.00

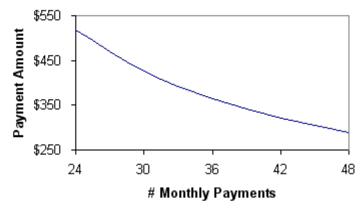
4-129

See Excel output below:

	A	B	C
1	11000	loan amount	
2	12	# payments/year	
3	3	# years of payments	
4	12.00%	nominal annual interest rate	
5	Calculated data		
6	1.00%	interest rate per period	
7	36	# payments	
8	$365.36	payment	=PMT(A6,A7,-A1)
9			
10	interest	payment	
11	5%	$329.68	=PMT(A11/12,$A$7,-$A$1)
12	6%	$334.64	
13	7%	$339.65	
14	8%	$344.70	
15	9%	$349.80	
16	10%	$354.94	
17	11%	$360.13	
18	12%	$365.36	
19	13%	$370.63	
20	14%	$375.95	
21	15%	$381.32	



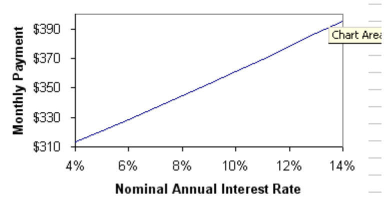
	A	B	C
26	# payment	payment	
27	24	$517.81	=PMT($A$6,A27,-$A$1)
28	30	$426.23	
29	36	$365.36	
30	42	$322.03	
31	48	$289.67	



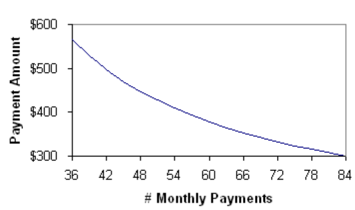
4-130

See Excel output below:

	A	B	C
1	17000	loan amount	
2	12	# payments/year	
3	5	# years of payments	
4	12.00%	nominal annual interest rate	
5	Calculated data		
6	1.00%	interest rate per period	
7	60	# payments	
8	$378.16	payment	=PMT(A6,A7,-A1)
9			
10	interest	payment	
11	4%	$313.08	=PMT(A11/12,$A$7,-$A$1)
12	5%	$320.81	
13	6%	$328.66	
14	7%	$336.62	
15	8%	$344.70	
16	9%	$352.89	
17	10%	$361.20	
18	11%	$369.62	
19	12%	$378.16	
20	13%	$386.80	
21	14%	$395.56	



	A	B	C
26	# payment	payment	
27	36	$564.64	=PMT($A$6,A27,-$A$1)
28	42	$497.69	
29	48	$447.68	
30	54	$408.96	
31	60	$378.16	
32	66	$353.10	
33	72	$332.35	
34	78	$314.92	
35	84	$300.10	



4-131

	5%	6%	10%	
Year	Salary	Interest	Deposit	Total
1	$50,000.00		$5,000.00	$5,000.00
2	52,500.00	$300.00	5,250.00	10,550.00
3	55,125.00	633.00	5,512.50	16,695.50
4	57,881.25	1,001.73	5,788.13	23,485.36
5	60,775.31	1,409.12	6,077.53	30,972.01
6	63,814.08	1,858.32	6,381.41	39,211.74
7	67,004.78	2,352.70	6,700.48	48,264.92
8	70,355.02	2,895.90	7,035.50	58,196.32
9	73,872.77	3,491.78	7,387.28	69,075.37
10	77,566.41	4,144.52	7,756.64	80,976.53
11	81,444.73	4,858.59	8,144.47	93,979.60
12	85,516.97	5,638.78	8,551.70	108,170.07
13	89,792.82	6,490.20	8,979.28	123,639.56
14	94,282.46	7,418.37	9,428.25	140,486.18
15	98,996.58	8,429.17	9,899.66	158,815.01
16	103,946.41	9,528.90	10,394.64	107,243.13
17	109,143.73	6,434.59	10,914.37	124,592.09
18	114,600.92	7,475.53	11,460.09	143,527.71
19	120,330.96	8,611.66	12,033.10	164,172.47
20	126,347.51	9,850.35	12,634.75	186,657.57
21	132,664.89	11,199.45	13,266.49	211,123.51
22	139,298.13	12,667.41	13,929.81	237,720.73
23	146,263.04	14,263.24	14,626.30	266,610.28
24	153,576.19	15,996.62	15,357.62	297,964.51
25	161,255.00	17,877.87	16,125.50	331,967.88
26	169,317.75	19,918.07	16,931.77	368,817.73
27	177,783.63	22,129.06	17,778.36	408,725.16
28	186,672.82	24,523.51	18,667.28	451,915.95
29	196,006.46	27,114.96	19,600.65	498,631.55
30	205,806.78	29,917.89	20,580.68	549,130.13
31	216,097.12	32,947.81	21,609.71	603,687.64
32	226,901.97	36,221.26	22,690.20	662,599.10
33	238,247.07	39,755.95	23,824.71	726,179.75
34	250,159.43	43,570.79	25,015.94	794,766.48
35	262,667.40	47,685.99	26,266.74	868,719.21
36	275,800.77	52,123.15	27,580.08	948,422.44
37	289,590.81	56,905.35	28,959.08	1,034,286.87
38	304,070.35	62,057.21	30,407.03	1,126,751.11
39	319,273.86	67,605.07	31,927.39	1,226,283.57
40	335,237.56	73,577.01	33,523.76	1,333,384.34

4-132

Year	$200,000 15%	Potential Lost Profit –3%	Incremental Cash Flow (B (1 − C)	PW (10%)
1	$200,000	1.00	$0.00	$0.00
2	230,000	0.9700	6,900.00	5,702.48
3	264,500	0.9409	15,631.95	11,744.52
4	304,175	0.9127	26,562.69	18,142.67
5	349,801	0.8853	40,124.72	24,914.29
6	402,271	0.8587	56,827.27	32,077.51
7	462,612	0.8330	77,269.18	39,651.31
8	532,004	0.8080	102,153.89	47,655.54
9	611,805,	0.7837	132,333.42	56,111.00
10	703,575	0.7602	168,695.49	65,039.42
			PW_5	= $60,503.96
			PW_{10}	= $301,038.74

4-133

Payment = 120K · (*A/P,* 10/12%, 360) = 120K · .00877572 = $1053.08

Mo.	Interest	Principal		Mo.	Interest	Principal	
0			$120,000.00	50			$116,723.88
1	$1.000.00	$53.09	119,946.91	51	$972.70	$80.39	116,643.49
2	999.56	53.53	119,893.399	52	972.03	81.06	116,562.43
3	999.11	53.97	119,839.411	53	971.35	81.73	116,480.70
4	998.66	54.42	119,784.99	54	970.67	82.41	116,398.29
5	998.21	54.88	119,730.11	55	969.99	83.10	116,315.19
6	997.75	55.33	119,674.77	56	969.29	83.79	116,231.40
7	997.29	55.80	119,618.98	57	968.60	84.49	116,146.91
8	996.82	56.26	119,562.72	58	967.89	85.20	116,061.71
9	996.36	56.73	119,505.99	59	967.18	85.91	115,975.81
10	995.88	57.20	119,448.79	60	966.47	86.62	115,889.18
11	995.41	57.68	119,391.11	61	965.74	87.34	115,801.84
12	994.93	58.16	119,332.95	62	965.02	88.07	115,713.77
13	994.44	58.64	119,274.30	63	964.28	88.80	115,624.97
14	993.95	59.13	119,215.17	64	963.54	89.54	115,535.42
15	993.46	59.63	119,155.54	65	962.80	90.29	115,445.13
16	992.96	60.12	119,095.42	66	962.04	91.04	115,354.09
17	992.46	60.62	119,034.79	67	961.28	91.80	115,262.29
18	991.96	61.13	118,973.67	68	960.52	92.57	115,169.72
19	991.45	61.64	118,912.03	69	959.75	93.34	115,076.38
20	990.93	62.15	118,849.87	70	958.97	94.12	114,982.27
21	990.42	62.67	118,787.20	71	958.19	94.90	114,887.37
22	989.89	63.19	118,724.01	72	957.39	95.69	114,791.67
23	989.37	63.72	118,660.29	73	956.60	96.49	114,695.19
24	988.84	64.25	118,596.04	74	955.79	97.29	114,597.89
25	988.30	64.79	118,531.26	75	954.98	98.10	114,499.79
26	987.76	65.33	118,465.93	76	954.16	98.92	114,400.87
27	987.22	65.87	118,400.06	77	953.34	99.75	114,301.12
28	986.67	66.42	118,333.64	78	952.51	100.58	114,200.55
29	986.11	66.97	118,266.67	79	951.67	101.41	114,099.13
30	985.56	67.53	118,199.14	80	950.83	102.26	113,996.87
31	984.99	68.09	118,131.05	81	949.97	103.11	113,893.76
33	983.85	69.23	117,993.15	83	948.25	104.84	113,684.95
34	983.28	69.81	117,923.34	84	947.37	105.71	113,579.24
35	982.69	70.39	117,852.95	85	946.49	106.59	113,472.65
36	982.11	70.98	117,781.98	86	945.61	107.48	113,365.17
37	981.52	71.57	117,710.41	87	944.71	108.38	113,256.79
38	980.92	72.17	117,638.24	88	943.81	109.28	113,147.51
39	980.32	72.77	117,565.47	89	942.90	110.19	113,037.32

40	979.71	73.37	117,492.10	90	941.98	111.11	112,926.21
41	979.10	73.99	117,418.11	91	941.05	112.03	112,814.18
42	978.48	74.60	117,343.51	92	940.12	112.97	112,701.21
43	977.86	75.22	117,268.29	93	939.18	113.91	112,587.30
44	977.24	75.85	117,192.44	94	938.23	114.86	112,472.44
45	976.60	76.48	117,115.96	95	937.27	115.82	112,356.63
46	975.97	77.12	117,038.84	96	936.31	116.78	112,239.85
47	975.32	77.76	116,961.07	97	935.33	117.75	112,122.09
48	974.68	78.41	116,882.66	98	934.35	118.74	112,003.36
49	974.02	79.06	116,803.60	99	933.36	119.72	111,883.63
50	973.36	79.72	116,723.88	100	932.36	120.72	111,762.91
100			$111,762.91	150			$104,250.62
101	$931.36	$121.73	111,641.18	151	$868.76	$184.33	104,066.29
102	930.34	122.74	111,518.44	152	867.22	185.87	103,880.42
103	929.32	123.77	111,394.68	153	865.67	187.42	103,693.01
104	928.29	124.80	111,269.88	154	864.11	188.98	103,504.03
105	927.25	125.84	111,144.04	155	862.53	190.55	103,313.48
106	926.20	126.89	111,017.16	156	860.95	192.14	103,121.34
107	925.14	127.94	110,889.21	157	859.34	193.74	102,927.60
108	924.08	129.01	110,760.20	158	857.73	195.36	102,732.24
109	923.00	130.08	110,630.12	159	856.10	196.98	102,535.26
110	921.92	131.17	110,498.95	160	854.46	198.63	102,336.63
111	920.82	132.26	110,366.69	161	852.81	200.28	102,136.35
112	919.72	133.36	110,233.33	162	851.14	201.95	101,934.40
113	918.61	134.47	110,098.85	163	849.45	203.63	101,730.77
114	917.49	135.60	109,963.26	164	847.76	205.33	101,525.44
115	916.36	136.73	109,826.53	165	846.05	207.04	101,318.40
116	915.22	137.86	109,688.67	166	844.32	208.77	101,109.63
117	914.07	139.01	109,549.65	167	842.58	210.51	100,899.13
118	912.91	140.17	109,409.48	168	840.83	212.26	100,686.87
119	911.75	141.34	109,268.14	169	839.06	214.03	100,472.84
120	910.57	142.52	109,125.62	170	837.27	215.81	100,257.03
121	909.38	143.71	108,981.92	171	835.48	217.61	100,039.42
122	908.18	144.90	108,837.01	172	833.66	219.42	99,819.99
123	906.98	146.11	108,690.90	173	831.83	221.25	99,598.74
124	905.76	147.33	108,543.58	174	829.99	223.10	99,375.64
125	904.53	148.56	108,395.02	175	828.13	224.96	99,150.69
126	903.29	149.79	108,245.23	176	826.26	226.83	98,923.86
127	902.04	151.04	108,094.18	177	824.37	228.72	98,695.14
128	900.78	152.30	107,941.88	178	822.46	230.63	98,464.51
129	899.52	153.57	107,788.31	179	820.54	232.55	98,231.96
130	898.24	154.85	107,633.46	180	818.60	234.49	97,997.48
131	896.95	156.14	107,477.32	181	816.65	236.44	97,761.04
132	895.64	157.44	107,319.88	182	814.68	238.41	97,522.62
133	894.33	158.75	107,161.13	183	812.69	240.40	97,282.23

134	893.01	160.08	107,001.05	184	810.69	242.40	97,039.83
135	891.68	161.41	106,839.64	185	808.67	244.42	96,795.41
136	890.33	162.76	106,676.88	186	806.63	246.46	96,548.95
137	888.97	164.11	106,512.77	187	804.57	248.51	96,300.44
138	887.61	165.48	106,347.29	188	802.50	250.58	96,049.85
139	886.23	166.86	106,180.43	189	800.42	252.67	95,797.18
140	884.84	168.25	106,012.18	190	798.31	254.78	95,542.41
141	883.43	169.65	105,842.53	191	796.19	256.90	95,285.51
142	882.02	171.06	105,671.47	192	794.05	259.04	95,026.47
143	880.60	172.49	105,498.98	193	791.89	261.20	94,765.27
144	879.16	173.93	105,325.05	194	789.71	263.38	94,501.90
145	877.71	175.38	105,149.67	195	787.52	265.57	94,236.33
146	876.25	176.84	104,972.84	196	785.30	267.78	93,968.54
147	874.77	178.31	104,794.52	197	783.07	270.01	93,698.53
148	873.29	179.80	104,614.72	198	780.82	272.26	93,426.26
149	871.79	181.30	104,433.43	199	778.55	274.53	93,151.73
150	870.28	182.81	104,250.62	200	776.26	276.82	92,874.91
201	$773.96	$279.13	92,595.78	251	$630.41	$422.68	75,226.21
202	771.63	281.45	92,314.32	252	626.89	426.20	74,800.01
203	769.29	283.80	92,030.52	253	623.33	429.75	74,370.26
204	766.92	286.16	91,744.36	254	619.75	433.33	73,936.92
205	764.54	288.55	91,455.81	255	616.14	436.94	73,499.98
206	762.13	290.95	91,164.86	256	612.50	440.59	73,059.39
207	759.71	293.38	90,871.48	257	608.83	444.26	72,615.14
208	757.26	295.82	90,575.65	258	605.13	447.96	72,167.18
209	754.80	298.29	90,277.37	259	601.39	451.69	71,715.48
210	752.31	300.77	89,976.59	260	597.63	455.46	71,260.03
211	749.80	303.28	89,673.31	261	593.83	459.25	70,800.77
212	747.28	305.81	89,367.50	262	590.01	463.08	70,337.70
213	744.73	308.36	89,059.14	263	586.15	466.94	69,870.76
214	742.16	310.93	88,748.22	264	582.26	470.83	69,399.93
215	739.57	313.52	88,434.70	265	578.33	474.75	68,925.17
216	736.96	316.13	88,118.57	266	574.38	478.71	68,446.46
217	734.32	318.76	87,799.81	267	570.39	482.70	67,963.77
218	731.67	321.42	87,478.39	268	566.36	486.72	67,477.04
219	728.99	324.10	87,154.29	269	562.31	490.78	66,986.27
220	726.29	326.80	86,827.49	270	558.22	494.87	66,491.40
221	723.56	329.52	86,497.96	271	554.10	498.99	65,992.41
222	720.82	332.27	86,165.69	272	549.94	503.15	65,489.26
223	718.05	335.04	85,830.65	273	545.74	507.34	64,981.92
224	715.26	337.83	85,492.82	274	541.52	511.57	64,470.35
225	712.44	340.65	85,152.18	275	537.25	515.83	63,954.52
226	709.60	343.48	84,808.69	276	532.95	520.13	63,434.38
227	706.74	346.35	84,462.35	277	528.62	524.47	62,909.92
228	703.85	349.23	84,113.11	278	524.25	528.84	62,381.08

229	700.94	352.14	83,760.97	279	519.84	533.24	61,847.84
230	698.01	355.08	83,405.89	280	515.40	537.69	61,310.15
231	695.05	358.04	83,047.86	281	510.92	542.17	60,767.98
232	692.07	361.02	82,686.84	282	506.40	546.69	60,221.30
233	689.06	364.03	82,322.81	283	501.84	551.24	59,670.06
234	686.02	367.06	81,955.74	284	497.25	555.84	59,114.22
235	682.96	370.12	81,585.62	285	492.62	560.47	58,553.75
236	679.88	373.21	81,212.42	286	487.95	565.14	57,988.61
237	676.77	376.32	80,836.10	287	483.24	569.85	57,418.77
238	673.63	379.45	80,456.65	288	478.49	574.60	56,844.17
239	670.47	382.61	80,074.04	289	473.70	579.38	56,264.79
240	667.28	385.80	79,688.23	290	468.87	584.21	55,680.57
241	664.07	389.02	79,299.22	291	464.00	589.08	55,091.49
242	660.83	392.26	78,906.96	292	459.10	593.99	54,497.50
243	657.56	395.53	78,511.43	293	454.15	598.94	53,898.56
244	654.26	398.82	78,112.61	294	449.15	603.93	53,294.63
245	650.94	402.15	77,710.46	295	444.12	608.96	52,685.67
246	647.59	405.50	77,304.96	296	439.05	614.04	52,071.63
247	644.21	408.88	76,896.08	297	433.93	619.16	51,452.47
248	640.80	412.29	76,483.80	298	428.77	624.32	50,828.16
249	637.37	415.72	76,068.08	299	423.57	629.52	50,198.64
250	633.90	419.19	75,648.89	300	418.32	634.76	49,563.88
300			$49,563.88	330			$27,851.01
301	$413.03	$640.05	48,923.82	331	$232.09	$820.99	27,030.01
302	407.70	645.39	48,278.43	332	225.25	827.84	26,202.18
303	402.32	650.77	47,627.67	333	218.35	834.73	25,367.44
304	396.90	656.19	46,971.48	334	211.40	841.69	24,525.75
305	391.43	661.66	46,309.82	335	204.38	848.70	23,677.05
306	385.92	667.17	45,642.65	336	197.31	855.78	22.821.27
307	380.36	672.73	44,969.92	337	190.18	862.91	21,958.36
308	374.75	678.34	44,291.59	338	182.99	870.10	21,088.26
309	369.10	683.99	43,607.60	339	175.74	877.35	20,210.91
310	363.40	689.69	42,917.91	340	168.42	884.66	19,326.25
311	357.65	695.44	42,222.47	341	161.05	892.03	18,434.22
312	351.85	701.23	41,521.24	342	153.62	899.47	17,534.75
313	346.01	707.08	40,814.16	343	146.12	906.96	16,627.79
314	340.12	712.97	40.101.20	344	138.56	914.52	15,713.27
315	334.18	718.91	39.382.29	345	130.94	922.14	14,791.12
316	328.19	724.90	38,657.39	346	123.26	929.83	13,861.30
317	322.14	730.94	37,926.45	347	115.51	937.58	12,923.72
319	309.91	743.17	36,446.24	349	99.82	953.27	11,025.07
320	303.72	749.37	35,696.87	350	91.88	961.21	10,063.86
321	297.47	755.61	34,941.26	351	83.87	969.22	9,094.64
322	291.18	761.91	34,179.35	352	75.79	977.30	8,117.34
323	284.83	768.26	33,411.09	353	67.64	985.44	7,131.90

324	278.43	774.66	32,636.43	354	59.43	993.65	6,138.25
325	271.97	781.12	31,855.32	355	51.15	1001.93	5,136.31
326	265.46	787.62	31,067.69	356	42.80	1010.28	4.126.03
327	258.90	794.19	30,273.50	357	34.38	1018.70	3,107.33
328	252.28	800.81	29,472.70	358	25.89	1027.19	2,080.13
329	245.61	807.48	28,665.22	359	17.33	1035.75	1,044.38
330	238.88	814.21	27,851.01	360	8.70	1044.38	0.00

4-134

There are several ways to solve this, but one of the easiest is to simply calculate the PW for years 0 to 1, 0 to 2, 0 to 3, etc. This is the cumulative PW in the last column below. Note that if the average monthly cash flow savings of $85 are used, the furnace is paid off sooner, since the savings occur throughout the year rather than at the end of the year. The period with monthly figures is 34 months rather than the 35 months indicated below.

Year	Cash Flow	PW 9%	Cumulative PW
0	−$2,500	−$2,500	−$2,500
1	$1,020.00	$935.78	−$1,564.22
2	$1,020.00	$858.51	−$705.71
3	$1,020.00	$787.63	$81.92

4-135

(a) See Excel output below:

	A	B	C	D
1	15000	loan amount		
2	12	# payments/year		
3	3	# years of payments		
4	8.90%	nominal annual interest rate		
5	Calculated data			
6	0.74%	interest rate per period		
7	36	# payments		
8	$476.30	payment		
9				
10	period	interest	principal	rem.bal.
11	0			$ 15,000.00
12	1	$ 111.25	$365.05	$14,634.95
13	2	108.54	367.76	14,267.20
14	3	105.82	370.48	13,896.71
15	4	103.07	373.23	13,523.48
16	5	100.30	376.00	13,147.48
17	6	97.51	378.79	12,768.70
18	7	94.70	381.60	12,387.10
19	8	91.87	384.43	12,002.67
20	9	89.02	387.28	11,615.39
21	10	86.15	390.15	11,225.24
22	11	83.25	393.04	10,832.20
23	12	80.34	395.96	10,436.24
24	13	77.40	398.90	10,037.34
25	14	74.44	401.85	9,635.49
26	15	71.46	404.83	9,230.65
27	16	68.46	407.84	8,822.82
28	17	65.44	410.86	8,411.95
29	18	62.39	413.91	7,998.04
30	19	59.32	416.98	7,581.06
31	20	56.23	420.07	7,160.99
32	21	53.11	423.19	6,737.80
33	22	49.97	426.33	6,311.48
34	23	46.81	429.49	5,881.99
35	24	43.62	432.67	5,449.32
36	25	40.42	435.88	5,013.43
37	26	37.18	439.12	4,574.32
38	27	33.93	442.37	4,131.95
39	28	30.65	445.65	3,686.29
40	29	27.34	448.96	3,237.34
41	30	24.01	452.29	2,785.05
42	31	20.66	455.64	2,329.41
43	32	17.28	459.02	1,870.38
44	33	13.87	462.43	1,407.96
45	34	10.44	465.86	942.10
46	35	6.99	469.31	472.79
47	36	3.51	472.79	(0.00)

(b) See Excel output below:

	A	B	C	D
1	15000	loan amount		
2	12	# payments/year		
3	5	# years of payments		
4	8.90%	nominal annual interest rate		
5	Calculated data			
6	0.74%	interest rate per period		
7	60	# payments		
8	$310.65	payment		
9	*after 1st year mid-year payments are hidden*			
10	period	interest	principal	rem.bal.
11	0			$15,000.00
12	1	$ 111.25	$199.40	$14,800.60
13	2	109.77	200.88	14,599.73
14	3	108.28	202.37	14,397.36
15	4	106.78	203.87	14,193.49
16	5	105.27	205.38	13,988.11
17	6	103.75	206.90	13,781.21
18	7	102.21	208.44	13,572.77
19	8	100.66	209.98	13,362.79
20	9	99.11	211.54	13,151.25
21	10	97.54	213.11	12,938.14
22	11	95.96	214.69	12,723.45
23	12	94.37	216.28	12,507.17
35	24	74.31	236.34	9,783.19
47	36	52.40	258.25	6,806.65
59	48	28.45	282.20	3,554.12
71	60	2.29	308.36	(0.00)

4-136

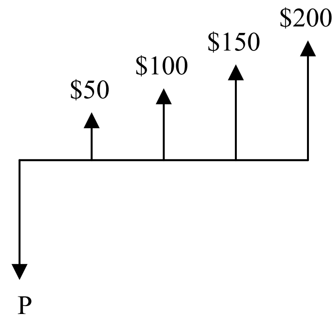
See Excel output below:

	A	B	C	D
3	15	# years of payments		
4	7.50%	nominal annual interest rate		
5	Calculated data			
6	0.63%	interest rate per period		
7	180	# payments		
8	$927.01	payment		
9	*after 1st year mid-year payments are hidden*			
10	period	interest	principal	rem.bal.
11	0			$ 100,000.00
12	1	$ 625.00	$302.01	$99,697.99
13	2	623.11	303.90	99,394.09
14	3	621.21	305.80	99,088.29
15	4	619.30	307.71	98,780.58
16	5	617.38	309.63	98,470.94
17	6	615.44	311.57	98,159.38
18	7	613.50	313.52	97,845.86
19	8	611.54	315.48	97,530.38
20	9	609.56	317.45	97,212.94
21	10	607.58	319.43	96,893.50
22	11	605.58	321.43	96,572.08
23	12	603.58	323.44	96,248.64
35	24	578.47	348.55	92,206.05
47	36	551.41	375.60	87,849.63
59	48	522.25	404.76	83,155.00
71	60	490.83	436.19	78,095.92
83	72	456.96	470.05	72,644.09
95	84	420.47	506.54	66,769.01
107	96	381.15	545.86	60,437.85
119	108	338.77	588.24	53,615.17
131	120	293.10	633.91	46,262.84
143	132	243.89	683.12	38,339.72
155	144	190.86	736.15	29,801.51
167	156	133.71	793.30	20,600.46
179	168	72.12	854.89	10,685.11
191	180	5.76	921.25	0.00

	A	B	C	D
1	100000	loan amount		
2	12	# payments/year		
3	30	# years of payments		
4	7.50%	nominal annual interest rate		
5	Calculated data			
6	0.63%	interest rate per period		
7	360	# payments		
8	$699.21	payment		
9	*after 1st year mid-year payments are hidden*			
10	period	interest	principal	rem.bal.
11	0			$ 100,000.00
12	1	$ 625.00	$74.21	$99,925.79
13	2	624.54	74.68	99,851.11
14	3	624.07	75.15	99,775.96
15	4	623.60	75.61	99,700.35
16	5	623.13	76.09	99,624.26
17	6	622.65	76.56	99,547.70
18	7	622.17	77.04	99,470.66
19	8	621.69	77.52	99,393.13
20	9	621.21	78.01	99,315.13
21	10	620.72	78.49	99,236.63
22	11	620.23	78.99	99,157.64
23	12	619.74	79.48	99,078.17
35	24	613.57	85.65	98,084.77
47	36	606.92	92.30	97,014.25
59	48	599.75	99.46	95,860.62
71	60	592.03	107.19	94,617.44
83	72	583.71	115.51	93,277.74
95	84	574.74	124.47	91,834.04
107	96	565.08	134.14	90,278.26
119	108	554.66	144.55	88,601.70
131	120	543.44	155.77	86,794.99
143	132	531.35	167.87	84,848.01
155	144	518.32	180.90	82,749.89
167	156	504.27	194.94	80,488.89
179	168	489.14	210.07	78,052.35
191	180	472.83	226.38	75,426.67
203	192	455.26	243.96	72,597.14
215	204	436.32	262.90	69,547.95
227	216	415.91	283.31	66,262.04
239	228	393.91	305.30	62,721.04
251	240	370.21	329.00	58,905.15
263	252	344.67	354.54	54,793.01
275	264	317.15	382.07	50,361.64
287	276	287.49	411.73	45,586.25
299	288	255.52	443.69	40,440.14
311	300	221.08	478.14	34,894.52
323	312	183.96	515.25	28,918.37
335	324	143.96	555.25	22,478.29
347	336	100.85	598.36	15,538.24
359	348	54.40	644.81	8,059.42
371	360	4.34	694.87	0.00

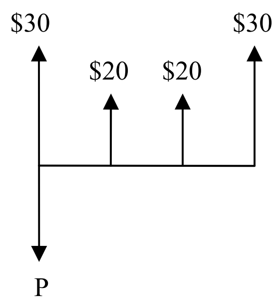
Chapter 5: Present Worth Analysis

5-1



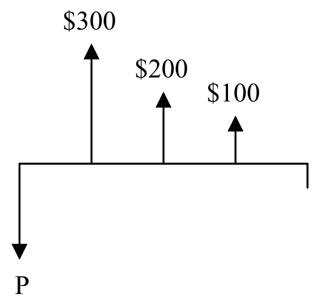
P= \$50 (P/A, 10%, 4) + \$50 (P/G, 10%, 4)
= \$50 (3.170) + \$50 (4.378)
= <u>\$377.40</u>

5-2



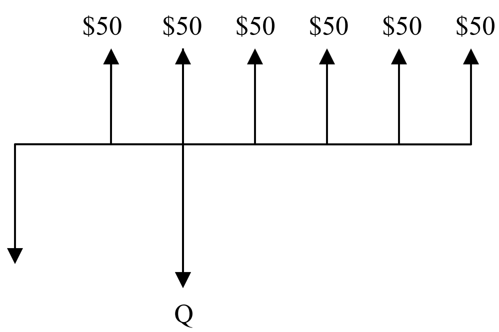
P= \$30 + \$20 (P/A, 15%, 2) + \$30 (P/F, 15%, 3)
= \$30 + \$20 (1.626) + \$30 (0.6575)
= <u>\$82.25</u>

5-3



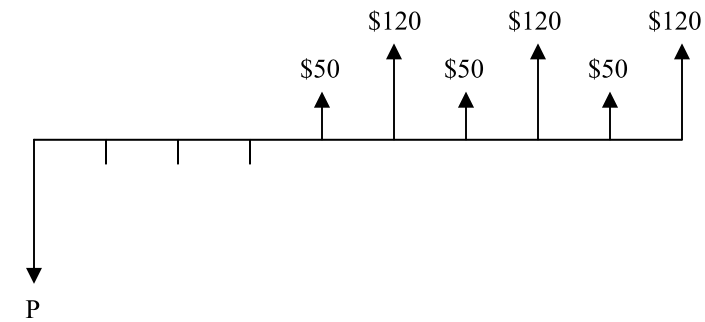
P = \$300 (P/A, 12%, 3) − \$100 (P/G, 12%, 3)
= \$300 (2.402) − \$100 (2.221)
= <u>\$498.50</u>

5-4



Q = \$50 (P/A, 12%, 6) (F/P, 12%, 2)
= \$50 (4.111) (1.254)
= <u>\$257.76</u>

5-5

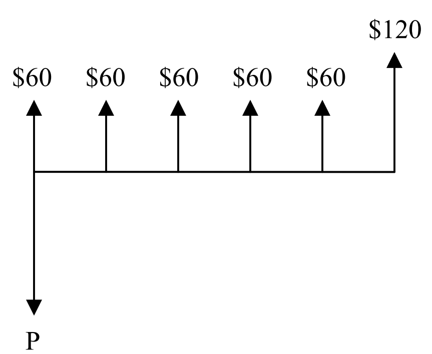


P = \$50 (P/A, 10%, 6) (P/F, 10%, 3) + \$70 (P/F, 10%, 5) + \$70 (P/F, 10%, 7) + \$70 (P/F, 10%, 9)
= \$50 (4.355) (0.7513) + \$70 (0.6209 + 0.5132 + 0.4241)
= \$272.67

Alternative Solution

P = [\$50 (P/A, 10%, 6) + \$70(P/F, 10%, 2) + \$70 (P/F, 10%, 4) + \$70 (P/F, 10%, 6)](P/F, 10%, 3)
= [\$50 (4.355) + \$70 (0.8264 + 0.6830 + 0.5645)] (0.7513)
= \$272.66

5-6

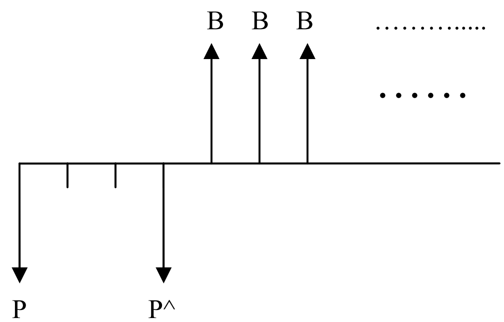


P = \$60 + \$60 (P/A, 10%, 4) + \$120 (P/F, 10%, 5)
= \$60 + \$60 (3.170) + \$120 (0.6209)
= \$324.71

5-7

$P = A_1$ (P/A, q, i, n)
$= A_1 [(1 - (1.10)^4 (1.15)^{-4})/(0.15 - 0.10)]$
= \$200 (3.258)
= <u>\$651.60</u>

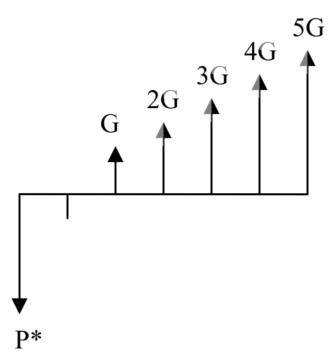
5-8



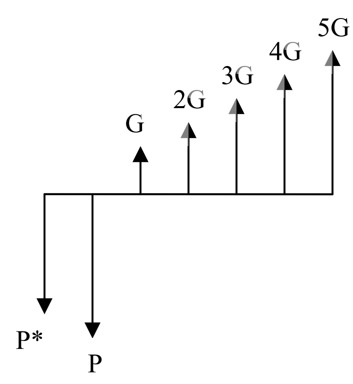
P^ = B/0.10 = 10 B
P = P^ (P/F, 10%, 3) = 10 B (0.7513) = <u>7.51 B</u>

5-9

Carved Equation



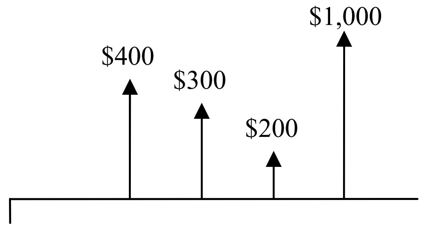
Carved Diagram



P^* = G (P/G, *i*%, 6)
The original equation by stonecutter place the present value P in year t= –1. So we need to move it forward one year using the F/P factor:
P = P^* (F/P, *i*%, 1)
Thus:
P = G (P/G, *i*%, 6) (F/P, *i*%, 1)

5-10

Assuming that the cycle repeats with a cash flow as below:

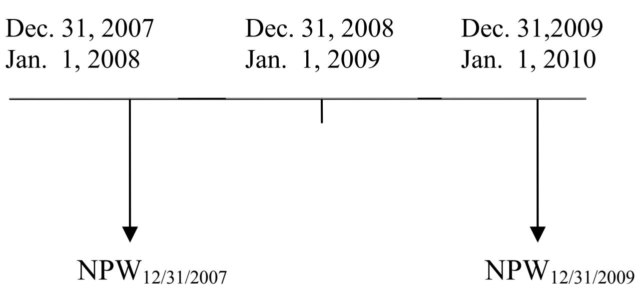


EUAC for repeating cash flow = \$400 − \$100 (A/G, 8%, 4) + \$900 (A/F, 8%, 4)
EUAC = \$400 – \$100 (1.404) + \$900 (0.2219) = \$459.31
P (year 5) = \$1,000 + EUAC/0.08 = \$1,000 + \$459.31/0.08 = \$6,741.38
P (year 0) = \$6,741.38 (P/F, 8%, 5) = \$6,741.38 (0.6806) = \$4,588.18

Alternative Solution: An alternate solution may be appropriate if one assumes that the \$1,000 cash flow is a repeating annuity from time 13 to infinity (rather than indicating the repeating decreasing gradient series cycles).

In this case P is calculated as
P= [\$500 − \$100 (A/G, 8%, 4)](P/A, 8%, 8)(P/F, 8%, 4) + \$500 (P/F, 8%, 5) +
\$500 (P/F, 8%, 9) + \$1,000 (P/A, 8%, ∞) (P/F, 8%, 12)
= <u>\$7,073</u>

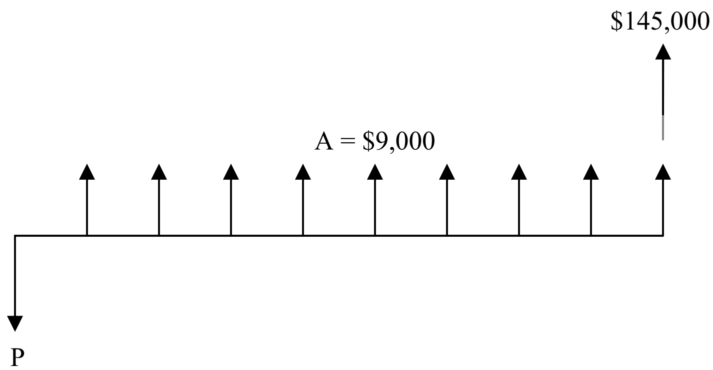
5-11



$NPW_{12/31/2009}$ = –\$140

$NPW_{12/31/2007}$ = –\$140 (P/F, 10%, 2) = –\$140 (0.8264) = <u>–\$115.70</u>

5-12

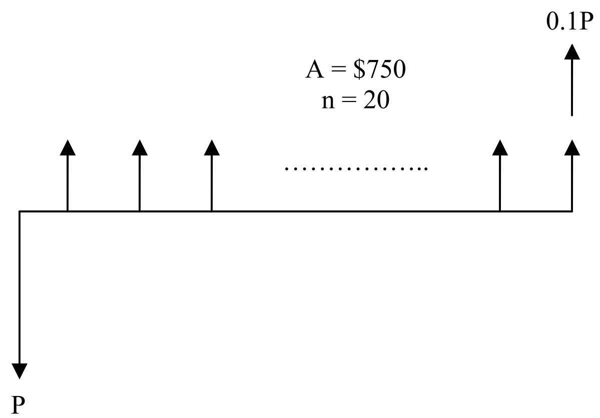


P = \$9,000 (P/A, 18%, 10) + \$145,000 (P/F, 18%, 10)
= \$9,000 (4.494) + \$145,000 (0.1911)
= <u>\$68,155.50</u>

5-13

P = \$100 (P/A, 6%, 6) + \$100 (P/G, 6%, 6)
= \$100 (4.917) + \$100 (11.459)
= <u>\$1,637.60</u>

5-14



PW of Cost = PW of Benefits
P = $750 (P/A, 7%, 20) + 0.1P (P/F, 7%, 20)
= $750 (10.594) + 0.1P (0.2584)
= $7945 + 0.02584P

P = $7945/(1− 0.02584)
= $7945/0.97416
= <u>$8156</u>

5-15

Determine the cash flow:

Year	Cash Flow
0	−$4,400
1	$220
2	$1,320
3	$1,980
4	$1,540

NPW = PW of Benefits – PW of Cost
= $220 (P/F, 6%, 1) + $1,320 (P/F, 6%, 2) + $1,980 (P/F, 6%, 3)
+ $1,540 (P/F, 6%, 4) − $4,400
= $220 (0.9434) + $1,320 (0.8900) + $1,980 (0.8396)
+ $1,540 (0.7921) − $4,400
= −$135.41

<u>NPW is negative. Do not purchase equipment.</u>

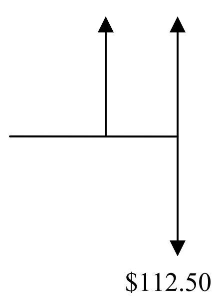
5-16

The market value of the bond is the present worth of the future interest payments and the face value on the current 6% yield on bonds.

A= \$1,000 (0.08%)/(2 payments/year) = \$40
P= \$40 (P/A, 3%, 40) + \$1,000 (P/F, 3%, 40)
= \$924.60 + \$306.60
= <u>\$1,231.20</u>

5-17

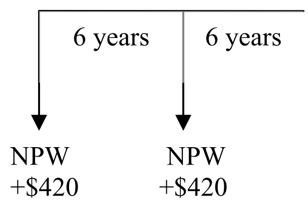
The interest the investor would receive is
i = \$5,000 (0.045/2) = \$112.50 per 6 months
Probably the simplest approach is to resolve the \$112.50 payments every 6 months into equivalent payments every 3 months:



A = \$112.50 (A/F, 2%, 2) = \$112.50 (0.4951) = \$55.70

PW of Bond = \$55.70 (P/A, 2%, 40) + \$5,000 (P/F, 2%, 40)
= \$55.70 (27.355) + \$5,000 (0.4529) = <u>\$3,788</u>

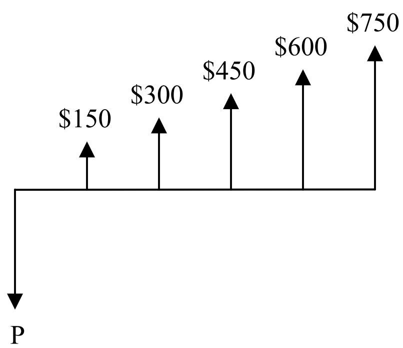
5-18



The replacement equipment will have to the same NPW = +$420 as the original equipment.

$NPW_{12\ years}$ = \$420 + \$420 (P/F, 10%, 6)
= +\$657.09

5-19



P = \$150 (P/A, 3%, 5) + \$150 (P/G, 3%, 5)
= \$150 (4.580) + \$150 (8.889)
= \$2,020.35

5-20

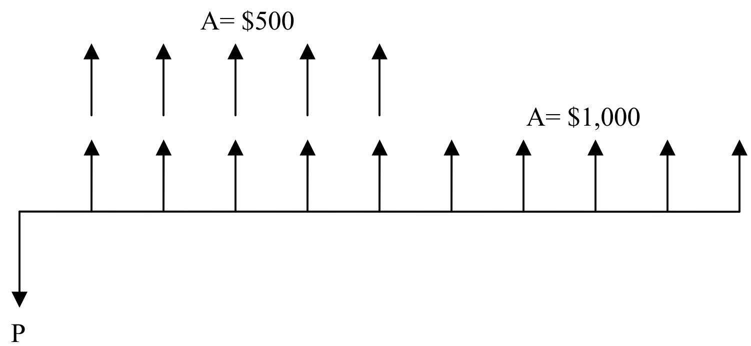
P = the first cost = \$980,000
F = the salvage value = \$20,000
AB = the annual benefit = \$200,000

Remember our convention of the costs being negative and the benefits being positive. Also, remember the *P* occurs at time = 0.

NPW = –P + AB (P/A, 12%, 13) + F (P/F, 12%, 13)
= –\$980,000 + \$200,000 (6.424) + \$20,000 (0.2292)
= \$309,384

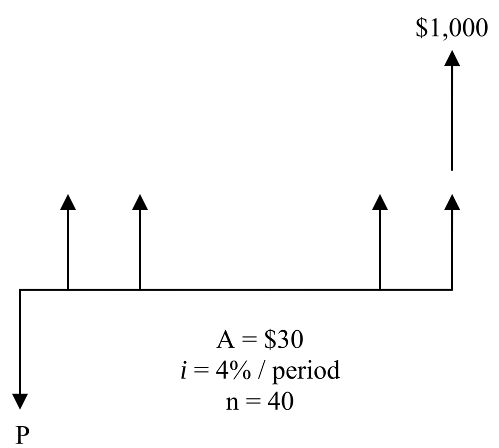
Therefore, purchase the machine, as NPW is positive.

5-21



Maximum investment = Present Worth of Benefits
= $1,000 (P/A, 4%, 10) + $500 (P/A, 4%, 5)
= $1,000 (8.111) + $500 (4.452)
= <u>$10,337</u>

5-22



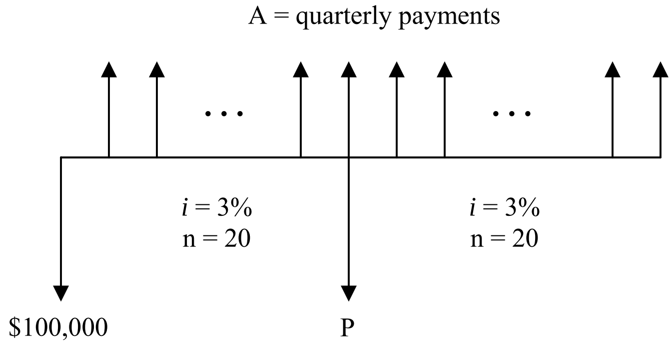
P = $30 (P/A, 4%, 40) + $1,000 (P/F, 4%, 40)
= $30 (19.793) + $1,000 (0.2083)
= $802

5-23

Maximum the contractor would pay equals the PW of Benefits:
= ($5.80 – $4.30) ($50,000) (P/A, 10%, 5) + $40,000(P/F, 10%, 5)
= ($1.50) ($50,000) (3.791) + $40,000 (0.6209)
= <u>$309,200</u>

5-24

(a)



A = $100,000 (A/P, 3%, 40) = $100,000 (0.0433) = <u>$4,330</u>

P = $4,330 (P/A, 3%, 20) = $4,330 (14.877) = $64,417

(b) Service Charge = 0.05 P
Amount of new loan = 1.05 ($64,417) = $67,638

Quarterly Payment on new loan = $67,638 (A/P, 2%, 80)
= $67,638 (0.0252)
= <u>$1,704</u>

Difference in quarterly payments = $4,330 – $1,704 = <u>$2,626</u>

5-25

The objective is to determine if the Net Present Worth is non-negative.

NPW of Benefits = $50,000 (P/A, 10%, 10) + $10,000 (P/F, 10%, 10)
= $50,000 (6.145) + $10,000 (0.3855)
= $311,105

PW of Costs = $200,000 + $9,000 (P/A, 10%, 10)
= $200,000 + $9,000 (6.145)
= $255,305

NPW = $311,105 – $255,305 = <u>$55,800</u>
<u>Since NPW is positive, the process should be automated.</u>

5-26

(a) PW Costs = $700,000,000 + $10,000,000 (P/A, 9%, 80)
= $811,000,000

PW Receipts = ($550,000) (90) (P/A, 9%, 10) + ($50,000) (90) (P/G, 9%, 10) + ($1,000,000) (90) (P/A, 9%, 70) (P/F, 9%, 10)
= $849,000,000

NPW = $849,000,000 – $811,000,000 = <u>$38,000,000</u>
This project meets the 9% minimum rate of return as NPW is positive.

(b) Other considerations:
Engineering feasibility
Ability to finance the project
Effect on trade with Brazil
Military/national security considerations

5-27

P = ?, *n* = 36 months, *i* = 1.50% /month, *A* = $250
P = $250 (P/A, 1.5%, 36) = $250 (27.661) = <u>$6,915</u>

5-28

P = $12,000, *n* = 60 months, *i* = 1.0% /month, *A* = ?
A = $12,000 (A/P, 1%, 60) = $12,000 (0.0222) = $266
<u>$266 > $250 and therefore she cannot afford the new car.</u>

5-29

Find *i*:
(A/P, *i,* 60) = A/P = $250/$12,000 = 0.0208
From tables, *i* = ¾% per month = <u>9% per year</u>

5-30

$i_{month} = (1 + (0.045/365))^{30} - 1 = 0.003705$
$P = A[((1 + i)^n - 1)/(i(1 + i)^n)]$
$= \$199\ [((1.003705)^{60} - 1)/(0.003705\ (1.003705)^{60})]$
= <u>$10,688</u>

5-31

(a) PW of Cost = (\$26,000 + \$7,500) (P/A, 18%, 6)
= \$117,183

(b) PW of Cost = [(\$26,000 + \$7,500)/12] (P/A, 1.5%, 72)
= \$122,400

(c) Part (a) assumes end-of-year payments. Part (b) assumes earlier payments, hence its PW of Cost is greater (effects of compounding comes into play).

5-32

For end-of-year disbursements,
PW of wage increases = (\$0.40 × 8 hr × 250 days) (P/A, 8%, 10) +
(\$0.25 × 8 hr × 250 days) (P/G, 8%, 10)
= \$800 (6.710) + \$500 (25.977)
= \$18,356

This <u>\$18,356</u> is the increased justifiable cost of the equipment.

5-33

PW of $Cost_A$ = \$1,300
PW of $Cost_B$ = \$100 (P/A, 6%, 5) + \$100 (P/G, 6%, 5)
= \$100 (4.212 + 7.934)
= \$1,215
<u>To minimize PW of Cost, choose B.</u>

5-34

PW of $Cost_{wheel}$ = \$50,000 – \$2,000 (P/F, 8%, 5) = \$48,640
PW of $Cost_{track}$ = \$80,000 – \$10,000 (P/F, 8%, 5) = \$73,190
<u>The wheel-mounted backhoe, with its smaller PW of Cost, is preferred.</u>

5-35

NPW_A = –$50,000 – $2,000 (P/A, 9%, 10) + $9,000 (P/A, 9%, 10) + $10,000 (P/F, 9%, 10)
= –$50,000 – $2,000 (6.418) + $9,000 (6.418) + $10,000 (0.4224)
= –$850

NPW_B = –$80,000 – $1,000 (P/A, 9%, 10) + $12,000 (P/A, 9%, 10) + $30,000 (P/F, 9%, 10)
= –$80,000 – $1,000 (6.418) + $12,000 (6.418) + $30,000 (0.4224)
= +$3,270

(a) Buy Model B because it has a positive NPW.

(b) Select null option. The NPW of Model A is negative therefore it is better to do nothing or look for more alternatives.

5-36

Machine A
NPW = −First Cost + Annual Benefit (P/A, 12%, 5) − Maintenance & Operating Costs (P/A, 12%, 5) + Salvage Value (P/F, 12%, 5)
= −$250,000 + $89,000 (3.605) – $4,000 (3.605) + $15,000 (0.5674)
= $64,936

Machine B
NPW = −First Cost + Annual Benefit (P/A, 12%, 5) − Maintenance & Operating Costs (P/A, 12%, 5) + Salvage Value (P/F, 12%, 5)
= −$205,000 + $86,000 (3.605) − $4,300 (3.605) + $15,000 (0.5674)
= $98,040
Choose Machine B because it has a greater NPW.

5-37

Since the necessary waste treatment and mercury recovery is classed as "Fixed Output," choose the alternative with the least Present Worth of Cost.

Foxhill
PW of Cost = \$35,000 + (\$8,000 − \$2,000) (P/A, 7%, 20) − \$20,000 (P/F, 7%, 20)
= \$35,000 + \$6,000 (10.594) − \$20,000 (0.2584)
= \$93,396

Quicksilver
PW of Cost = \$40,000 + (\$7,000 − \$2,200) (P/A, 7%, 20)
= \$40,000 + \$4,800 (10.594)
= \$90,851

Almeden
PW of Cost = \$100,000 + (\$2,000 − \$3,500) (P/A, 7%, 20)
= \$100,000 − \$1,500 (10.594)
= \$84,109

<u>Select the Almaden bid.</u>

5-38

Here minimize cost so choose the alternative having the least cost. To write as a single equation subtract the two individual equations and call it ∆PWC. Then if ∆PWC > 0 choose the second and if ∆PWC < 0 choose the first.

PWCA = 500,000 + 25,000 (P/A, 7%, 18) = \$751,475
PWCB = 640,000 + 10,000 (P/A, 7%, 18) = \$740,590

∆PWC = PWCA – PWCB = –140,000 + 15,000 (P/A, 7%, 18) = \$10,885 > 0 so choose option B.

5-39

Revenues are common; the objective is to minimize cost.

(a) Present Worth of Cost for Option 1:

PW of Cost = \$200,000 + \$15,000 (P/A, 10%, 30)
= \$341, 400

Present Worth of Cost for Option 2:

PW of Cost = \$150,000 + \$150,000 (P/F, 10%, 10) + \$10,000 (P/A, 10%, 30) + \$10,000 (P/A, 10%, 20) (P/F, 10%, 10)
= \$150,000 + \$150,000 (0.3855) + \$10,000 (9.427) + \$10,000 (8.514) (0.3855)
= \$334,900

Select option 2 because it has a smaller Present Worth of Cost.

(b) The cost for option 1 will not change. The cost for option 2 will now be higher.

PW of Cost = \$150,000 + \$150,000 (P/F, 10%, 5) + \$10,000 (P/A, 10%, 30) + \$10,000 (P/A, 10%, 25) (P/F, 10%, 5)
= \$394,300

Therefore, the answer will change to option 1.

5-40

Compute the PW of Cost for a 25-year analysis period.
Note that in both cases the annual maintenance is \$100,000 per year after 25 years.
Thus after 25 years all costs are identical.

Single Stage Construction

PW of Cost = \$22,400,000 + \$100,000 (P/A, 4%, 25)
= \$22,400,000 + \$100,000 (15.622)
= \$23,962,000

Two Stage Construction

PW Cost = \$14,200,000 + \$75,000 (P/A, 4%, 25) + \$12,600,000 (P/F, 4%, 25)
= \$14,200,000 + \$75,000 (15.622) + \$12,600,000 (0.3751)
= \$20,098,000

Choose two stage construction.

5-41

Full Capacity Tunnel
Capitalized Cost = \$556,000 + (\$40,000 (A/F, 7%, 10))/0.07
= \$556,000 + (\$40,000 (0.0724))/0.07
= \$597,400

First Half Capacity Tunnel
Capitalized Cost = \$402,000 + [(\$32,000 (0.0724))/0.07] + [\$2,000/0.07]
= \$463,700

Second Half-Capacity Tunnel
20 years hence the capitalized cost of the second half-capacity tunnel equals the present capitalized cost of the first half.
Capitalized Cost = \$463,700 (P/F, 7%, 20)
= \$463,700 (0.2584)
= \$119,800
Capitalized Cost for two half-capacity tunnels = \$463,700 + \$119,800
= \$583,500

<u>Build the full capacity tunnel.</u>

5-42

NPW = PW of Benefits – PW of Cost

NPW of 8 years of alternate A
= \$1,800 (P/A, 10%, 8) − \$5,300 − \$5,300 (P/F, 10%, 4)
= \$1,800 (5.335) − \$5,300 − \$5,300 (0.6830)
= \$683.10

NPW of 8 years of alternate B
= \$2,100 (P/A, 10%, 8) − \$10,700
= \$2,100 (5.335) − \$10,700
= \$503.50

<u>Select Alternate A.</u>

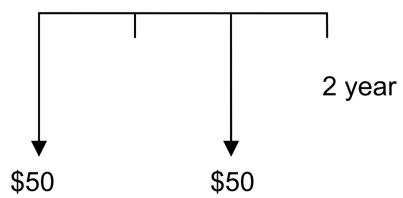
5-43

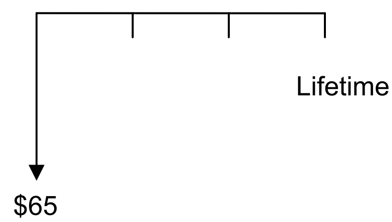
Cap. $Cost_A$ = \$500,000 + \$35,000/0.12 + [\$350,000(A/F, 12%, 10)]/0.12
= \$500,000 + \$35,000/0.12 + [\$350,000 (0.0570)]/0.12
= \$957,920

Cap. $Cost_B$ = \$700,000 + \$25,000/0.12 + [\$450,000 (A/F, 12%, 15)]/0.12
= \$700,000 + \$25,000/0.12 + [\$450,000 (0.0268)]/0.12
= \$1,008,830

<u>Type A with its smaller capitalized cost is preferred.</u>

5-44



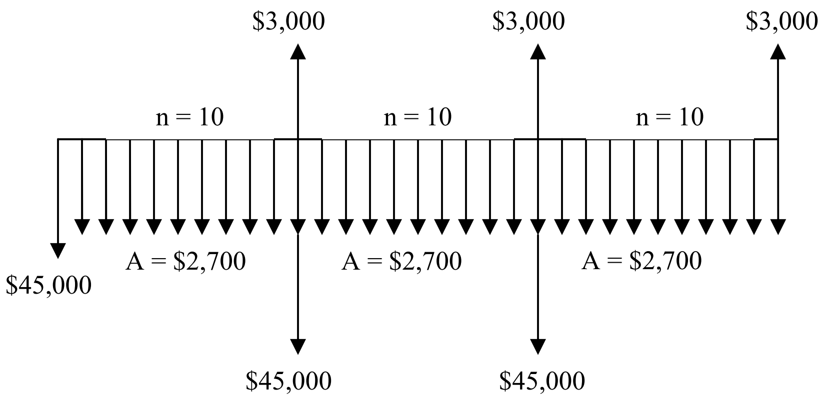


By buying the "lifetime" muffler the car owner will avoid paying \$50 two years hence. Compute how much he is willing to pay now to avoid the future \$50 disbursement.

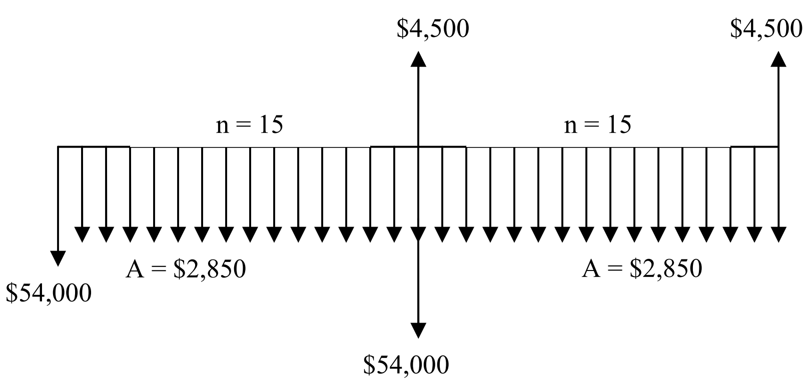
P = \$50 (P/F, 20%, 2) = \$50 (0.6944) = \$34.72

Since the lifetime muffler costs an additional \$15, it appears to be the desirable alternative.

5-45



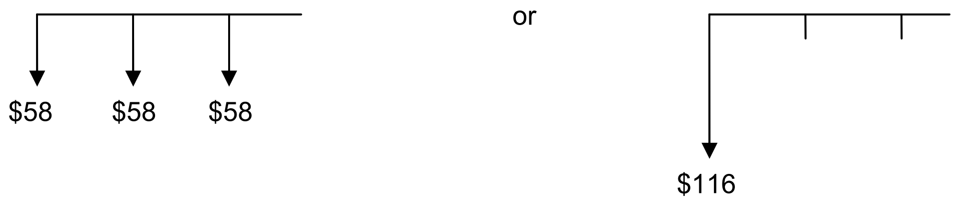
PW of Cost of 30 years of Westinghome
= $45,000 + $2,700 (P/A, 10%, 30) + $42,000 (P/F, 10%, 10) +
$42,000 (P/F, 10%, 20) − $3,000 (P/F, 10%, 30)
= $45,000 + $2,700 (9.427) + $42,000 (0.3855) + $42,000 (0.1486) −
$3,000 (0.0573)
= <u>$92,713</u>



PW of Cost of 30 years of Itis
= $54,000 + $2,850 (P/A, 10%, 30) + $49,500 (P/F, 10%, 15) −
$4,500 (P/F, 10%, 30)
= $54,000 + $2,850 (9.427) + $49,500 (0.2394) − $4,500 (0.0573)
= <u>$92,459</u>

<u>The Itis bid has a slightly lower cost.</u>

5-46



Three One-Year Subscriptions
PW of Cost = \$58 + \$58 (P/F, 20%, 1) + \$58 (P/F, 20%, ,2)
= \$58 (1 + 0.8333 + 0.6944)
= \$146.61

One Three-Year Subscription
PW of Cost = \$116
<u>Choose three-year subscription.</u>

5-47

Capitalized Cost = \$2,000,000 + \$15,000/0.05 = <u>\$2.3 million</u>

5-48

Effective annual interest rate $= (1.025)^2 - 1$
$= 0.050625$
$= 5.0625\%$

Annual Withdrawal A = P*i* = \$25,000 (0.05062) = <u>\$1,265.60</u>

5-49

The amount of money needed now to begin the perpetual payments is
P' = A/i = \$10,000/0.08 = \$125,000

The amount of money that would need to have been deposited 50 years ago at 8% interest is
P = \$125,000 (P/F, 8%, 50) = \$125,000 (0.0213) = <u>\$2,662</u>

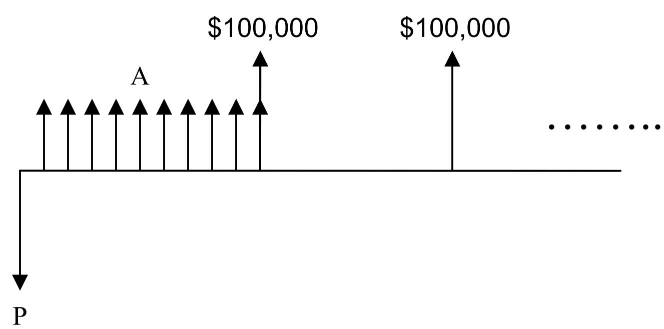
5-50

P = A/*i* = $67,000/0.08 = $837,500

5-51

P = ?, *n* = ∞, *i* = 10%, *A* = $100,000
P = A/*i* = $100,000/0.10 = <u>$1,000,000</u>

5-52



Compute an A that is equivalent to $100,000 at the end of 10 years.
A = $100,000 (A/F, 5%, 10) = $100,000 (0.0795) = $7,950

For an infinite series,
P = A/*i* = $7,950/0.05 = <u>$159,000</u>

5-53

Capitalized Cost = PW of an infinite analysis period
When n = ∞ or P = A/*i*
PW = $5,000/0.08 + $150,000 (A/P, 8%, 40)/0.08
= $62,500 + $150,000 (0.0839)/0.08
= <u>$219,800</u>

5-54

Two assumptions are needed:
(1) Value of an urn of cherry blossoms (plus the cost to have the bank administer the trust) – say \$50.00 / year
(2) A "conservative" interest rate—say 5%
P = A/*i* = \$50.00/0.05 = <u>\$1,000</u>

5-55

To provide \$1,000 a month she must deposit:
P = A/i = \$1,000/0.005 = \$200,000

5-56

The trust fund has three components:
(1) P = \$1 million
(2) For n = ∞ P= A/*i* = \$150,000/0.06 = \$2.5 million
(3) \$100,000 every 4 years: First compute equivalent A. Solving one portion of the perpetual series for A:

A= \$100,000 (A/F, 6%, 4) = \$100,000 (0.2286) = \$22,860
P= A/*i* = \$22,860/0.06 = \$381,000
Required money in trust fund
= \$1 million + \$2.5 million + \$381,000 = <u>\$3,881 million</u>

5-57

Fred is not responsible for the initial \$500 cost in year zero. That cost was paid by the Audubon Society.
i = 5%
P= \$50/0.05 + [\$500 (A/F, 5%, 5)]/0.05
= \$50/0.05 + [\$500 (0.1810)]/0.05
= \$2,810

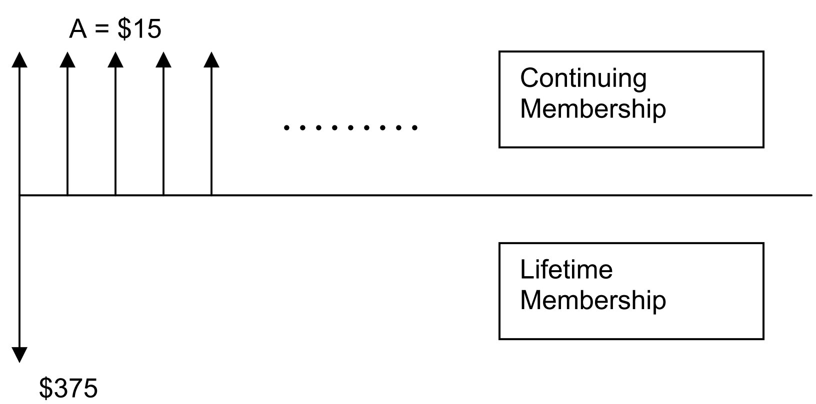
5-58

(a) P= \$5,000 + \$200/0.08 + \$300 (A/F, 8%, 4)/0.08
= \$5,000 + \$2,500 + \$300 (0.1705)/0.08
= <u>\$8,139</u>

(b) P= \$5,000 + \$200 (P/A, 8%, 75) + \$300 (A/F, 8%, 5) (P/A, 8%, 75)
= \$5,000 + \$200 (12.461) + \$300 (0.1705) (12.461)
= <u>\$8,130</u>

5-59

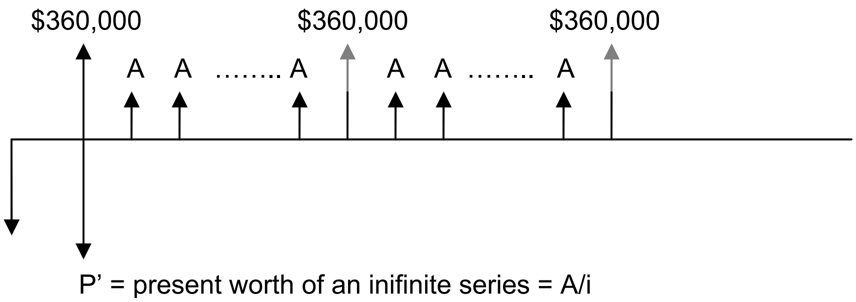
\$375 invested at 4% interest produces a perpetual annual income of \$15.
A = P*i* = \$375 (0.04) = \$15
But this is not quite the situation here.



An additional \$360 now instead of *n* annual payments of \$15 each. Compute *n.*
P= A (P/A, 4%, *n*)
\$360 = \$15 (P/A, 4%, *n*)
(P/A, 4%, *n*) = \$360/\$15
= 24
From the 4% interest table, *n* = 82.

Lifetime (patron) membership is not economically sound unless one expects to be active for 82 + 1 = <u>83 years.</u> (But that's probably not why people buy patron memberships or avoid buying them.)

5-60



A = 6 ($60,000) (A/F, 4%, 25)
= $360,000 (0.0240)
= <u>$8640</u>

P' = A/i
= $8640/0.04
= <u>$216,000</u>

P = ($216,000 + $360,000) (P/F, 4%, 10)
= $576,000 (0.6756)
= <u>$389,150</u>

5-61

Here minimize capitalized cost (CC).

Concrete: CC = $\frac{50M(A/P,5\%,70)}{0.05}+\frac{0.25M}{0.05}=51.70M+5.00M$ = $56.70M

Steel: CC = $\frac{40M(A/P,5\%,50)}{0.05}+\frac{1M}{0.05}=43.84M+20.00M$ = $63.84M

Choose concrete. It is the cheaper alternative.

5-62

Here minimize capitalized cost (CC).

Concrete: CC = $\frac{25M(A/P,6\%,80)}{0.06}+\frac{0.2M}{0.06}=25.25M+3.33M$ = \$28.58M

Steel: CC = $\frac{21M(A/P,6\%,60)}{0.06}+\frac{1M}{0.06}=21.675M+16.667M$ = \$38.33M

Choose concrete. It is the cheaper alternative.

5-63

Use a 20 year analysis period:
Alt. A NPW = \$1,625 (P/A, 6%, 20) − \$10,000 − \$10,000 (P/F, 6%, 10)
= \$1,625 (11.470) − \$10,000 − \$10,000 (0.5584)
= \$3,055

Alt. B NPW = \$1,530 (P/A, 6%, 20) − \$15,000
= \$1,530 (11.470) − \$15,000
= \$2,549

Alt. C NPW = \$1,890 (P/A, 6%, 20) − \$20,000
= \$1,890 (11.470) − \$20,000
= \$1,678

<u>Choose Alternative A.</u>

5-64

Fuel	Installed Cost	Annual Fuel Cost
Natural Gas	\$30,000	\$7,500 > Fuel Oil
Fuel Oil	\$55,000	
Coal	\$180,000	\$15,000 > Fuel Oil

For fixed output, minimize PW of Cost:

Natural Gas
PW of Cost = \$30,000 + \$7,500 (P/A, 8%, 20) + PW of Fuel Oil Cost
= \$30,000 + \$7,500 (9.818) + PW of Fuel Oil Cost
= \$103,635 + PW of Fuel Oil Cost

Fuel Oil
PW of Cost = \$55,000 + PW of Fuel Oil Cost

Coal
PW of Cost = \$180,000 − \$15,000 (P/A, 8%, 20) + PW of Fuel Oil Cost
= \$180,000 − \$15,000 (9.818) + PW of Fuel Oil Cost
= \$32,730 + PW of Fuel Oil Cost

Install coal-fired steam boiler.

5-65

Company A
NPW = −\$15,000 + (\$8,000 − \$1,600)(P/A, 15%, 4) + \$3,000 (P/F, 15%, 4)
= −\$15,000 + \$6,400 (2.855) + \$3,000 (0.5718)
= \$4,987

Company B
NPW = −\$25,000 + (\$13,000 − \$400) (P/A, 15%, 4) + \$6,000 (P/F, 15%, 4)
= −\$25,000 + \$12,600 (2.855) + \$6,000 (0.5718)
= \$14,404

Company C
NPW = −\$20,000 + (\$11,000 − \$900) (P/A, 15%, 4) + \$4,500 (P/F, 15%, 4)
= −\$20,000 + \$10,100 (2.855) + \$4,500 (0.5718)
= \$11,409

To maximize NPW select Company B's office equipment.

5-66

The least common multiple life is 12 years, so this will be used as the analysis period.

Machine A

$NPW_4 = -\$52{,}000 + (\$38{,}000 - \$15{,}000)(P/A, 12\%, 4) + \$13{,}000(P/F, 12\%, 4)$
$= -\$52{,}000 + \$69{,}851 + \$8{,}262$
$= \$26{,}113$

$NPW_{12} = NPW_4\ [1 + (P/F, 12\%, 4) + (P/F, 12\%, 8)]$
$= \$26{,}113\ [1 + (1.12)^{-4} + (1.12)^{-8}]$
$= \$53{,}255$

Machine B

$NPW_6 = -\$63{,}000 + (\$31{,}000 - \$9{,}000)(P/A, 12\%, 6) + \$19{,}000(P/F, 12\%, 6)$
$= -\$63{,}000 + \$90{,}442 + \$9{,}625$
$= \$37{,}067$

$NPW_{12} = NPW_6\ [1 + (P/F, 12\%, 6)]$
$= \$37{,}067\ [1 + (1.12)^{-6}]$
$= \$55{,}846$

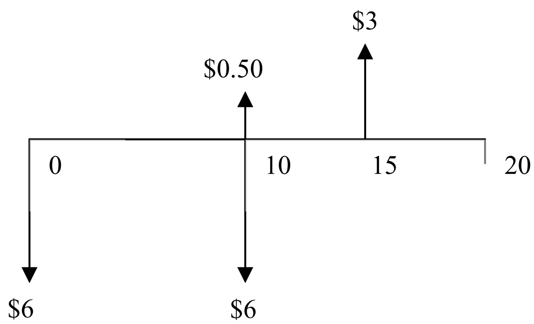
Machine C

$NPW_{12} = -\$67{,}000 + (\$37{,}000 - \$12{,}000)(P/A, 12\%, 12) + \$22{,}000(P/F, 12\%, 12)$
$= -\$67{,}000 + \$154{,}850 + \$5{,}647$
$= \$93{,}497$

<u>Machine C is the correct choice.</u>

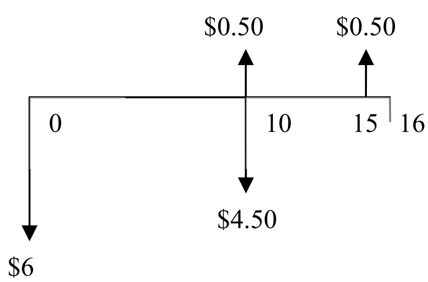
5-67

It appears that there are four alternative plans for the ties:
(1) Use treated ties initially and as the replacement



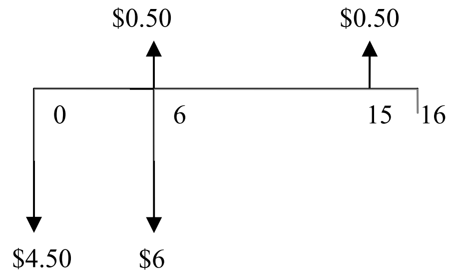
PW of Cost = \$6 + \$5.50 (P/F, 8%, 10) − \$3 (P/F, 8%, 15)
= \$6 + \$5.50 (0.4632) − \$3 (0.3152)
= \$7.60

(2) Use treated ties initially. Replace with untreated ties.



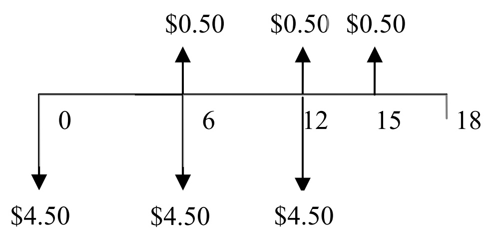
PW of Cost = \$6 + \$4 (P/F, 8%, 10) − \$0.50 (P/F, 8%, 15)
= \$6 + \$4 (0.4632) − \$0.50 (0.3152)
= \$7.70

(3) Use untreated ties initially. Replace with treated ties.



PW of Cost = \$4.50 + \$5.50 (P/F, 8%, 6) − \$0.50 (P/F, 8%, 15)
= \$4.50 + \$5.50 (0.6302) − \$0.50 (0.3152)
= \$7.81

(4) Use untreated ties initially, then two replacements with untreated ties.



PW of Cost = \$4.50 + \$4 (P/F, 8%, 6) + \$4 (P/F, 8%, 12) − \$0.50 (P/F, 8%, 15)
= \$4.50 + \$4 (0.6302) + \$4 (0.3971) − \$0.50 (0.3152)
= \$8.45

<u>Choose Alternative 1 to minimize cost.</u>

5-68

For fixed output, minimize the Present Worth of Cost.

Quick Paving

PW of Cost = \$42,500 + \$21,250 (P/F, 1%, 6) + \$21,250 (P/F, 1%, 12)
= \$42,500 + \$21,250 (0.9420) + \$21,250 (0.8874)
= \$81,375

Tartan Paving

PW of Cost = \$82,000

Faultless Paving

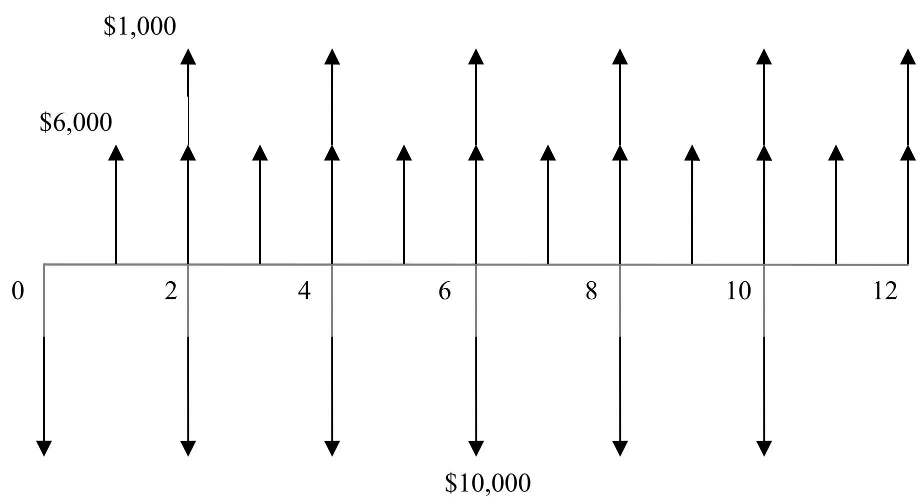
PW of Cost = \$21,000 + \$63,000 (P/F, 1%, 6)
= \$21,000 + \$63,000 (0.9420)
= \$80,346

<u>Award the job to Faultless Paving.</u>

5-69

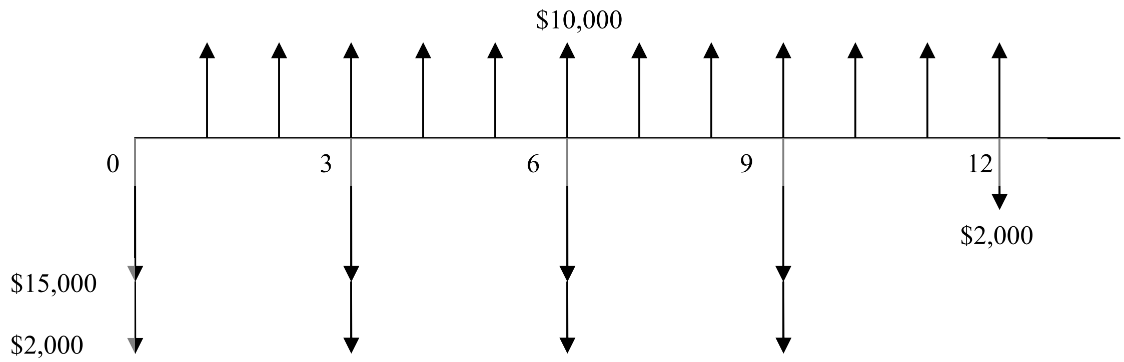
Using the PW Method the study period is a common multiple of the lives of the alternatives. Thus we use 12 years and assume repeatability of the cash flows.

Alternative A



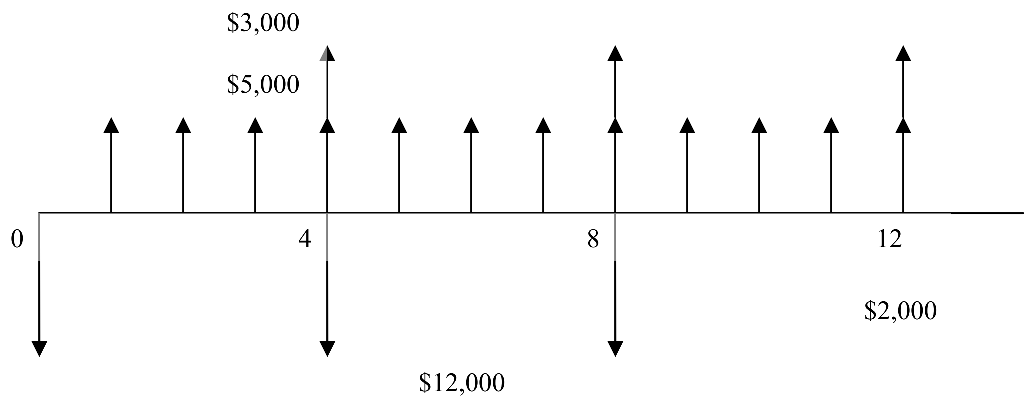
NPW = \$6,000 (P/A, 10%, 12) + \$1,000 (P/G, 10%, 12) − \$10,000 −
(\$10,000 − \$1,000) [(P/F, 10%, 2) + (P/F, 10%, 4) + (P/F, 10%, 6) +
(P/F, 10%, 8) + (P/F, 10%, 10)]
= \$40,884 + \$319 − \$10,000 − \$26,331
= <u>\$4,872</u>

Alternative B



NPW = $10,000 (P/A, 10%, 12) − $2,000 (P/F, 10%, 12) − $15,000 −
($15,000 + $2,000)[(P/F, 10%, 3) + (P/F, 10%, 6) + (P/F, 10%, 9)]
= $68,140 − $637 − $15,000 − $29,578
= <u>$22,925</u>

Alternative C



NPW = $5,000 (P/A, 10%, 12) + $3,000 (P/F, 10%, 12) − $12,000 −
($12,000 − $3,000) [(P/F, 10%, 4) + (P/F, 10%, 8)]
= $34,070 + $956 − $12,000 − $10,345
= <u>$12,681</u>

<u>Choose Alternative B.</u>

5-70

NPW = PW of Benefits – PW of Cost
NPW_A = 0
NPW_B = \$12 (P/A, 10%, 5) − \$50 = \$12 (3.791) − \$50 = −\$4.51
NPW_C = \$4.5(P/A, 10%, 10) − \$30 = \$4.5 (6.145) − \$30 = −\$2.35
NPW_D = \$6 (P/A, 10%, 10) − \$40 = \$6 (6.145) − \$40 = −\$3.13

<u>Select alternative A with NPW = 0.</u>

5-71

Choose the alternative to maximize NPW.
(a) 8% interest
NPW_1 = \$135 (P/A, 8%, 10) − \$500 − \$500 (P/F, 8%, 5)
= +\$65.55

NPW_2 = (\$100 + \$250) (P/A, 8%, 10) − \$600 − \$350 (P/F, 9%, 5)
= −\$51.41

NPW_3 = \$100 (P/A, 8%, 10) − \$700 + \$180 (P/F, 8%, 10)
= +\$54.38

NPW_4 = \$0

<u>Choose Alternative 1.</u>

(b) 12% interest
NPW_1 = \$135 (P/A, 12%, 10) − \$500 − \$500 (P/F, 12%, 5)
= −\$20.95

NPW_2 = (\$100 + \$250) (P/A, 12%, 10) − \$600 − \$350 (P/F, 12%, 5)
= −\$153.09

NPW_3 = \$100 (P/A, 12%, 10) − \$700 + \$180 (P/F, 12%, 10)
= −\$77.04

NPW_4 = \$0

<u>Choose Alternative 4.</u>

5-72

This is a situation of Fixed Input. Therefore, maximize PW of benefits. By inspection, one can see that C, with its greater benefits, is preferred over A and B. Similarly, E is preferred over D. The problem reduces to choosing between C and E.

Alternative C

PW of Benefits = \$100 (P/A, 10%, 5) + \$110 (P/A, 10%, 5) (P/F, 10%, 5)
= \$100 (3.791) + \$110 (3.791) (0.6209)
= \$638

Alternative E

PW of Benefits = \$150 (P/A, 10%, 5) + \$50 (P/A, 10%, 5) (P/F, 10%, 5)
= \$150 (3.791) + \$50 (3.791) (0.6209)
= \$686.40

<u>Choose Alternative E.</u>

5-73

Compute the Present Worth of Benefit for each stock.
From the 10% interest table: (P/A, 10%, 4)= 3.170
(P/F, 10%, 4) = 0.683

	PW of Future Price	PW of Dividends		PW of Benefit
Western House	\$32 × 0.683	+ 1.25 × 3.170	= 21.86 + 3.96	= \$25.82
Fine Foods	\$45 × 0.683	+ 4.50 × 3.170	= 30.74 + 14.26	= \$45.00
Mobile Motors	\$42 × 0.683	+ 0 × 3.170	= 28.69 + 0	= \$28.69
Spartan Products	\$20 × 0.683	+ 0 × 3.170	= 13.66 + 0	= \$13.66
U.S. Tire	\$40 × 0.683	+ 2.00 × 3.170	= 27.32 + 6.34	= \$33.66
Wine Products	\$60 × 0.683	+ 3.00 × 3.170	= 40.98 + 9.51	= \$50.49

	PW of Benefit	PW of Cost	NPW per share	NPW per \$1 invested
Western House	\$25.82	\$23.75	+2.07	+0.09
Find Foods	\$45.00	\$45.00	0	0
Mobile Motors	\$28.69	\$30.62	−1.93	−0.06
Spartan Products	\$13.66	\$12.00	+1.66	+0.14
U.S. Tire	\$33.66	\$33.37	+0.29	+0.01
Wine Products	\$50.49	\$52.50	−2.01	−0.04

In this problem, choosing to Maximize NPW per share leads to Western House. But the student should recognize that this is a faulty criterion.

An investment of some lump sum of money (like \$1,000) will purchase different numbers of shares of the various stock. It would buy 83 shares of Spartan Products, but only 42 shares of Western House. The criterion, therefore, is to Maximize NPW for the amount invested. This could be stated as Maximize NPW per \$1 invested.

<u>Buy Spartan Products.</u>

5-74

NPW_A = \$6 (P/A, 8%, 6) − \$20 = +\$7.74
NPW_B = \$9.25 (P/A, 8%, 6) − \$35 = +\$7.76
NPW_C = \$13.38 (P/A, 8%, 6) − \$55 = +\$6.86
NPW_D = \$13.78 (P/A, 8%, 6) − \$60 = +\$32.70
NPW_E = \$24.32 (P/A, 8%, 6) − \$80 = +\$32.43
NPW_F = \$24.32 (P/A, 8%, 6) − \$100 = +\$12.43

Choose E.

5-75

Eight mutually exclusive alternatives:

Plan	Initial Cost	Net Annual Benefit × (P/A, 10%, 10) 6.145	PW of Benefit	NPW = PW of Benefit minus Cost
1	\$265	\$51	\$313.40	\$48.40
2	\$220	\$39	\$239.70	\$19.70
3	\$180	\$26	\$159.80	−\$20.20
4	\$100	\$15	\$92.20	−\$7.80
5	\$305	\$57	\$350.30	\$45.30
6	\$130	\$23	\$141.30	\$11.30
7	\$245	\$47	\$288.80	\$43.80
8	\$165	\$33	\$202.80	\$37.80

To maximize NPW, choose Plan 1.

5-76

Using the Excel function = −PV (B3,B2,B1) for Present Worth, obtain:

	A	B
1	Payment	\$500
2	N	48
3	Interest rate	0.50%
4	PW	\$21,290

5-77

Using the Excel function = −PV (B3,B2,B1) for Present Worth, obtain:

	A	B
1	Payment	$6,000
2	N	4
3	Interest rate	6%
4	PW	$20,791

5-78

Using the Excel function = −PV (B3,B2,B1) for Present Worth, obtain:

	A	B
1	Payment	$6,000
2	N	4
3	Interest rate	6.168%
4	PW	$20,711

5-79

Problem 5-76 will repay the largest loan because the payments start at the end of the month, rather than waiting until the end of the year.

Problem 5-78 has the same effective interest rate as 5-76, but the rate on 5-77 is lower.

5-80

Using the Excel function = −PV (B3,B2,B1) for Present Worth, obtain:

	A	B
1	Payment	1,000
2	N	360
3	Interest rate	0.50%
4	PW	$166,792

5-81

Using the Excel function = −PV (B3,B2,B1) for Present Worth, obtain:

	A	B
1	Payment	12,000
2	N	30
3	Interest rate	6%
4	PW	$165,178

5-82

Using the Excel function = −PV (B3,B2,B1) for Present Worth, obtain:

	A	B
1	Payment	12,000
2	N	30
3	Interest rate	6.168%
4	PW	$162,251

5-83

Problem 5-80 involves monthly compounding so the effective rate is higher than problem 5-81 resulting in a greater PW value. 5-80 will repay the largest loan because the payments start at the end of the first month, rather than waiting until the end of the year. Problem 5-82 has the same effective interest rate as 5-80, but the rate on 5-81 is lower.

5-84

At a 15% rate of interest, use the excel function = PV ($A$1, A3, −1) for Present Worth.

<$$>	A	B	C	D
1	Year	Net Cash	(P/F,i,n)	PW
2	0	0	1.0000	0
3	1	−120,000	0.8696	−104,348
4	2	−60,000	0.7561	−45,369
5	3	20,000	0.6575	13,150
6	4	40,000	0.5718	22,870
7	5	80,000	0.4972	39,774
8	6	100,000	0.4323	43,233
9	7	60,000	0.3759	22,556
10		Total	−8133	

So don't do. This problem can also be solved by using NPV function:
PW = −$8,133 = NPV (A1, B4:B10) + B3
Notice that NPV function starts with year 1, and year 0 is added in separately.

5-85

Using a 10% interest rate, solve for PW using the function = PV ($A$1, A3, −1)

<$$>	A	B	C	D	E	F	G
1	Year	Annual Sales	Cost/Unit	Price/Unit	Net Revenue	(P/F,i,n)	PW
2	0	0			−42,000	1.0000	−42,000
3	1	5,000	3.50	6.00	12,500	0.9091	11,364
4	2	6,000	3.25	5.75	15,000	0.8264	12,397
5	3	9,000	3.00	5.50	22,500	0.7513	16,905
6	4	10,000	2.75	5.25	25,000	0.6830	17,075
7	5	8,000	2.50	4.50	16,000	0.6209	9,935
8	6	4,000	2.25	3.00	3,000	0.5645	1,693
9						total	27,368

So do.
Can also solve without P/F column by using NPV function:
PW = $27,368 = NPV(A1, E4:E9) + E3
Notice that NPV function starts with year 1, and year 0 is added in separately.

5-86

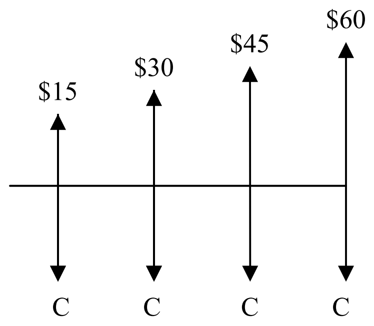
Using interest =15%, solve for PW using the function = PV ($A$1, A3,−1)

<$$>	A	B	C	D	E	F	G
1	Year	Annual Prod.	Cost/unit	Price/Unit	Net Revenue	(P/F,i,n)	PW
2	0	0			−8,000,000	1.0000	−8,000,000
3	1	70,000	25	35	700,000	0.8696	608,696
4	2	90,000	20	34	1,260,000	0.7561	952,741
5	3	120,000	22	33	1,320,000	0.6575	867,921
6	4	100,000	24	34	1,000,000	0.5718	571,753
7	5	80,000	26	35	720,000	0.4972	357,967
8	6	60,000	28	36	640,000	0.4323	276,690
9	7	40,000	30	37	420,000	0.3759	157,894
10						total	−4,206,338

So do.
Can also solve without P/F column by using NPV function:
PW = −$4,206,338 = NPV(A1, E4:E10) + E3
Notice that NPV function starts with year 1, and year 0 is added in separately.

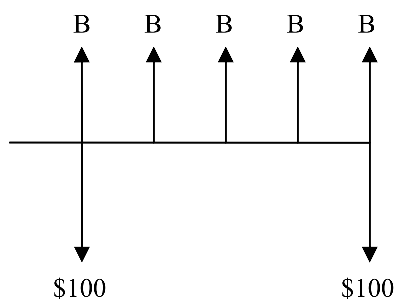
Chapter 6: Annual Cash Flow Analysis

6-1



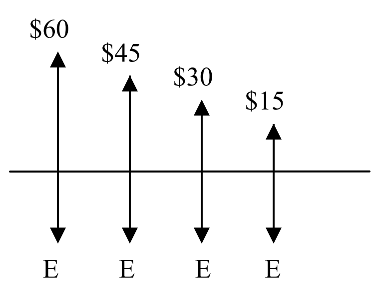
C = \$15 + \$15 (A/G, 10%, 4)
= \$15 + \$15 (1.381) = <u>\$35.72</u>

6-2



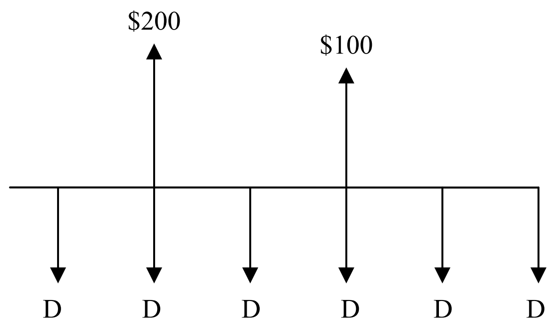
B = [\$100 + \$100 (F/P, 15%, 4)] (A/F, 15%, 5)
= [\$100 + \$100 (1.749)] (0.1483) = <u>\$40.77</u>

6-3



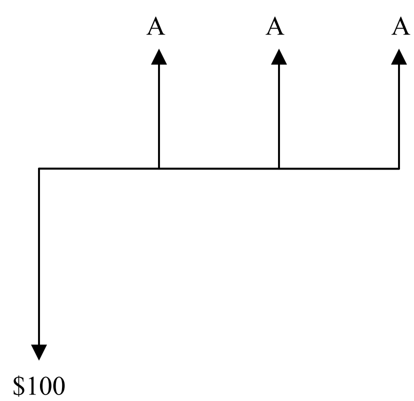
E = \$60 − \$15 (A/G, 12%, 4)
= \$60 − \$15 (1.359) = <u>\$39.62</u>

6-4



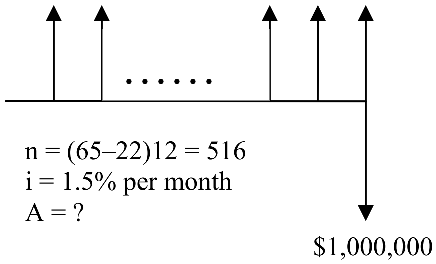
D = [$100 (F/P, 6%, 2) + $200 (F/P, 6%, 4)] (A/F, 6%, 6)
= [$100 (1.124) + $200 (1.262)] (0.1434)
= <u>$52.31</u>

6-5

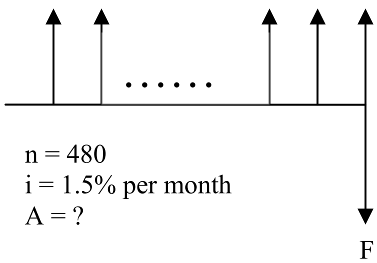


A = $100 (A/P, 3.5%, 3)
= $100 (0.3569)
= $35.69

6-6



The 1.5% interest table does not contain n = 516. The problem must be segmented to use the 1.5% table.

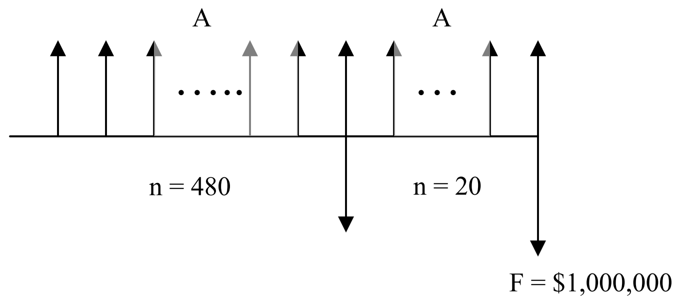


Compute the future value F of a series of A's for 480 interest periods.
F = A (F/A, 1.5%, 480) = A (84,579) = 84,579 A

Then substitute 84,579 A for the first 480 interest periods and solve for A.
84,579 A (F/P, 1.5%, 36) + A (F/A, 1.5%, 36) = $1,000,000
84,579 A (1.709) + A (42.276) = $1,000,000

A = <u>$6.92 monthly investment</u>

6-7



Assuming you only had the tables available it would be:

F = A (F/A, 1.25%, 480) (F/P, 1.25%, 20) + A (F/A, 1.25%, 20)
= A [(31,017) (1.282) + 22.6]
= A (39,786)

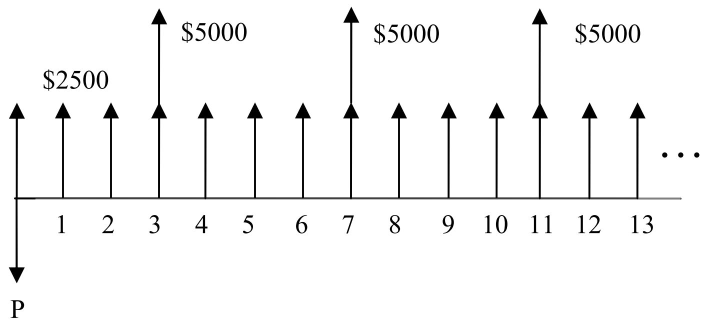
A = $1,000,000/39,786
= $25.13

Using the formula A/F factor we can compute it directly:
A = $1,000,000 (0.0125 / (1.0125^{500} – 1)) = $25.13

6-8

A = F[(e^r −1)/(e^m − 1)]
= $5 × 10^6 [(e$^{0.15}$ − 1)/(e$^{(0.15)(40)}$ − 1)]
= $5 × 10^6 [0.161834/402.42879]
= $2,011

6-9



(a) The repeating cycle of beginning of year payments has the EUAC over four years of student's college career:

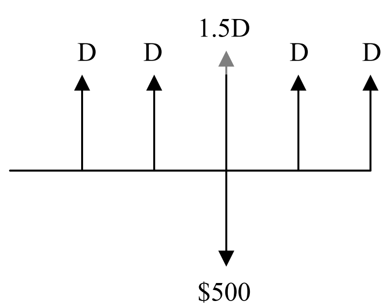
EUAC = \$2,500 + \$5,000 (A/F, 8%, 4)
= \$2,500 + \$5,000 (0.2219)
= \$3,609.50

(b) The capitalized cost of this series of cash flows is computed below, but positioned in year "–1" (not year 0) since the cycle starts in year 0.

P(year "–1") = A/*i* = \$3,609.50/0.08 = \$45,118.75
P(year 0) = \$45,119 (F/P, 8%, 1) = \$48,729

Students often benefit by seeing this answer verified using an Excel table that shows yearly expenditures and interest based on the starting balance. They are able to visualize that the \$48,729 value reappears at the end of years 3, 7, etc.

6-10



\$500 = D (F/A, 12%, 3) + 0.5D + D (P/A, 12%, 2)
= D (3.374 + 0.5 + 1.690)

D = \$500/5.564
= <u>\$89.86</u>

6-11

EUAC = \$60,000 (0.10) + \$3,000 + \$1,000 (P/F, 10%, 1) (A/P, 10%, 4)
= \$6,000 + \$3,000 + \$1,000 (0.9091) (0.3155)
= <u>\$9,287</u>

This is the relatively unusual situation where Cost = Salvage Value. In this situation the annual capital recovery cost equals interest on the investment. If anyone doubts this, they should compute:

\$60,000 (A/P, 10%, 4) − \$60,000 (A/F, 10%, 2).
This equals $P^{*}i$ = \$60,000 (0.10) = \$6,000.

6-12

Prospective Cash Flow:

Year	Cash Flow
0	-\$30,000
1-8	+A
8	+\$35,000

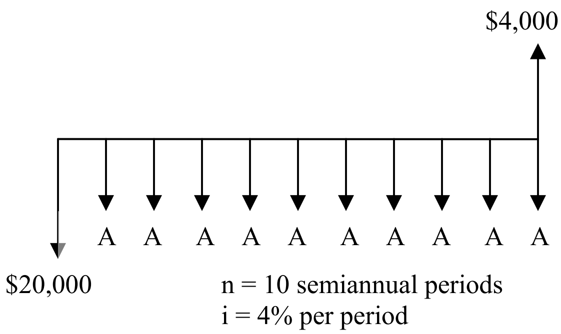
EUAC = EUAB
\$30,000 (A/P, 15%, 8) = A + \$35,000 (A/F, 15%, 8)
\$30,000 (0.2229) = A + \$35,000 (0.0729)
\$6,687 = A + \$2,551.50
A = <u>\$4,135.50</u>

6-13

EUAC = \$30,000 (A/P, 8%, 8) − \$1,000 − \$40,000 (A/F, 8%, 8)
= \$30,000 (0.1740) − \$1,000 − \$40,000 (0.0940)
= <u>\$460</u>

The equipment has an annual cost that is \$460 greater than the benefits. The equipment purchase did not turn out to be desirable.

6-14



First, compute A:

A= ($20,000 − $4,000) (A/P, 4%, 10) + $4,000 (0.04)
= $16,000 (0.1233) + $160
= $2,132.80 per semiannual period

Now, compute the equivalent uniform annual cost:

EUAC = A (F/A, i%, n)
= $2,132.80 (F/A, 4%, 2)
= $2,132.80 (2.040)
= <u>$4,350.91</u>

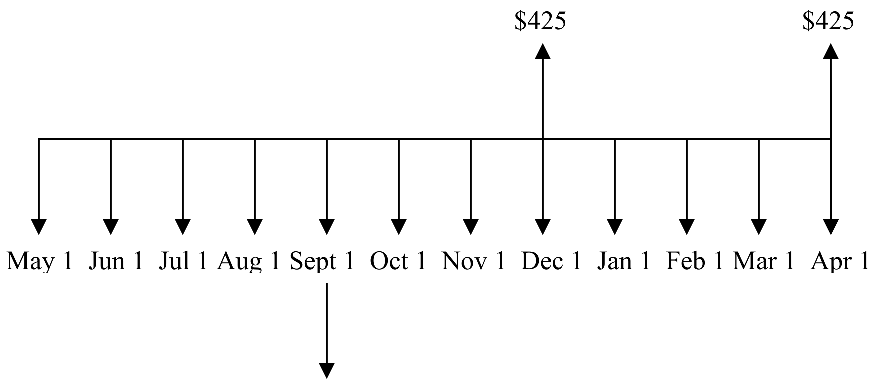
6-15

$i_{month} = (1 + (0.1075/52))^4 - 1 = 0.008295$

P= 0.9 ($178,000) = $160,200

$A = P [(i (1 + i)^n)/((1 + i)^n - 1)]$
$= \$160,200 [(0.008295 (1.008295)^{300})/((1.008295)^{300} - 1)]$
= <u>$1,450.55</u>

6-16



Equivalent total taxes if all were paid on April 1st:
= \$425 + \$425 (F/P, ¾%, 4)
= \$425 + \$425 (1.030)
= \$862.75

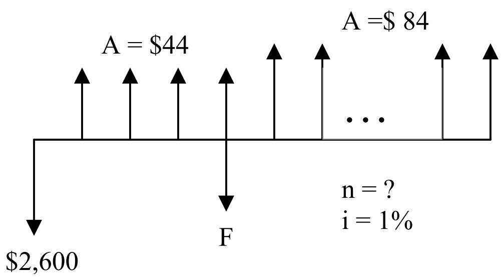
Equivalent uniform monthly payment:
= \$862.75 (A/P, ¾%, 12)
= \$862.75 (0.0800)
= \$69.02

Therefore the monthly deposit is \$69.02.

Amount to deposit September 1:
= Future worth of 5 months deposits (May − Sept)
= \$69.02 (F/A, ¾%, 5)
= \$69.02 (5.075)
= \$350.28

Notes:
1. The fact that the tax payments are for the fiscal year, July 1 through June 30, does not affect the computations.
2. Quarterly interest payments to the savings account could have an impact on the solution, but they do not in this problem.
3. The solution may be verified by computing the amount in the savings account on Dec. 1 just before making the payment (about \$560.03) and the amount on April 1 after making that payment (\$0).

6-17



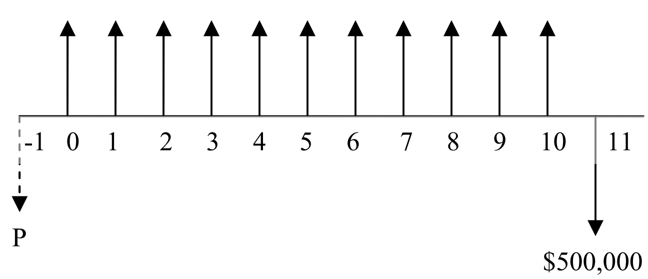
Compute the equivalent future sum for the $2,600 and the four $44 payments at F.
F = $2,600 (F/P, 1%, 4) − $44 (F/A, 1%, 4)
= $2,600 (1.041) − $44 (4.060) = $2,527.96

This is the amount of money still owed at the end of the four months. Now solve for the unknown n.

$2,527.96 = $84 (P/A, 1%, n)
(P/A, 1%, n) = $2,572.96/$84 = 30.09
From the 1% interest table n is almost exactly 36. Thus <u>36 payments of $84</u> will be required.

6-18

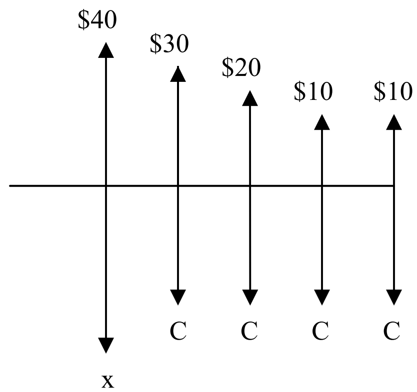
A diagram is essential to properly see the timing of the 11 deposits:



These are beginning of period deposits, so the compound interest factors must be adjusted for this situation.

P_{now-1} = $500,000 (P/F, 1%, 12) = $500,000 (0.8874) = $443,700
A = P_{now-1} (A/P, 1%, 11) = $443,700 (0.0951) = $42,196
<u>Quarterly beginning of period deposit = $42,196</u>

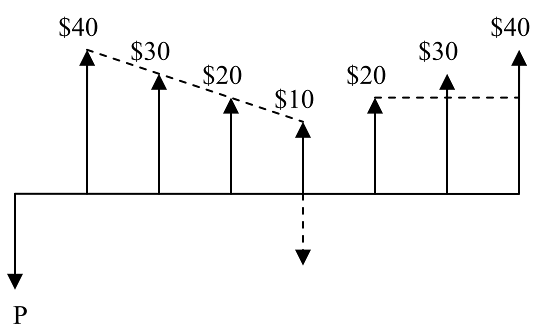
6-19



X = \$40 + \$10 (P/A, 10%, 4) + \$20 (P/F, 10%, 1) + \$10 (P/F, 10%, 2)
= \$40 + \$10 (3.170) + \$20 (0.9091) + \$10 (0.8264)
= \$98.15

C = \$98.15 (A/P, 10%, 4)
= \$98.15 (0.3155)
= <u>\$30.97</u>

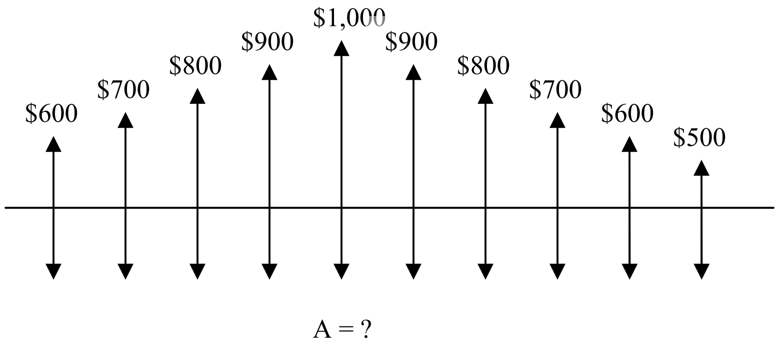
6-20



P = \$40 (P/A, 10%, 4) – \$10 (P/G, 10%, 4) + [\$20 (P/A, 10%, 3) + \$10 (P/G, 10%, 3)] (P/F, 10%, 4)
= \$40 (3.170) − \$10 (4.378) + [\$20 (2.487) + \$10 (2.329)] (0.6830)
= \$132.90

A = \$132.90 (A/P, 10%, 7)
= \$132.90 (0.2054)
= <u>\$27.30</u>

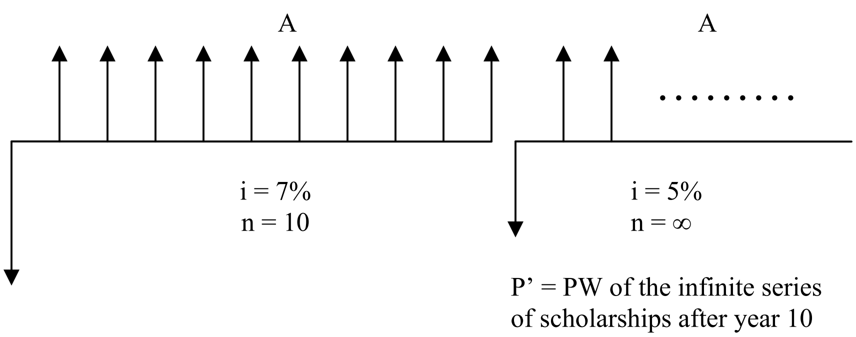
6-21



This problem is much harder than it looks!

EUAC = {$600 (P/A, 8%, 5) + $100 (P/G, 8%, 5) + [$900 (P/A, 8%, 5) − $100 (P/G, 8%, 5)][(P/F, 8%, 5)]}{(A/P, 8%, 10)}
= {$600 (3.993) + $100 (7.372) + [$900 (3.993) − $100 (7.372)][0.6806]}{0. 1490}
= <u>$756.49</u>

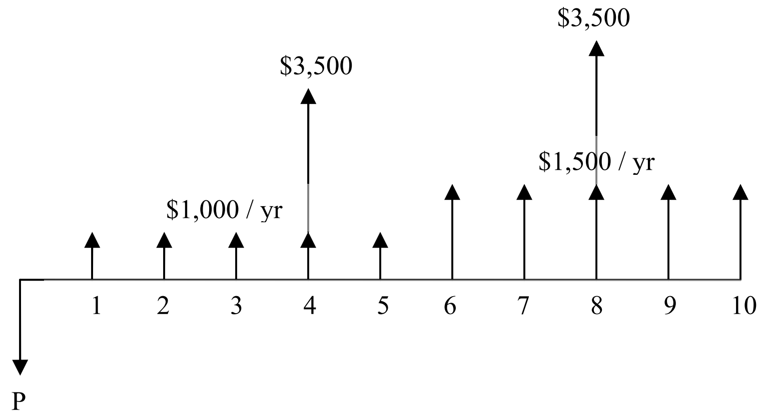
6-22



P' = A/I = A/0.05
$30,000 = PW of all future scholarships
= A (P/A, 7%, 10) + P'(P/F, 7%, 10)
= A (7.024) + A(0.5083/0.05)

A = $30,000/17.190
= <u>$1,745.20</u>

6-23



P = \$1,000 (P/A, 6%, 5) + \$3,500 (P/F, 6%, 4) + \$1,500 (P/A, 6%, 5) (P/F, 6%, 5) + \$3,500 (P/F, 6%, 8)
= \$1,000 (4.212) + \$3,500 (0.7921) + \$1,500 (4.212) (0.7473) + \$3,500 (0.6274)
= \$4,212 + \$2,772 + \$4,721 + \$2,196
= \$13,901

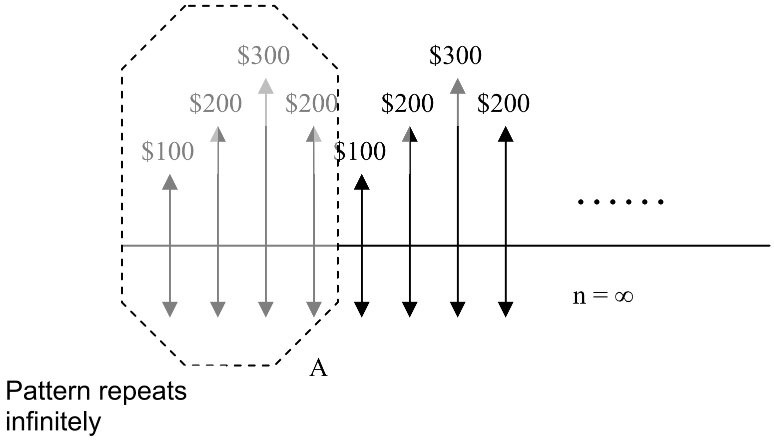
Equivalent Uniform Annual Amount = \$13,901 (A/P, 6%, 10) = <u>\$1,889</u>

6-24

Given:
P = −\$150,000
A = −\$2,500
F_4 = −\$20,000
F_5 = −\$45,000
F_8 = −\$10,000
F_{10} = +\$30,000

EUAC = \$150,000(A/P, 5%, 10) + \$2,500 + \$20,000(P/F, 5%, 4)(A/P, 5%, 10) + \$45,000(P/F, 5%, 5)(A/P, 5%, 10) + \$10,000(P/F, 5%, 8) (A/P, 5%, 10) – \$30,000 (A/F, 5%, 10)
= \$19,425 + \$2,500 + \$2,121 + \$4,566 + \$876 − \$2,385
= <u>\$27,113</u>

6-25

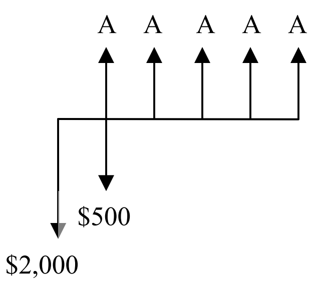


Pattern repeats infinitely
There is a repeating series: 100 − 200 − 300 − 200. Solving this series for A gives us the A for the infinite series.

A= \$100 + [\$100 (P/F, 10%, 2) + \$200 (P/F, 10%, 3) +
\$100 (P/F, 10%, 4)] (A/P, 10%, 4)
= \$100 + [\$100 (0.8254) + \$200 (0.7513) + \$100 (0.6830)] (0.3155)
= \$100 + [\$301.20] (0.3155)
= <u>\$195.03</u>

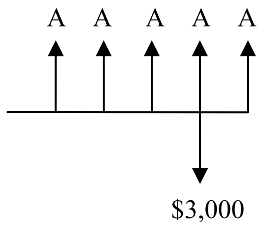
6-26

Alternative A



EUAC = A = [\$2,000 + \$500 (P/F, 12%, 1)] (A/P, 12%, 5)
= [\$2,000 + \$500 (0.8929)] (0.2774)
= \$678.65

Alternative B



EUAC = A = $3,000 (F/P, 12%, 1) (A/F, 12%, 5)
= $3,000 (1.120) (0.1574)
= $528.86

<u>To minimize EUAC, select B.</u>

6-27

(a) EUAC = $6,000 (A/P, 8%, 30) + $3,000 (labor) + $200 (material)
− 500 bales ($2.30/bale) − 12 ($200/month trucker)
= $182.80

Therefore, baler is not economical.

(b) The need to recycle materials is an important intangible consideration. While the project does not meet the 8% interest rate criterion, it would be economically justified at a 4% interest rate. The baler probably should be installed.

6-28

(a) EUAC = $5,000 + $35,000 (A/P, 6%, 20)
= $5,000 + $35,000 (0.0872)
= $8,052

(b) Since the EUAC of the new pipeline is less than the $5,000 annual cost of the existing pipeline, it should be constructed.

6-29

Compute equivalent uniform monthly cost for each alternative.

(a) Purchase for cash

Equivalent Uniform Monthly Cost = (\$13,000 − \$4,000) (A/P, 1%, 36) + \$4,000 (0.01)
= \$338.80

(b) Lease at a monthly cost = \$350.00

(c) Lease with repurchase option = \$360.00 − \$500 (A/F, 1%, 36)
= \$348.40

<u>Alternative (a) has the least equivalent monthly cost, but nonmonetary considerations might affect the decision.</u>

6-30

Original Loan

Annual Payment = \$80,000 (A/P, 10%, 25) = \$8,816

Balance due at end of 10 years:

<u>Method 1:</u> Balance = \$8,816 (P/A, 10%, 15) = \$67,054

<u>Method 2:</u> The payments would repay:
= \$8,816 (P/A, 10%, 10) = \$54,170
making the unpaid loan at Year 0:
= \$80,000 – \$54,170 = \$25,830

At year 10 this becomes:
= \$25,830 (F/P, 10%, 10) = \$67,000

Note: The difference is due to four place accuracy in the compound interest tables. The exact answer is \$67,035.80

New Loan

(Using \$67,000 as the existing loan)

Amount = \$67,000 + 2% (\$67,000) + \$1,000 = \$69,340

New Pmt. = \$69,340 (A/P, 9%, 15) = \$69,340 (0.1241) = \$8,605

<u>New payment < Old payment, therefore refinancing is desirable.</u>

6-31

Provide Autos
P = \$18,000, F = \$7,000, A = \$600/yr + 0.12/mile, n = 4 years

Pay Salesmen
0.30 x where x = miles driven
0.30 x = (\$18,000 − \$7,000) (A/P, 10%, 4) + \$7,000 (0.10) + \$600 + \$0.12 x
0.18 x = (\$11,000) (0.3155) + \$700 + \$600
= \$4,770

Miles Driven (x) = \$4,770/\$0.18/mile = 26,500 mile

6-32

EUAC Comparison
Gravity Plan
Initial Investment: = \$2.8 million (A/P, 10%, 40)
= \$2.8 million (0.1023) = \$286,400
Annual Operation and maintenance = \$10,000
Annual Cost = \$296,400

Pumping Plan
Initial Investment: = \$1.4 million (A/P, 10%, 20)
= \$1.4 million (0.1023) = \$143,200

Additional investment in 10^{th} year:
= \$200,000 (P/F, 10%, 10) (A/P, 10%, 40)
= \$200,000 (0.3855) (0.1023) = \$7,890

Annual Operation and maintenance = \$25,000
Power Cost: = \$50,000 for 40 years = \$50,000

Additional Power Cost in last 30 years:
= \$50,000 (F/A, 10%, 30) (A/F, 10%, 40)
= \$50,000 (164.494) (0.00226) = \$18,590

Annual Cost = \$244,680

Select the Pumping Plan.

6-33

New Machine
EUAC = $3,700 (A/P, 8%, 4) − $500 − $200
= $3,700 (0.3019) − $700
= $417.03

Existing Machine
EUAC = $1,000 (A/P, 8%, 4)
= $1,000 (0.3019)
= $301.90

The new machine should not be purchased.

6-34

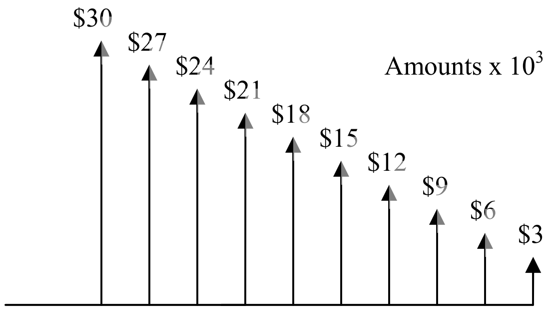
	Around the Lake	Under the Lake
First Cost	$75,000	$125,000
Maintenance	$3,000/yr	$2,000/yr
Annual Power Loss	$7,500/yr	$2,500/yr
Property Taxes	$1,500/yr	$2,500/yr
Salvage Value	$45,000	$25,000
Useful Life	15 years	15 years

Around the Lake
EUAC = $75,000 (A/P, 7%, 15) + $12,000 − $45,000 (A/F, 7%, 15)
= $75,000 (0.1098) + $12,000 – $45,000 (0.0398)
= $18,444

Under the Lake
EUAC = $125,000 (A/P, 7%, 15) + $7,000 − $25,000 (A/F, 7%, 15)
= $125,000 (0.1098) + $7,000 − $25,000 (0.0398)
= $19,730

Go around the lake.

6-35

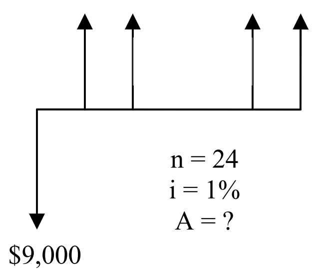


EUAC = $30,000 − $3,000 (A/G, 8%, 10)
= $30,000 − $3,000 (3.871)
= $18,387

Hyro-clean's offer of $15,000/yr is less costly.

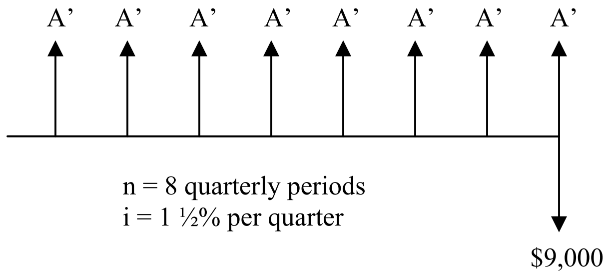
6-36

(a)



A = $9,000 (A/P, 1%, 24)
= $9,000 (0.0471)
= $423.90/month

(b)



Note that interest is compounded quarterly
A' = $9,000 (A/F, 1.5%, 8)
= $9,000 (0.1186)
= $1,067.40

Monthly Deposit = ½ of A' = ($1,067.40)/3 = <u>$355.80/month</u>

(c) In part (a) Bill Anderson's monthly payment includes an interest payment on the loan. The sum of his 24 monthly payments will exceed $9,000.

In part (b) Doug James' savings account monthly deposit earns interest for him that helps to accumulate the $9,000. The sum of Doug's 24 monthly deposits will be less than $9,000.

6-37

With neither input nor output fixed, maximize (EUAB – EUAC)
Continuous compounding capital recovery:
$A = P\ [(e^{m}\ (e^{r} - 1))/(e^{m} - 1)]$

For r = 0.15 and n = 5,
$[(e^{m}\ (e^{r} - 1))/(e^{m} - 1)] = [(e^{(0.15)(5)}\ (e^{0.15} - 1))/(e^{(0.15)(5)} - 1)]$
$= 0.30672$

Alternative A
EUAB – EUAC = $845 – $3,000 (0.30672) = –$75.16

Alternative B
EUAB – EUAC = $1,400 − $5,000 (0.30672) = −$133.60

To maximize (EUAB – EUAC), choose alternative A (less negative value).

6-38

Machine X
EUAC = $5,000 (A/P, 8%, 5)
= $5,000 (0.2505)
= $1,252

Machine Y
EUAC = ($8,000 − $2,000) (A/P, 8%, 12) + $2,000 (0.08) + $150
= $1,106

Select Machine Y.

6-39

Machine A
EUAC = $1,000 + $10,000 (A/P, 10%, 4) − $10,000 (A/F, 10%, 4)
= $1,000 + $1,000
= $2,000

Machine B
EUAC = ($20,000 − $10,000) (A/P, 10%, 10) + $10,000 (0.10)
= $1,627 + $1,000
= $2,627

Choose Machine A.

6-40

Because we may assume identical replacement, we may compare 20 years of B with an infinite life for A by EUAB – EUAC.

Alternative A
EUAB – EUAC (for an inf. period) = $16 − $100 (A/P, 10%, ∞)
= $16 − $100 (0.10)
= +$6.00

Alternative B
EUAB – EUAC (for 20-yr period) = $24 − $150 (A/P, 10%, 20)
= $24 − $150 (0.1175)
= +$6.38

Choose Alternative B.

6-41

$EUAC_{gas}$ = (P − S) (A/P, *i*%, n) + SL + Annual Costs
= ($2,400 − $300) (A/P, 10%, 5) + $300 (0.10) + $1,200 + $300
= $2,100 (0.2638) + $30 + $1,500
= $2,084

$EUAC_{electr}$ = ($6,000 − $600) (A/P, 10%, 10) + $600 (0.10) + $750 + $50
= $5,400 (0.1627) + $60 + $800
= $1,739

<u>Select the electric motor.</u>

6-42

Annual Cost of Diesel Fuel = [$50,000 km/(35 km/liter)] × $0.48/liter = $685.71
Annual Cost of Gasoline = [$50,000 km/(28 km/liter)] × $0.51/1liter = $910.71

$EUAC_{diesel}$ = ($13,000 − $2,000) (A/P, 6%, 3) + $2,000 (0.06) + $685.71 fuel + $300 repairs + $500 insurance
= $11,000 (0.3741) + $120 + $1,485.71
= $5,720.81

$EUAC_{gasoline}$ = ($12,000 − $3,000) (A/P, 6%, 4) + $3,000 (0.06) + $910.71 fuel + $200 repairs + $500 insurance
= $4,388.11

<u>The gasoline taxi is more economical.</u>

6-43

Machine A
EUAB – EUAC = − First Cost (A/P, 12%, 7)
− Maintenance & Operating Costs + Annual Benefit + Salvage Value (A/F, 12%, 7)
= −$15,000 (0.2191) − $1,600 + $8,000 + $3,000 (0.0991)
= $3,411

Machine B
EUAB – EUAC = − First Cost (A/P, 12%, 10)
− Maintenance & Operating Costs + Annual Benefit + Salvage Value (A/F, 12%, 10)
= −$25,000 (0.1770) − $400 + $13,000 + $6,000 (0.0570)
= $8,517
<u>Choose Machine B to maximize (EUAB – EUAC).</u>

6-44

Alternative A

EUAB – EUAC = $10 − $100 (A/P, 8%, ∞) = $10 − $100 (0.08)
= +$2.00

Alternative B

EUAB – EUAC = $17.62 − $150 (A/P, 8%, 20) = $17.62 − $150 (0.1019)
= +$2.34

Alternative C

EUAB – EUAC = $55.48 − $200 (A/P, 8%, 5) = $55.48 – $200 (0.2505)
= +$5.38

Select C.

6-45

Machine A

EUAB – EUAC = −$700,000 (A/P, 15%, 10) − $18,000 + $154,000 − $900 (A/G, 15%, 10) + $142,000 (A/F, 15%, 10)
= −$139,510 – $18,000 + $154,000 – $3,044.70 + $7,001
= $446

Machine B

EUAB – EUAC = −$1,700,000 (A/P, 15%, 20) − $29,000 + $303,000 − $750 (A/F, 15%, 20) + $210,000 (A/F, 15%, 20)
= −$271,660 − $29,000 + $303,000 − $4,024 + $2,050
= $366

Thus, the choice is Machine A but note that there is very little difference between the alternatives.

6-46

Choose alternative with minimum EUAC.

(a) 12-month tire EUAC = $39.95 (A/P, 10%, 1) = $43.95
(b) 24-month tire EUAC = $59.95 (A/P, 10%, 2) = $34.54
(c) 36-month tire EUAC = $69.95 (A/P, 10%, 3) = $28.13
(d) 48-month tire EUAC = $90.00 (A/P, 10%, 4) = $28.40

Buy the 36-month tire.

6-47

It is important to note that the customary "identical replacement" assumption is not applicable here.

Alternative A
EUAB – EUAC = \$15 − \$50 (A/P, 15%, 10) = \$15 − \$50 (0.1993)
= +\$5.04

Alternative B
EUAB – EUAC = \$60 (P/A, 15%, 5) (A/P, 15%, 10) − \$180 (A/P, 15%, 10)
= +\$4.21

<u>Choose A.</u>

Check solution using NPW:
Alternative A
NPW = \$15 (P/A, 15%, 10) − \$50 = <u>+\$25.28</u>

Alternative B
NPW = \$60 (P/A, 15%, 5) − \$180 = +\$21.12

6-48

Seven year analysis period:
Alternative A
EUAB – EUAC = \$55 − [\$100 + \$100 (P/F, 10%, 3) +
\$100 (P/F, 10%, 6)] (A/P, 10%, 7)
= \$55 − [\$100 + \$100 (0.7513) + \$100 (0.5645)] (0.2054)
= +\$7.43

Alternative B
EUAB – EUAC = \$61 − [\$150 + \$150 (P/F, 10%, 4)] (A/P, 10%, 7)
= \$61 − [\$150 + \$150 (0.683)] (0.2054)
= <u>+\$9.15</u>

<u>Choose B.</u>

Note: The analysis period is seven years, hence one cannot compare three years of A vs. four years of B, If one does, the problem is constructed so he will get the wrong answer.

6-49

Use 20-year analysis period.

Net Present Worth Approach

$NPW_{Mas.}$ = −$250 − ($250 − $10) [(P/F, 6%, 4) + (P/F, 6%, 8) + (P/F, 6%, 12) + (P/F, 6%, 16)] + $10 (P/F, 6%, 20) − $20 (P/A, 6%, 20)
= −$250 − $240 [0.7921 + 0.6274 + 0.4970 + 0.3936) + $10 (0.3118) − $20 (11.470)
= −$1,031

NPW_{BRK} = −$1,000 − $10 (P/A, 6%, 20) + $100 (P/F, 6%, 20)
= −$1,000 − $10 (11.470) + $100 (0.3118)
= −$1,083

Choose Masonite to save $52 on Present Worth of Cost.

Equivalent Uniform Annual Cost Approach

$EUAC_{Mas.}$ = $20 + $250 (A/P, 6%, 4) − $10 (A/F, 6%, 4)
= $20 + $250 (0.2886) − $10 (0.2286)
= $90

$EUAC_{BRK}$ = $10 + $1,000 (A/P, 6%, 20) − $100 (A/F, 6%, 20)
= $10 + $1,000 (0.872) − $100 (0.0272)
= $94

Choose Masonite to save $4 per year.

6-50

Note: r = 6%, i = $\frac{6\%}{12}$ = 0.5%, 60 monthly payments (5 years)

Monthly payment = 18000 (A/P, 0.5%, 60) = (18,000) (0.0193)
= $347.40 (from tables)
(exact $347.99 from capital recovery formula)

After 24 months (2 years) have 60 – 24 = 36 months of payments left.

Amount owed = (347.40) (P/A, 0.5%, 36) = $11,419 (from tables)
(exact $11,439 using $347.99 and present worth formula)

Spreadsheet solution shown below after problem 51.

6-51

Same as Problem 6-50 except an extra $1,500 is paid at month 1. To find when the final payment is made, first find the principal at the end of the first month.

First month's interest = (18,000) (0.005) = $90
Principal reduction = 347.99 – 90 = $257.99
Principal at first month = 18,000 – 257.99 – 1500 = $16,242.01

To determine n (counted from the first month), have

$(P/A, 0.5\%, n) = \frac{16242.01}{347.99} = 46.674$. Interpolate to determine n using the tables

$n = 52 + (8)\left[\frac{46.674 - 45.690}{51.726 - 45.690}\right] = 53.3$.

Thus, the last payment is made in month = 54 +1 = 55. To find the amount of the last payment, first, find the present worth of the 53 remaining full payments at the first month.

P = (347.99) (P/A, 0.5%, 53) = (347.99) $\left[\frac{(1+0.005)^{53} - 1}{0.005(1+0.005)^{53}}\right]$ = $16166.73.

The extra amount at the first month is 16242.01 – 16166.73 = $75.28 and the amount due at the last month (55 – 1 = 54) is F = 75.28 $(1+ 0.005)^{54}$ = $98.55 ($98.52 from spread sheet). So, $98.52 is due at the end of the 55th month.

Spreadsheet solution shown next page.

	A	B	C	D	E	F
1	**Problem 6-50**					
2						
3	Nom. Interest Rate	6%				
4	Compounding Per/Yr	12				
5	Num Periods of Loan	60	months			
6	Loan Amount	$ 18,000				
7	Monthly Payment	$347.99	**=PMT(B3/12,B5,-B6)**			
8						
9	Num Payments Made	24				
10	Num Payments Left	36	**=B5-B9**			
11	Remaining Balance	$11,438.80	**=PV(B3/B4,B10,-B7)**			
12						
13	**Problem 6-51**					
14						
15	Extra Payment Amount	$ 1,500				
16	Period of Extra Payment	1				
17	Remaining Loan Balance (without extra payment)	$17,742.01	**=PV(B3/B4,B5-B17,-B7)**			
18	Num Periods Remaining	53.28	**=NPER(B3/B4,-B7,B18-B16)**			
19	Final Payment Period	55	**=ROUNDUP(B19+B17,0)**			
20	PV remaining payments (excludes last partial per)	$16,166.75	**=PV(B3/B4,ROUNDDOWN(B19,0),-B7)**			
21	Final Payment Amount	$98.52	**=FV(B3/B4,ROUNDUP(B19,0),,-(B18-B21-B16))**			

6-52

Payment = PMT (0.75%, 48, −12000) = $298.62
Owed = PV (0.75%, 18, −298.62) = $5,010.60

6-53

Payment = PMT (0.75%, 60, −15000) = $311.38
Owed = PV (0.75%, 48, −311.38) = $12,512.74
She will have to pay $513 more than she receives for the car.

6-54

(a) Payment = PMT (0.75%, 360, −78,000) = \$627.61

(b) $\text{Owed}_{\text{yr 1}}$ = PV (0.75%, 348, −627.61) = \$77,467.64

(c) Interest_{13} = 0.0075 (77,467.64) = \$581.01
or
Interest_{13} = IPMT (0.75%, 13,360, −\$78,000) = \$581.00
Principal_{13} = \$627.61 − \$581.01 = \$46.60
or
Principal_{13} = PPMT(0.75%, 13360, −\$78,000) = \$46.60

6-55

(a) Payment = PMT (0.75%, 360, −92,000) = \$740.25

(b) $\text{Owed}_{\text{yr 1}}$ = PV (0.75%, 348, −740.25) = \$91,371.11
so % paid = 629/92,000 = 0.68%

(c) $\text{Owed}_{\text{yr 10}}$ = PV (0.75%, 240, −740.25) = \$82,275.05

(d) Interest_{25} = IPMT (0.75%, 25,360, −\$92,000) = \$680.13
Principal_{25} = PPMT(0.75%, 25,360, −\$92,000) = \$60.12

6-56

(a) Payment = PMT (0.75%, 360, −95,000) = \$764.39

(b) NPER (0.75%, 1000, −95,000) = 168.8 months = 14.07 years

(c) NPER (0.75%, 2(764.39), −95,000) = 84.0 montos = 7 years

6-57

(a) Payment = PMT (0.5%, 360, −145,000) = \$869.35

(b) NPER (0.5%, 1000, −145,000) = 258.8 months = 21.57 years

(c) NPER (0.5%, 2(869.35), −145,000) = 238.1 months = 19.84 years

6-58

	Around the Lake	Under the Lake
2ALTEUAW (modified)	MARR	7,00%
Length, km	16	5
First Cost/km	$5,000	$25,000
Maintenance/km/yr	$200	$400
Yearly Power Loss/km	$500	$500
Salvage Value/km	$3,000	$5,000
Property tax/0.02*First Cost/yr	$1,500	$2,500
USEFUL LIFE	15	15
INITIAL COST	$75,000	$125,000
ANNUAL COSTS	$12,000	$7,000
ANNUAL REVENUE	$0	$0
SALVAGE VALUE	$45,000	$25,000
EUAB	$0	$0
EUAC (CR) + EUAC (O&M)	$18,444	$19,729
EUAW	−$18,444	−$19,279

Input Data in Shaded Cells **Breakeven Analysis**

	Around the Lake	Under the Lake
2ALTEUAW (modified)	MARR	7.00%
Length, km	15	5
First Cost/km	$5,000	$23,019
Maintenance/km/yr	$200	$400
Yearly Power Loss/km	$500	$500
Salvage Value/km	$3,000	$5,000
Property Tax/0.02*first cost/yr	$1,500	$2,302
USEFUL LIFE	15	15
INITIAL COST	$75,000	$115,095
ANMUAL COSTS	$12,000	$6,802
ANNUAL REVENUE	$0	$0
SALVAGE VALUE	$45,000	$25,000
EUAB	$0	$0
EUAC (CR) + EUAC (O&M)	$18,444	$18,444
EUAW	−$18,444	−$18,444

6-59

Input Data in Shaded Cells

	Diesel	Gasoline
2ALTEUAW (modified)	MARR	6.00%
Km per Year	50,000	50,000
First Cost	$13,000	$12,000
Fuel Cost per Liter	$0.48	$0.51
Mileage, km/liter	35	28
Annual Repairs	$300	$200
Annual Insurance Premium	$500	$500
USEFUL LIFE	4	3
INITIAL COST	$13,000	$12,000
ANNUAL COSTS	$1,486	$1,611
ANNUAL REVENUE	$0	$0
SALVAGE VALUE	$2,000	$3,000
EUAB	$0	$0
EUAC (CR) + EUAC (O&M)	$4,780	$5,158
EUAW	−$4,780	−$5,158

Mileage (km)		
10,000	$4,232	$4,429
20,000	$4,369	$4,611
40,000	$4,643	$4,976
60,000	$4,917	$5,340
80,000	$5,192	$5,704

6-60

Input Data in Shaded Cells

MARR	8.00%
Current Trucking Cost per Month	$200.00
Labor Cost per Year	$3,000
Strapping Material cost Per Bale	$0,40
Revenue per Bale	$2,30
Bales per Year Produced	500
USEFUL LIFE	30
Initial Cost for Baler	$6,000
ANNUAL COSTS	$3,200
Annual Benefits	$3,550
SALVAGE VALUE	$0
Salvage Value as a Reduced Cost	
EUAB	3,550
EUAC (CR) + EUAC (O%M)	$3,733
EUAW	−$183

6-61

Input Data in Shaded Cells

	MARR	10.00%
	Gravity Plan	Pumping
USEFUL LIFE	40	40
COMMON MULTIPLE	40	40
INITIAL COST	$2,800,000	$1,400,000
ANNUAL COSTS	$10,000	$25,000
Additional Cost, 10th Year		$200,000,
Additional Power Cost, yr 1−10		$50,000
Additional Power Cost, yr 11−40		$100,000
ANNUAL REVENUE	$0.00	$0.00
SALVAGE VALUE	$0.00	$0.00
NET ANNUAL CASH FLOW	−$10,000.00	−$75,000.00
Net Annual Cash Flow (NACF) Method		
PWB	$0.00	$0.00
PWC	$2,907,800	$2,467,262
NPW = PWB − PWC	−$2,907,800	$2,467,262
EUAC	$297,467	$252,401

Year	Gravity	Pumping
0	–$2,800,000	–$1,400,000
1	–$10,000	–$75,000
2	–$10,000	–$75,000
3	–$10,000	–$75,000
4	–$10,000	–$75,000
5	–$10,000	–$75,000
6	–$10,000	–$75,000
7	–$10,000	–$75,000
8	–$10,000	–$75,000
9	–$10,000	–$75,000
10	–$10,000	–$275,000
11	–$10,000	–$125,000
12	–$10,000	–$125,000
13	–$10,000	–$125,000
14	–$10,000	–$125,000
15	–$10,000	–$125,000
16	-$10,000	–$125,000
17	–$10,000	–$125,000
18	–$10,000	–$125,000
19	–$10,000	–$125,000
20	–$10,000	–$125,000
21	–$10,000	–$125,000
22	–$10,000	–$125,000
23	–$10,000	–$125,000
24	–$10,000	–$125,000
25	–$10,000	–$125,000
26	–$10,000	–$125,000
27	–$10,000	–$125,000
28	–$10,000	–$125,000
29	–$10,000	–$125,000
30	–$10,000	–$125,000
31	–$10,000	–$125,000
32	–$10,000	–$125,000
33	–$10,000	–$125,000
34	–$10,000	–$125,000
35	–$10,000	–$125,000
36	–$10,000	–$125,000
37	–$10,000	–$125,000
38	–$10,000	–$125,000
39	–$10,000	–$125,000
40	–$10,000	–$125,000

Chapter 7: Rate of Return Analysis

7-1

$100 = $27 (P/A, i%, 10)
(P/A, i%, 10) = 3.704

Performing Linear Interpolation:

(*P/A,* i%, 10)	*I*
4.192	20%
3.571	25%

Rate of Return = 20% + (5%) [(4.192 − 3.704)/(4.912 − 3.571)]
= 23.9%

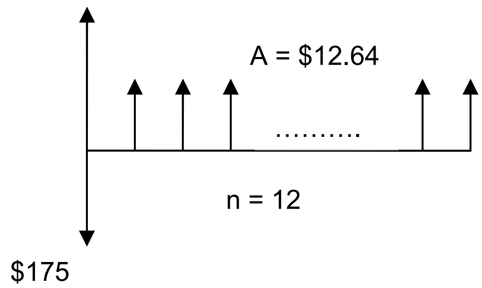
7-2

(a) *F* = $5, *P* = $1, *n* = 5
$F = P (1 + i)^n$
$\$5 = \$1 (1 + i)^5$
$(1 + i) = 5^{0.20} = 1.38$
$i^* = 38\%$

(b) For a 100% annual rate of return
$F = \$1 (1 + 1.0)^5 = \$32$, not $5!

Note that the prices diagonal charges do not necessarily reflect what anyone will pay a collector for his/her stamps.

7-3



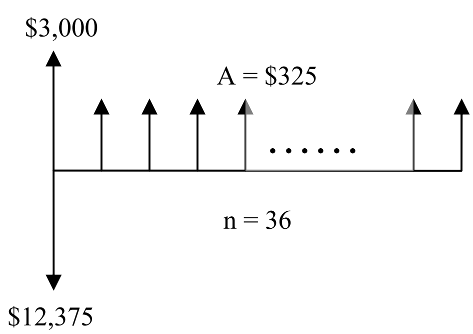
(\$175 − \$35) = \$12.64 (P/A, i%, 12)
(P/A, i%, 12) = \$140/\$12.64 = 11.08

i = 1 ¼%
Nominal interest rate = 12 (1 ¼%) = 15%

7-4

The rate of return exceeds 60% so the interest tables are not useful.
$F = P (1 + i)^n$
$\$25,000 = \$5,000 (1 + i)^3$
$(1 + i) = (\$25,000/\$5,000)^{1/3} = 1.71$
$i^* = 0.71$
Rate of Return = 71%

7-5



\$9,375 = \$325 (P/A, i%, 36)
(P/A, i%, 36) = \$9,375/\$325 = 28.846

From compound interest tables, i = 1.25%
Nominal Interest Rate = 1.25 × 12 = 15%
Effective Interest Rate = $(1 + 0.0125)^{12} - 1$ = 16.08%

7-6

1991 − 1626 = 365 years = n
F = P $(1 + i)^n$
$12 \times 10^9 = 24(1 + i)^{365}$
$(1 + i)^{365} = 12 \times 10^0/24 = 5.00 \times 10^8$
This may be immediately solved on most hand calculators:
$i^* = 5.64\%$

Solution based on compound interest tables:
(F/P, i%, 365) = 5.00×10^8
= (F/P, i%, 100) (F/P, i%, 100) (F/P, i%, 100) (F/P, i%, 65)

Try i = 6%
(F/P, 6%, 365) = $(339.3)^3$ (44.14) = 17.24×10^8 (i too high)

Try i = 5%
(F/P, 5%, 365) = $(131.5)^3$ (23.84) = 0.542×10^8 (i too low)

Performing linear interpolation:
$i^* = 5\% + (1\%) [((5 - 0.54) (10^8))/((17.24 - 0.54) (10^8))]$
= 5% + 4.46/16.70
= <u>5.27%</u>

The linear interpolation is inaccurate.

7-7

(F/A, i, 35) = $\frac{10^6}{5800}$ = 172.414 and is very close to 8% from tables. (Exact = 8.003%)

7-8

(F/A, i, 20) = $\frac{10^7}{2.5 \times 10^5}$ = 40 and interpolating
i = 6% + (1%) $\left[\frac{36.786 - 40}{36.786 - 40.996}\right]$ = 6.76% (exact value 6.774%)

7-9

Year	Cash Flow
0	−\$1,000
3	+\$1,094.60
6	+\$1,094.60

\$1,000 = \$1,094 [(P/F, i%, 3) + (P/F, i%, 6)]

Try i = 20%
\$1,094 [(0.5787) + (0.3349)] = \$1,000
Rate of Return = 20%

7-10

3,000 = 30 (P/A, i^*, 120)
(P/A, i^*, 120) = 3,000/30 = 100

Performing Linear Interpolation:

(P/A, i%, 120)	*I*
103.563	¼%
100	i^*
90.074	½%

i^* = 0.0025 + 0.0025 [(103.562 − 100)/(103.562 − 90.074)]
= 0.00316 per month

Nominal Annual Rate = 12 (0.00316) = 0.03792 = 3.79%

7-11

\$3,000 = \$119.67 (P/A, i%, 30)
(P/A, i%, 30) = \$3,000/\$119.67 = 25.069

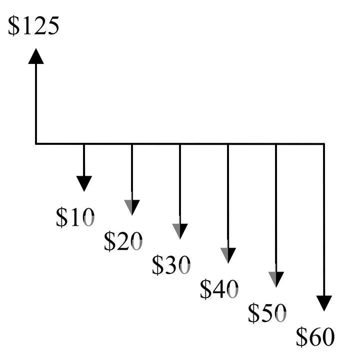
Performing Linear Interpolation:

(*P/A, i*% 30)	*i*
25.808	1%
24.889	1.25%

i = 1% + (0.25%)((25.808 − 25.069)/(25.808 − 24.889))
= 1.201%

(a) Nominal Interest Rate = 1.201 × 12 = 14.41%
(b) Effective Interest Rate = $(1 + 0.01201)^{12} - 1$ = 0.154 = 15.4%

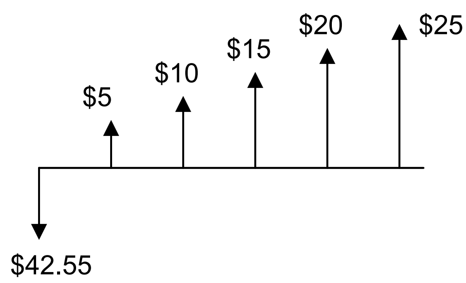
7-12



\$125 = \$10 (P/A, i%, 6) + \$10 (P/G, i%, 6)
at 12%, \$10 (4.111) + \$10 (8.930) = \$130.4
at 15%, \$10 (3.784) + \$10 (7.937) = \$117.2

i^* = 12% + (3%) ((130.4 − 125)/(130.4 −117.2)) = 13.23%

7-13



$42.55 = $5 (P/A, i%, 5) + $5 (P/G, i%, 5)
Try i = 15%, $5 (3.352) + $5 (5.775) = $45.64 > $42.55
Try i = 20%, $5 (2.991) + $5 (4.906) = $39.49 < $42.55

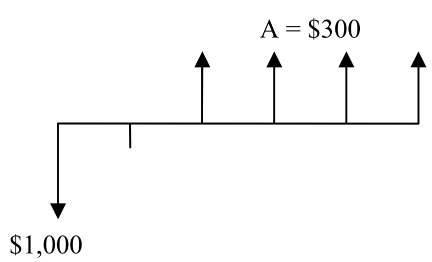
Rate of Return = 15% + (5%) [($45.64 − $42.55)/($45.64 − $39.49)]
= 17.51%

Exact Answer: 17.38%

7-14

The algebraic sum of the cash flows equals zero. Therefore, the rate of return is 0%.

7-15



Try i = 5%
$1,000 = (?) $300 (3.546) (0.9524)
= (?) $1,013.16
Try i = 6%
$1,000 = (?) $300 (3.465) (0.9434)
= (?) $980.66

Performing Linear Interpolation:
i^* = 5% + (1%) (($1,013.6 − $1,000)/($1,013.6 − $980.66))
= 5.4%

7-16

Since the rate of return exceeds 60%, the tables are useless.
$F = P (1 + i)^n$
$\$4,500 = \$500 (1 + i)^4$
$(1 + i)^4 = \$4,500/\$500 = 0$
$(1 + i) = 9^{1/4} = 1.732$
$i^* = 0.732 =$ <u>73.2%</u>

7-17

(a) Using Equation 4-39:
$F = Pe^{rn}$
$\$4,000 = \$2,000e^{r(9)}$
$2 = e^{r(9)}$
$9r = \ln 2 = 0.693$
r = <u>7.70%</u>

(b) Equation 4-34
$i_{eff} = e^r - 1 = e^{0.077} - 1 = 0.0800 =$ <u>8.00%</u>

7-18

Year	Cash Flow
0	−$640
1	0
2	+$100
3	+$200
4	+$300
5	+$300

$640 = $100 (P/G, i%, 4) + $300 (P/F, i%, 5)

Try i = 9%
$100 (4.511) + $300 (0.6499) = $646.07 > $640

Try i = 10%
$100 (4.378) + $300 (0.6209) = $624.07 < $640

Rate of Return = 9% + (1%) [(%646.07 − $640)/($646.07 − $624.07)]
= <u>9.28%</u>

7-19

Year	Cash Flow
0	−\$223
1	−\$223
2	−\$223
3	−\$223
4	−\$223
5	−\$223
6	+\$1,000
7	+\$1,000
8	+\$1,000
9	+\$1,000
10	+\$1,000

The rate of return may be computed by any conventional means. On closer inspection one observes that each \$223 increases to \$1,000 in five years.

\$223 = \$1,000 (P/F, i%, 5)
(P/F, i%, 5) = \$223/\$1,000 = 0.2230
From interest tables, Rate of Return = <u>35%</u>

7-20

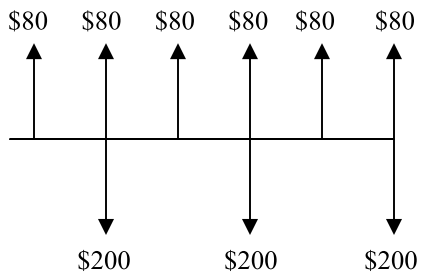
Do nothing has a cash flow of zero, thus, the difference between alternatives is just the Leaseco cash flow.

Year	Leaseco – Do Nothing
0	–\$1,000
1	\$200
2	\$200
3	\$1,200
4	\$1,200
5	\$1,200

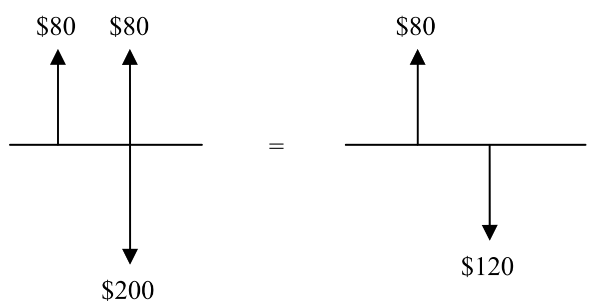
NPW = 0 = −1000 + 200 (P/A, ROR, 5) + 1000 (P/F, i, 2) (P/A, i, 3) and interpolating

ROR = 45% + (5%) $\left[\frac{85.271}{85.271 + 27.329}\right]$ = 48.8%. Obviously, do nothing is rejected.

7-21



The easiest solution is to solve one cycle of the repeating diagram:



$120 = $80 (F/P, i%, 1)
$120 = $80 (1 + i)
(1 + i) = $120/$80 = 1.50
i^* = 0.50 = 50%

Alternative Solution:
EUAB = EUAC
$80 = [$200 (P/F, i%, 2) + $200 (P/F, i%, 4) + $200 (P/F, i%, 6)] (A/P, i%, 6)

Try i = 50%
$80 = [$200 (0.4444) + $200 (0.1975) + $200 (0.0878)] (0.5481) = $79.99

Therefore i^* = 50%

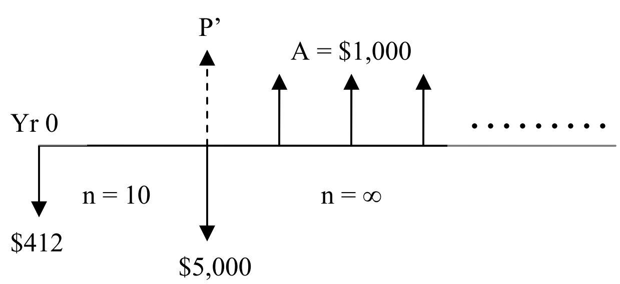
7-22

For infinite series: A = Pi
EUAC = EUAB
\$3,810 (i) = \$250 + \$250 (F/P, i%, 1) (A/F, i%, 2)*

Try i = 10%
\$250 + \$250 (1.10) (0.4762) = \$381
\$3,810 (0.10) = \$381
i = 10%

*Alternate Equations:
\$3,810 (i) = \$250 + \$250 (P/F, i%, 1) (A/P, i%, 2)
\$3,810 (i) = \$500 − \$250 (A/G, i%, 2)

7-23



At Year 0, PW of Cost = PW of Benefits
\$412 + \$5,000 (P/F, i%, 10) = (\$1000/i) (P/F, i%, 10)

Try i = 15%
\$412 + \$5,000 (0.2472) = (\$1,000/0.15) (0.2472)
\$1,648 = \$1,648

ROR = 15%

7-24

\$400 = [\$200 (P/A, i%, 4) − \$50 (P/G, i%, 4)] (P/F, i%, 1)

Try i = 7%
[\$200 (3.387) − \$50 (4.795)] (0.9346) = 409.03

Try i = 8%
[\$200 (3.312) − \$50 (4.650)] (0.9259) = \$398.08
i^* = 7% + (1%) [(\$409.03 − \$400)/(\$409.03 − \$398.04)]
= 7.82%

7-25

The one-time \$2,000 life membership fee avoids the 40-year series of beginning-of-year membership dues that start at \$200 and increase 3% annually.

(a) The equation for determining the rate of return for the life membership is the difference of the present worth of the two cash flows set to zero:

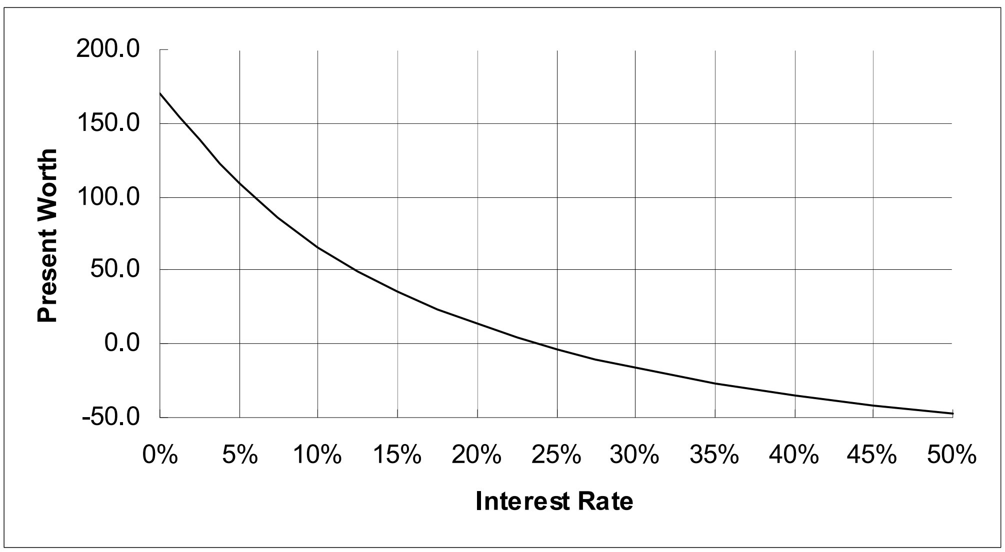
2000 – 200 – 206 (P/A, 3%, ROR, 39) = 0 (39 since beginning-of-year payments)

(b) Use Excel where Result = $\frac{1800}{206} - \left[\frac{1-(1+0.03)^{39}(1+i)^{-39}}{i-0.03}\right]$ and vary i = ROR until zero is obtained. ROR = 14.243%

7-26

Year	Cash Flow		i	PW
0	–100		0%	170.0
1	27		5%	108.5
2	27		10%	65.9
3	27		15%	35.5
4	27		20%	13.2
5	27		25%	–3.6
6	27		30%	–16.5
7	27		35%	–26.7
8	27		40%	–34.8
9	27		45%	–41.5
10	27		50%	–46.9

PW = –100 + 27*(P/A, i, 10) ; use NPV in for (P/A, i, 10) in Excel.

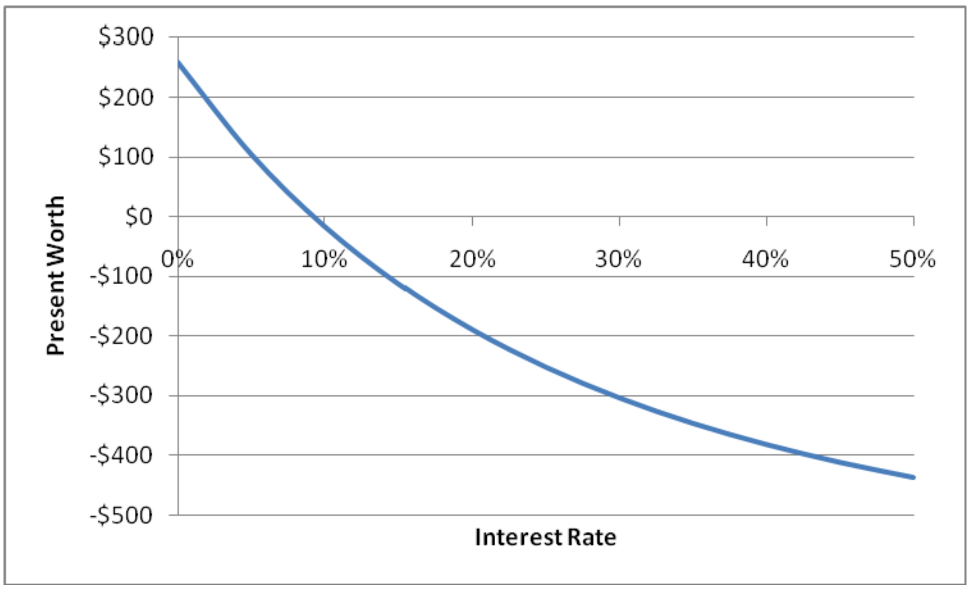


This is a typical PW graph for an investment.

7-27

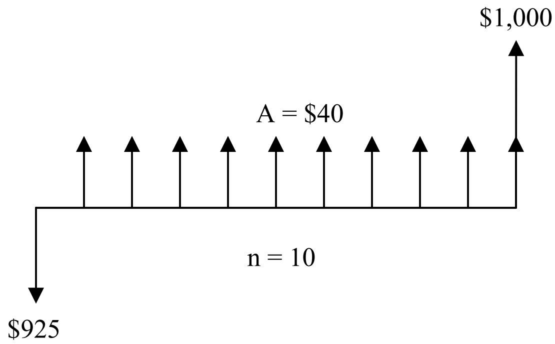
Year	Cash Flow		i	PW
0	–640		0%	$260.00
1	0		5%	$105.34
2	100		10%	–$15.91
3	200		15%	–$112.20
4	300		20%	–$189.58
5	300		25%	–$252.42
			30%	–$303.96
			35%	–$346.62
			40%	–$382.22
			45%	–$412.16
			50%	–$437.53

PW = –640 + 100*(P/G, i, 5) – 100*(P/F, i, 5) ; use NPV for years 1–5 in Excel.



Yes, this is the typical graph for an investment.

7-28

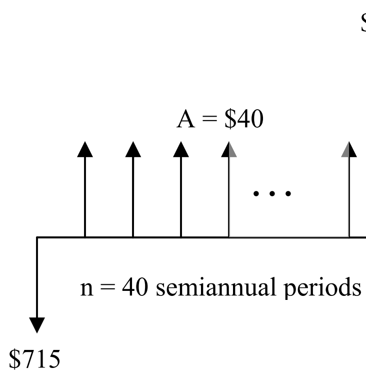


PW of Cost = PW of Benefits
\$925 = \$40 (P/A, i%, 10) + \$1,000 (P/F, i%, 10)

Try i = 5%
\$925 = \$40 (7.722) + \$1,000 (0.6139) = \$922.78 (i too high)

Try i = 4.5%
\$925 = \$40 (7.913) + \$1,000 (0.6439) = \$960.42 (i too low)
i^* = <u>4.97%</u>

7-29



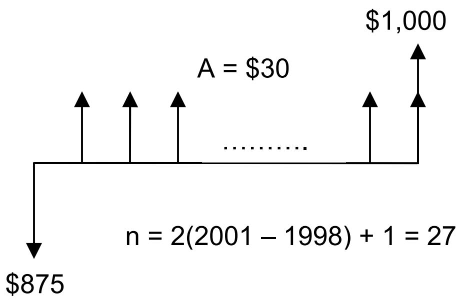
PW of Benefits – PW of Costs = 0
\$20 (P/A, i%, 40) + \$1,000 (P/F, i%, 40) − \$715 = 0

Try i = 3%
\$20 (23.115) + \$1,000 (0.3066) − \$715 = \$53.90 i too low

Try i = 3.5%
\$20 (21.355) + \$1,000 (0.2526) − \$715 = −\$35.30 i too high

Performing linear interpolation:
i^* = 3% + (0.5%) [53.90/(53.90 − (−35.30))] = 3.30%
Nominal i^* = <u>6.60%</u>

7-30



PW of Benefits – PW of Cost = $0
$30 (P/A, i%, 27) + $1,000 (P/F, i%, 27) − $875 = $0

Try i = 3 ½%
$30 (17.285) + $1,000 (0.3950) − $875 = $38.55 >$0

Try i = 4%
$30 (16.330) + $1,000 (0.3468) − $875 = −$38.30 < $0

$i^* = \underline{3.75\%}$
Nominal rate of return = 2 (3.75%) = 7.5%

7-31

(a) For the cash flow of the bond have i = $\frac{6.8\%}{2}$ = 3.4%, so (0.034) (1000) = $34 is paid semiannually and $1,000 is paid at the end of the 10th year (20th pay period).

NPW = 0 = +1000 – 34 (P/A, i, 20) – 1000 (P/F, i, 20) and interpolating
i = $3\% + (0.5\%)\left[\frac{59.518}{59.518 + 14.192}\right]$ = 3.404% (exact value = 3.400%),
r = (2) (3.404%) = 6.808%, and $i_a = (1 + 0.03404)^2 - 1 = 0.06924$ or 6.924%.

(b) The fee is $1,000 x 0.0075 = $7.50. So ABC Corp. receives $1,000 – $7.50 = $992.50.

NPW = 0 = 992.5 – 34 (P/A, i, 20) – 1000 (P/F, i, 20) and interpolating
i = $3\% + (0.5\%)\left[\frac{67.018}{67.018 + 6.692}\right]$ = 3.4546% (exact value = 3.453%),
r = (2) (3.4546%) = 6.909%, and $i_a = (1 + 0.034546)^2 - 1 = 0.07029$ or 7.029%.

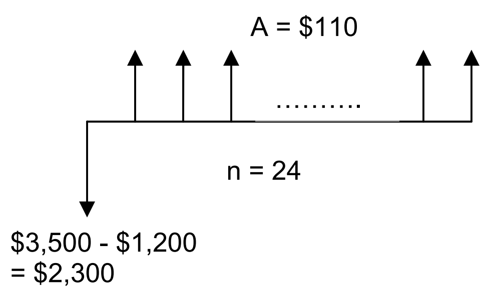
7-32

(a) NPW = 0 = –3118 + 10000 (P/F, i, 20), so, (P/F, i, 20) = 0.3118. Next you can solve $(1+i)^{-20} = 0.3118$ for i or look in the tables to find i = 0.06 or 6.0%. Next, because it is paid annually, the effective annual interest rate is 6.0%.

(b) The fee is \$10,000 x 0.01 = \$100. So ABC Corp. receives \$3,118 – \$100 = \$3,018.

NPW = 0 = 3018 – 10000 (P/F, i, 20), so, (P/F, i, 20) = 0.3018. Next solve $(1+i)^{-20} = 0.3018$ and find i = 0.06173 or 6.173%. As above i_a = 6.173%.

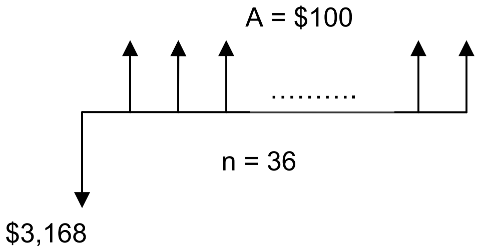
7-33



\$2,300 = \$110 (P/A, i%, 24)
(P/A, i%, 24) = \$2,300/\$110 = 20.91

From tables: 1 % < i < 1.25%
On Financial Calculator: i = 1.13% per month
Effective interest rate = $(1 + 0.0113)^{12} - 1$ = 0.144 = 14.4%

7-34



PW of Cost = PW of Benefits
$100 (P/A, i%, 36) = $3,168
(P/A, i%, 36) = $3,168/$100 = 31.68

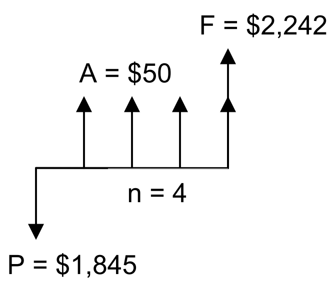
Performing Linear Interpolation:

(*P/A,* 1%, 36)	*i*
32.871	½%
31.447	¾%

i^* = (1/2%) + (1/4%) [(32.87 − 31.68)/(32.87 − 31.45)] = <u>0.71%</u>

Nominal Interest Rate = 12 (0.71%) = <u>8.5%</u>

7-35



Set PW of Cost = PW of Benefits
$1,845 = $50 (P/A, i%, 4) + $2,242 (P/F, i%, 4)

Try i = 7%
450 (3.387) + $2,242 (0.7629) = $1,879 > $1,845

Try i = 8%
450 (3.312) + $2,242 (0.7350) = $1,813 < $1,845
Rate of Return = 7% + (1%) [($1,879 − $1,845)/($1,879 − $1,813)]
= <u>7.52% for 6 months</u>

Nominal annual rate of return = 2 (7.52%) = 15.0%

Equivalent annual rate of return = $(1 + 0.0752)^2 - 1$ = 15.6%

Performing Linear Interpolation:

(*P/A,* 1%, 36)	*i*
32.871	½%
21.447	¾%

i^* = (1/2%) + (1/4%) [(32.87 − 31.68)/(32.87 − 31.45)]
= 0.71%

Nominal Interest Rate = 12 (0.71%) = 8.5%

7-36

(a) The monthly payments are $\frac{6000}{36} = \$166.67$ (over 3 years).

NPW = 0 = 6000 – 250 – 166.67 (P/A, i, 36), so, (P/A, i, 36) = 34.50 . The tables don't go to a low enough interest rate so must solve:

$\left[\frac{(1+i)^{36}-1}{i(1+i)^{36}}\right] = 34.50$ by trial and error or Excel using the IRR function. Excel yields

i = 0.00232, so, $i_a = (1 + 0.00232)^{12} - 1$ = 0.0282 or 2.82%.

(b) The fact that the dealer would accept $5,200 cash for the car indicates its true worth so the extra $800 is a hidden finance charge. Your payments are still based on the original $6,000 cost but you only receive a car worth only $5,200!

NPW = 0 = 5200 – 250 – 166.67 (P/A, i, 36), so, (P/A, i, 36) = 29.70 and interpolating

i = 1% + (0.25%) $\left[\frac{30.107-29.70}{30.107-28.847}\right]$ = 1.081% (exact value = 1.079%), so,
$i_a = (1 + 0.01081)^{12} - 1$ = 13.77% (exact value = 13.75%).

7-37

(a) The foregone cash rebate is like a hidden finance charge. You pay $12,000 for the car but receive a car only worth $12,000 – $3,000 = $9,000. The monthly payments = $\frac{12000}{4\times 12}$ = $250 for 48 months. NPW = 0 = 9000 – 250 (P/A, i, 48), so (P/A, i, 48) = 36.0 and interpolating

i = 1% + (0.25%) $\left[\frac{37.974-36.0}{37.974-35.932}\right]$ = 1.242%, so r = (12) (1.242%) = 14.90% and i_a = (1 + 0.01242)^{12} –1 = 0.15965 or 15.97%.

(b) Worth of car = Cost – Rebate = $18,000 – $3,000 = $15,000.
The monthly payments = $\frac{18000}{4\times 12}$ = $375 for 48 months.
NPW = 0 = 15,000 – 375 (P/A, i, 48), so, (P/A, i, 48) = 40.0 and interpolating
i = 0.75% + (0.25%) $\left[\frac{40.185-40.0}{40.185-37.974}\right]$ = 0.771%, so r = (12)(0.771%) = 9.65%
and i_a = (1 + 0.00771)^{12} –1 = 0.0965 or 9.65%.

(c) Worth of car = Cost – Rebate = $24,000 – $3,000 = $21,000.
The monthly payments = $\frac{24000}{4\times 12}$ = $500 for 48 months.
NPW = 0 = 21000 – 500 (P/A, i, 48), so, (P/A, i, 48) = 42.0 and interpolating
i = 0.50% + (0.25%) $\left[\frac{42.580-42.0}{42.580-40.185}\right]$ = 0.561%, so r = (12)(0.561%) = 6.73%
and i_a = (1 + 0.00561)^{12} –1 = 0.0694 or 6.94%.

7-38

First determine the monthly payments for the loan where i = $\frac{4\%}{12}$ = 0.3333%, so

A = 6,000 (A/P, 0.3333%, 36) = $\left[\frac{(0.003333)(1+0.003333)^{36}}{(1+0.003333)^{36}-1}\right]$ = $177.14 .

(a) NPW = 0 = 6000 – 250 – 177.14 (P/A, i, 36), so, (P/A, i, 36) = 32.46 and interpolating
i = 0.50% + (0.25%) $\left[\frac{32.871-32.46}{32.871-31.447}\right]$ = 0.572%, so
r = (12) (0.572%) = 6.86% and i_a = (1 + 0.00572)^{12} −1 = 0.0709 or 7.09%.

(b) Worth of the car = \$6,000 − \$800 = \$5,200 but the payments are determined by the actual cost to buyer, here \$6,000. Thus, the payments are the same as above.

NPW = 0 = 5200 – 250 – 177.14 (P/A, i, 36), so, (P/A, i, 36) = 27.944 and interpolating i = 1.25% + (0.25%) $\left[\frac{28.847 - 27.944}{28.847 - 27.661}\right]$ = 1.440%, so,
r = (12) (1.440%) = 17.28% and $i_a = (1 + 0.01440)^{12} - 1$ = 0.1872 or 18.72%.

(c) The actual value of the car seems to be the most important factor!

7-39

The amount of cash paid will be \$75,000 – \$50,000 = \$25,000 with \$50,000 financed, so, the monthly payments will be 50,000 (A/P, 8%, 4) = (50,000) (0.3019) = \$15,095. The reduction in cost if one pays entirely in cash is \$75,000 x 0.10 = \$7,500, so, a 100% cash payment would be \$75,000 − \$7,500 = \$67,500 (true value of equipment).

Year	Pay Cash	Borrow from Manufacturer	Incremental Difference
0	–\$67,500	−\$25,000	−\$42,500
1		−15,095	15,095
2		−15,095	15,095
3		−15,095	15,095
4		−15,095	15,095

NPW = 0 = –42500 + 15095 (P/A, IRR, 4), so (P/A, IRR, 4) = 2.816. Interpolating IRR = 15% + (3%) $\left[\frac{2.855 - 2.816}{2.855 - 2.690}\right]$ = 15.72%.

7-40

The loan value is \$120,000 − \$12,000 (10% down payment) = \$108,000. The loan origination fee is \$108,000 x 0.02 = \$2,160, so the loan becomes \$108,000 + \$2,160 = \$110,160.

(a) Number of months is 30 x 12 = 360. The monthly interest rate, i = $\frac{6\%}{12}$ = 0.5%.

The monthly payment = 110,160 (A/P, 0.5%, 360) but to get accuracy use

$$\frac{110160}{(P/A, 0.5\%, 360)} = \frac{110160}{166.792} = \$660.46$$.

(b) The actual value received is \$108,000, thus, to find the effective interest rate solve

NPW = 0 = 108,000 – 660.46 (P/A, i, 360).

(P/A, i, 360) = $\frac{108000}{660.46}$ = 163.522. Interpolating

i_{mo} = ½% + (¼%)[(163.522 – 166.792)/(124.282 – 166.792]
= 0.51923% per month

i_a = $(1 + 0.0051923)^{12}$ – 1 = 0.0641 or 6.41%

(c) In ten years there are still 20 years left on the original loan, so, value of remaining loan at year ten
= 660.46 (P/A, 0.5%, 240) = (660.46)(139.581)
= \$92,187.67. To find the effective interest rate solve

NPW = 108,000 – 660.46 (P/A, i, 120) – 92,187.67 (P/F, i, 120). Interpolating

i_{mo} = ½% + (¼%)[2156.62/(2156.62 + 18258.62]
= 0.5264% (exact value 0.5236%)

i_a = $(1 + 0.005264)^{12}$ – 1 = 0.0650 or 6.50% (exact value 6.467%)

(d) In three years there are still 27 years left on the original loan, so, value of remaining loan at year three = 660.46 (P/A, 0.5%, 324)

$$= 660.46\left[\frac{(1+0.005)^{324}-1}{0.005(1+0.005)^{324}}\right] = (660.46)(160.26) = \$105,845 .$$

NPW = 108,000 – 660.46 (P/A, i, 36) – 105,845 (P/F, i, 36) . Interpolating

i_{mo} = ½% + (¼%)[2154.06/(2154.06 + 6,354.35)]
= 0.5633% (exact value 0.5614%)

i_a = $(1 + 0.005633)^{12}$ – 1 = 0.0697 or 6.97% (exact value 6.949%)

7-41

\$2,000 = \$91.05 (P/A, i^*, 30)
(P/A, i^*, 30) = \$2,000/\$91.05 = 21.966

(*P/A,* 1%, 30)	*i*
22.396	2
20.930	2½

i_{mo} = 2% + (½%) [(22.396 − 21.966)/(22.396 − 20.930)]
= <u>2.15% per month</u>

Nominal ROR received by finance company = 12 (2.15%) = <u>25.8%</u>

7-42

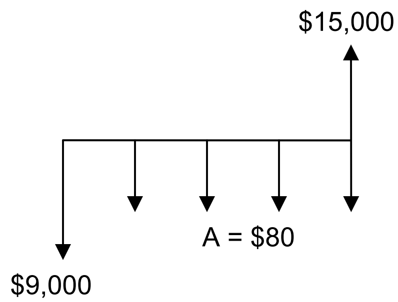
$\$3,000 = \$118.90\ (P/A, i^*, 36)$
$(P/A, i^*, 36) = \$3,000/\$118.90 = 26.771$

(P/A, i%, 36)	*i*
27.661	1½%
26.543	1¾%

$i_{mo} = 1½\% + ¼\%\ [(27.661 − 26.771)/(27.661 − 26.543)]$
$= \underline{1.699\% \text{ per month}}$

Nominal Annual ROR = 12 (1.699%) = $\underline{20.4\%}$

7-43



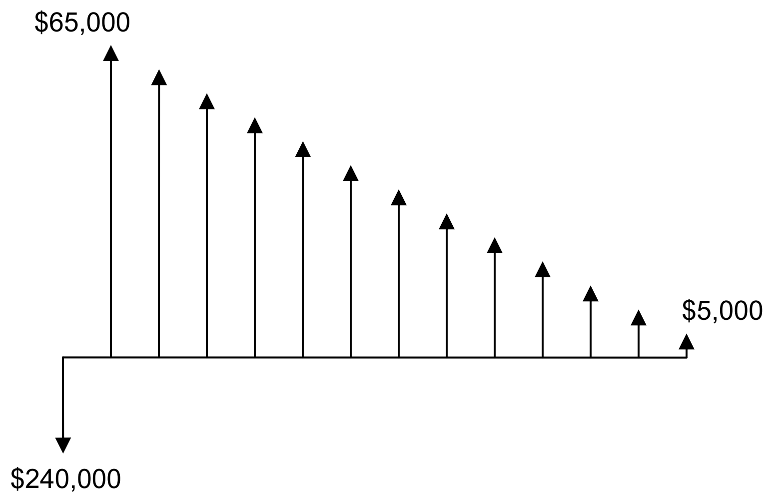
PW of Benefits – PW of Cost = $0
$\$15,000\ (P/F, i\%, 4) − \$9,000 − \$80\ (P/A, i\%, 4) = \$0$

Try i = 12%
$\$15,000\ (0.6355) − \$9,000 − \$80\ (3.037) = +\$289.54$

Try i = 15%
$\$15,000\ (0.5718) − \$9,000 − \$80\ (2.855) = −\$651.40$

Performing Linear Interpolation:
$i^* = 12\% + (3\%)\ [289.54/(289.54 + 651.40)]$
$= \underline{12.92\%}$

7-44



$240,000 = $65,000 (P/A, i%, 13) − $5,000 (P/G, i%, 13)

Try i = 15%
$65,000 (5.583) −$5,000 (23.135) = $247,220 > $240,000

Try i = 18%
$65,000 (4.910) −$5,000 (18.877) = $224,765 < $240,000

Rate of Return = 15% + 3% [($247,220 − $240,000)/($247,220 − $224,765)]
= <u>15.96%</u>

7-45

(a) Total Annual Revenues = $500 (12 months) (4 apt.) = $24,000
Annual Revenues – Expenses = $24,000 − $8,000 = $16,000
To find Internal Rate of Return the Net Present Worth must be $0.
NPW = $16,000 (P/A, i^*, 5) + $160,000 (P/F, i^*, 5) − $140,000

At i = 12%, NPW = $8,464
At i = 15%, NPW = −$6,816

IRR = 12% + (3%) [$8,464/($8,464 + $6,816)]
= <u>13.7%</u>

(b) At 13.7% the apartment building is more attractive than the other options.

7-46

NPW = −\$300,000 + \$20,000 (P/F, i^*, 10)
+ (\$67,000 − \$3,000) (P/A, i^*, 10) − \$600 (P/G, i^*, 10)

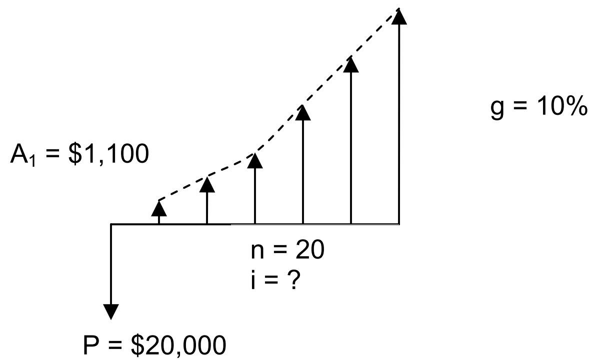
Try i = 10%
NPW = −\$300,000 + \$20,000 (0.3855) + (\$64,000) (6.145)
− \$600 (22.891)
= \$87,255 > \$0
The interest rate is too low.

Try i = 18%
NPW = −\$300,000 + \$20,000 (0.1911) + (\$64,000) (4.494)
− \$600 (14.352)
= −\$17,173 < \$0
The interest rate is too high.

Try i =15%
NPW = −\$300,000 + \$20,000 (0.2472) + (\$64,000) (5.019)
− \$600 (16.979)
= \$9,130 > \$0

Thus, the rate of return (IRR) is between 15% and 18%. By linear interpolation:
i^* = 15% + (3%) [\$9,130/(\$9,130 − \$17,173)]
= <u>16.0%</u>

7-47



The payment schedule represents a geometric gradient.
There are two possibilities:
$i \neq g$ and $i = g$

Try the easier i = g computation first:
$P = A_1 n (1 + i)^{-1}$, where g = i = 0.10
$\$20,000 = \$1,100 (20) (1.10)^{-1} = \$20,000$
Rate of Return $i^* = g =$ <u>10%</u>

7-48

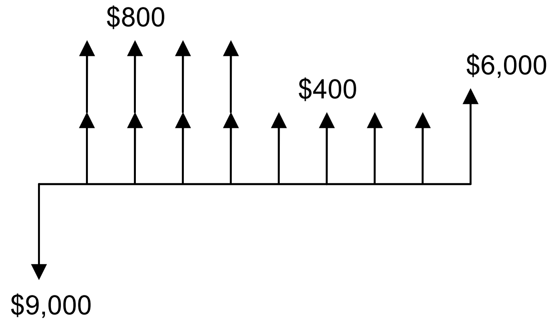
(a) When n = ∞, i = A/P = $3,180/$100,000 = <u>3.18%</u>

(b) (A/P, i%, 100) = $3,180/$100,000 = 0.318
From interest tables, i^* = <u>3%</u>

(c) (A/P, i%, 50) = $3, 180/$100,000 = 0.318
From interest tables, i^* = <u>2%</u>

(d) The saving in water truck expense is just a small part of the benefits of the pipeline. Convenience, improved quality of life, increased value of the dwellings, etc., all are benefits. Thus, the pipeline appears justified.

7-49



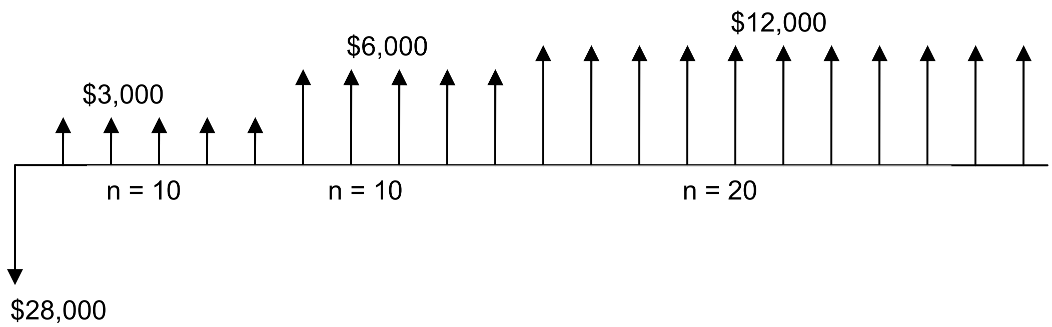
Year	Cash Flow
0	−$9,000
1−4	+$800
5−8	+$400
9	+$6,000

PW of Cost = PW of Benefits
$9,000 = $400 (P/A, i%, 8) + $400 (P/A, i%, 4) + $6,000 (P/F, i%, 9)

Try i = 3%
$400 (7.020) + $400 (3.717) + $6,000 (0.7664) = $8,893 < $9,000

Try i = 2 ½%
$400 (7.170) + $400 (3.762) + $6,000 (0.8007) = $9,177 > $9,000
Rate of Return = 2½% + (1/2%) [($9,177 − $9,000)/($9,177 − $8,893)]
= <u>2.81%</u>

7-50



PW of Cost = PW of Benefits
\$28,000 = \$3,000 (P/A, i%, 10) + \$6,000 (P/A, i%, 10) (P/F, i%, 10) +
\$12,000 (P/A, i%, 20) (P/F, i%, 20)

Try i = 12%
\$3,000 (5.650) + \$6,000 (5.650) (0.3220) + \$12,000 (7.469) (0.1037)
= \$37,160 > \$28,000

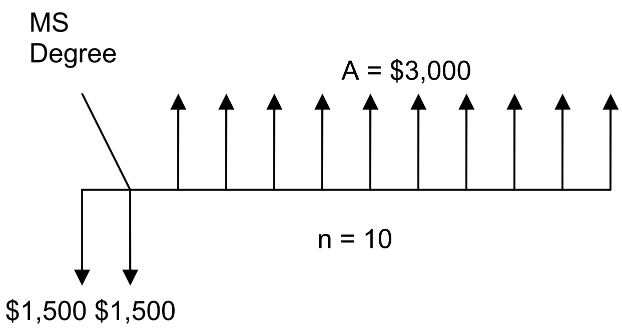
Try i = 15%
\$3,000 (5.019) + \$6,000 (5.019) (0.2472) + \$12,000 (6.259) (0.0611)
= \$27,090 < \$28,000

Performing Linear Interpolation:
i^* = 15% − (3%) [(\$28,000 − \$27,090)/(\$37,160 − \$27,090)]
= 15% − (3%) (910/10,070)
= <u>14.73%</u>

7-51

This is a thought-provoking problem for which there is no single answer. Two possible solutions are provided below.

(a) Assuming the MS degree is obtained by attending graduate school at night while continuing with a full-time job:



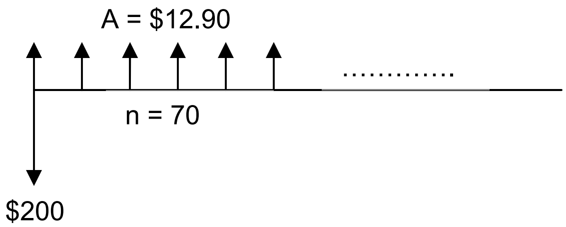
Cost: $1,500 per year for 2 years
Benefit: $3,000 per year for 10 years
Computation as of award of MS degree:
$1,500 (F/A, i%, 2) = $3,000 (P/A, i%, 10)
$\underline{i^* > 60}$

(b) Assuming the MS degree is obtained by one of year of full-time study
Cost: Difference between working & going to school. Whether working or at school there are living expenses. The cost of the degree might be $24,000.
Benefit: $3,000 per year for 10 years
$24,000 = $3,000 (P/A, i%, 10)
$\underline{i^* = 4.3\%}$

7-52

The problem requires an estimate for n, the expected life of the infant. Seventy or seventy-five years might be the range of reasonable estimates. Here we will use 71 years.

The purchase of a $200 life subscription avoids the series of *beginning-of-year* payments of $12.90. Based on 71 beginning-of-year payments,



$200 – $12.90 = $12.90 (P/A, i%, 70)
(P/A, i%, 70) = $187.10/$ 12.90 = 14.50
<u>6% < i^* < 8%. By Calculator: i^* = 6.83%</u>

7-53

Assumptions:
1. The market value of the car for the seller is $39,264 (the sticker price of 43,658 is irrelevant). By the buyer taking possession of it, the seller is foregoing the opportunity to receive $39,264 for the vehicle.
2. The car is not driven more than 36,000 miles during the 36 months
3. Payments begin are due at the beginning of period.
3. The purchaser buys the vehicle at the end of the lease period for $27,854.
4. Cash flow is from the perspective of the seller.

Month	Case 1 (incl. Deposit)
0	−$39,264.00 + 599 + 625 = $38,040
1	+$599.00
2	+$599.00
3	+$599.00
4	+$599.00
5	+$599.00
6	+$599.00
7	+$599.00
8	+$599.00
9	+$599.00
10	+$599.00
11	+$599.00
12	+$599.00
…	+$599.00

33	+$599.00
34	+$599.00
35	+$599.00
36	+$27,854.00 −$625.00 = +$27,229.00

IRR = 0.86%
Nominal IRR = 10.32%
Effective IRR =10.83%

7-54

The number of months between August 15 and January 15 is 5.

Month	Annual Permit	Semester Permit
0	–$100	–$65
1	0	0
2	0	0
3	0	0
4	0	0
5	0	–65

To solve for the monthly interest rate set the two PWs equal to each other, so,
–100 = –65 – 65 (P/F, i, 5) . Thus, $(1+i)^{-5} = \frac{100-65}{65} = 0.53846$.
Solving get i = 0.1318 or 13.18% and i_a = $(1 + 0.1318)^{12}$ – 1 = 3.418 or 342%.
Unless the student is graduating in January or just doesn't have the $100, it is clearly better to buy the permit a year at a time.

7-55

Details will vary by university, but is solved like Problem 7-54.

7-56

Quarter	Annual Payment	Quarter Payment
0	–$65,000	–$18,000
1	0	–18,000
2	0	–18,000
3	0	–18,000

To solve for the monthly interest rate, set the two PWs equal to each other, so –65,000 = –18,000 – 18,000 (P/A, i, 3) . Thus, (P/A, i, 3) = 2.611 and interpolating

$i = 7\% + (1\%) \left[\frac{2.624 - 2.611}{2.624 - 2.577}\right] = 7.28\%$, so r = 4 x 0.0728 = 0.2912 or 29.1% and

$i_a = (1 + 0.0728)^4 - 1 = 0.3246$ or 32.5%. This is a high rate of return, but some firms use an even higher hurdle rate for projects.

7-57

–$65,000 = –$18,000(1 + (P/A, i , 3))

The amount that the series of future payments is worth is
–65,000 + 18,000 = –47,000 = –18,000*(P/A, i , 3)

Using the end-of-period designation (default) in RATE (Excel) yields
RATE(3,18,000,–47,000) = 7.2766%

One could also solve with quarterly payments at the beginning of the period:
RATE(4,18,000,–65,000,0,1) = 7.2766%

7-58

Insurance payments must be paid in advance, here on the first of the month or year.

Month	Annual Basis	Monthly Basis
0	–$1,650	–$150
1	0	–150
2	0	–150
3	0	–150
4	0	–150
5	0	–150
6	0	–150
7	0	–150
8	0	–150
9	0	–150
10	0	–150
11	0	–150

To solve for the monthly interest rate set the PWs of the two cash flows equal to each other. Thus, −1650 = −150 – 150 (P/A, i, 11), so, (P/A, i, 11) = 10.0.
Interpolating
i = 1.5% + (0.25%) $\left[\dfrac{10.071-10.0}{10.071-9.928}\right]$ = 1.624%. Next, i_a = (1 + 0.01624)^{12} −1 = 0.2133 or 21.3%. This is a relatively high rate of return, but the student might prefer to pay monthly if there is a significant chance of wrecking the car before the year is up.

7-59

Details will vary by student, but solved like Problem 7-58.

7-60

Year	A	B	(B– A)
0	−$2,000	−$2,800	−$800
1−3	+$800	+$1,100	+$300
Computed ROR	9.7%	8.7%	6.1%

The rate of return on the increment (B – A) exceeds the Minimum Attractive Rate of Return (MARR), therefore the higher cost alternative B should be selected.

7-61

Year	X	Y	X – Y
0	−$100	−$50	−$50
1	+$35	+$16.5	+$18.5
2	+$35	+$16.5	+$18.5
3	+$35	+$16.5	+$18.5
4	+$35	+$16.5	+$18.5
Computed ROR	15.0%	12.1%	17.8%

The ΔROR on X – Y is greater than 10%. Therefore, the increment is desirable.
<u>Select X.</u>

7-62

Year	A	B	(B – A)
0	−$100.00	−$50.00	−$50.00
1–10	+$19.93	+$11.93	+$8.00
Computed ROR	15%	20%	9.61%

ΔROR = 9.61% > MARR.
<u>Select A.</u>

7-63

Year	X	Y	X– Y
0	−\$5,000	−\$5,000	\$0
1	−\$3,000	+\$2,000	−\$5,000
2	+\$4,000	+\$2,000	+\$2,000
3	+\$4,000	+\$2,000	+\$2,000
4	+\$4,000	+\$2,000	+\$2,000
Computed ROR	16.9%	21.9%	9.7%

Since X – Y difference between alternatives is desirable, <u>select Alternative X.</u>

7-64

(a) Present Worth Analysis – Maximize NPW

NPW_A = \$746 (P/A, 8%, 5) − \$2,500
= \$746 (3.993) − \$2,500 = +\$479

NPW_B = \$1,664 (P/A, 8%, 5) − \$6,000 = +\$644
<u>Select B.</u>

(b) Annual Cash Flow Analysis – Maximize (EUAB – EUAC)

$(EUAB – EAUC)_A$ = \$746 − \$2,500 (A/P, 8%, 5)
= \$746 − \$2,500 (0.2505)
= +\$120

$(EUAB − EUAC)_B$ = \$1,664 − \$6,000 (A/P, 8%, 5)
= +\$161
<u>Select B.</u>

(c) Rate of Return Analysis: Compute the rate of return on the B – A increment of investment and compare to 8% MARR.

Year	A	B	B – A
0	−\$2,500	−\$6,000	−\$3,500
1−5	+\$746	+\$1,664	+\$918

\$3,500 = \$918 (P/A, i%, 5)

Try i = 8%, \$918 (3.993) = \$3,666 > \$3,500

Try i = 10%, \$918 (3.791) = \$3,480 < \$3,500

Δ Rate of Return = 9.8%
Since ΔROR > MARR, B – A increment is desirable. <u>Select B.</u>

7-65

Using incremental analysis, computed the internal rate of return for the difference between the two alternatives.

Year	A–B
0	–$9,000
1	$3,000
2	$3,000
3	$3,000
4	$3,000
5	$3,000
6	$3,000
7	$3,000
8	$1,200

Note: Internal Rate of Return (IRR) equals the interest rate that makes the PW of costs minus the PW of Benefits equal to zero.

$9,000 – $3,000 (P/A, i^*, 7) − $1,200 (P/F, i^*, 8) = $0
Try i = 25%
$9,000 − $3,000 (3.161) − $1,200 (0.1678) = −$684.36 < $0
Try i = 30%
$9,000 − $3,000 (2.802) − $1,200 (0.1226) = $446.88 > $0
i^* = 25% + (5%) [$684.36/($446.88 + $684.36)]
= <u>28.0% (actual value is 27.9%)</u>

<u>The contractor should choose Alternative A and lease because 28% > 15% MARR.</u>

7-66

	B	A	A– B
First Cost	$300,000	$615,000	$315,000
Maintenance & Operating Costs	$25,000	$10,000	−$15,000
Annual Benefit	$92,000	$158,000	$66,000
Salvage Value	−$5,000	$65,000	$70,000

NPW = −$315,000 + [$66,000 − (−$15,000)] (P/A, i^*, 10) + $70,000 (P/F, i^*, 10) = $0
Try i = 15%
−$315,000 + [$66,000 − (−$15,000)] (5.019) + $70,000 (0.2472) = $108,840
ΔROR > MARR (15%)

<u>The higher cost alternative A is the more desirable alternative.</u>

7-67

Year	(A) Gas Station	(B) Ice Cream Stand	(B – A)
0	−$80,000	−$120,000	−$40,000
1−20	+$8,000	+$11,000	+$3,000
Computed ROR	7.75%	6.63%	4.22%

The rate of return in the incremental investment (B – A) is less than the desired 6%. In this situation the lower cost alternative (A) Gas Station should be selected.

7-68

MARR = 5%   *P* = $30,000 *n* = 35 years

Alternative 1: Withdraw $15,000 today and lose $15,000
Alternative 2: Wait, leave your fund in the system until retirement.
Equivalency seeks to determine what future amount is equal to $15,000 now.

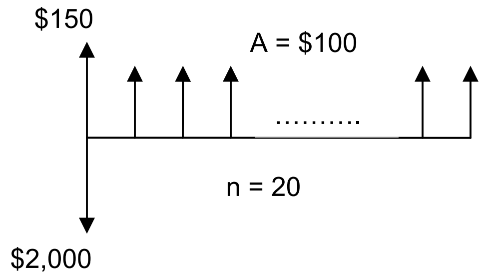
$F = P(1 + i)^n$
$= \$30,000\ (1.05)^{35}$
$= \$30,000\ (5.516015)$
$= \$165,480.46$

Therefore:
$\$15,000 = \$165,480.46\ (1 + i)^{-35}$
$\$15,000\ (1 + i)^{35} = \$165,480.46$
$(1 + i) = [(165,480.46/\$15,000)]^{1/35}$
$i = 1.071 - 1 = \underline{7.1002\%} > 5\%$

Unless $15,000 can be invested with a return higher than 7.1%, it is better to wait for 35 years for the retirement fund. $15,000 now is only equivalent to $165,480.46 35 years from now if the interest rate now is 7.1% instead of the quoted 5%.

7-69

(a)



($2,000 − $150) = $100 (P/A, i%, 20)
(P/A, i%, 20) = $1,850/$100 = 18.5
I = ¾% per month
The alternatives are equivalent at a nominal 9% annual interest.

(b) Take Alt 1— the $2,000 — and invest the money at a higher interest rate.

7-70

(a) Salvage = 0.15 x $380,000 = $57,000 and firm's interest rate = 12%.

Year	Purchase	Lease	Purchase – Lease
0	–$380,000	–$60,000	–$320,000
1	0	–60,000	60,000
2	0	–60,000	60,000
3	0	–60,000	60,000
4	0	–60,000	60,000
5	0	–60,000	60,000
6	57,000	0	57,000

NPW = 0 = –320,000 + 60,000 (P/A, IRR , 5) + 57,000 (P/A, IRR, 6) and interpolating

IRR = 3% + (0.5%) $\left[\frac{2538}{2538+2730}\right]$ = 3.24% (also 3.24% from Excel). The IRR is well below the firm's interest rate on the borrowed amount ($320,000) from leasing, so lease the bulldozer.

(b) The firm receives $65,000 more than it spends on operating and maintenance costs.

Year	Purchase	Lease	Purchase – Lease
0	–$380,000	–$60,000	–$320,000
1	65,000	–60,000	125,000
2	65,000	–60,000	125,000
3	65,000	–60,000	125,000
4	65,000	–60,000	125,000
5	65,000	–60,000	125,000
6	65,000 57,000	0	122,000

NPW = 0 = –320,000 + 125,000 (P/A, IRR, 5) + 122,000 (P/F, IRR, 6) and interpolating

IRR = 30% + (5%) $\left[\frac{9778}{9778+22346}\right]$ = 31.5% (31.42% from Excel). Clearly, the situation has changed. The interest rate on the borrowed amount is now well above the firm's interest rate, so, buy the bulldozer. The rate of return for the bulldozer will clearly be largest for this cash flow and is given by

PW = 0 = −380,000 + 65,000 (P/A, ROR, 6) + 57,000 (P/F, ROR, 6) and interpolating

ROR = 4% + (0.5%) $\left[\frac{5777}{5777+960}\right]$ = 4.43% (4.43% from Excel).

Note that the author has failed to give a practical scenario for how the $65,000 benefit can be realized if the bulldozer is purchased instead of leased!

7-71

(a) Salvage = $50,000 and community's interest rate = 8%.

Year	Purchase	Lease	Purchase – Lease
0	–$480,000	–$70,000	–$410,000
1	0	–70,000	70,000
2	0	–70,000	70,000
3	0	–70,000	70,000
4	0	–70,000	70,000
5	0	–70,000	70,000
6	0	–70,000	70,000
7	0	–70,000	70,000
8	0	–70,000	70,000
9	0	–70,000	70,000
10	50,000	0	50,000

NPW = 0 = −410,000 +70,000 (P/A, IRR , 9) + 50,000 (P/A, IRR, 10) and interpolating

IRR = 10% + (2%) $\left[\frac{12405}{12405+20940}\right]$ = 10.74% (10.71% Excel). The IRR is above the community's interest rate on the borrowed amount ($410,000) from leasing, so buy the generator.

(b) The community spends $80,000 less on fuel and maintenance than it spends on buying power.

Year	Purchase	Lease	Purchase – Lease
0	–$480,000	–$70,000	–$410,000
1	80,000	–70,000	150,000
2	80,000	–70,000	150,000
3	80,000	–70,000	150,000
4	80,000	–70,000	150,000
5	80,000	–70,000	150,000
6	80,000	–70,000	150,000
7	80,000	–70,000	150,000
8	80,000	–70,000	150,000
9	80,000	–70,000	150,000
10	80,000 50,000	0	130,000

NPW = 0 = −410,000 + 150,000 (P/A, IRR, 9) + 130,000 (P/F, IRR, 10) and interpolating

IRR = 30% + (5%) $\left[\frac{52275}{52275+3789}\right]$ = 34.66% (34.63% from Excel). The interest rate on the borrowed amount is now well above the firm's interest rate, so buy the generator. The rate of return for the generator will clearly be largest for this cash flow and is given by

PW = 0 = −480,000 + 80,000 (P/A, ROR, 10) + 50,000 (P/F, ROR, 10) and interpolating

ROR = 10% + (2%) $\left[\frac{30875}{30875+11900}\right]$ = 11.44% (11.42% from Excel).

Note that the author has failed to give a practical scenario for how the $80,000 benefit can be realized if the generator is purchased instead of leased!

7-72

Year	A	B	A– B	NPW at 7%	NPW at 9%
0	−$9,200	−$5,000	−$4,200	−$4,200	−$4,200
1	+$1,850	+$1,750	+$100	+$93	+$92
2	+$1,850	+$1,750	+$100	+$87	+$84
3	+$1,850	+$1,750	+$100	+$82	+$77
4	+$1,850	+$1,750 −$5,000	+$5,100	+$3,891	+$3,613
5	+$1,850	+$1,750	+$100	+$71	+$65
6	+$1,850	+$1,750	+$100	+$67	+$60
7	+$1,850	+$1,750	+$100	+$62	+$55
8	+$1,850	+$1,750	+$100	+$58	+$50
			Sum	+$211	−$104

<u>Δ ROR ≈ 8.3%</u>
<u>Choose Alternative A.</u>

7-73

Year	Zappo	Kicko	Kicko − Zappo
0	−$56	−$90	−$34
1	−$56	$0	+$56
2	$0	$0	$0

Compute the incremental rate of return on (Kicko − Zappo)
PW of Cost = PW of Benefit
$34 = $56 (P/F, i%, 1)
(P/F, i%, 1) = $34/$56 = 0.6071

From interest tables, incremental rate of return > 60% (ΔROR = 64.7%), hence the increment of investment is desirable.
Buy Kicko.

7-74

Year	A	B	A– B
0	−$9,200	−$5,000	−$4,200
1	+$1,850	+$1,750	+$100
2	+$1,850	+$1,750	+$100
3	+$1,850	+$1,750	+$100
4	+$1,850	+$1,750 −$5,000	+$100 +$5,000
5	+$1,850	+$1,750	+$100
6	+$1,850	+$1,750	+$100
7	+$1,850	+$1,750	+$100
8	+$1,850	+$1,750	+$100
			Sum

Rates of Return
A: $9,200 = $1,850 (P/A, i%, 5)
Rate of Return = 11.7%

B: $5,000 = $1,750 (P/A, i%, 4)
Rate of Return = 15%

A − B: $4,200 = $100 (P/A, i%, 8) + $5,000 (P/F, i%, 4)
$\Delta ROR_{A\text{-}B}$ = 8.3%
Select A.

7-75

Year	A	B	A – B
0	−$150	−$100	−$50
1- 10	+$25	+$22.25	+$2.75
11- 15	+$25	$0	+$25
15	+$20	$0	+$20
Computed ROR	14.8%	18%	11.6%

Rate of Return (A– B):
$50 = $2.75 (P/A, i%, 10) + $25 (P/A, i%, 5) (P/F, i%, 10) + $20 (P/F, i%, 15)
Rate of Return = 11.65
Select A.

7-76

This is an unusual problem with an extremely high rate of return. Available interest tables obviously are useless.

One may write:
PW of Cost = PW of Benefits
$\$0.5 = \$3.5\,(1 + i)^{-1} + \$0.9\,(1 + i)^{-2} + \$3.9\,(1 + i)^{-3} + \$8.6\,(1 + i)^{-4} + \ldots$
For high interest rates only the first few terms of the series are significant:

Try i = 650%
PW of Benefits $= \$3.5/(1 + 6.5) + \$0.9/(1 + 6.5)^2 + \$3.9/(1 + 6.5)^3 + \$8.6/(1 + 6.5)^4 + \ldots$
$= 0.467 + 0.016 + 0.009 + 0.003$
$= 0.495$

Try i = 640%
PW of Benefits $= \$3.5/(1 + 6.4) + \$0.9/(1 + 6.4)^2 + \$3.9/(1 + 6.4)^3 + \$8.6/(1 + 6.4)^4 + \ldots$
$= 0.473 + 0.016 + 0.010 + 0.003$
$= 0.502$

$i^* = 642\%$
(Calculator Solution: i = 642.9%)

7-77

$52,000.00	Income		
3.00%	Income gradient		
10.00%	% Deposit		
15	Horizon (years)		
4.00%	Savings rate		
Year	Salary	Deposit	Cumulative Savings
1	$52,000.00	$5,200.00	$5,200.00
2	53,560.00	5,356.00	10,764.00
3	55,166.80	5,516.68	16,711.24
4	56,821.80	5,682.18	23,061.87
5	58,526.46	5,852.65	29,836.99
6	60,282.25	6,028.23	37,058.70
7	62,090.72	6,209.07	44,750.12
8	63,953.44	6,395.34	52,935.46
9	65,872.04	6,587.20	61,640.09
10	67,848.21	6,784.82	70,890.51
11	69,883.65	6,988.37	80,714.50
12	71,980.16	7,198.02	91,141.09
13	74,139.57	7,413.96	102,200.69
14	76,363.75	7,636.38	113,925.10
15	78,654.67	7,865.47	126,347.57

For any row: Salary = (1 + 0.03)*(Previous year's Salary)
Deposit = (Percent Deposit)*(Current year's Salary)
Savings = (1 + 0.04)*(Previous year's Savings) + Current year's Deposit
Amount saved is $126,347.57 in 15 years.

7-78

$55,000.00	Income		
2.00%	Income gradient		
10.00%	% Deposit		
40	Horizon (years)		
5.00%	Savings rate		
Year	Salary	Deposit	Cumulative Savings
1	$55,000.00	$5,500.00	$5,500.00
2	56,100.00	5,610.00	11,385.00

3	57,222.00	5,722.20	17,676.45
4	58,366.44	5,836.64	24,396.92
5	59,533.77	5,953.38	31,570.14
6	60,724.44	6,072.44	39,221.09
7	61,938.93	6,193.89	47,376.04
8	63,177.71	6,317.77	56,062.61
9	64,441.27	6,444.13	65,309.87
10	65,730.09	6,573.01	75,148.37
11	67,044.69	6,704.47	85,610.26
12	68,385.59	6,838.56	96,729.33
13	69,753.30	6,975.33	108,541.13
14	71,148.36	7,114.84	121,083.02
15	72,571.33	7,257.13	134,394.30
16	74,022.76	7,402.28	148,516.30
17	75,503.21	7,550.32	163,492.43
18	77,013.28	7,701.33	179,368.38
19	78,553.54	7,855.35	196,192.15
20	80,124.61	8,012.46	214,014.22
21	81,727.11	8,172.71	232,887.65
22	83,361.65	8,336.16	252,868.19
23	85,028.88	8,502.89	274,014.49
24	86,729.46	8,672.95	296,388.16
25	88,464.05	8,846.40	320,053.97
26	90,233.33	9,023.33	345,080.01
27	92,038.00	9,203.80	371,537.81
28	93,878.76	9,387.88	399,502.57
29	95,756.33	9,575.63	429,053.33
30	97,671.46	9,767.15	460,273.15
31	99,624.89	9,962.49	493,249.29
32	101,617.38	10,161.74	528,073.49
33	103,649.73	10,364.97	564,842.14
34	105,722.73	10,572.27	603,656.52
35	107,837.18	10,783.72	644,623.07
36	109,993.93	10,999.39	687,853.61
37	112,193.80	11,219.38	733,465.67
38	114,437.68	11,443.77	781,582.72
39	116,726.43	11,672.64	832,334.50
40	119,060.96	11,906.10	885,857.33

For any row: Salary = (1 + 0.02)*(Previous year's Salary)
Deposit = (Percent Deposit)*(Current year's Salary)
Savings = (1 + 0.05)*(Previous year's Savings) + Current year's Deposit
Amount saved is $885,857.33 in 40 years.

7-79

$55,000.00	Income		
2.00%	Income gradient		
11.29%	% Deposit		
40	Horizon (years)		
5.00%	Savings rate		
Year	Salary	Deposit	Cumulative Savings
1	$55,000.00	$6,209.50	$6,209.50
2	56,100.00	6,333.69	12,853.67
3	57,222.00	6,460.36	19,956.71
4	58,366.44	6,589.57	27,544.12
5	59,533.77	6,721.36	35,642.69
6	60,724.44	6,855.79	44,280.61
7	61,938.93	6,992.91	53,487.55
8	63,177.71	7,132.76	63,294.69
9	64,441.27	7,275.42	73,734.84
10	65,730.09	7,420.93	84,842.51
11	67,044.69	7,569.35	96,653.98
12	68,385.59	7,720.73	109,207.41
13	69,753.30	7,875.15	122,542.93
14	71,148.36	8,032.65	136,702.73
15	72,571.33	8,193.30	151,731.17
16	74,022.76	8,357.17	167,674.90
17	75,503.21	8,524.31	184,582.96
18	77,013.28	8,694.80	202,506.90
19	78,553.54	8,868.70	221,500.94
20	80,124.61	9,046.07	241,622.06
21	81,727.11	9,226.99	262,930.15
22	83,361.65	9,411.53	285,488.19
23	85,028.88	9,599.76	309,362.36
24	86,729.46	9,791.76	334,622.23
25	88,464.05	9,987.59	361,340.94
26	90,233.33	10,187.34	389,595.33
27	92,038.00	10,391.09	419,466.18
28	93,878.76	10,598.91	451,038.40
29	95,756.33	10,810.89	484,401.21
30	97,671.46	11,027.11	519,648.38
31	99,624.89	11,247.65	556,878.45
32	101,617.38	11,472.60	596,194.97
33	103,649.73	11,702.05	637,706.78
34	105,722.73	11,936.10	681,528.21

35	107,837.18	12,174.82	727,779.44
36	109,993.93	12,418.31	776,586.73
37	112,193.80	12,666.68	828,082.74
38	114,437.68	12,920.01	882,406.90
39	116,726.43	13,178.41	939,705.66
40	119,060.96	13,441.98	1,000,132.92

For any row: Salary = (1 + 0.02)*(Previous year's salary)
Deposit = (Percent deposit)*(Current year's salary)
Savings = (1 + 0.05)*(Previous year's savings) + Current year's deposit
To solve, just vary the percent deposit to get $1M in savings for year 40.
Amount saved is $1,000,132.92 in 40 years at 11.29%.

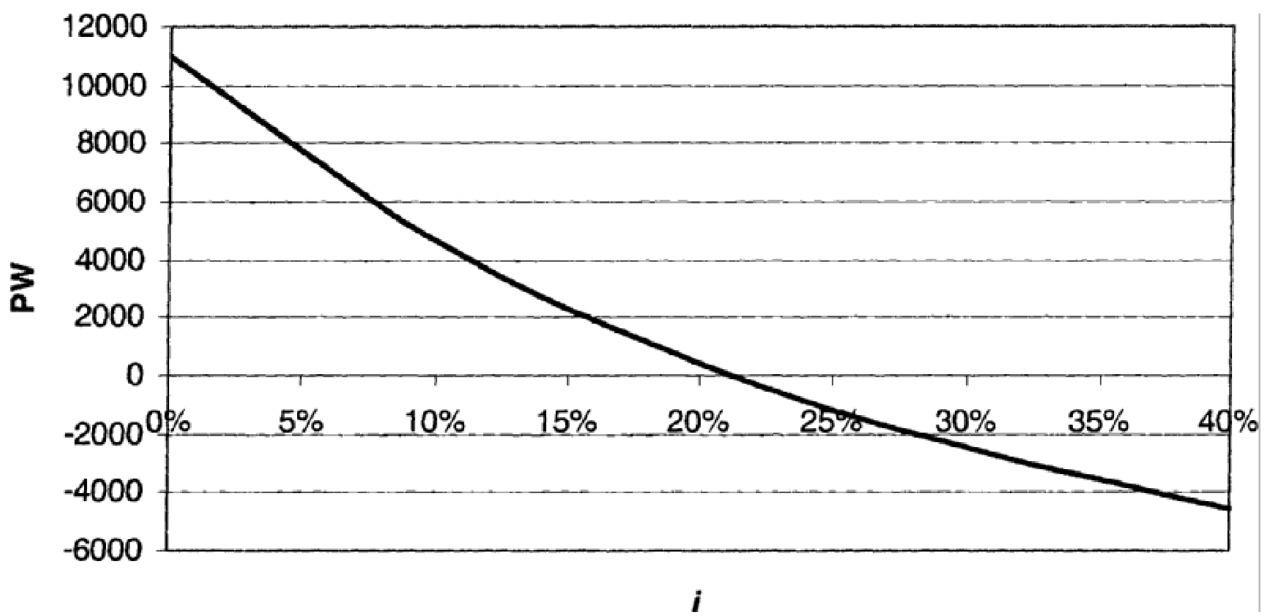
7-80

Details will vary by student, but solved like Problem 7-79.

Chapter 7A: Difficulties Solving for an Interest Rate

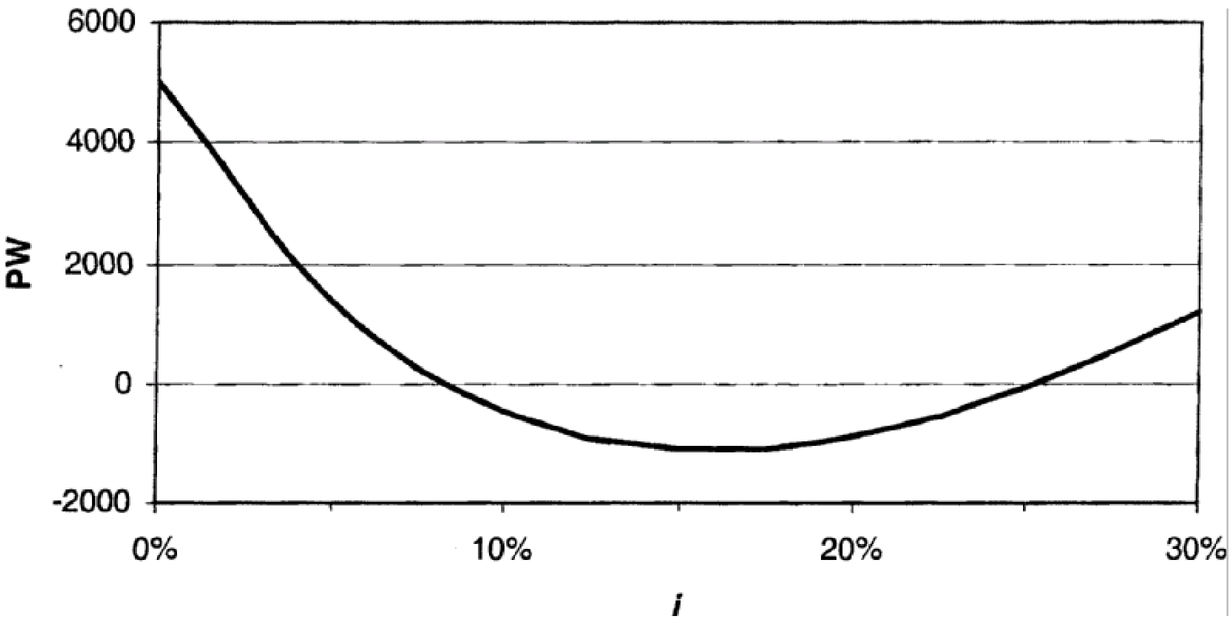
7A-1

Year	Cash Flow	i	PW			
0	−15,000	0%	11,000	=$B$2+NPV(D2,$B$3:$B$6)		
1	10,000	10%	4,623			
2	−8,000	20%	413			
3	11,000	30%	−2,483			
4	13,000	40%	−4,546		21.22%	IRR
					Unique IRR	
3 sign changes => 3 roots possible						



7A-2

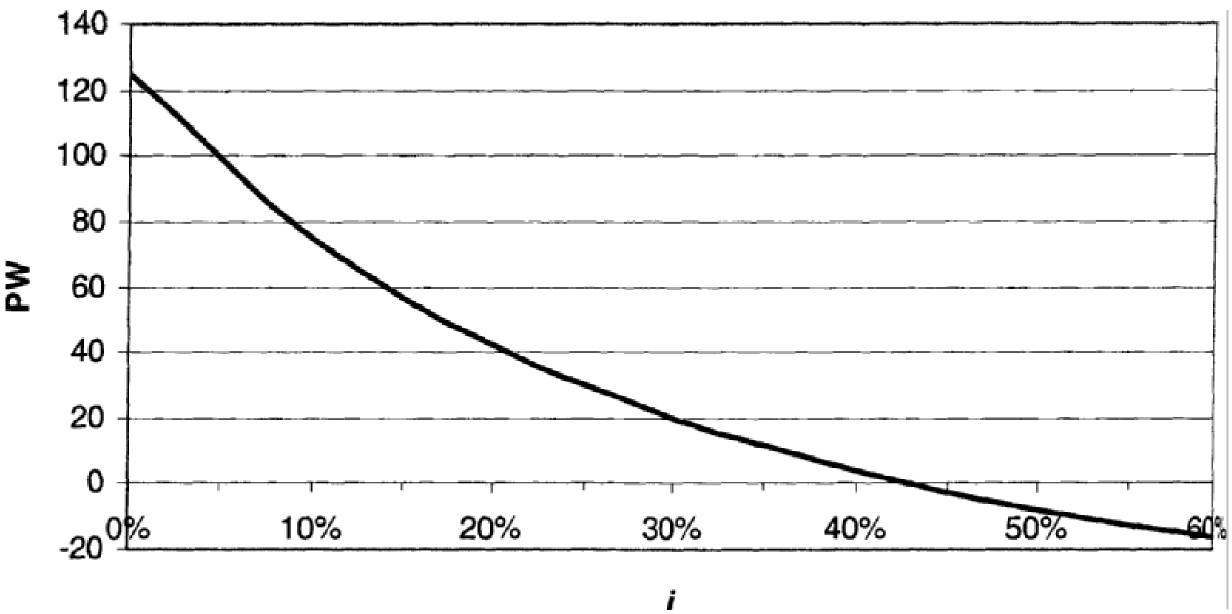
Year	Cash Flow	I	PW			
0	80,000	0%	5,000	=$B$2+NPV(D2,$B$3:$B$6)		
1	−85,000	5%	1,372			
2	−70,000	10%	−483			
3	0	15%	−1,103		2 roots possible	
4	80000	20%	−864		Root	8.26%
	5000	25%	−32		Root	25.15%
		30%	1,205			



6%	external financing rate						
12%	external investing rate						
9.6%	MIRR						

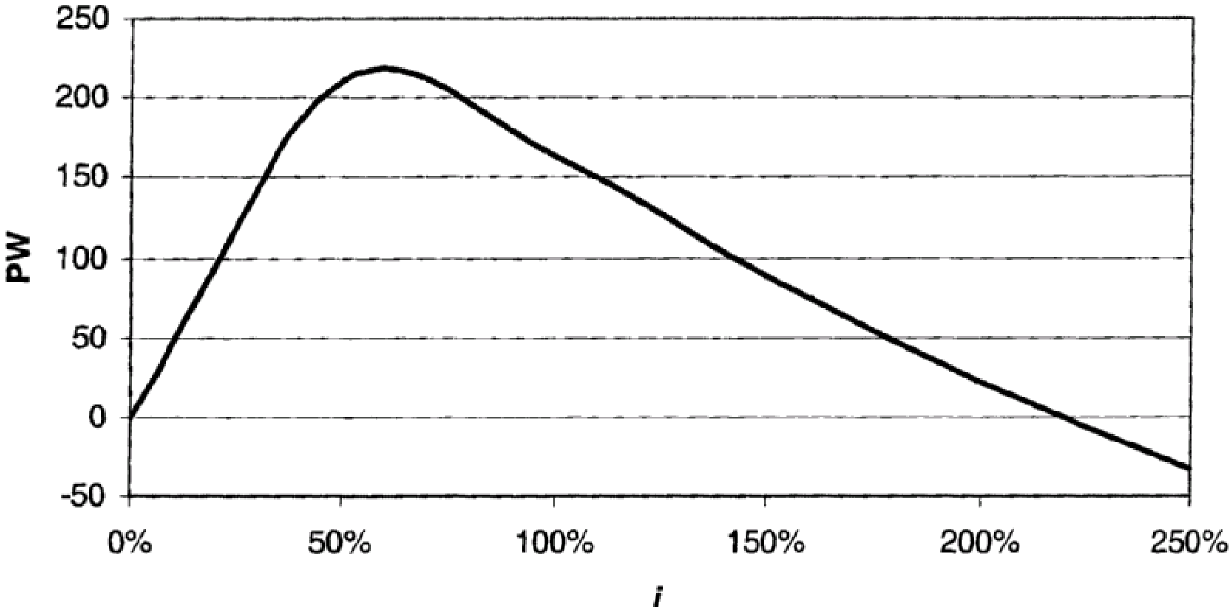
7A-3

Year	Cash Flow		I	PW			
0	−75		0%	125	=$B$2+NPV(D2,$B$3:$B$6)		
1	75		10%	75			
2	−50		20%	42			
3	50		30%	20		3 roots possible; but only 1 exists	
4	125		40%	4		42.99%	IRRquarter
	125		50%	−8		172%	IRRnominal
			60%	−16		318%	IRReffective
While the rates of return are very high, no value included for time spent operating the machine.							



7A-4

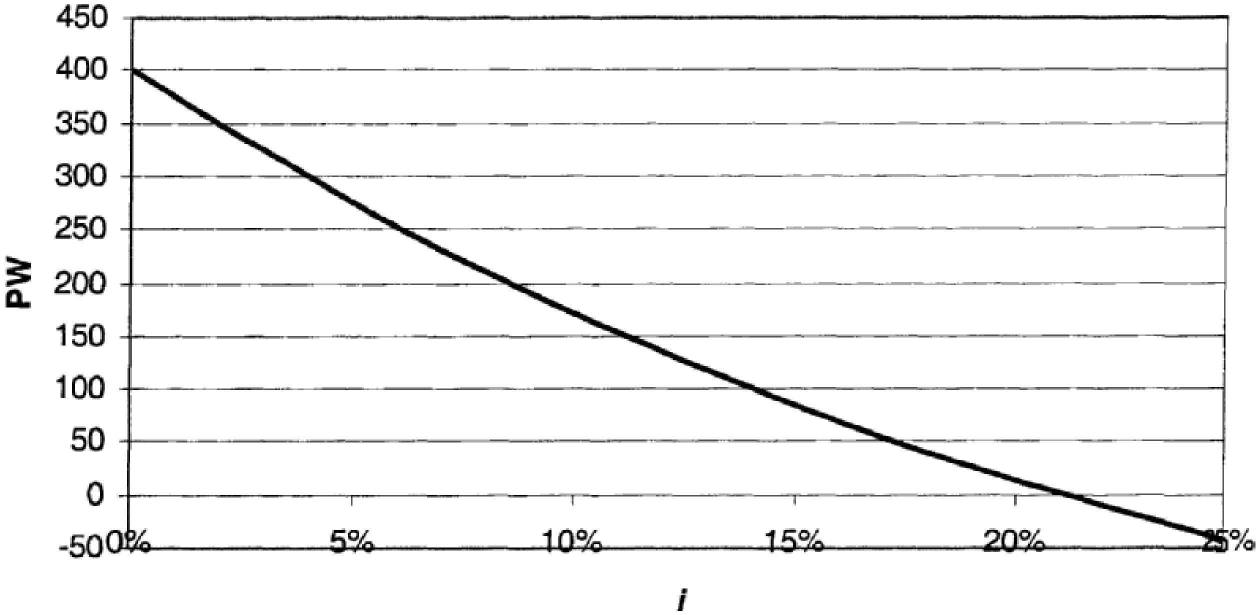
Year	Cash Flow	i	PW			
0	−500	0%	0	=$B$2+NPV(D2,$B$3:$B$6)		
1	2,000	50%	211			
2	−1,200	100%	163			
3	−300	150%	89		2 roots possible	
4		200%	22		Root	0.00%
	0	250%	−34		Root	218.82%



6%	External financing rate						
12%	External investing rate						
11.3%	MIRR						

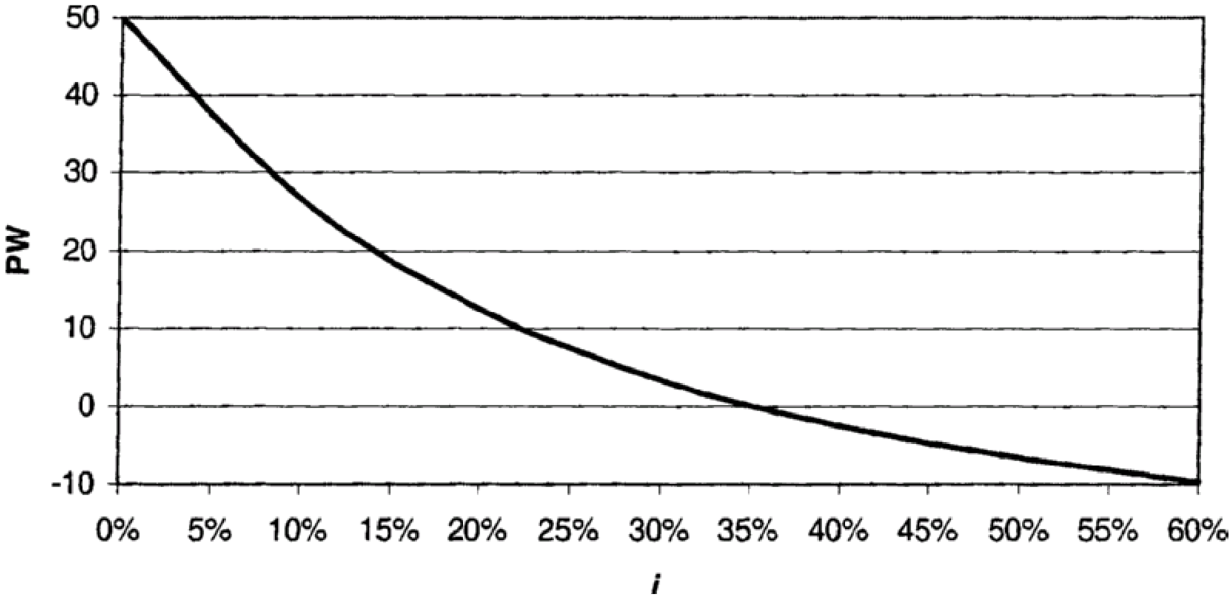
7A-5

Year	Cash Flow	i	PW			
0	−500	0%	400	=$B$2+NPV(D2,$B$3:$B$6)		
1	200	5%	274			
2	−500	10%	170			
3	1,200	15%	85		3 roots possible	
4		20%	14		21.09%	IRR
	400	25%	−46			



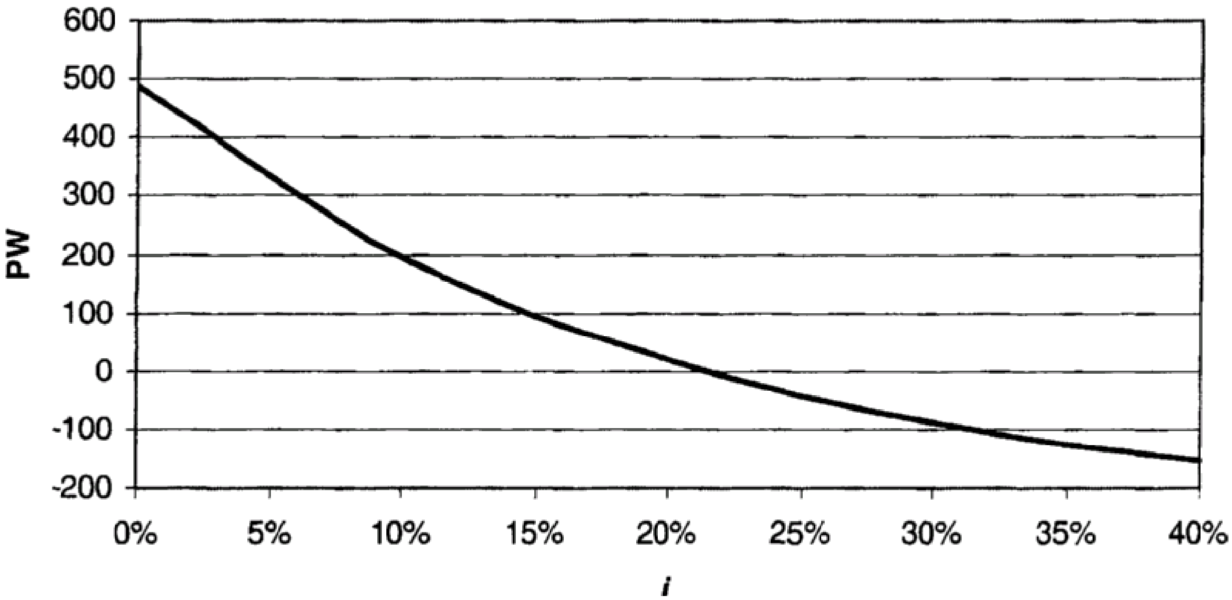
7A-6

Year	Cash Flow		i	PW			
0	−100		0%	50	=$B$2+NPV(D2,$B$3:$B$6)		
1	360		10%	26.67			
2	−570		20%	12.50			
3	360		30%	3.50		3 roots possible	
			40%	−2.48		35.39%	IRR
			50%	−6.67			
			60%	−9.77			



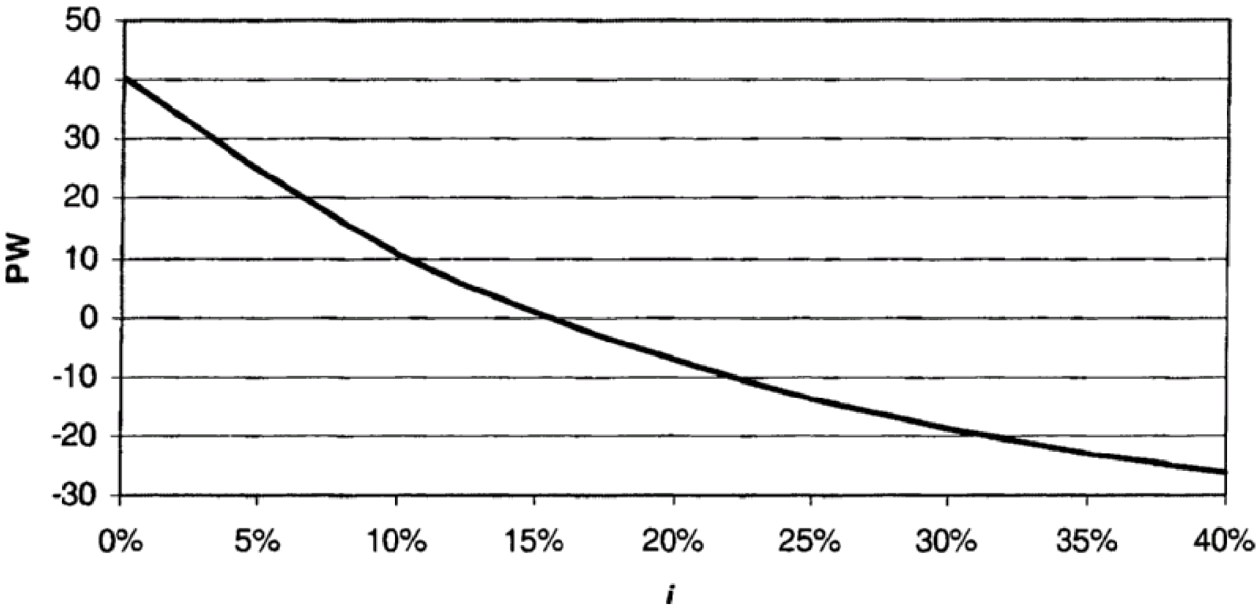
7A-7

Year	Cash Flow		i	PW			
0	−110		0%	490	=$B$2+NPV(D2,$B$3:$B$7)		
1	−500		10%	192			
2	300		20%	18			
3	−100		30%	−88			
4	400		40%	−153		21.37%	IRR
5	500					Unique IRR	
3 sign changes => 3 roots possible							



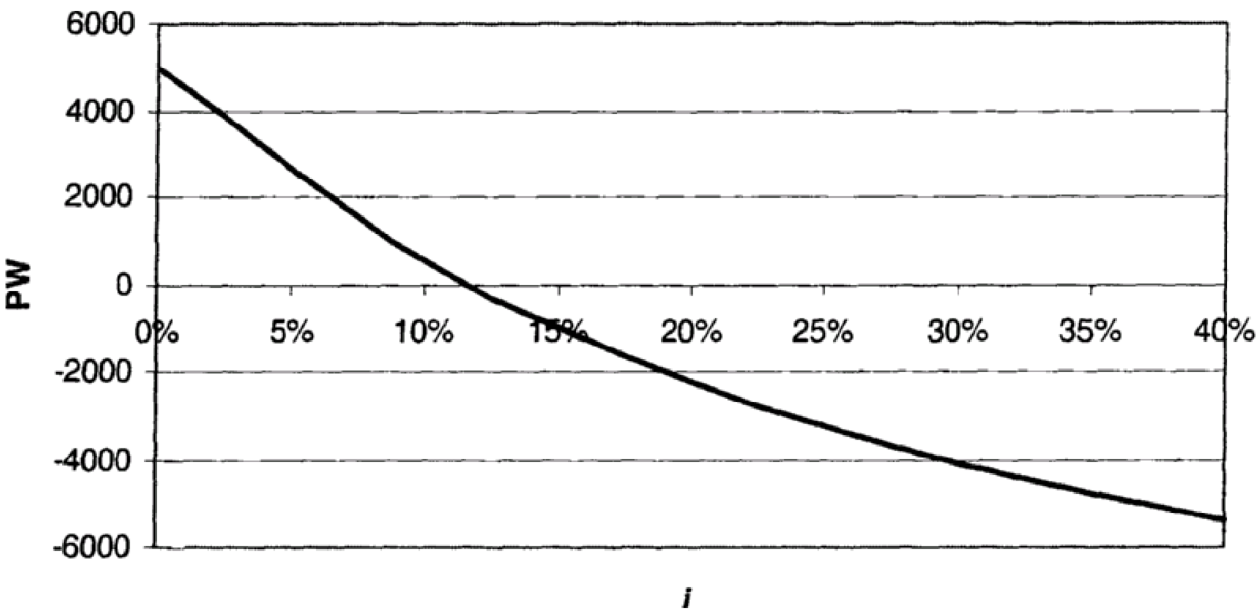
7A-8

Year	Cash Flow		i	PW			
0	−50		0%	40	=$B$2+NPV(D2,$B$3:$B$7)		
1	20		10%	11			
2	−40		20%	−7			
3	36.8		30%	−19			
4	36.8		40%	−26		15.38%	IRR
5	36.8					Unique IRR	
3 sign changes => 3 roots possible							



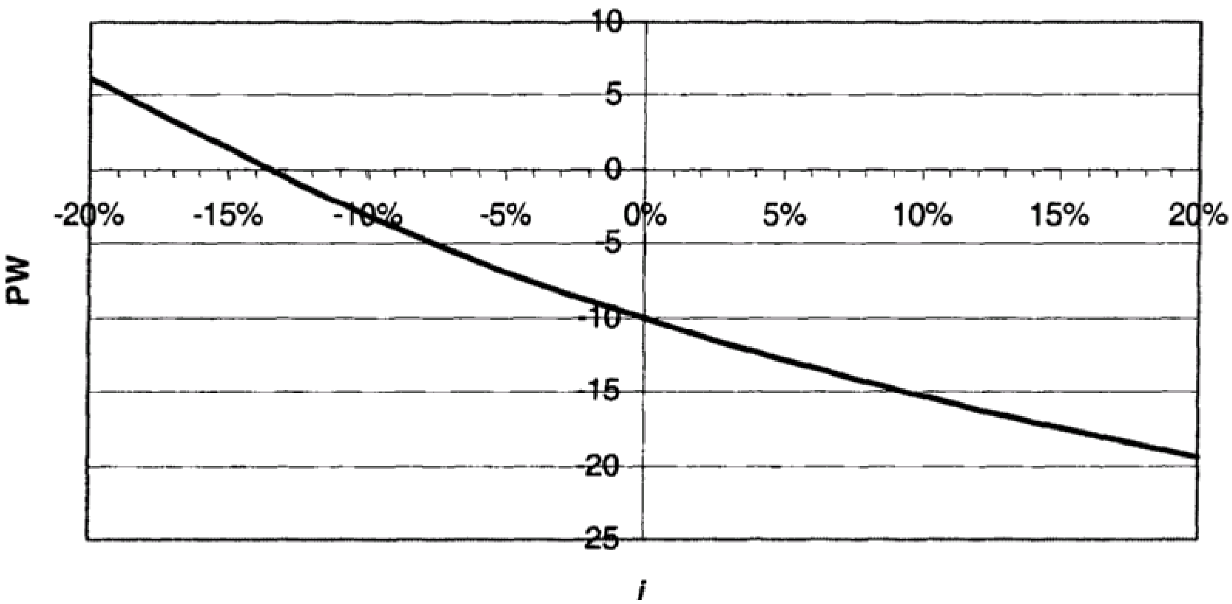
7A-9

Year	Cash Flow		i	PW			
0	−15,000		0%	5,000	=$B$2+NPV(D2,$B$3:$B$8)		
1	10,000		10%	513			
2	6,000		20%	−2,254			
3	−8,000		30%	−4,092			
4	4,000		40%	−5,395		11.54%	IRR
5	4,000					Unique IRR	
6	4000						
3 sign changes => 3 roots possible							



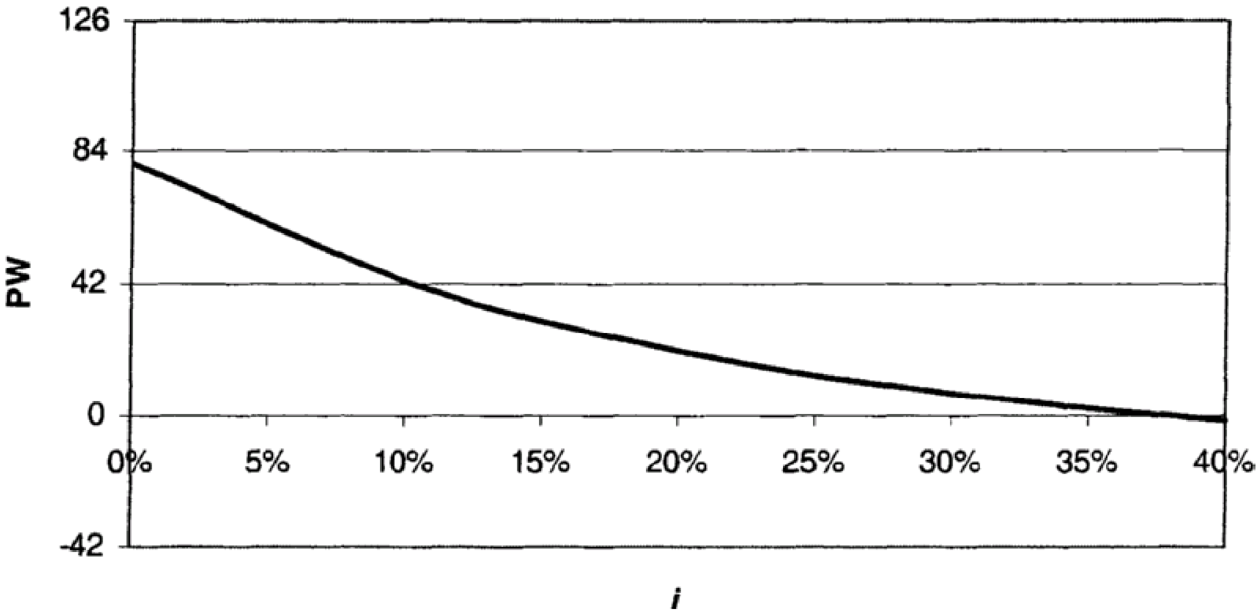
7A-10

Year	Cash Flow		i	PW			
0	−50		−20%	6	=$B$2+NPV(D2,$B$3:$B$4)		
1	20		−10%	−3			
2	20		0%	−10			
			10%	−15			
			20%	−19		−13.67%	IRR
						Unique IRR	



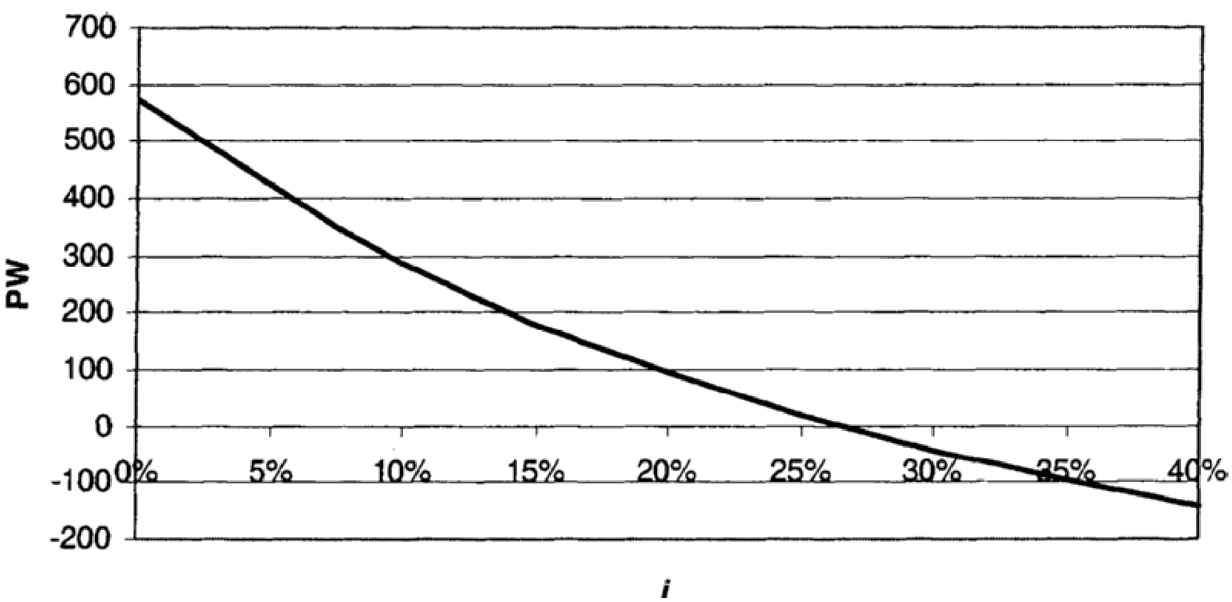
7A-11

Year	Cash Flow		i	PW			
0	−20		0%	80	=$B$2+NPV(D2,$B$3:$B$7)		
1	0		10%	42			
2	−10		20%	20			
3	20		30%	7			
4	−10		40%	−2		37.44%	IRR
5	100					Unique IRR	
3 sign changes => 3 roots possible							



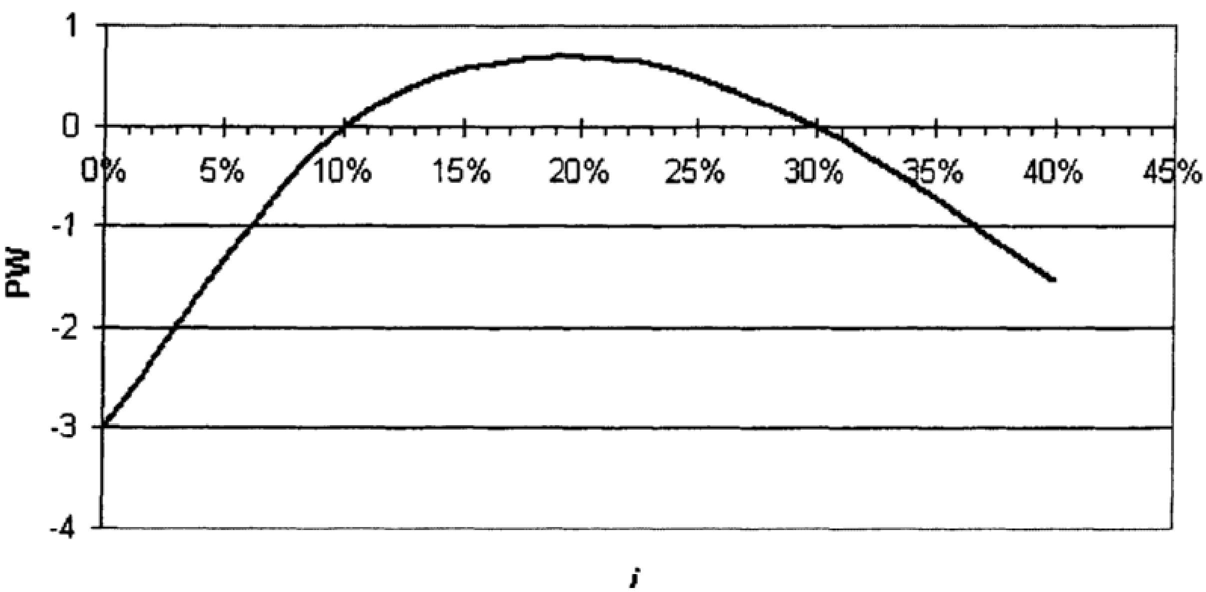
7A-12

Year	Cash Flow		i	PW				
0	−800		0%	575	=$B$2+NPV(D2,$B$3:$B$7)			
1	500		10%	286				
2	500		20%	94				
3	−300		30%	−42				
4	400		40%		−142		26.55%	IRR
5	275					Unique IRR		
3 sign changes => 3 roots possible								



7A-13

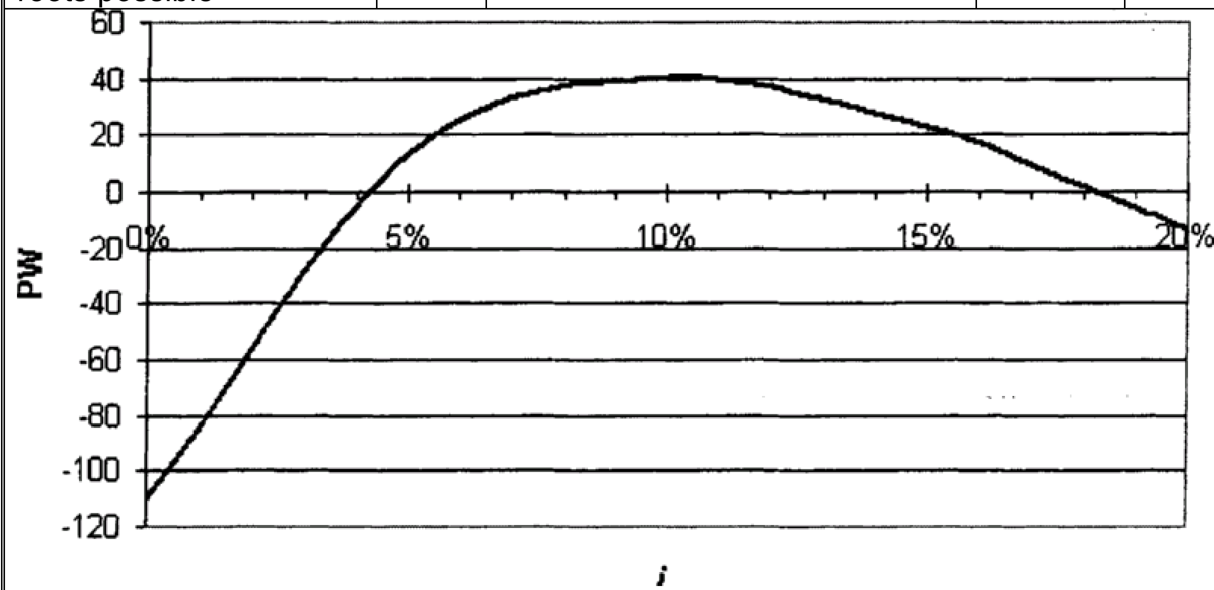
Year	Cash Flow		i	PW			
0	−100		0%	−3	=$B$2+NPV(D2,$B$3:$B$4)		
1	240		10%	0			
2	−143		20%	1			
			30%	0			
			40%	−2		10.00%	Root
						30.00%	Root
2 sign changes => 2 roots possible							



6%	External financing rate					
12%	External investing rate					
8.8%	MIRR		Value is less than external investing rate => not attractive			

7A-14

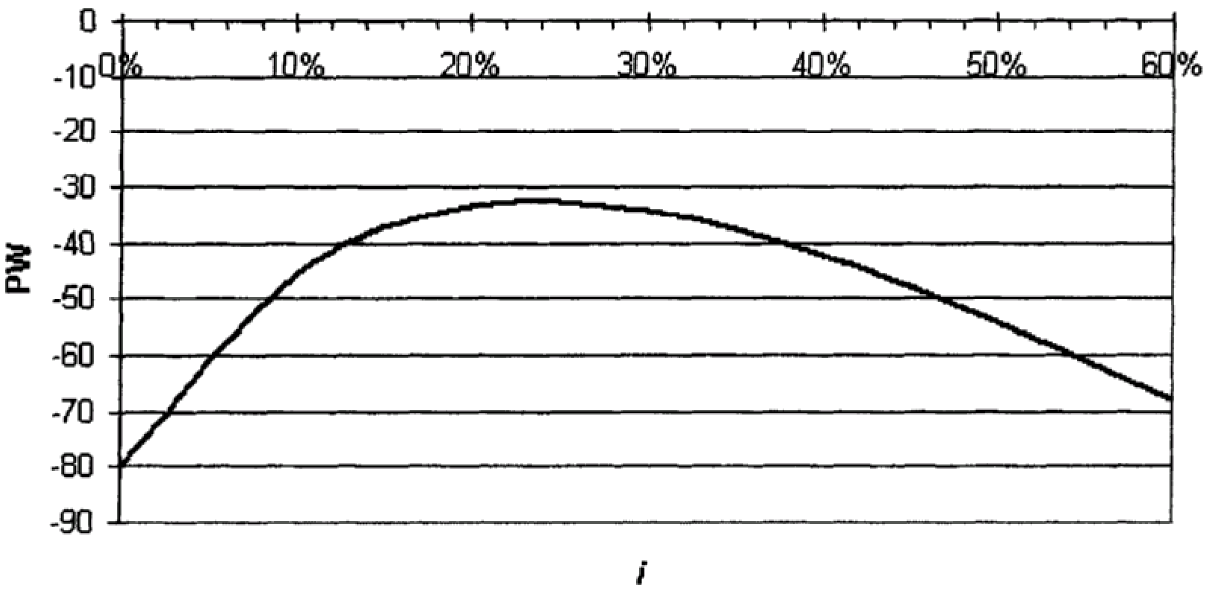
Year	Cash Flow		i	PW			
0	−610		0%	−110	=$B$2+NPV(D2,$B$3:$B$12)		
1	200		5%	13			
2	200		10%	41			
3	200		15%	23			
4	200		17%	10		4.07%	Root
5	200		19%	−6		18.29%	Root
6	200		20%	−14			
7	200		25%	−57			
8	200						
9	200						
10	−1300						
2 sign changes => 2 roots possible							



6%	External financing rate						
12%	External investing rate						
9.5%	MIRR		Value is less than external investing rate => not attractive				

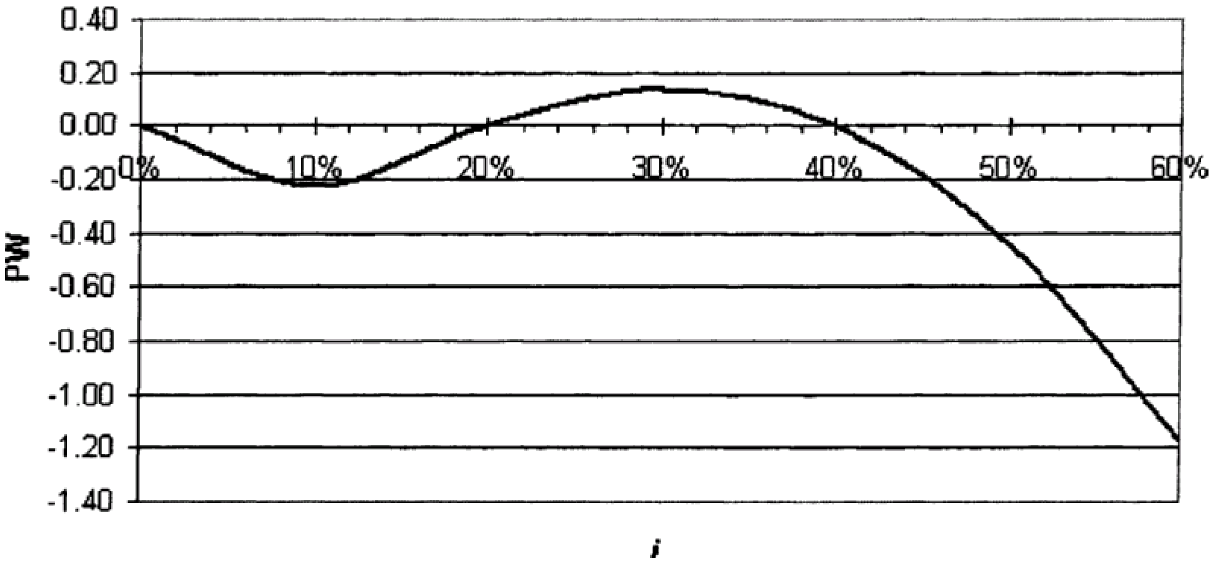
7A-15

Year	Cash Flow		i	PW			
0	−500		0%	−80	=$B$2+NPV(D2,$B$3:$B$5)		
1	800		10%	−45			
2	170		20%	−34			
3	−550		30%	−34			
			40%	−42		#NUM!	Root
			50%	−54		#NUM!	Root
			60%	−68		No roots exist	
2 sign changes => 2 roots possible							



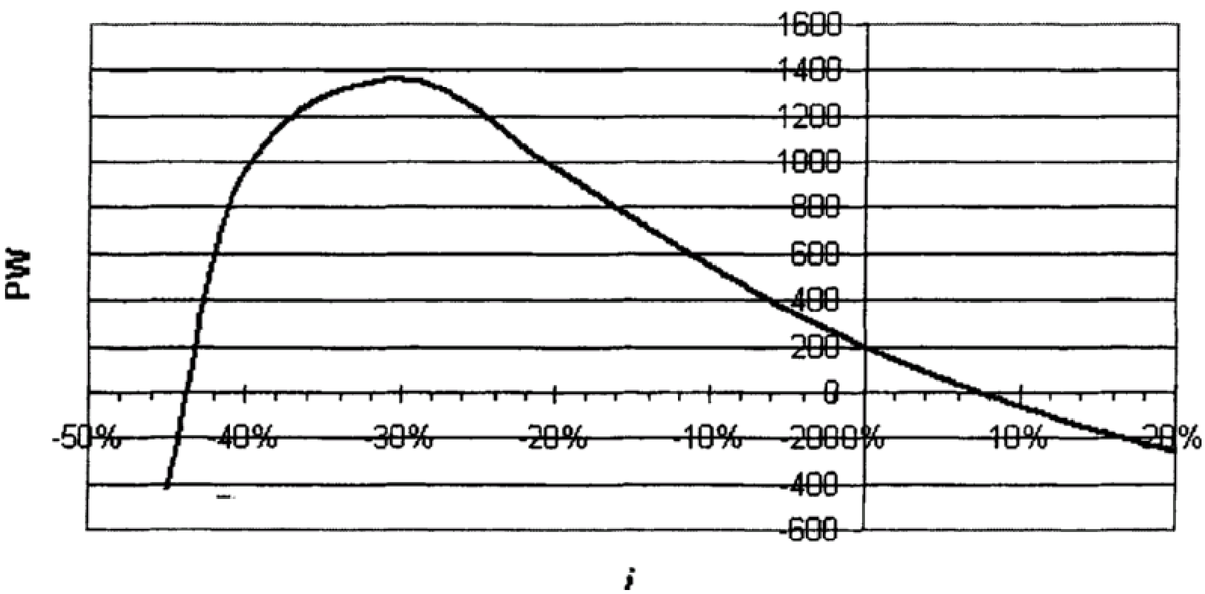
6%	External financing rate						
12%	External investing rate						
7.5%	MIRR		Value is less than external investing rate => not attractive				

7A-16

Year	Cash Flow		I	PW			
0	−100		0%	0.00 =$B$2+NPV(D2,$B$3:$B$5)			
1	360		10%	−0.23			
2	−428		20%	0.00			
3	168		30%	0.14			
			40%	0.00		0.00%	Root
			50%	−0.44		20.00%	Root
			60%	−1.17		40.00%	Root
3 sign changes <EQ1> 3 roots possible				All PW values = 0 given significant digits of cash flows			
							
6%	External financing rate						
12%	External investing rate						
8.8%	MIRR		Value is less than external investing rate <EQ1> not attractive				

7A-17

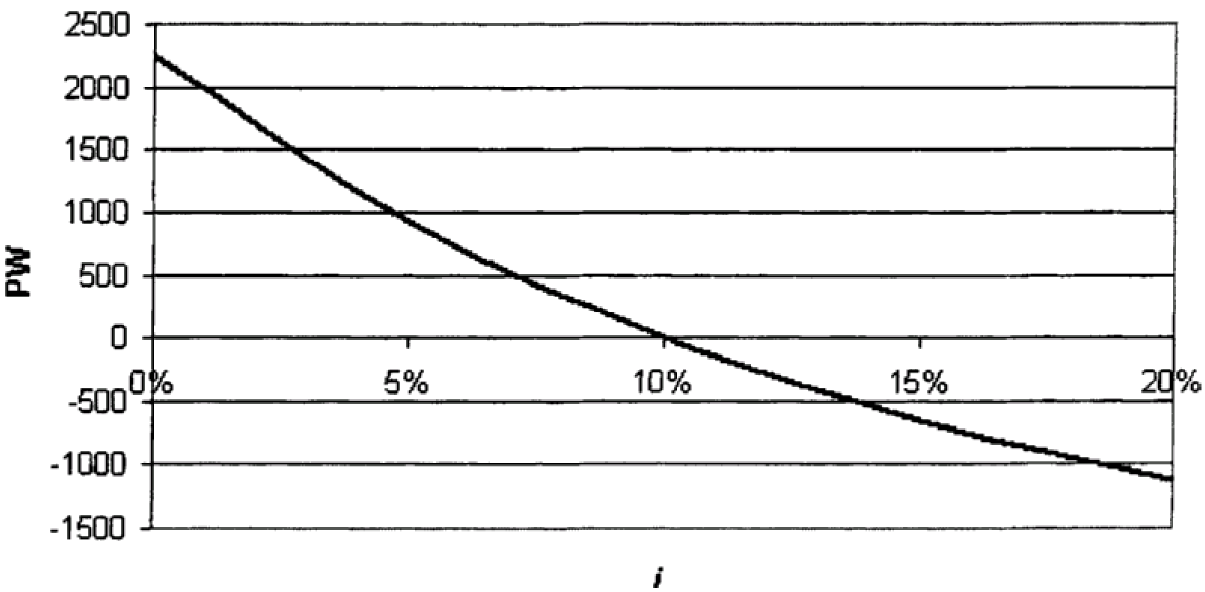
Year	Cash Flow		i	PW			
0	−1,200		−45%	−422	=$B$2+NPV(D2,$B$3:$B$8)		
1	358		−40%	970			
2	358		−30%	1358			
3	358		−20%	970			
4	358		−10%	541		7.22%	Root
5	358		0%	196		−43.96%	Root
6	−394		10%	−65			
			20%	−261			
2 sign changes => 2 roots possible							



6%	External financing rate						
12%	External investing rate						
9.5%	MIRR		Value is less than external investing rate => not attractive				

7A-18

Year	Cash flow		i	PW			
0	−3,570		0%	2260	=$B$2+NPV(D2,$B$3:$B$10)		
1	1,000		5%	921			
2	1,000		10%	−1			
3	1,000		15%	−651			
4	−3,170		20%	−1120		10.00%	IRR
5	1,500					unique IRR	
6	1,500						
7	1,500						
8	1,500						
3 sign changes => 3 roots possible							



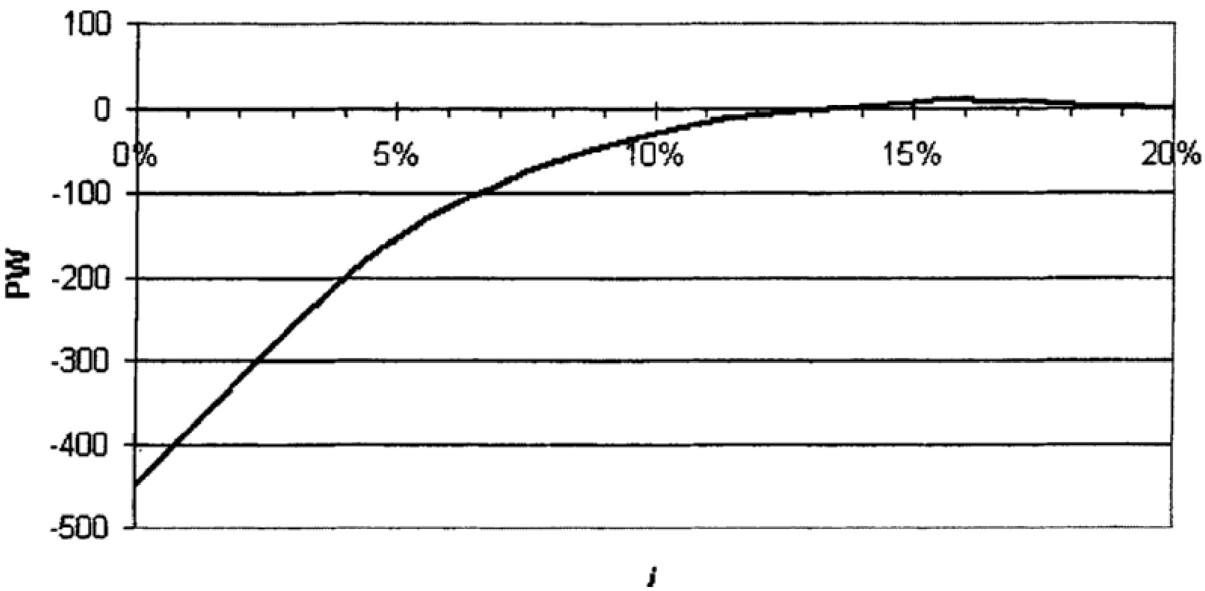
7A-19

800	Down payment	
55	Monthly payment	1 sign change <EQ1> 1 root possible
40	# payment	
2,500	Final receipt	
−0.75%	IRR monthly	= RATE(A3, –A2, –A1, A4)
−8.62%	Effective annual rate =(1+A6)^ 12–	

	1	

7A-20

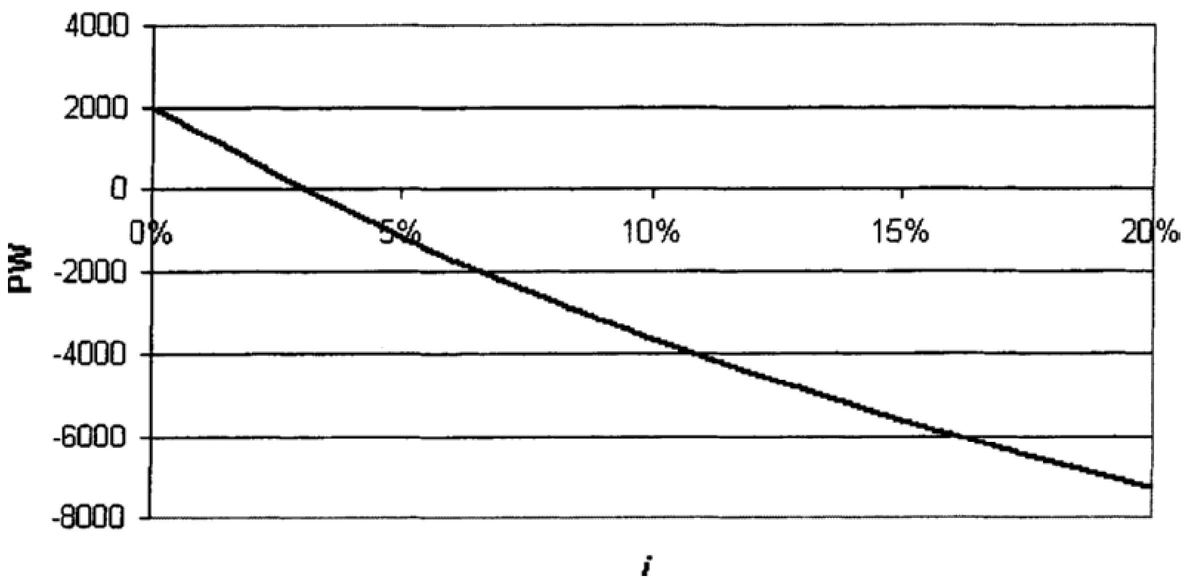
Year	Cash Flow		i	PW			
0	−850		0%	−450	=$B$2+NPV(D2,$B$3:$B$12)		
1	600		5%	−153			
2	200		10%	−29			
3	200		15%	7			
4	200		17%	8		12.99%	Root
5	200		19%	3		19.72%	Root
6	200		20%	−1			
7	200		25%	−31			
8	200						
9	200						
10	−1,800						
2 sign changes = > 2 roots possible							



6%	External financing rate					
12%	External investing rate					
9.1%	MIRR		value is less than external investing rate = > not attractive			

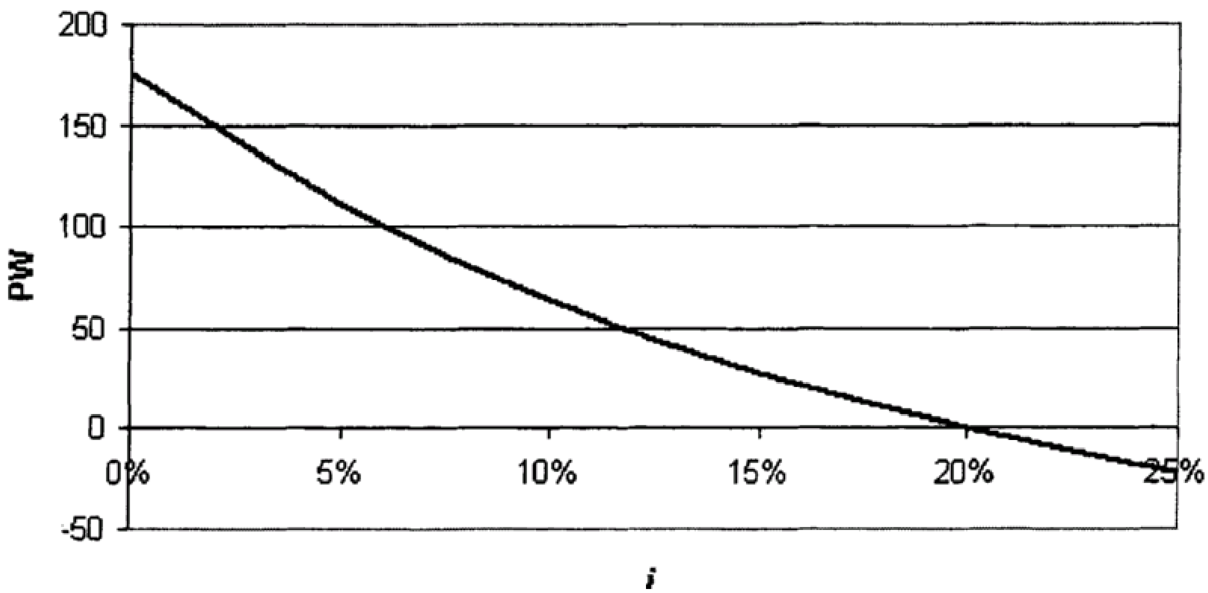
7A-21

Year	Cash Flow		i	PW			
0	−16,000		0%	1,950	=$B$2+NPV(D2,$B$3:$B$7)		
1	−8,000		5%	−1,158			
2	11,000		10%	−3,639			
3	13,000		15%	−5,644			
4	−7,000		20%	−7,284		3.00%	IRR
5	8,950					unique IRR	
3 sign changes = > 3 roots possible							



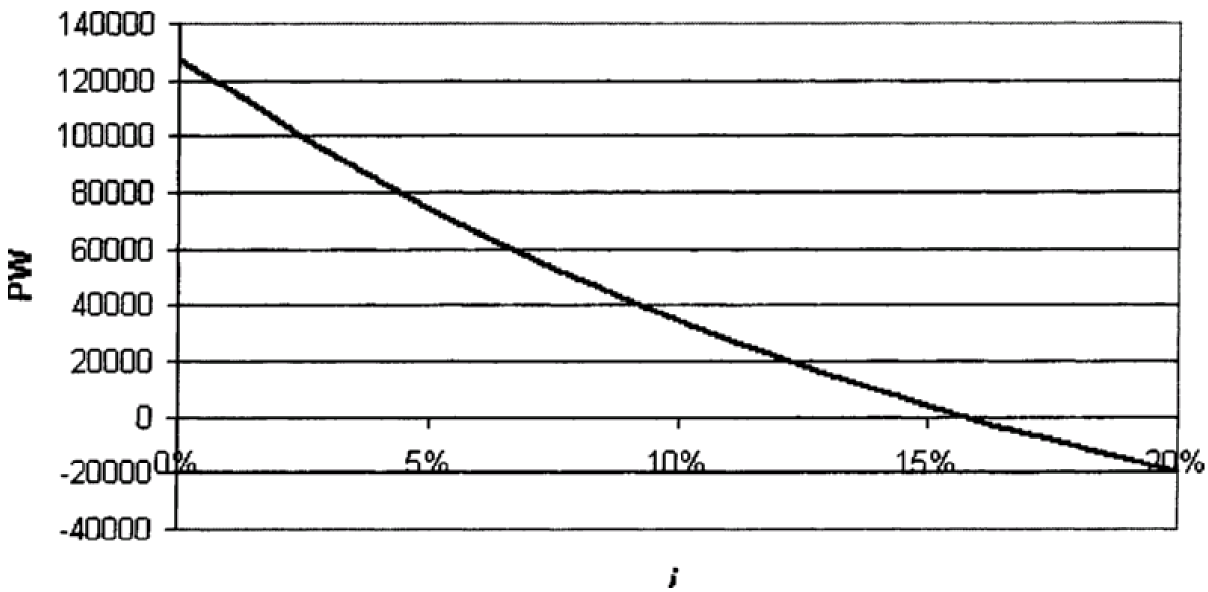
7A-22

Year	Cash Flow		i	PW			
0	−200		0%	176	=$B$2+NPV(D2,$B$3:$B$10)		
1	100		5%	111			
2	100		10%	63			
3	100		15%	27			
4	−300		20%	0		20.00%	IRR
5	100		25%	−21		Unique IRR	
6	200						
7	200						
8	−124.5						
4 sign changes = > 4 roots possible							



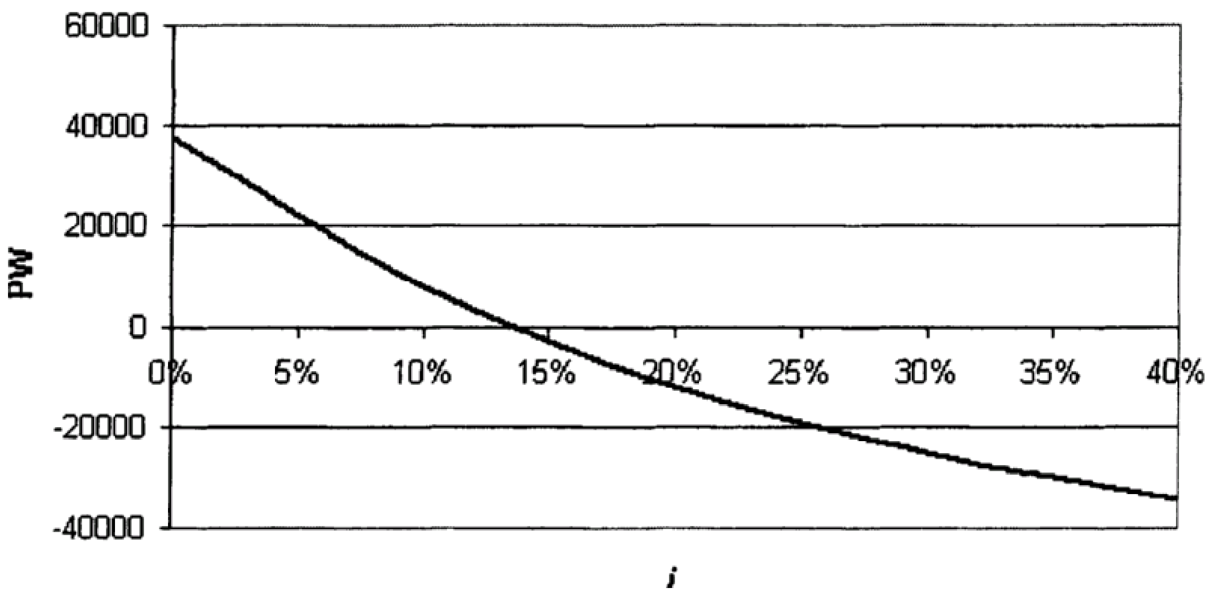
7A-23

Year	Cash Flow		i	PW			
0	−210,000		0%	127000	=$B$2+NPV(D2,$B$3:$B$9)		
1	88,000		5%	74284			
2	68,000		10%	34635			
3	62,000		15%	4110			
4	−31,000		20%	−19899		15.78%	IRR
5	30,000					Unique IRR	
6	55,000						
7	65,000						
3 sign changes = > 3 roots possible							



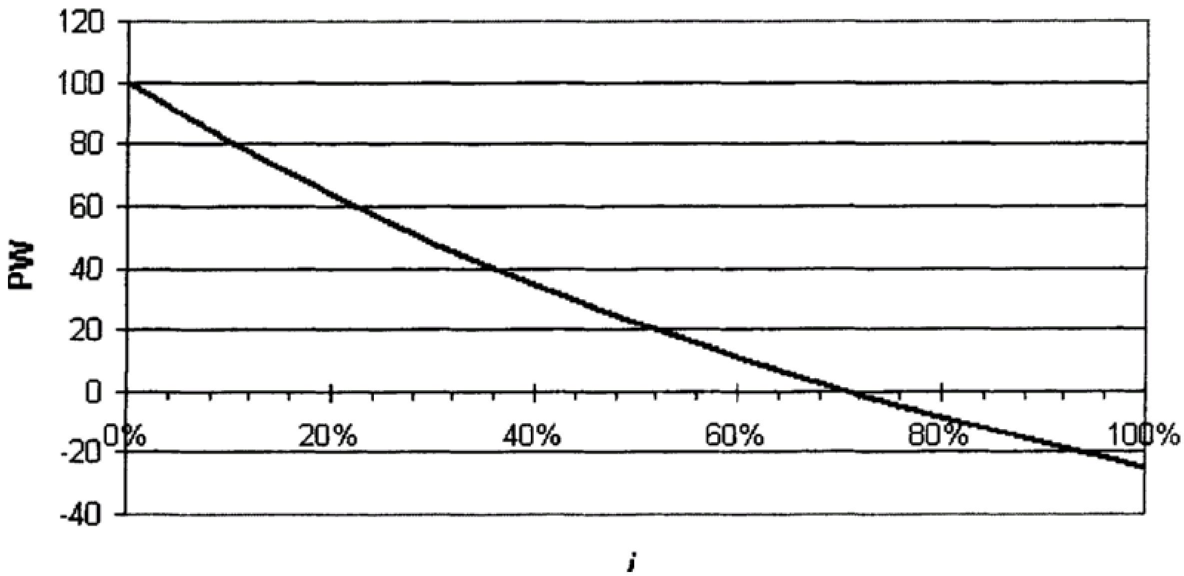
7A-24

Year	Cash flow		i	PW			
0	−103,000		0%	37,400	=$B$2+NPV(D2,$B$3:$B$7)		
1	102,700		10%	7,699			
2	−87,000		20%	−11,676			
3	94,500		30%	−25,003			
4	−8,300		40%	−34,594		13.51%	IRR
5	38,500					Unique IRR	
5 sign changes = > 3 roots possible							



7A-25

Year	Cash flow		i	PW			
0	–200		0%	100	=$B$2+NPV(D2,$B$3:$B$4)		
1	400		20%	64			
2	–100		40%	35			
			60%	11			
			80%	–9		70.71%	IRR
			100%	–25		Unique IRR	
2 sign changes => 2 roots possible							

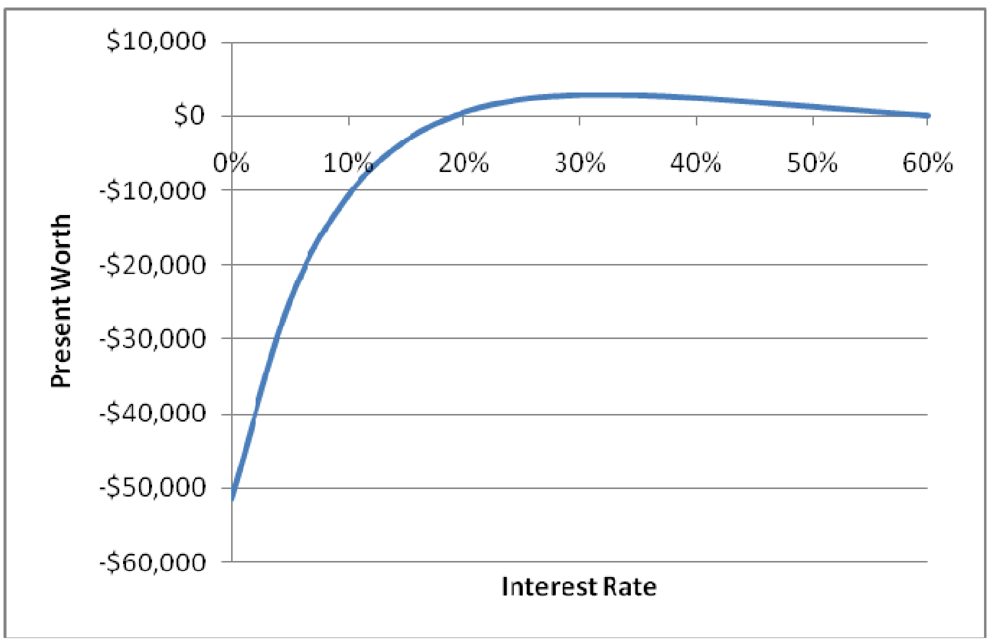


6%	External financing rate						
12%	External investing rate						
24.5%	MIRR		Value is more than external investing rate => attractive				

7A-26

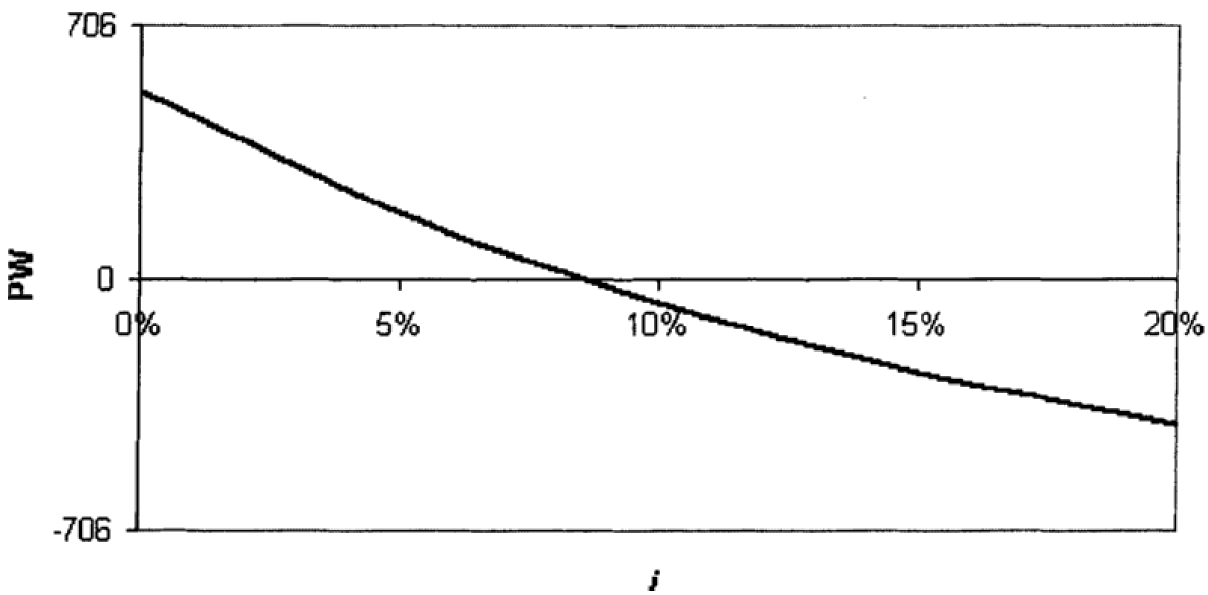
Year	A	B	A–B		i	PW
0	-$58,500	-$48,500	-$10,000		0%	-$51,520
1	$6,648	$0	$6,648		5%	-$24,969
2	$6,648	$0	$6,648		10%	-$10,790
3	$6,648	$0	$6,648		15%	-$3,331
4	$6,648	$0	$6,648		20%	$429
5	$6,648	$0	$6,648		25%	$2,140
6	$6,648	$0	$6,648		30%	$2,718
7	$6,648	$0	$6,648		35%	$2,678
8	$6,648	$0	$6,648		40%	$2,312
9	$6,648	$0	$6,648		45%	$1,785
10	$36,648	$138,000	-$101,352		50%	$1,193
IRR	8.0%	11.0%	19.2%		55%	$587
					60%	-$3

2 sign changes -> 2 roots possible. Graph shows(19.2% and 60%)



7A-27

Year	Cash Flow		i	PW			
0	-1,000		0%	520	=$B$2+NPV(D2,$B$3:$B$7)		
1	60		5%	181			
2	60		10%	–71			
3	-340		15%	–261			
4	0		20%	–406		8.44%	IRR
5	1,740					Unique IRR	
3 sign changes => 3 roots possible							



7A-28

Year	Pump 1	Pump 2	Increment 2 − 1
0	−\$100	−\$110	−\$10
1	+\$70	+\$115	+\$45
2	\$70	\$30	−\$40

Transformation: x(1 + 0.10) = \$40
Solve for x: x = \$40/1.1 = \$36.36

Year	Transformed Increment 2 − 1
0	−\$10
1	+\$8.64
2	\$0

This is obviously an undesirable increment as ΔROR < 0%.
<u>Select Pump 1.</u>

Chapter 8: Choosing the Best Alternative

8-1

Year	X	Y	Y – X
0	−\$10	−\$20	−\$10
1	\$15	\$28	+\$13
Computed ROR	50%	40%	30%

Choice table:
If 0 < MARR ≤ 30% Select Y
If 30 < MARR ≤ 100 Select X

8-2

Compute Rates of Return of the individual alternatives
Alternative X: \$100 = \$31.5 (P/A, i%, 4)
(P/A, i%, 4) = \$100/\$31.5 = 3.17
ROR_X = 9.9%

Alternative Y: \$50 = \$16.5 (P/A, i%, 4)
(P/A, i%, 4) = \$50/\$16.5 = 3.03
ROR_Y = 12.1%

Incremental Analysis

Year	X − Y
0	−\$50
1-4	+\$15

\$50 = \$15 (P/A, i%, 4)
Δ ROR_{X-Y} = 7.7%

Choice table:
If 0 < MARR ≤ 7.7 Select X
If 7.7 < MARR ≤ 12.1 Select Y
If 12.1 < MARR ≤ 100 Do Nothing

8-3

Compute Rates of Return

Alternative A: \$100 = \$30 (P/A, i%, 5)
(P/A, i%, 5) = \$100/\$30 = 3.33
ROR_A = 15.2%

Alternative B: \$150 = \$43 (P/A, i%, 5)
(P/A, i%, 5) = \$150/\$43 = 3.49
ROR_B = 13.3%

Incremental Analysis

Year	B − A
0	−\$50
1-5	+\$13

\$50 = \$13 (P/A, i%, 5)
Δ ROR_{B-A} = 9.4%

Choice table:
If 0 < MARR ≤ 9.4 Select B
If 9.4 < MARR ≤ 15.2 Select A
If 15.2 < MARR ≤ 100 Do Nothing

8-4

Year	A	B	A − B
0	−\$10,700	−\$5,500	−\$5,200
1 – 4	+\$2,100	\$1,800	+\$300
4		-\$5,500	+\$5,500
5 – 8	+\$2,100	+\$1,800	+\$300
Computed ROR	11.3%	11.7%	10.8%

(a) Choice table: (Assuming "Do-Nothing" is not an alternative)
If 0 < MARR ≤ 10.8 Select A
If 10.8 < MARR ≤ 100 Select B

(b) Since ΔROR_{A-B} > MARR, the increment is desirable. Select A.

8-5

Using Equivalent Uniform Annual Cost:
$EUAC_{Th}$ = \$5 + \$20 (A/P, 12%, 3) = \$5 + \$20 (0.4163) = \$13.33
$EUAC_{SL}$ = \$2 + \$40 (A/P, 12%, 5) = \$2 + \$40 (0.2774) = \$13.10
Fred should choose **slate** over thatch to save \$0.23/yr in costs.

To find incremental ROR, find *i* such that $EUAC_{SL} - EUAC_{TH} = 0$.
\$0 = \$2 + \$40 (A/P, i^*, 5) − [\$5 + \$20 (A/P, i^*, 3)]
= −\$3 + \$40 (A/P, i^*, 5) − \$20 (A/P, i^*, 3)

At i = 12%
−\$3 + \$40 (0.2774) − \$20 (0.4163) = −\$0.23 < \$0 so 12% too low

At i = 15%
−\$3 + \$40 (0.2983) − \$20 (0.4380) = \$0.172 > \$0 so 15% too high

Using Linear Interpolation: ΔROR = 12 + 3[–0.23/(–0.23 − 0.172)] = 13.72%

Choice table:

If	0	< MARR ≤ 13.72	Select Slate
If	13.72	< MARR ≤ 100	Select Thatch

8-6

(a) For the Atlas mower, the cash flow table is

Year	Net Cash Flow (Atlas)
0	−\$6,700
1	\$2,500
2	\$2,500
3	\$3,500

NPW = −\$6,700 +\$2,500 (P/A, i^*, 2) + \$3,500 (P/F, i^*, 3) = \$0

To solve for i^*, construct a table as follows:

i	NPW
12%	+\$16
i^*	\$0
15%	−\$334

Use linear interpolation to determine ROR:
ROR = 12% + 3% (\$16 − \$0)/(\$16 + \$334) = 12.1%

(b) For the Zippy mower, the cash flow table is

Year	Net Cash Flow (Zippy)
0	−$16,900
1−5	$3,300
6	$6,800

NPW = −$16,900 + $3,300 (P/A, i%, 5) + $6,800 (P/F, i%, 6)
At MARR = 8% NPW = −$16,900 + $3,300 (3.993) + $6,800 (0.6302)
= +$562

Since NPW is positive at 8%, the ROR > MARR.

(c) The incremental cash flow is

Year	Net Cash Flow (Zippy)	Net Cash Flow (Atlas)	Difference (Zippy – Atlas)
0	−$16,900	−$6,700	−$10,200
1	$3,300	$2,500	+$800
2	$3,300	$2,500	+$800
3	$3,300	$3,500 − $6,700	+$6,500
4	$3,300	$2,500	+$800
5	$3,300	$2,500	+$800
6	$6,800	$3,500	+$3,300

NPW = −$10,200 +$800(P/A, i^*, 5) + $5,700(P/F, i^*, 3) + $3,300(P/F, i^*, 6)

Compute the ΔROR
Try i = 6%
NPW = −$10,200 +$800(4.212) + $5,700(0.8396) + $3,300(0.7050)
= +$282

Try i = 7%
NPW = −$10,200 +$800(4.100) + $5,700(0.8163) + $3,300(0.6663)
= −$68

Using Linear Interpolation:
ΔROR= 6% + 1 % ($282 − $0)/($282 + $68) = 6.8%

Choice table:

If	0	< MARR ≤ 6.8	Select Zippy
If	6.8	< MARR ≤ 100	Select Atlas

8−7

	South End	Both Stores	North End
New Store Cost		−$500,000	
Annual Profit	$170,000	$260,000	+$90,000
Salvage Value			+$500,000

Where the investment ($500,000) is fully recovered, as is the case here:
Rate of Return = A/P = $90,000/$500,000 = 0.18 = 18%

a) Choice table:

If 0 < MARR ≤ 18 Open North End
If 18 < MARR ≤ 100 Do not open new store

b) <u>Open The North End.</u>

8−8

Year	Neutralization	Precipitation	Neut. − Precip.
0	−$700,000	−$500,000	−$200,000
1−5	−$40,000	−$110,000	+$70,000
5	+$175,000	+$125,000	+$50,000

Solve (Neut. − Precip.) for rate of return.
$200,000 = $70,000 (P/A, i%, 5) + $50,000 (P/F, i%, 5)

Try i = 25%
$200,000 = $70,000 (2.689) + $50,000 (0.3277) = $204,615
Therefore, ROR > 25%. Computed rate of return = 26%

a) Choice table:

If 0 < MARR ≤ 26 Select Neutralization
If 26 < MARR ≤ 100 Select Precipitation

b) <u>Choose Neutralization.</u>

8−9

Year	Gen. Dev.	RJR	RJR − Gen Dev.
0	−$480	−$630	−$150
1−15	+$94	+$140	+$46
15	+$1,000	+$1,000	$0
Computed ROR	21.0%	22.9%	30.1%

a) Choice table:

If 0 < MARR ≤ 22.9 Select RJR
If 22.9 < MARR ≤ 100 Do Nothing

b) If the MARR is 25% then neither bond is desirable. Do nothing.
Note that simply examining the (RJR − Gen Dev) increment might lead one to the wrong conclusion.

8-10

Year	A	B	C	B – C
0	−$300	−$600	−$200	−$400
1−10	$41	$98	$35	$63
Computed ROR	6.1%	10.1%	11.7%	9.2%
Decision	ROR_A < MARR-reject.	Ok	Ok	$ROR_{\Delta B-C}$ > MARR. Select B.

a) Choice table:

If 0 < MARR ≤ 9.24 Select B
If 9.24 < MARR ≤ 11.7 Select C
If 11.7 < MARR ≤ 100 Do Nothing

b) If the MARR is 8% then select alternative B.

8-11

Looking at Alternatives B & C it is apparent that B dominates C. Since at the same cost B produces a greater annual benefit, it will always be preferred over C. C may, therefore, be immediately discarded.

Year	B	A	D	A–B	D–B	D–A
0	–50	–75	–85	–25	–35	–10
1	$12	$16	$17	4	5	1
2	$12	$16	$17	4	5	1
3	$12	$16	$17	4	5	1
4	$12	$16	$17	4	5	1
5	$12	$16	$17	4	5	1
6	$12	$16	$17	4	5	1
7	$12	$16	$17	4	5	1
8	$12	$16	$17	4	5	1
9	$12	$16	$17	4	5	1
10	$12	$16	$17	4	5	1
ROR	20.2%	16.8%	15.1%	9.6%	7.1%	0.0%

a) Choice table:
If 0 < MARR ≤ 9.6 Select A
If 9.6 < MARR ≤ 20.2 Select B
If 20.2 < MARR ≤ 100 Do Nothing

b) If the MARR is 8% then select alternative A.

8-12

Like all situations where neither input nor output is fixed, the key to the solution is incremental rate of return analysis.

Alternative:	A	B	C
Cost	$200	$300	$600
Annual Benefit	$59.7	$77.1	$165.2
Useful Life	5 yr	5 yr	5 yr
Rate of Return	15%	9%	11.7%

	B − A	C − B	C − A
Δ Cost	\$100	\$300	\$400
Δ Annual Benefit	\$17.4	\$88.1	\$105.5
Δ Rate of Return	< 0%	14.3%	10%

Choice table: (Assuming "Do-Nothing" is not an alternative)
If 0 < MARR ≤ 10 Select C
If 10 < MARR ≤ 100 Select A

8-13

Incremental Rate of Return Solution

	A	B	C	D	C − D	B − C	A − C
Cost	\$1,000	\$800	\$600	\$500	\$100	\$200	\$400
Uniform Annual Benefit	\$122	\$120	\$97	\$122	−\$25	\$23	\$25
Salvage Value	\$750	\$500	\$500	\$0	\$500	\$0	\$250
Rate of Return	10.0%	11.9%	15.0%	17.8%	10.0%	< 0%	1.9%

Rank of alternatives: D - C - B - A (Do nothing is not an alternative)
The C − D increment is desirable if MARR ≤ 10.0%
The B − C increment is never desirable.
The A − C increment is desirable if MARR ≤ 1.9%.

a) Choice table:
If 0 < MARR ≤ 1.9 Select A
If 1.9 < MARR ≤ 10.0 Select C
If 10.0 < MARR ≤ 100 Select D

b) If the MARR is 8% then select alternative C

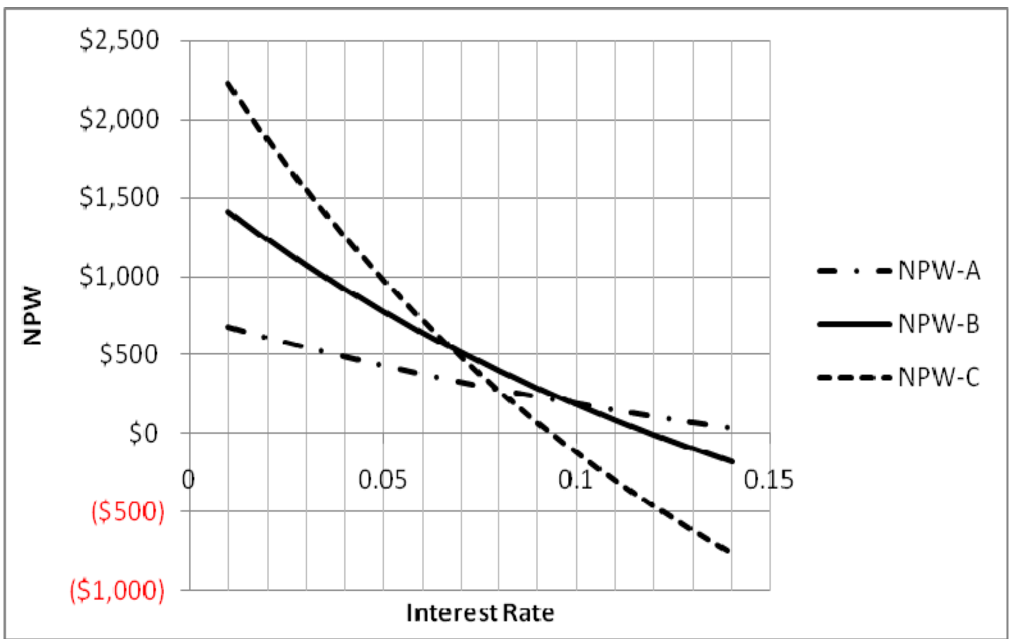
Net Present Worth Solution
Net Present Worth = Uniform Annual Benefit (P/A, 8%, 8) + Salvage Value (P/F, 8%, 8) – First Cost
NPW_A = \$122 (5.747) + \$750 (0.5403) − \$1,000= +\$106.36
NPW_B = \$120 (5.747) + \$500 (0.5403) − \$800 = +\$159.79
NPW_C = \$97 (5.747) + \$500 (0.5403) − \$600 = +\$227.61
NPW_D = \$122 (5.747) − \$500 = +\$201.13
NPW_c is greatest, so it is the best alternative if MARR is 8%.

8-14

Year	A	B	B − A	C	C − B	C – A
0	−$1,000	−$2,000	−$1,000	−$3,000	−$1,000	−$2,000
1	+$150	+$150	$0	$0	−$150	−$150
2	+$150	+$150	$0	$0	−$150	−$150
3	+$150	+$150	$0	$0	−$150	−$150
4	+$150	+$150	$0	$0	−$150	−$150
5	+$150 +$1,000	+$150	−$1,000	$0	$150	$1,150
6		+$150 +$2,700	+$2,850	$0	−$2,850	−$2,850
7				$5,600	+$5,600	+$5,600
Rate of Return	15.0%	11.8%	9.8%	86.7%	6.7%	7.9%

Rank of alternatives is: A - B - C. Do Nothing is not an alternative.

The B − A increment is desirable if MARR ≤ 9.8%.
The C − B increment is desirable if MARR ≤ 6.7%.
The C − A increment is desirable if MARR > 9.8%.



a) Choice table:

If	0	< MARR ≤ 6.7	Select C
If	6.7	< MARR ≤ 9.8	Select B
If	9.8	< MARR ≤ 100	Select A

b) If the MARR is 8% then select alternative B

Check solution by NPW
NPW_A = \$150 (P/A, 8%, 5) + \$1,000 (P/F, 8%, 5) − \$1,000 = +\$279.55
NPW_B = \$150 (P/A, 8%, 6) + \$2,700 (P/F, 8%, 6) − \$2,000 = +\$397.99**
NPW_C = \$5,600 (P/F, 8%, 7) − \$3,000 = +\$267.60

NPW_B is greatest, so it is the best alternative if MARR is 8%.

8-15

Since B has a higher initial cost and higher rate of return, it dominates A with the result that there is no interest rate at which A is the preferred alternative. Assuming this is not recognized, one would first compute the rate of return on the increment B − A and then C − B. The problem has been worked out to make the computations relatively easy.

Year	A	B	B − A
0	−\$770	−\$1,406.3	−\$636.30
1	+\$420	+\$420	\$0
2	+\$420	+\$420	\$0
	−\$770	\$0	+\$770
3	+\$420	+\$420	\$0
4	+\$420	+\$420	\$0

Cash flows repeat for the next four years.
Rate of Return on B − A: \$636.30 = \$770 (P/F, i%, 2)
$\Delta ROR_{B\text{-}A}$ = 10%

Year	B	C	C – B
0	−\$1,406.3	−\$2,563.3	−\$1,157.0
1-3	+\$420	+\$420	\$0
4	+\$420	+\$420	\$0
	−\$1,406.3	\$0	+\$1,406.30
5-8	+\$420	+\$420	\$0

Rate of Return on B − A: \$1,157.00 = \$1,406.30 (P/F, i%, 4)
$\Delta ROR_{C\text{-}B}$ = 5%

Summary of Rates of Return

A	B − A	B	C – B	C
6.0%	10%	7.5%	5%	6.4%

Choice Table

Value of MARR	Decision
If 0 < MARR ≤ 5.0	C is preferred
If 5.0 < MARR ≤ 10.0%	B is preferred
If 10.0 < MARR ≤ 100%	A is preferred

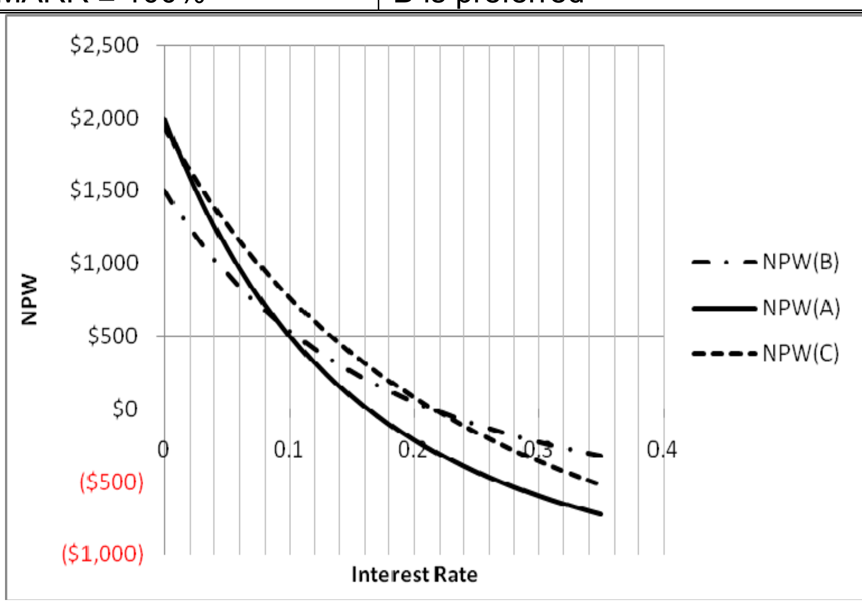
8-16

	A	B	A − B	C	C − B	C − A
Cost	−$1,500	−$1,000	−$500	−$2,035	−$1,035	−$1,035
Annual Benefit, first 5 years	+$250	+$250	$0	+$650	+$400	+$400
Annual Benefit, next 5 years	+$450	+$250	+$200	+$145	–$105	–$105
Rate of return	16.3%	21.4%	9.2%	21.0%	22.0%*	22.0%*

*Two sign changes in C − B cash flow. Used IRR function in Excel.

Choice Table

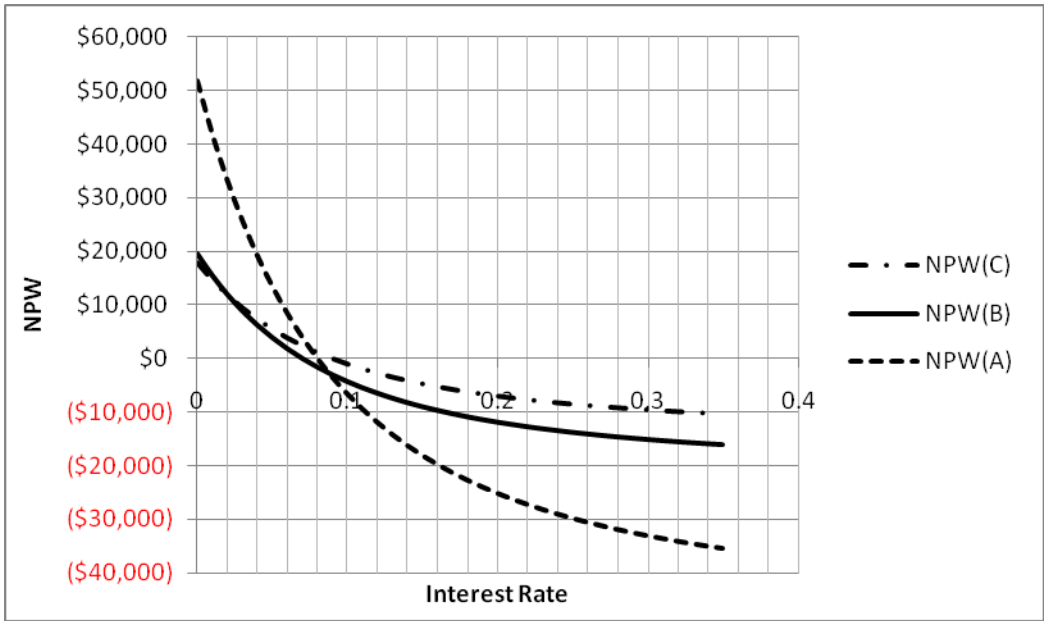
Value of MARR	Decision
If 0 < MARR ≤ 1.0	A is preferred
If 1.0 < MARR ≤ 22.0%	C is preferred
If 22.0 < MARR ≤ 100%	B is preferred



8−17

The ROR of each alternative ≥ MARR. Proceed with incremental analysis. Examine increments of investment.

	C	B	A	B − C	A − C
Initial investment	$15,000	$22,000	$50,000	$7,000	$35,000
Annual Income	$1,643	$2,077	$5,093	$434	$3,450
ROR	9.0%	7.0%	8.0%	2.1%	7.6%



a) **Choice Table**

Value of MARR	Decision
If 0 < MARR ≤ 7.6%	A is preferred
If 7.6 < MARR ≤ 100%	C is preferred

b) If the MARR is 7% then choose alternative A based on the choice table.

Alternative solution: Using incremental analysis we first evaluation B–C
$7,000 = $434 (P/A, i%, 20)
(P/A, i%, 20) = $7,000/$434 = 16.13
ΔROR_{B-c} = 2.1%
Since ΔROR_{B-c} < 7%, reject B.

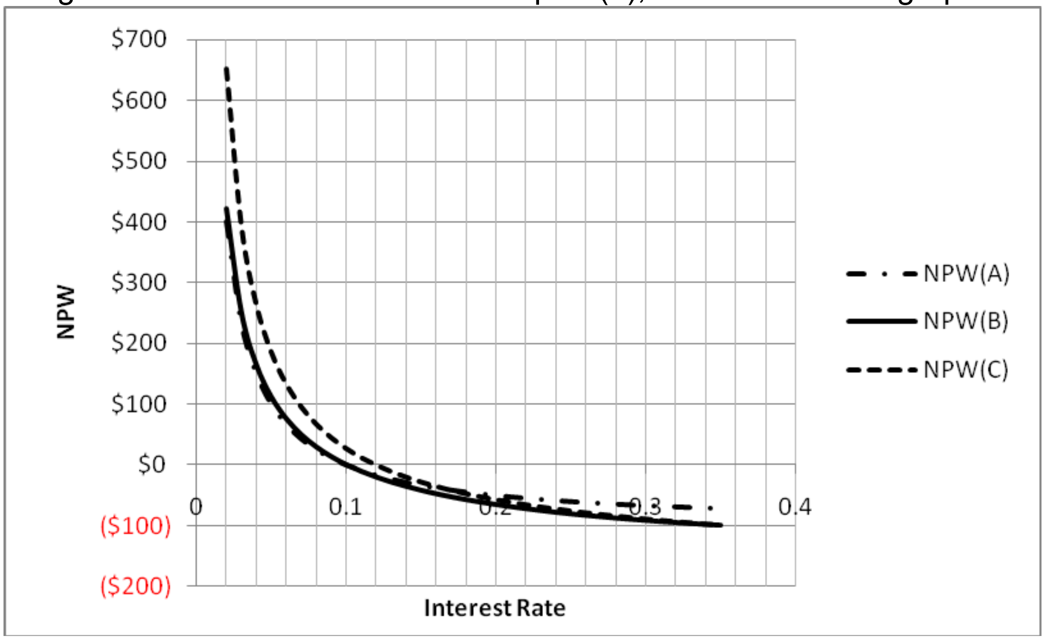
Consider A–C:
$35,000 = $3,450 (P/A, i%, 20)
(P/A, i%, 20) = $35,000/$3,450 = 10.14
ΔROR_{A-c} = 7.6%
Since ΔROR_{A-c} > 7%, reject C and select A.

8-18

a) Choice table:

Value of MARR	Decision
If 0 < MARR ≤ 17%	C is preferred
If 17 < MARR ≤ 100%	A is preferred

Using the same formulas as shown in part (b), we can draw the graph.



b) At an MARR of 8% we can compare the NPW of each alternative.

$NPW_{A:}$

NPW_A = (UAB/i) – PW of Cost = \$10/0.08 − \$100 = +\$25.00

NPW_B:

EUAC = \$150 (A/P, 8%, 20) = \$15.29

EUAB = \$17.62 (Given)

NPW_B = (EUAB – EUAC)/i = (\$17.62 − \$15.29)/0.08 = + \$29.13

NPW_C uses same method as Alternative B:

EUAC = \$200 (A/P, 8%, 5) = \$50.10

NPW_C = (EUAB – EUAC)/i = (\$55.48 − \$50.10)/0.08 = +\$67.25

<u>Select C.</u>

8−19

Year	A	B	C	A− B	C− B	C− A
0	−$20,000	−$20,000	−$20,000	$0	$0	$0
1	$10,000	$10,000	$5,000	$0	−$5,000	−$5,000
2	$5,000	$10,000	$5,000	−$5,000	−$5,000	$0
3	$10,000	$10,000	$5,000	$0	−$5,000	−$5,000
4	$6,000	$0	$15,000	$6,000	$15,000	$9,000
ROR	21.3%	23.4%	15.0%	9.5%	0%	< 0%

a) Choice table:

Value of MARR	Decision
If 0 < MARR ≤ 9.5%	A is preferred
If 9.5 < MARR ≤ 100%	B is preferred

b) If the MARR is 12%, then choose Alternative B.

Using incremental analysis, the order of alternatives is: B - A - C
A – B: Since ROR = 9.5% which is less than MARR, select B
C – B: Since ROR = 0% which is less than MARR, select B.

8-20

Year	Plan A Cash Flow	Plan B Cash Flow	Plan B Rather than Plan A	Plan C Cash flow	Plan C rather than Plan B
0	−$10,000	−$15,000	−$5,000	−$20,000	−$5,0000
1−10	+$1,625	$1,625	$0	+$1,890	+$265
10	−$10,000	$0	+$10,000	$0	$0
11−20	+$1,625	+$1,625	$0	+$1,890	+$265
Rate of Return	10%*	8.8%	7.2%**	7%	0.6%***

*The computation may be made for a 10-year period:
$10,000 = $1,625 (P/A, i%, 10) i = 10%
The second 10-year period has the same return.

**The computation is
$5,000 = $10,000 (P/F, i%, 10)
(P/F, i%, 10) = $5,000/$10,000 = 0.5 i = 7.2%

***The computation is:
$5,000 = $265 (P/A, i%, 20) i = 0.6%
The table above shows two different sets of computations.

(a) Choice table: (Assuming “Do-Nothing” is not an alternative)

If 0 < MARR ≤ 0.6 Select C
If 0.6 < MARR ≤ 7.2 Select B
If 7.2 < MARR ≤ 100 Select A

(b) If the MARR = 6%, then select Plan B Se

8-21

Monthly payment on new warehouse loan = \$350,000 (A/P, 1.25%, 60)
= \$8,330

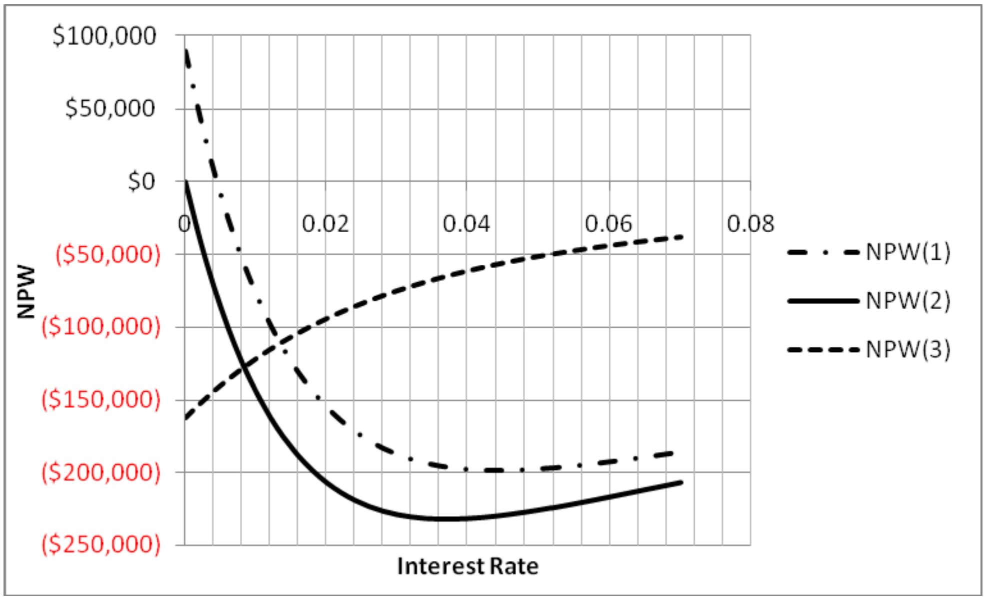
Month	Alt. 1	Alt. 2	Alt. 3
0	−\$100,000	−\$100,000	\$0
1- 60	−\$8,330 +\$2,500 −\$1,000	−\$8,330 \$0 \$0	−\$2,700
60	+\$600,000	+\$600,000	\$0

Month	1–2	1–3
0	\$0	−\$100,000
1- 60	+\$1,500	−\$4,130
60	\$0	+\$600,000
Decision	By inspection, this increment is desirable. Reject 2. Keep 1.	Δ ROR = 1.34%/mo Nom. ROR = (1.34%)12 = 16.1% Eff. ROR = $(1 + 0.0134)^{12} - 1$ = 17.3%

Being less desirable than Alternative 1, Alternative 2 may be rejected.
The 1-3 increment reveals that 1 is preferred only if the MARR is less than 1.34%/month.

Choice table:

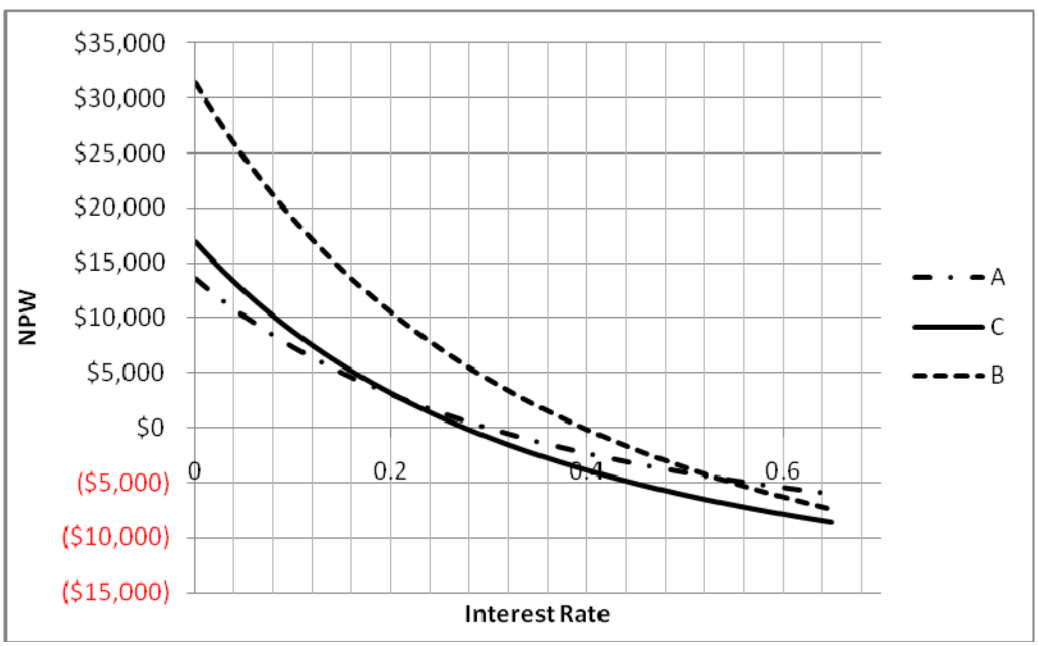
Value of MARR	Decision
If 0 < MARR ≤ 1.34%/month	1 is preferred
If 1.34 < MARR ≤ 100%	3 is preferred



8-22

(a) Choice Table:

Value of MARR	Decision
If 0 < MARR ≤ 53.8%	B is preferred
If 53.8 < MARR ≤ 100%	A is preferred



(b) Incremental analysis for MARR = 15%
(1) Arrange the alternatives in ascending order of investment.

	Company A	Company C	Company B
First Cost	\$15,000	\$20,000	\$25,000

(2) Compute the rate of return for the least cost alternative (Company A) or at least insure that the ROR_A > MARR. At i = 15%:

NPW_A = −\$15,000+(\$8,000 − \$1,600)(P/A, 15%, 4) + \$3,000(P/F, 15%, 4)
= −\$15,000 + \$6,400 (2.855) + \$3,000 (0.5718) = \$4,987

Since NPW_A at i = 15% is positive, ROR_A > 15%.

(3) Consider the increment (Company C – Company A)

	C − A
First Cost	\$5,000
Maintenance & Operating Costs	−\$700
Annual Benefit	\$1,000
Salvage Value	\$1,500

Determine whether the rate of return for the increment (C–A) is more or less than the 15% MARR. At i = 15%:

$NPW_{C\text{-}A}$ = −\$5,000 + [\$1,000 − (−\$700)](P/A, 15%, 4) + \$1,500(P/F, 15%, 4)
= −\$5,000 + \$1,700 (2.855) + \$1,500(0.5718) = \$711

Since $NPW_{C\text{-}A}$ is positive at MARR%, it is desirable. Reject Company A.

(4) Consider the increment (Company B – Company C)

	B−C
First Cost	\$5,000
Maintenance & Operating Costs	−\$500
Annual Benefit	\$4,000
Salvage Value	\$1,500

Determine whether the rate of return for the increment (B – C) is more or less than the 15% MARR. At i = 15%:

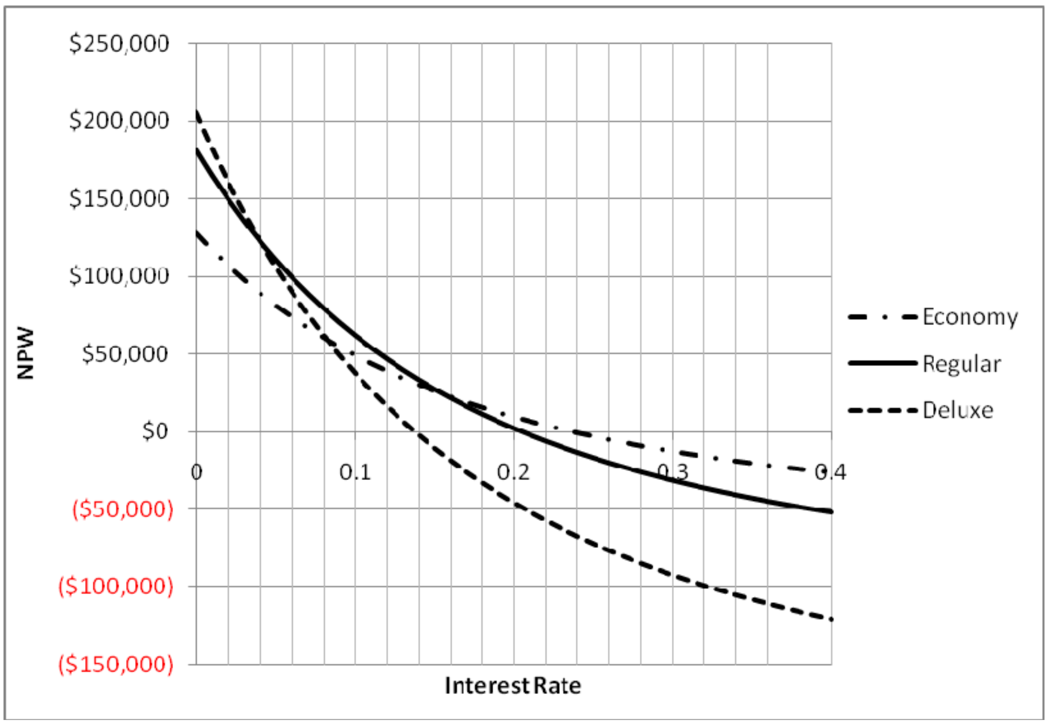
$NPW_{B\text{-}C}$ = −\$5,000 + [\$4,000 − (−\$500)](P/A, 15%, 4) + \$1,500(P/F, 15%, 4)
= −\$5,000 + \$4,500 (2.855) + \$1,500(0.5718)= \$8,705

Since $NPW_{B\text{-}C}$ is positive at MARR%, it is desirable. Select Company B.

8-23

(a) Choice Table:

Value of MARR	Decision
If 0 < MARR ≤ 4.1%	Deluxe is preferred
If 4.1 < MARR ≤ 15.6%	Regular is preferred
If 15.6 < MARR ≤ 100%	Economy is preferred



(b) Incremental analysis for MARR = 15% and n = 10 years
RANKING: Do Nothing < Economy < Regular < Deluxe

Δ (Economy − Do Nothing)
NPW = −$75,000 + ($28,000 – $8,000) (P/A, i^*, 10) + $3,000 (P/F, i^*, 10)

i^*	NPW
0	$128,000
0.15	$26,122
∞	−$75,000

i^* > MARR (actual ROR = 23.6%), so Economy is better than doing nothing.

Δ (Regular – Economy)

NPW = − ($125,000 − $75,000) + [($43,000 − $28,000)
− ($13,000 − $8,000)](P/A, i^*, 10) + ($6,900 − $3,000) (P/F, i^*, 10)

i^*	NPW
0	$53,900
0.15	$1,154
∞	−$50,000

i^* > MARR (actual ROR = 15.6%), so Regular is better than Economy.

Δ (Deluxe – Regular)

NPW = − ($220,000 − $125,000) + [($79,000 – $43,000)
− ($38,000 − $13,000)](P/A, i^*, 10) + ($16,000 − $6,900) (P/F, i^*, 10)

i^*	NPW
0	$24,100
0.15	−$37,541
∞	−$95,000

i^* < MARR (actual ROR = 4.1%), so Deluxe is less desirable than Regular.
The correct choice is the Regular model.

8-24

(a) Develop a choice table using IRR

Year	U-Sort-M	Ship-R - U-Sort-M	Sort-Of - Ship-R	Sort-Of - U-Sort-M
0	–$180,000	–$4,000	–$51,000	–$55,000
1	$56,000	–$1,700	$13,700	$12,000
2	$56,000	–$1,700	$13,700	$12,000
3	$56,000	–$1,700	$13,700	$12,000
4	$56,000	–$1,700	$13,700	$12,000
5	$56,000	–$1,700	$13,700	$12,000
6	$56,000	–$1,700	$13,700	$12,000
7	$70,400	$22,200	$19,400	$41,600
IRR	25.0%	9.98%	19.98%	18.0%

Choice Table:

Value of MARR	Decision
If 0 < MARR ≤ 18.0%	Sort-Of is preferred
If 18.0 < MARR ≤ 25.0%	U-Sort-M is preferred
If 25.0 < MARR ≤ 100%	Do Nothing is preferred

(b) Incremental analysis for MARR = 15%
Put the four alternatives in order of increasing cost:
Do nothing < U-Sort-M < Ship-R < Sort-Of

U-Sort-M: Do Nothing

First Cost	$180,000
Annual Benefit	$68,000
Maintenance & Operating Costs	$12,000
Salvage Value	$14,400

$NPW_{15\%}$ = −$180,000 + ($68,000 − $12,000)(P/A, 15%, 7) + $14,400(P/F, 15%, 7)
= −$180,000 + $232,960 + $5,413
= $58,373
ROR > MARR: Reject Do Nothing

Ship-R – U-Sort-M

First Cost	$4,000
Annual Benefit	$7,300
Maintenance & Operating Costs	$9,000
Salvage Value	$23,900

$$\begin{aligned} NPW_{15\%} &= -\$4,000 + (\$7,300 - \$9,000)(P/A, 15\%, 7) + \$23,900(P/F, 15\%, 7) \\ &= -\$4,000 - \$7,072 + \$8,984 \\ &= -\$2,088 \end{aligned}$$

ROR < MARR: Reject Ship-R

Sort-Of – U-Sort-M

First Cost	$55,000
Annual Benefit	$21,000
Maintenance & Operating Costs	$9,000
Salvage Value	$29,600

$$\begin{aligned} NPW_{15\%} &= -\$55,000 + (\$21,000 - \$9,000)(P/A, 15\%, 7) + \$29,600(P/F, 15\%, 7) \\ &= -\$55,000 + \$49,920 + \$11,127 \\ &= \$6,047 \end{aligned}$$

ROR > MARR: Reject U-Sort-M, Select Sort-of

8-25

(a) Develop a choice table using IRR

Yr	1	2	3	4	2-1	3-2	4-2
0	-$100,000	-$130,000	-$200,000	-$330,000	-$30,000	-$70,000	-$200,000
1	$26,380	$38,780	$47,480	$91,550	$12,400	$8,700	$52,770
2	$26,380	$38,780	$47,480	$91,550	$12,400	$8,700	$52,770
3	$26,380	$38,780	$47,480	$91,550	$12,400	$8,700	$52,770
4	$26,380	$38,780	$47,480	$91,550	$12,400	$8,700	$52,770
5	$26,380	$38,780	$47,480	$91,550	$12,400	$8,700	$52,770
IRR	10.0%	15.0%	6.0%	12.0%	30.35%	-14.02%	10.01%

Choice Table:

Value of MARR	Decision
If 0 < MARR ≤ 10.0%	4 is preferred
If 10.0 < MARR ≤ 15.0%	2 is preferred
If 15.0 < MARR ≤ 100%	Do Nothing is preferred

(b) Incremental analysis for MARR = 8%

Since there are alternatives with ROR > 8% MARR, Alternative 3 may be immediately rejected as well as Alternative 5 (Do-Nothing). Note also that Alternative 2 dominates Alternative 1 since its ROR > ROR Alt. 1 and its investment cost is greater. Thus ΔROR_{2-1} > 15% (actual ROR = 30.4%). So Alternative 1 can be rejected. This leaves alternatives 2 and 4. Examine (4 − 2) increment.

	2	4	4 – 2
Initial Investment	$130.00	$330.00	$200.00
Uniform Annual Benefit	$38.78	$91.55	$52.77

$200 = $52.77 (P/A, i%, 5)
(P/A, i%, 5) = $200/$52.77 = 3.79
ΔROR_{4-2} = 10%
Since ΔROR_{4-2} > 8% MARR, <u>select Alternative 4.</u>

8-26

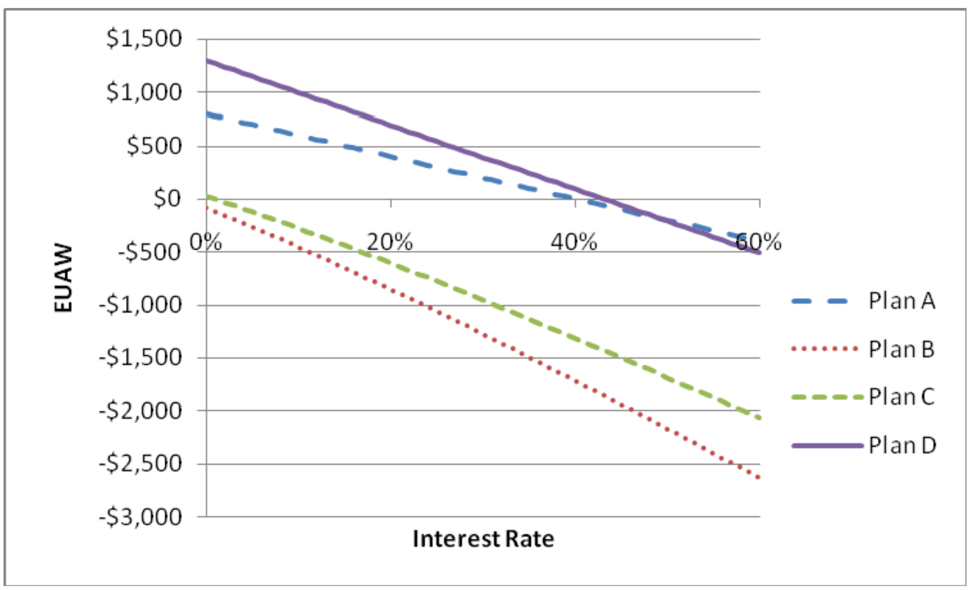
As with Example 8-5, we will use the EUAW as the basis for determining the best alternative. This was done using Excel to compute EUAW with the formula:

= Annual Beneft + PMT(interest rate, life, initial cost) –
PMT(interest rate, life, , Salvage Value)

The last term in this equation computes the EUAW of the salvage value. The first few rows of the table are shown below.

Interest	Plan A	Plan B	Plan C	Plan D
0	$800.00	($83.33)	$28.57	$1,300.00
0.01	$780.00	($118.92)	($0.43)	$1,270.00
0.02	$760.00	($154.84)	($29.73)	$1,240.00
0.03	$740.00	($191.09)	($59.32)	$1,210.00
0.04	$720.00	($227.67)	($89.18)	$1,180.00
0.05	$700.00	($264.56)	($119.33)	$1,150.00

The plot of EUAW for each alternative as a function of interest rate is used to define the choice table.



Note that plans C and B (lower two lines) are never an option.

Choice table: (Assuming “Do-Nothing” is not an alternative)

If 0 < MARR ≤ 50 Select D

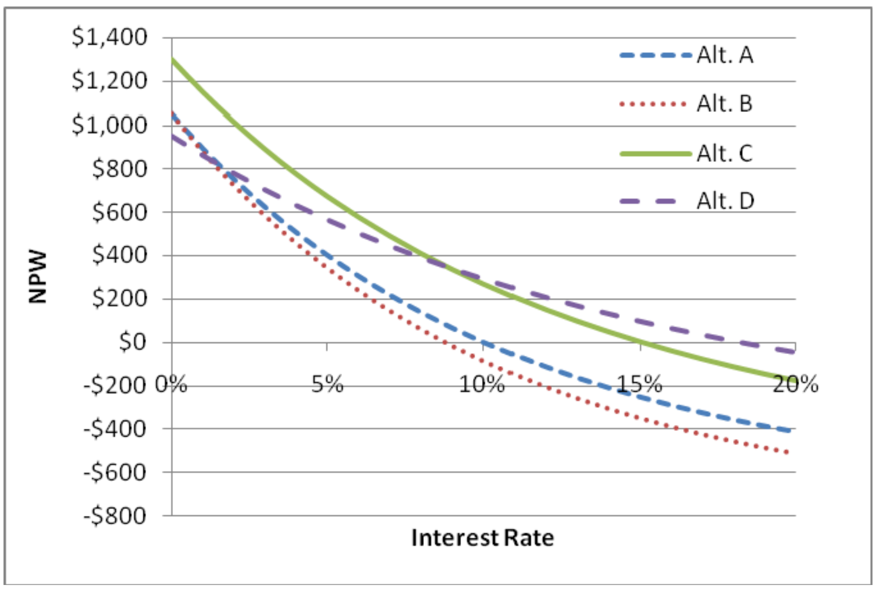
If 50 < MARR ≤ 100 Select A

8-27

As with Example 8-6, we will use the NPW as the basis for determining the best alternative. This was done using Excel and NPV function. The first few rows of the table are shown below.

i	Alt. A	Alt. B	Alt. C	Alt. D
0	$1,050.00	$1,050.00	$1,300.00	$950.00
0.01	$893.50	$878.11	$1,151.03	$861.30
0.02	$751.87	$723.12	$1,015.17	$778.83
0.03	$623.58	$583.24	$891.12	$702.09
0.04	$507.24	$456.90	$777.72	$630.61
0.05	$401.65	$342.69	$673.95	$563.98
0.06	$305.73	$239.36	$578.89	$501.81
0.07	$218.53	$145.81	$491.71	$443.77

Graphing the NPW for each alternative versus the interest rate we get the figure below.



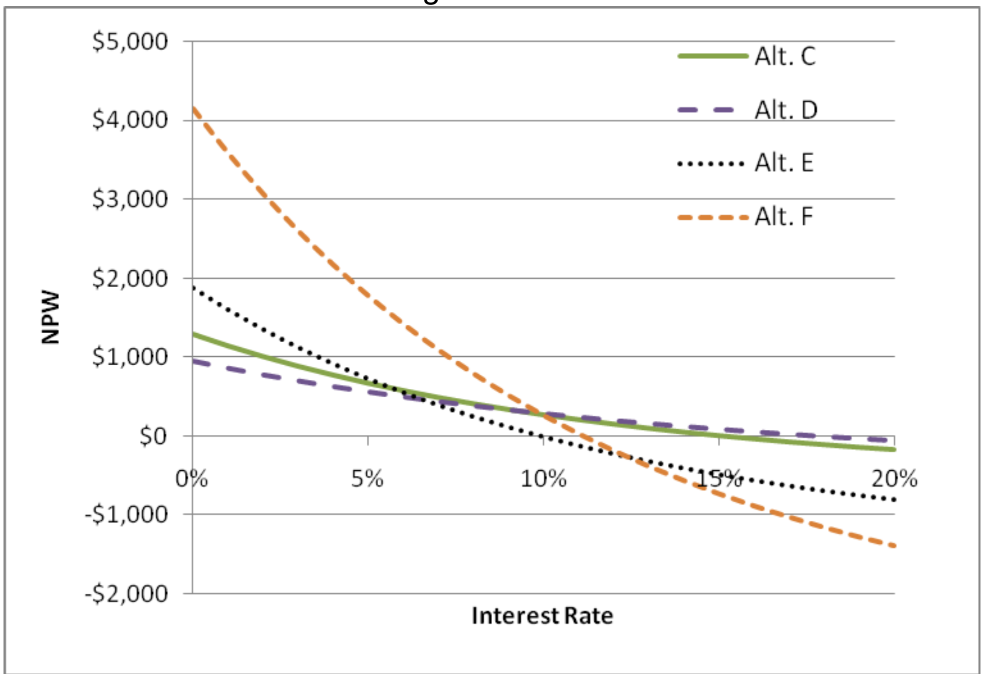
(a) From this we get the choice table: (Assuming "Do-Nothing" is not an alternative)

If	0	< MARR < 9	Select C
If	9	≤ MARR ≤ 100	Select D

(b) IF MARR = 8%, then select alternative C.

8-28

Due to the results in problem 8-27, we can eliminate alternatives A and B from further consideration. Therefore, our focus will be on alternatives C, D, E, and F. Plotting NPW as a function of interest rate we get:



(a) The resulting choice table is:

If	0	< MARR ≤ 9	Select F
If	9	< MARR ≤ 100	Select D

(b) If the MARR is 8% then select alternative F.

8-29

	NPW						
rate	Sell	Keep	1 Story	2 Story	3 Story	4 Story	5 Story
0.000	$0	$330,000	$900,000	$1,080,000	$1,500,000	$1,575,000	$1,800,000
0.010	$0	$277,301	$776,443	$921,333	$1,282,517	$1,334,511	$1,525,156
0.020	$0	$231,287	$668,162	$782,520	$1,092,187	$1,124,311	$1,284,926
0.030	$0	$191,007	$573,021	$660,765	$925,190	$940,112	$1,074,414
0.040	$0	$155,657	$489,209	$553,696	$778,287	$778,287	$889,471
0.050	$0	$124,556	$415,186	$459,300	$648,729	$635,754	$726,576
0.060	$0	$97,122	$349,641	$375,864	$534,174	$509,893	$582,735
0.070	$0	$72,863	$291,453	$301,927	$432,626	$398,471	$455,396
0.080	$0	$51,357	$239,665	$236,242	$342,379	$299,582	$342,379
0.090	$0	$32,243	$193,457	$177,738	$261,972	$211,593	$241,821
0.100	$0	$15,212	$152,122	$125,500	$190,152	$133,106	$152,122
0.110	$0	$0	$115,054	$78,740	$125,840	$62,920	$71,909
0.114	$0	-$5,628	$101,302	$61,414	$102,005	$36,933	$42,209
0.115	$0	-$6,997	$97,954	$57,198	$96,205	$30,611	$34,984
0.120	$0	-$13,622	$81,730	$36,779	$68,109	$0	$0
0.130	$0	-$25,850	$51,699	-$969	$16,156	-$56,546	-$64,624
0.140	$0	-$36,853	$24,569	-$35,010	-$30,711	-$107,488	-$122,843
0.150	$0	-$46,779	$0	-$65,783	-$73,092	-$153,493	-$175,421
0.160	$0	-$55,755	-$22,302	-$93,668	-$111,509	-$195,141	-$223,018
0.170	$0	-$63,890	-$42,593	-$118,996	-$146,415	-$232,933	-$266,209
0.180	$0	-$71,282	-$61,099	-$142,055	-$178,205	-$267,308	-$305,495
0.190	$0	-$78,014	-$78,014	-$163,098	-$207,224	-$298,647	-$341,310
0.200	$0	-$84,159	-$93,509	-$182,343	-$233,774	-$327,283	-$374,038
0.210	$0	-$89,780	-$107,736	-$199,985	-$258,118	-$353,509	-$404,011
0.220	$0	-$94,935	-$120,826	-$216,192	-$280,489	-$377,581	-$431,522
0.230	$0	-$99,671	-$132,895	-$231,113	-$301,091	-$399,724	-$456,828
0.240	$0	-$104,034	-$144,047	-$244,879	-$320,104	-$420,136	-$480,155
0.250	$0	-$108,059	-$154,371	-$257,606	-$337,685	-$438,991	-$501,704

a) Choice Table

Using the NPW values from the table above, we can identify the decision for each interest value.

Value of MARR	Decision
If 0 < MARR ≤ 7.7%	5 Story is preferred
If 7.7 < MARR ≤ 11.4%	3 Story is preferred
If 11.4 < MARR ≤ 15.0%	1 Story is preferred
If 15.0 < MARR ≤ 100%	Sell is preferred

b) Incremental Analysis at MARR = 10%

This problem is one of neither fixed input nor fixed output. When the estimated resale value equals the present total investment, we have the special case where A = Pi or i = A/P (comes from: EUAC = (P – S)(A/F, i, n) + Pi where P = S).

Alternative	P*	A*	i	ΔP	ΔA	Δi	Decision
Sell Parking Lot	0	0	0%				
Keep Parking Lot	$200	$22	11.0%	$200	$22	11.0%	Keep Lot
1 Story Building	$400	$60	15.0%	$200	$38	19.0%	1 Story
2 Story Building	$555	$72	12.9%	$155	$12	7.7%	1 Story
3 Story Building	$750	$100	13.3%	$350	$40	11.4%	3 story
4 Story Building	$875	$105	12.0%	$125	$5	4.0%	3 Story
5 Story Building	$1,000	$120	12.0%	$250	$20	8.0%	3 Story

*All values in thousands.

Conclusion: <u>Build 3 story building.</u>

8-30

Year	Denver	Dallas	San Antonio	Los Angeles	Cleveland	Atlanta
0	–$300	–$550	–$450	–$750	–$150	–$200
1	$52	$137	$117	$167	$18	$49
2	$52	$137	$117	$167	$18	$49
3	$52	$137	$117	$167	$18	$49
4	$52	$137	$117	$167	$18	$49
5	$52	$137	$117	$167	$18	$49
6	$52	$137	$117	$167	$18	$49
7	$52	$137	$117	$167	$18	$49
8	$52	$137	$117	$167	$18	$49
IRR	7.9%	18.5%	19.9%	15.0%	–0.9%	18.0%

Year	San - Atl	Dal - San	San - Dal	LA - San	LA - Dal
0	–$250	–$100	–$200	–$250	$200
1	$68	$20	$30	$85	–$30
2	$68	$20	$30	$85	–$30
3	$68	$20	$30	$85	–$30
4	$68	$20	$30	$85	–$30
5	$68	$20	$30	$85	–$30
6	$68	$20	$30	$85	–$30
7	$68	$20	$30	$85	–$30
8	$68	$20	$30	$85	–$30
IRR	21.5%	11.8%	4.2%	29.8%	4.2%

a) Choice Table

Using the IRR values from the table above, we can identify the decision for each interest value. Order of evaluation: Chicago - Cleveland - Atlanta - Denver - San Antonio - Dallas - Los Angeles.

Value of MARR	Decision
If 0 < MARR ≤ 4.2%	Los Angeles is preferred
If 4.2 < MARR ≤ 12.0%	Dallas is preferred
If 12.0 < MARR ≤ 19.9%	San Antonio is preferred
If 19.9 < MARR ≤ 100%	Chicago is preferred

b) Incremental Analysis at MARR = 10%

Plant Location	P	A	(P/A, i%, 8) = P/A	Computed i	
1. Denver	$300	$52	5.77	7.9%	Reject
2. Dallas	$550	$137	4.01	18.5%	
3. San Antonio	$450	$117	3.85	19.9%	
4. Los Angeles	$750	$167	4.49	15.0%	
5. Cleveland	$150	$18	8.33	< 0%	Reject
6. Atlanta	$200	$49	4.08	18.0%	
7. Chicago	0	0	0	0%	Reject

Rearrange remaining alternatives in order of increasing cost:

Plant Location	P	A	Inc.	ΔP	ΔA	Δi	
6. Atlanta	$200	$49					
3. San Antonio	$450	$117	3−6	$250	$68	21.5%	Retain 3
2. Dallas	$550	$137	2−3	$100	$20	11.8%	Retain 2
4. Los Angeles	$750	$167	4−2	$200	$20	4.2%	Retain 2

Decision: <u>Select Dallas</u>

8-31

Yr	A	B	C	D	A - C	D - A	B - A
0	-$145,000	-$300,000	-$100,000	-$200,000	-$45,000	-$55,000	$155,000
1	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
2	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
3	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
4	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
5	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
6	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
7	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
8	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
9	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
10	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
11	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
12	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
13	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
14	$23,300	$44,300	$10,000	$27,500	$13,300	$4,200	-$21,000
15	$93,300	$114,300	$80,000	$97,500	$13,300	$4,200	-$21,000
IRR	15.0%	12.8%	9.0%	12.0%	28.90%	1.75%	10.53%

a) Choice Table
Using the IRR values from the table above, we can identify the decision for each interest value. Order of evaluation: Chicago - Cleveland - Atlanta - Denver - San Antonio - Dallas - Los Angeles.

Value of MARR	Decision
If 0 < MARR ≤ 10.5%	B is preferred
If 10.5 < MARR ≤ 15.0%	A is preferred
If 15.0 < MARR ≤ 100%	Do Nothing is preferred

b) Incremental Analysis at MARR = 10%

Plan	Cost of Improvements and Land	Net Annual Income	Salvage Value	Computed Rate of Return	Decision
A	$145,000	$23,300	$70,000	15%	Accept
B	$300,000	$44,300	$70,000	12.9%	Accept
C	$100,000	$10,000	$70,000	9%	Reject: fails to meet the 10% criterion
D	$200,000	$27,500	$70,000	12%	Accept

Rank the three remaining projects in order of cost and examine each separable increment of investment.

Plan D rather than Plan A

Δ Investment	Δ Annual Income	Δ Salvage Value
\$55,000	\$4,200	\$0

\$55,000 = \$4,200 (P/A, i%, 15)
(P/A, i%, 15) = \$55,000/\$4,200 = 13.1
From interest tables: i = 1.75%
This is an unacceptable increment of investment. Reject D and retain A.

Plan B rather than Plan A

Δ Investment	Δ Annual Income	Δ Salvage Value
\$155,000	\$21,000	\$0

\$155,000 = \$21,000 (P/A, i%, 15)
(P/A, i%, 15) = \$155,000/\$21,000 = 7.38
From interest tables: i = 10.5%

This is a desirable increment of investment. Reject A and accept B.
Conclusion: Select Plan B.

8-32

(a) It seems best to annualize each cash flow over some life expectancy, say 20 years (can be variable in Excel), and then use Excel to analyze for various interest rates. Note that for the life annuity, payments would cease at death. The 10-year annuity would pay out all of its benefits even if death occurs before the tenth year. Thus,

EUAB(A) = 30976 (A/P, i, n), EUAB(B) = 12 x 359.60 = 4315.20, and
EUAB(C) = (12 x 513.80 = 6165.60) (P/A, i, 10) (A/P, i, n), where here n = 20 will be used. The Excel results for the choice table are (see below):

Choice B	0% ≤ i < 8.84%
Choice C	8.84% ≤ i < 14.97%
Choice A	14.97% ≤ i

$30,976.00	(A) Amount obtained now		
$4,315.20	(B) Amount per year for life		
$6,165.60	(C) Amount per year for 10 years		
20	Life expectancy in years		
The following table annualizes each cash flow at various interest rates.			
Interest Rate	A	B	C
0%	$1,548.80	$4,315.20	$3,082.80
2%	1,894.39	4,315.20	3,387.04
4%	2,279.27	4,315.20	3,679.72
6%	2,700.63	4,315.20	3,956.38
8%	3,154.97	4,315.20	4,213.80
9%	3,393.31	4,315.20	4,334.61
10%	3,638.43	4,315.20	4,449.95
12%	4,147.03	4,315.20	4,663.94
14%	4,676.94	4,315.20	4,855.78
15%	4,948.77	4,315.20	4,943.61
16%	5,224.63	4,315.20	5,026.23
20%	6,361.12	4,315.20	5,308.28
30%	9,341.96	4,315.20	5,748.61
40%	12,405.23	4,315.20	5,959.57
50%	15,492.66	4,315.20	6,060.50

An alternate method would be to use an incremental analysis for benefits as shown below:

Year	A	C	B	C–A	B–C
0	$30,976	0	0	–$30,976	0
1	0	$6,165.6	$4,315.2	6,165.6	–$1,850.4
2	0	6,165.6	4,315.2	6,165.6	–1,850.4
.	.	.	.	.	.
10	0	6,165.6	4,315.2	6,165.6	–1,850.4
11	0	0	4,315.2	0	4,315.2
.	.	.	.	.	.
20	0	0	4,315.2	0	4,315.2

Using IRR (Excel) obtain ROR(B – C) = 8.84% and ROR(C – A) = 14.97%. The results of the first method are confirmed.

(b) Here we want to determine which option has the largest yearly benefit. It seems reasonable to check the equivalent yearly value of the difference at the desired interest rate (here 9%) and accept the option with the highest initial benefit if negative and the lowest initial benefit if positive. Using the second chart in part (a):

B-C: [–1850.4 (P/A, 9%,10) + 4315.2 (P/A, 9%, 10) (P/F, 9%, 10)] (A/P, 9%, 20) = –19.4, so accept C. Next,

C-A: –30976 (A/P, 9%, 20) + 6165.6 (P/A, 9%, 10) (A/P, 9%, 20) = 941.1, so accept option C.

The result is consistent with the choice table developed in part (a).

8-33

Given that the values shown in the table for profit rate, one can see that the life of the investment is considered infinite (i = A/P). For example:

Alt. A: i = $30,000/$100,000 = 0.30
Alt. B: i = $66,000/$300,000 = 0.22

Rank the Alternatives: A - B - C. Evaluate at MARR = 20%.

Increment A over Do-Nothing
Alternative A produces a 30% profit rate > MARR. Therefore, it is worth investing over doing nothing.

Increment B over A
i = ($66,000 – $30,000)/($300,000 – $100,000) = 18% < MARR, so we will reject B and select A.

Increment C over A
i = ($80,000 – $30,000)/($500,000 – $100,000) = 12,5% < MARR, so we will reject C and select A.

Thus the best investment of $300,000, for example, would be Alternative A (annual profit = $30,000) plus $200,000 elsewhere (yielding 20% or $40,000 annually). This combination yields a $70,000 profit, which is better than Alternative B profit of $66,000. Alternative C did not even yield the minimum 20% and could be eliminated up front.

8-34

Incremental Analysis
Since no life is specified for the alternatives we will assume a 20 year life.

	A	B	C	D	B-A	C-A	D-C
Investment	\$10,000	\$18,000	\$25,000	\$30,000	\$8,000	\$15,000	\$5,000
Net Annual Income	\$2,000	\$3,000	\$4,500	\$5,000	\$1,000	\$2,500	\$500

Order of alternatives: Do Nothing → A → B → C → D

ΔIRR of (A - Do Nothing)
NPW = –\$10,000 + \$2,000(P/A, i%, 20) = 0
Solve using Excel function = RATE(20, 2000, –10000)
ΔIRR = 19.4% > MARR, so we prefer the high cost alternative A over doing nothing.

ΔIRR of (B - A)
NPW = –\$8,000 + \$1,000(P/A, i%, 20) = 0
Solve using Excel function = RATE(20, 1000, –8000)
ΔIRR = 10.9% < MARR, so we prefer the low cost alternative A.
ΔIRR of (C - A)
NPW = –\$15,000 + \$2,500(P/A, i%, 20) = 0
ΔIRR = 15.8% > MARR, so we prefer the high cost alternative C.

ΔIRR of (D - C)
NPW = –\$5,000 + \$500(P/A, 15%, 20) = 0
ΔIRR = 7.8% < MARR, so we prefer the low cost alternative C.

8-35

Lease: Pay \$267 per month for 24 months.
Purchase: A = \$9,400 (A/P, 1%, 24) = \$9,400 (0.0471) = \$442.74
Salvage (resale) value = \$4,700

(a) Purchase rather than lease
ΔMonthly payment = \$442.74 − \$267= \$175.74
ΔSalvage value = \$4,700 − \$0 = \$4,700
Δ Rate of return
PW of Cost = PW of Benefit

\$175.74 (F/A, i%, 24) = \$4,700
(F/A, i%, 24) = \$4,700/\$175.74 = 26.74
i = 0.93% per month

$i_{eff} = (1 + 0.0093)^{12} – 1 = 0.117$
Thus, the additional monthly payment of $175.74 would yield an 11.7% rate of return. Leasing is therefore preferred at all interest rates above 11.7%.

(b) Items that might make leasing more desirable:
1. One does not have, or does not want to spend, the additional $175.74 per month.
2. One can make more than 11.7% rate of return in other investment.
3. One does not have to be concerned about the resale value of the car at the end of two years.

8-36

(a) It will be assumed that the choice table is to be developed for the nominal annual interest rate. Note that the car can be sold in 5 years for $6,500 for the two cases where it is purchased. Also, i = r/12 and n = 12 x 5 = 60 payment periods. Determine EUAC for all three options and use Excel to develop the choice table by varying the interest rate.

EUAC(A) = 19,999(A/P, i, 60) – 6500 (A/F, i, 60)
EUAC(B) = 19,999(A/P, 0.5%, 60) – 6,500(A/F, i, 60) = 386.64 – 6,500(A/F, i, 60)
EUAC(C) = [1,000 + 299 + 299 (P/A, i, 59)] (A/P, i, 60)

The Excel results for the choice table (see below) are:

Choice A	0.0% ≤ i ≤ 6.0%
Choice B	6.0% ≤ i ≤ 30.0%
Choice C	30.0% ≤ i

$19,999.00	(A) Pay now for car (sell in 5 years)		
$6,500.00	Sell in 5 years		
6.00%	(B) Interest rate on 5-year loan with monthly payments (sell in 5 years)		
$299.00	(C) Monthly lease payment (at first of each month) plus $1000 at first payment		
5	Number of years		
60	Number of months		
The following table gives the monthly cost of each option for each interest rate.			
Interest Rate	A	B	C
0%	$224.98	$278.30	$315.67
2%	247.44	283.54	317.03

4%	270.27	288.60	318.41
6%	293.47	293.47	319.83
7%	305.21	295.85	320.55
8%	317.04	298.17	321.27
10%	340.98	302.70	322.74
12%	365.28	307.05	324.23
14%	389.93	311.23	325.76
16%	414.94	315.24	327.30
18%	440.29	319.08	328.88
20%	465.97	322.76	330.48
25%	531.63	331.27	334.58
29%	558.57	337.40	337.97
30%	599.24	338.84	338.83
35%	668.67	345.53	343.21
40%	739.78	351.42	347.72
45%	812.42	356.56	352.34
50%	886.45	361.04	357.06

(b) Using incremental rate of return analysis.

An alternative method would be to use an incremental analysis for costs as shown below:

Month	B	C	A	C – B	A – B
0	0	–$1,299	–$19,999	–$1,299	–$19,999
1	–$386	–299	0	87	386
.	.	.	.	.	.
59	–386	–299	0	87	386
60	–386 6,500	0	6,500	–6,114	386

Using IRR (Excel) obtain ROR(A – B) = 0.500%, so, r = (12)(0.500%) = 6.0% and ROR(C – B) = 2.498%, so, r = (12)(2.498%) = 29.976% or 30.0%. The results of the first method are confirmed. Note that knowledge of the choice order is required here.

Here we want to determine which option has the least effective monthly cost. It seems reasonable to check the equivalent monthly value of the difference in cost at the desired interest rate (here 9%/12 = 0.75%) and accept the option with the least initial cost if positive and the higher initial cost if negative. Here order in increasing original cost (i.e. B, C, A) then:

C-B: ∆Monthly Cost = –1299 (A/P, 0.75%, 60) – 87 +
(6113.36 + 87.64)(A/F, 0.75%, 60)
= +21.85, so accept option B. Next,

A-B: ΔMonthly Cost = –19999 (A/P, 0.75%, 60) – 386.64 = +29.34, so accept option B. The result is consistent with the choice table of part (a).

8-37

(a) It will be assumed that the choice table is to be developed for the nominal annual interest rate. Note that the car can be sold in 5 years for $4,500 for the two cases where it is purchased. Also, i = r/12 and n = 12 x 5 = 60 payment periods. Determine EUAC for all three options and use Excel to develop the choice table by varying the interest rate.

EUAC(A) = 15,999(A/P, i, 60) – 4,500 (A/F, i, 60)
EUAC(B) = 15,999 (A/P, 0.75%, 60) – 4,500 (A/F, i, 60)
= 332.11 – 4,500 (A/F, i, 60)
EUAC(C) = [500 + 269 + 269 (P/A, i, 59)] (A/P, i, 60)

The Excel results for the choice table (see below) are:

Choice A	0% ≤ i ≤ 9.0%
Choice B	9.0% ≤ i < 18.49%
Choice C	18.49% ≤ i

$15,999.00	(A) Pay now for car		
$4,500.00	Sell in 5 years		
9.00%	(B) Interest rate on 5-year loan with monthly payments		
$269.00	(C) Monthly lease payment (at first of each month) plus $500 for first payment		
5	Number of years		
60	Number of months		
The following table gives the monthly cost of each option.			
Interest Rate	A	B	C
0%	$191.65	$257.11	$277.33
2%	209.05	260.74	278.21
4%	226.77	264.24	279.10
6%	244.81	267.62	280.01
8%	263.16	270.87	280.93
9%	272.450	272.450	281.40
10%	281.82	274.00	281.87
12%	300.79	277.01	282.81
14%	320.06	279.91	283.77
16%	339.63	282.68	284.75
18%	359.50	285.34	285.73
18.5%	364.51	285.99	285.98

20%	379.65	287.89	286.73
25%	431.26	293.78	289.28
30%	484.53	299.02	291.90
35%	539.35	303.66	294.59
40%	595.61	307.73	297.34
45%	653.16	311.29	300.15
50%	711.91	314.39	303.01

(b) An alternate method would be to use an incremental analysis for costs as shown below:

Month	B	C	A	C – B	A – B
0	0	–$769	–$15,999	–$769	–$15,999
1	–$333	–269	0	64	333
.	.	.	.	.	.
59	–333	–269	0	64	333
60	–333 4,500	0	4,500	–4,167	333

Using IRR (Excel) obtain ROR(B − A) = 0.750%, so, r = (12)(0.750%) = 9.0% and ROR(C − B) = 1.541%, so, r = (12)(1.541%) = 18.49%. The results of the first method are confirmed. Note that knowledge of the choice order is required here.

(b) Here we want to determine which option has the least effective monthly cost. It seems reasonable to check the equivalent monthly value of the difference at the desired interest rate (here 9%/12 = 0.75%) and accept the option with the least initial cost if positive and the higher initial cost if negative. Here the order in increasing original cost (i.e. B, C, A) then:

C – B: ΔMonthly Cost = 769 (A/P, 0.75%, 60) – 63.11 + (4167.89 + 63.11) (A/F, 0.75%, 60)
= 9.16, so accept option B.

A – B: ΔMonthly Cost = 15999 (A/P, 0.75%, 60) – 332.11 = 0.67, so accept either option B or option A based on intangibles.

The result is consistent with the choice table of part (a).

8-38

(a) It will be assumed that the choice table is to be developed for the nominal annual interest rate. Note that the car can be sold in 10 years for $2,000 for all three cases. Also, i = r/12 and n = 6 x 10 = 60 payment periods for the loan case. For the lease case the car can be bought for $6,500 at the end of the 5th year. Determine EUAC for all three options and use Excel to develop the choice table by varying the interest rate.

EUAC(A) = 19,999(A/P, i, 120) – 2,000 (A/F, i, 120)
EUAC(B) = [19,999 (A/P, 0.5%, 60) (P/A, i, 60) – 2000 (P/F, i, 120)] (A/P, i, 120)
EUAC(C) = [1,000 + 299 + 299 (P/A, i, 59) +6,500 (P/F, i, 60) – 2,000 (P/F, i, 120)] (A/P, i, 120)

The Excel results for the choice table (see below) are:

Choice A	0.0% ≤ i ≤ 6.0%
Choice B	6.0% ≤ i ≤ 30.0%
Choice C	30.0% ≤ i

$19,999.00	(A) Pay now for car	
$2,000.00	Sell in 10 years for all three cases.	
6.00%	(B) Interest rate on 5-year loan with monthly payments	
$299.00	(C) Monthly lease payment (at first of each month) plus $1,000 at first payment; buy year 5 for $6,500.	
10	Number of years	
120	Number of months	
The following table gives the monthly cost of each option.		

Interest Rate	A	B	C
0%	$149.99	$176.65	$195.33
2%	168.95	187.90	205.48
4%	188.90	198.97	215.36
5%	199.24	204.43	220.20
6%	209.83	209.83	224.96
7%	220.65	215.16	229.64
8%	231.71	220.42	234.24
10%	254.52	230.71	243.18
12%	278.23	240.68	251.76
14%	302.80	250.28	259.97
16%	328.17	259.50	267.81
18%	354.32	268.31	275.26
20%	381.17	276.71	282.34
24%	436.84	292.23	295.37
25%	451.13	295.84	298.40
30%	524.49	312.31	312.30
35%	600.52	326.25	324.28
40%	678.60	337.88	334.63
45%	758.20	347.46	343.65
50%	838.93	355.28	351.62

(b) An alternate method would be to use an incremental analysis for costs as shown below:

Month	B	C	A	C–B	A–B
0	0	–\$1,299	–\$19,999	–\$1,299	–\$19,999
1	–\$386	–299	0	87	386
·	·	·	·	·	·
59	–386	–299	0	87	386
60	–386	–6,500	0	–6,114	386
61	0	0	0	0	0
·	·	·	·	·	·
119	0	0	0	0	0
120	2,000	2,000	2,000	0	0

The difference columns are identical with those in Problem 8-36, so the results will be the same, i.e., using IRR (Excel) obtain ROR(A − B) = 0.500%, so, r = (12)(0.500%) = 6.0% and ROR(C − B) = 2.498%, so, r = (12)(2.498%) = 29.976% or 30.0%. The results of the first method are confirmed. Note that knowledge of the choice order is required here.

Here we want to determine which option has the least effective monthly cost. It seems reasonable to check the equivalent monthly value of the difference in cost at the desired interest rate (here 9%/12 = 0.75%) and accept the option with the least initial cost if positive and the higher initial cost if negative. Here order in increasing original cost (i.e. B, C, A) and, since the value of the automobile cancels at month 120 the analysis can be done for 60 months, we obtain:

C − B: ΔMonthly Cost = 1299 (A/P, 0.75%, 60) – 87.64 + (6113.36 + 87.64) (A/F, 0.75%, 60) = +21.85, so accept option B. Next,

A − B: ΔMonthly Cost = 19999 (A/P, 0.75%, 60) – 386.64 = +29.34, so accept option B. The result is consistent with the choice table of part (a).

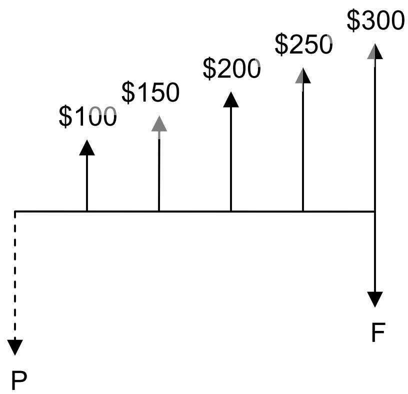
The answers are the same as in Problem 8-36 because the incremental cash flows are the same.

8-39

Results will depend on the student’s experience with the car dealer.

Chapter 9: Other Analysis Techniques

9-1



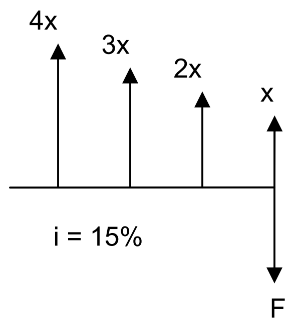
P = $100 (P/A, 12%, 5) + $50 (P/G, 12%, 5) = $100 (3.605) + $50 (6.397)
= $680.35

F = $680.35 (F/P, 12%, 5)
= $680.35 (1.762)
= $1,198.78

Alternate Solution

F = [$100 + $50 (A/G, 12%, 5)] (F/A, 12%, 5)
= [$100 + $50 (1.775)] (6.353)
= 1,199.13

9-2

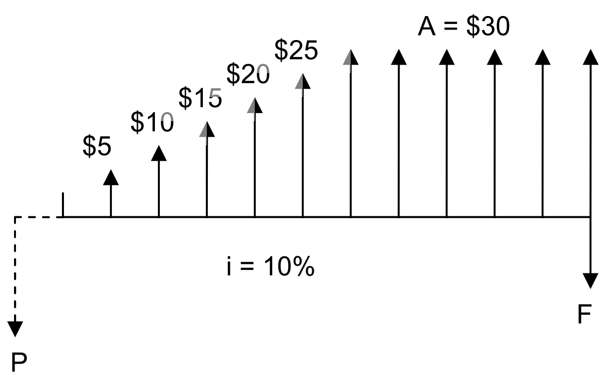


F = [4x − x(A/G, 15%, 4)] (F/A, 15%, 4)
= [4x − x(1.326)] (4.993)
= 13.35x

Alternate Solution

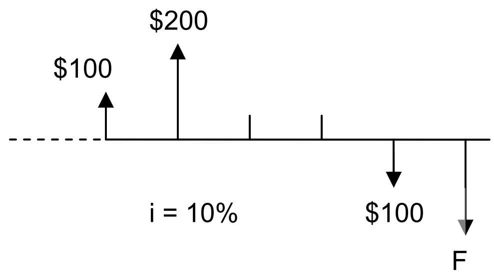
F = 4x (F/P, 15%, 3) + 3x (F/P, 15%, 2) + 2x (F/P, 15%, 1) + x
= 4x (1.521) + 3x (1.322) + 2x (1.150) + x
= 13.35x

9-3



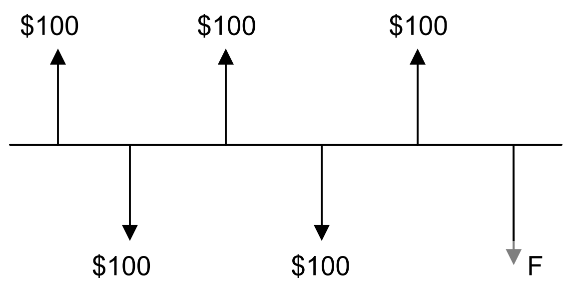
F = $5 (P/G, 10%, 6) (F/P, 10%, 12) + $30 (F/A, 10%, 6)
= $5 (9.684) (3.138) + $30 (7.716)
= 383.42

9-4



F = \$100 (F/P, 12%, 5) + \$200 (F/P, 12%, 4) − \$100 (F/P, 12%, 1)
= \$100 (1.762) + \$200 (1.574) − \$100 (1.120)
= 379.00

9-5



F = \$100 (F/P, 10%, 5) + \$100 (F/P, 10%, 3) + \$100 (F/P, 10%, 1)
− \$100 (F/P, 10%, 4) − \$100 (F/P, 10%, 2)
= \$100(1.611 + 1.331 + 1.100 − 1.464 − 1.210)
= \$136.80

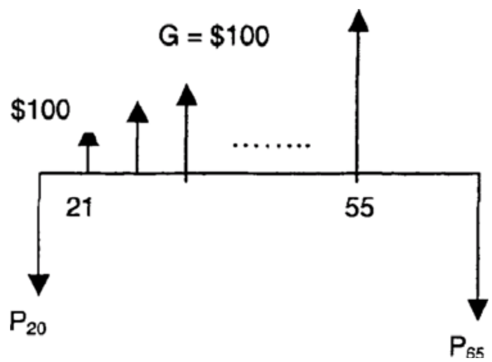
9-6

P = \$15,000 (P/A, 12%, 10) + \$1,200 (P/G, 12%, 10)
P = \$15,000 (5.650) + \$1,200 (20.254)
P = \$84,750 + \$24,305 = \$109,055
F = P (F/P, 12%, 10)= \$109,055 (3.106) = \$338,725

9-7

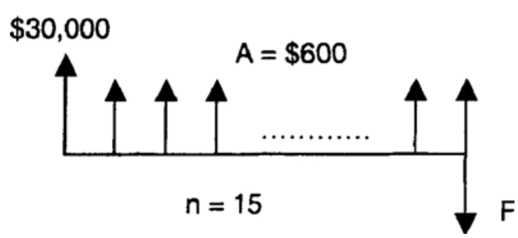
F = \$100 (F/A, ½%, 24) (F/P, ½%, 60)
= \$100 (25.432) (1.349)
= <u>\$3,430.78</u>

9-8



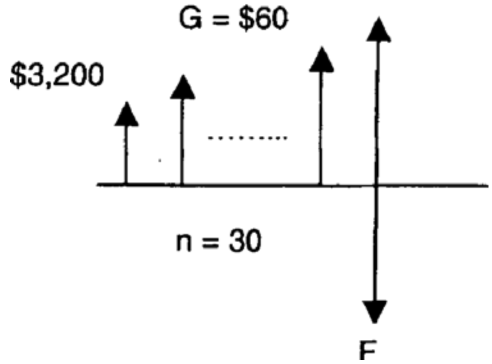
P_{20} = \$100 (P/A, 12%, 35) + \$100 (P/G, 12%, 35)
= \$100 (8.176) + \$100 (62.605)
= \$7,078.10
P_{65} = P_{20} (F/P, 12%, 45) = \$7,078.10 (163.988) = <u>\$1,160,723</u>
P_{65} = <u>\$1,160,700</u> (rounded to 5 siginificant digits)

9-9



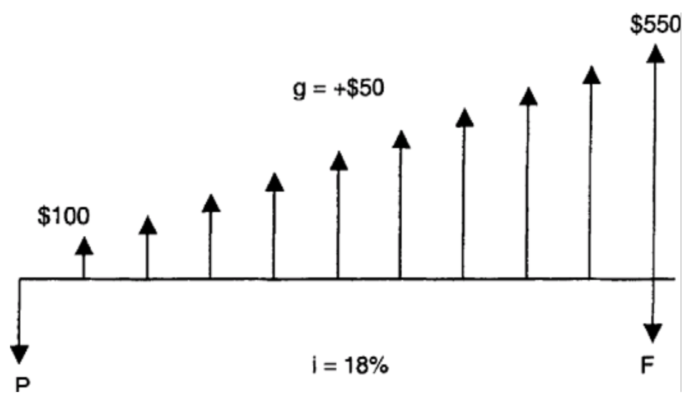
F = \$30,000 (F/P, 10%, 15) + \$600 (F/A, 10%, 15)
= \$30,000 (4.177) + \$600 (31.772)
= <u>\$144,373</u>

9-10



F = \$3,200 (F/A, 7%, 30) + \$60 (P/G, 7%, 30) (F/P, 7%, 30)
= \$3,200 (94.461) + \$60 (120.972) (7.612)
= <u>\$357,526</u>

9-11



P = \$100 (P/A, 18%, 10) + \$50 (P/G, 18%, 10)
= \$100 (4.494) + \$50 (14.352) = 1,167.00

F = \$1,167 (F/P, 18%, 10)
= \$1,167 (5.234) = <u>6.108.08</u>

9-12

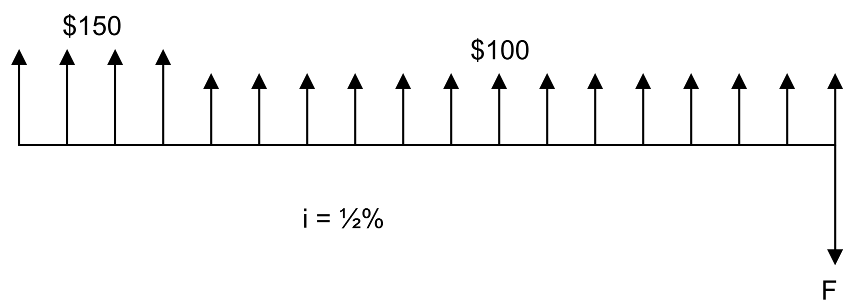
i_a = $(1 + r/m)^m - 1$
= $(1 + 0.16/48)^{48} - 1$
= 0.17320

F = $P (1 + i_a)^5$ = $50,000 $(1 + 0.17320)^5$ = <u>$111,130</u>

9-13

F = £100 $(1 + 0.10)^{800}$ = £1.3 x 10^{35}

9-14



F = $150 (F/A, ½%, 4) (F/P, ½%, 14) + $100 (F/A, ½%, 14)
= $150 (4.030) (1.072) + $100 (14.464)
= <u>2,094.42</u>

9-15

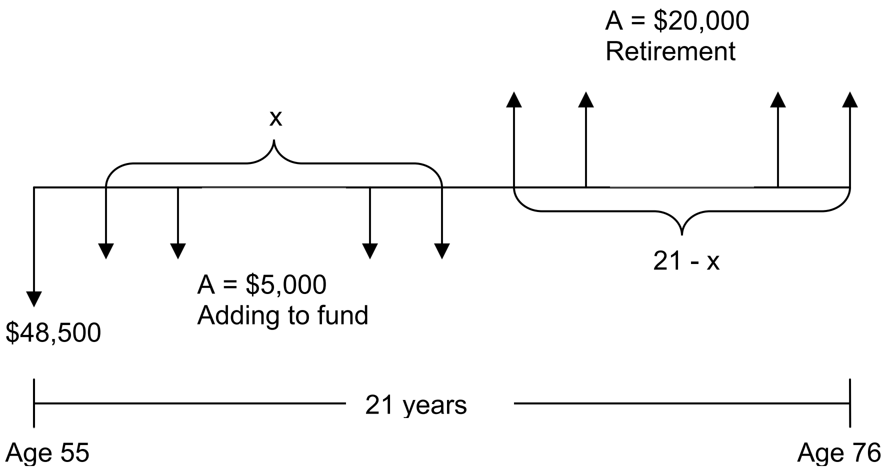
Using single-payment compound amount factors, we obtain

F = $1,000 [(F/P, 4%, 12) + (F/P, 4%, 10) + (F/P, 4%, 8) + (F/P, 4%, 6) + (F/P, 4%, 4) + (F/P, 4%, 2)]
= $1,000 [1.601 + 1.480 + 1.369 + 1.265 + 1.170 + 1.082]
= <u>$7,967</u>

Alternate Solution

A = $1,000 (A/P, 4%, 2) = $1,000 (0.5302) = $530.20
F = $530.20 (F/A, 4%, 12) = $530.20 (15.026) = <u>$7,966.80</u>

9-16



x = years to continue working
age to retire = 55 + x

Amount at Retirement = PW of needed retirement funds

$48,500 (F/P, 12%, x) + $5,000 (F/A, 12%, x) = $20,000 (P/A, 12%, 21− x)

Try x = 10

$48,500 (3.106) + $5,000 (17.549) = $238,386
$20,000 (5.938) = $118,760 so x can be < 10

Try x = 5

$48,500 (1.762) + $5,000 (6.353) = $117,222
$20,000 (6.974) = $139,480 so x > 5

Try x = 6

$48,500 (1.974) + $5,000 (8.115) = $136,314
$20,000 (6.811) = $136,220

Therefore, x = 6. The youngest age to retire is 55 + 6 = 61.

9-17

Geometric Gradient:

$n = 10$	$g = 100\%$	$A_1 = \$100$	$i = 10\%$

$$\begin{aligned} P &= A_1 [(1 - (1+g)^n (1+i)^{-n})/(i - g)] \\ &= \$100 [(1 - (1 + 1.0)^{10} (1 + 0.10)^{-10})/(0.10 - 1.0)] \\ &= \$100 [(1 - (1{,}024) (0.38554))/(-0.90)] \\ &= \$43{,}755 \end{aligned}$$

Future Worth = \$43,755 (F/P, 10%, 10) = \$43,755 (2.594) = <u>\$113,500</u>

9-18

$$\begin{aligned} i &= 0.0865/12 = 0.007208 \\ n &= 24 \end{aligned}$$

$$F = P (1 + i)^n = \$2{,}500 (1 + 0.007208)^{24} = \underline{\$2{,}970.30}$$

9-19

$$\begin{aligned} F_{56} &= \$25{,}000 (F/P, 6\%, 35) + \$1{,}000 (F/A, 6\%, 35) + \\ &\quad \$200 (P/G, 6\%, 35) (F/P, 6\%, 35)^* \\ &= \$25{,}000 (7.686) + \$1{,}000 (111.432) + \$200 (165.743) (7.686) \\ &= \underline{\$558{,}362} \end{aligned}$$

* The factor we want is (F/G, 6%, 35) but it is not tabulated in the back of the book. Instead we can substitute: (P/G, 6%, 35) (F/P, 6%, 35)

9-20

Assuming no disruption, the expected end-of-year deposits are:

A_1 = \$1,000,000 (A/F, 7%, 10)
= \$1,000,000 (0.0724)
= \$72,400

Compute the future worth of \$72,400 per year at the end of 7 years:

F_7 = \$72,400 (F/A, 7%, 7)
= \$626,550

Compute the future worth of \$626,550 in 3 years i.e., at the end of year 10:

F_{10} = \$626,550 (F/P, 7%, 3)
= \$767,524

Remaining two deposits = (\$1,000,000 − \$767,524) (A/F, 7%, 2)
= <u>\$112,309</u>

9-21

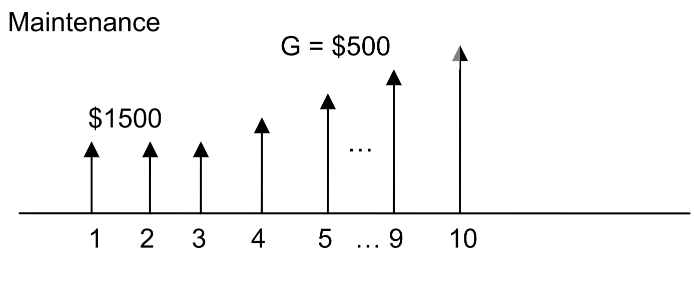
Assuming she also makes a deposit on her 65th birthday as well:

F = \$2,000 (F/A, 10%, 41) = \$2,000 (487.852) = <u>\$975,704</u>

Alternative solutions using interest table values:

F = \$2,000 (F/A, 10%, 40) + \$2,000 (F/P, 10%, 40)
= \$2,000 (442.593 + 45.259)
= <u>\$975,704</u>

9-22



FW (Costs) = \$150,000 (F/P, 10%, 10) + \$1,500 (F/A, 10%, 10) +
\$500 (P/G, 10%, 8) (F/P, 10%, 8) − (0.05) (\$150,000)
= \$150,000 (2.594) + \$1,500 (15.937) + \$500 (16.029) (2.144) − \$7,500
= <u>\$422,689</u>

9-23

Given: P = \$325,000
A_{1-120} = \$1,200
A_{84-120} = \$2,000 – \$1,200 = \$800
F_{60} = \$55,000 overhaul
n = 12 (10) = 120 months
i = 7.2/12 = 0.60% per month

Find: F_{120} = ?

F_P = (F/P, 0.60%, 120) (\$325,000)
= $(1 + 0.0060)^{120}$ (\$325,000)
= \$666,256

F_{A1-120} = (F/A, 0.60%, 120) (\$1,200)
= $[((1 + 0.006)^{120} - 1)/0.006]$ (\$1,200)
= \$210,004

$F_{A84-120}$ = (F/A, 0.60%, 36) (\$800)
= $[((1 + 0.006)^{36} - 1)/0.006]$ (\$800)
= \$32,040

F_{60} = (F/P, 0.60%, 60) (\$55,000)
= $(1 + 0.006)^{60}$ (\$55,000)
= \$78,748

F_{120} = \$666,256 + \$210,004 + \$32,040 + \$78,748
= <u>\$987,048</u>

9-24

Find F, assuming that they make a deposit on each birthday staring with his 8th and continuing up to and including his 18th.

F = \$150 (F/A, 9%, 11) + \$150 (P/G, 9%, 11) (F/P, 9%, 11)
F = \$150 (17.560) + \$150 (28.248) (2.580)
= <u>\$13,566</u>

Alternate Solution

Remembering that G must equal zero at the end of period 1, adjust the time scale where equation time zero = problem time − 1. Then:
F = \$150 (F/G, 9%, 12) = \$150 (P/G, 9%, 12) (F/P, 9%, 12)
= \$150 (32.159) (2.813) = <u>\$13,569</u>

9-25

$i_{semiannual} = (1 + 0.192/12)^6 - 1 = 0.10 = 10\%$
$F_{1/1/12} = F_A + F_G$

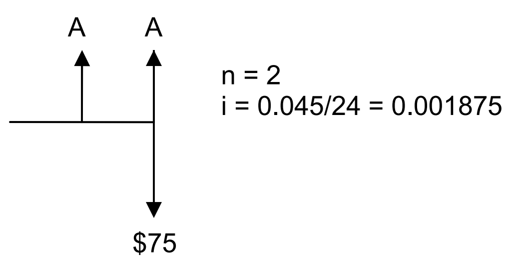
From the compound interest tables (i = 10%, n = 31):
F_A = \$5,000 (F/A, 10%, 31) = \$5,000 (181.944) = \$909,720
F_G = −\$ 150 (P/G, 10%, 31) (F/P, 10%, 31)
= −\$150 (78.640) (19.194)
= −\$226,412

$F_{1/1/12}$ = \$909,720 – \$226,412 = \$683,308
$F_{7/1/14}$ = \$683,308 (F/P, 10%, 5) = \$683,308 (1.611) = \$1,100,809

9-26

The monthly deposits to the savings account do not match the twice a month compounding period. To use the standard formulas we must either
(1) compute an equivalent twice a month savings account deposit, or
(2) compute an equivalent monthly interest rate.

Method 1

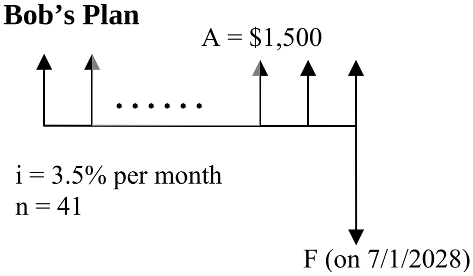


Equivalent twice a month deposit (A) = \$75 (A/F, i%, n)
$= \$75\ [0.001875/((1 + 0.001875)^2 - 1)]$
= \$37.4649
Between July 1 2007 and January 1 2025 there will be 211 deposits
Future Sum $F_{1/1/25}$ = A (F/A, i%, 211)
$= \$37.4649\ [((1 + 0.001875)^{422} - 1)/0.001875]$
= <u>\$24,068</u>

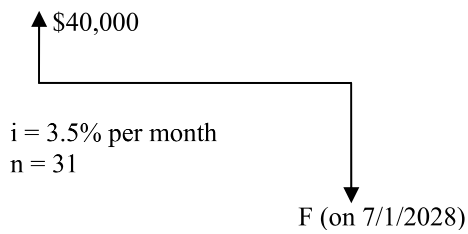
Method 2
Effective i per month (i_{month}) $= (1 + 0.045/24)^2 - 1 = 0.0037535$
Future Sum $F_{1/1/25}$ = A (F/A, i_{month}, 211)
$= \$75\ [((1 + 0.0037535)^{211} - 1)/0.0037535]$
= <u>\$24,068</u>

9-27

Bob's Plan



Joe's Plan

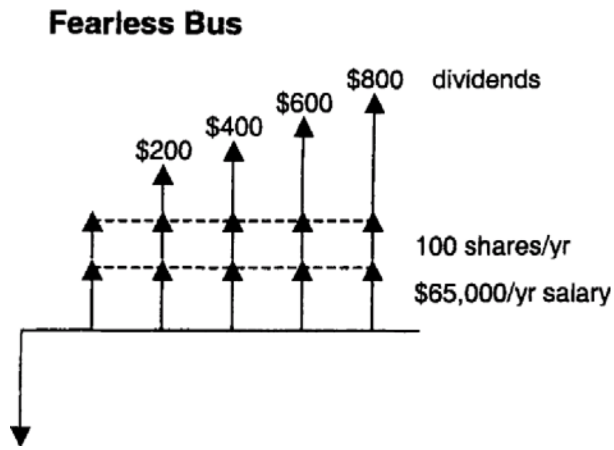


F (Bob) = $1,500 (F/A, 3.5%, 41) = $1,500 (88.510) = $132,764

F (Joe) = $40,000 (F/P, 3.5%, 31) = $40,000 (2.905) = $116,200

Joe's deposit will be insufficient.
He should deposit: $132,764 (P/F, 3.5%, 31) = $132,764 (0.3442) = <u>$45,697</u>

9-28



P = \$65,000 (P/A, 9%, 5) + \$200 (P/G, 9%, 5) + 100 shares of stock
= \$65,000 (3.890) + \$200 (7.111) + 100 shares of stock
= \$254,272 + 100 shares of stock

F = \$254,272 (F/P, 9%, 5) + 100 shares (\$60/share)
= \$254,272 (1.539) + \$6,000
= \$397,325

Generous Electric

F = \$62,000 (F/A, 9%, 5) + 600 shares of stock
= \$62,000 (5.985) + 600 shares of stock
= \$371,070 + 600 shares of stock

Set $F_{Fearless} = F_{GE}$
600 shares of GE stock + \$371,070 = \$397,325

Required Value of GE stock = (\$397,325 – \$371,070)/600
= \$26,255/600
= <u>\$43.76/share</u>

9-29

F = A (F/A, 9%, 40) =A (337.883)

If instead of buying a \$1 lottery ticket every week, the money is deposited into an investment account earning 9% interest compounded annually (here a total of \$52 per year), in 40 years you would have: F = (52)(337.883)= \$17,570.

9-30

	A	B	C
Cost	\$600	\$500	\$200
Uniform Annual Benefit	\$158.3	\$138.7	\$58.3

$B/C_{OF\ A}$ = \$158.3/[\$600 (A/P, 10%, 5)] = 1.00
$B/C_{OF\ B}$ = \$138.7/[\$500 (A/P, 10%, 5)] = 1.05
$B/C_{OF\ C}$ = \$58.3/[\$200 (A/P, 10%, 5)] = 1.11
All alternatives have a B/C ratio ≥ 1.00. Proceed with incremental analysis.

	B – C	A – B
Cost	\$300	\$100
Uniform Annual Benefit	\$80.4	\$19.6

$B/C_{OF\ B-C}$ = \$80.4/[\$300 (A/P, 10%, 5)] = 1.02
Desirable increment. Reject C.

$B/C_{OF\ A-B}$ = \$19.6/[\$100 (A/P, 10%, 5)] = 0.74
Undesirable increment. Reject A.

Conclusion: <u>Select B.</u>

9-31

B/C_A = (\$142 (P/A, 10%, 10))/\$800 = 1.09
B/C_B = (\$60 (P/A, 10%, 10))/\$300 = 1.23
B/C_C = (\$33.5 (P/A, 10%, 10))/\$150 = 1.37

Incremental Analysis

B – C Increment

	B – C
Δ Cost	\$150
Δ UAB	\$26.5

ΔB/ΔC = (\$26.5 (P/A, 10%, 10))/\$150 = 1.09
This is a desirable increment. Reject C.

A – B Increment

	A – B
Δ Cost	\$500
Δ UAB	\$82

ΔB/ΔC = (\$82 (P/A, 10%, 10))/\$500 = 1.01
This is a desirable increment. Reject B.
Conclusion: Select A.

9-32

	2 Stories	5 Stories	10 Stories
Cost (including land)	$400,000	$800,000	$2,100,000
Annual Income (A)	$70,000	$105,000	$256,000
Salvage Value (F)	$200,000	$300,000	$400,000

B/C Ratio Analysis

Cost	$400,000	$800,000	$2,100,000
– PW of Salvage Value = F(P/F, 8%, 20) = 0.2145F	$42,900	$64,350	$85,800
PW of Cost	$357,100	$735,650	$2,014,200
PW of Benefit = A (P/A, 8%, 20) = 9.818A	$687,260	$1,030,890	$2,513,408
B/C Ratio = PW of Benefit/PW of Cost	1.92	1.40	1.25

Incremental B/C Ratio Analysis

	5 Stories Rather than 2 Stories	10 Stories Rather than 2 Stories
Δ PW of Cost	$735,650 – $357,100 = $378,550	$2,014,200 – $357,100 = $1,657,100
Δ PW of Benefit	$1,030,890 – $687,260 = $343,630	$2,513,408 – $687,260 = $1,826,148
ΔB/ΔC = ΔPW of Benefits/ΔPW of Costs	$343,630/$378,550 = 0.91	$1,826,148/$1,657,100 = 1.10
Decision	< 1 Undesirable increment. Reject 5 stories	> 1 Desirable increment.

With ΔB/ΔC = 0.91, the increment of 5 stories rather than 2 stories is undesirable. The 10 stories rather than 2 stories is desirable since its ratio is greater than 1.

Conclusion: <u>Choose the 10-story alternative.</u>

9-33

Note that the three alternatives have been rearranged below in order of increasing cost.

	C	B	A
First Cost	$120	$340	$560
Uniform Annual Benefit	$40	$100	$140
Salvage Value	$0	$0	$40
Compute B/C Ratio	1.45	1.28	1.13

Incremental Analysis

	B – C	A – B
Δ First Cost	$220	$220
Δ Uniform Annual Benefit	$60	$40
Δ Salvage Value	$0	$40
Compute ΔB/ΔC value	1.19	0.88

Benefit- Cost Ratio Computations:

Alternative A: B/C = [$140 (P/A, 10%, 6)]/[$560 – $40 (P/F, 10%, 6)]
= [$140 (4.355)]/($560 – $40 (0.5645)]
= 1.13

Alternative B: B/C = [$100 (P/A, 10%, 6)]/$340
= 1.28

Alternative C: B/C = [$40 (P/A, 10%, 6)]/$120
= 1.45

Incremental Analysis:

B – C: ΔB/ΔC = [$60 (P/A, 10%, 6)]/$220 = 1.19
B – C is a desirable increment.

A – B: ΔB/ΔC = [$40 (P/A, 10%, 6)/[$220 – $40 (P/F, 10%, 6)] = 0.88
A – B is an undesirable increment.

Conclusion: Choose B.

The solution may be checked by Net Present Worth or Rate of Return

NPW Solution

NPW_A = \$140 (P/A, 10%, 6) + \$40 (P/F, 10%, 6) – \$560
= \$140 (4.355) + \$40 (0.5645) – \$560
= +\$72.28

NPW_B = \$100 (P/A, 10%, 6) – \$340
= +\$95.50

NPW_C = \$40 (P/A, 10%, 6) – \$120
= +\$54.20

Select B.

Rate of Return Solution

	B – C	A – B
Δ Cost	\$220	\$220
Δ Uniform Annual Benefit	\$60	\$40
Δ Salvage Value	\$0	\$40
Computed Δ ROR	16.2%	6.6%
Decision	> 10% Accept B. Reject C.	< 10%

Select B.

9-34

This is an above-average difficulty problem. An incremental Uniform Annual Benefit becomes a cost rather than a benefit.

Compute B/C for each alternative

Form of computation used:
(PW of B)/(PW of C) = (UAB (P/A, 8%, 8))/(Cost – S (P/F, 8%, 8))
= (UAB (5.747))/(Cost – S (0.5403))

B/C_A = (\$12.2 (5.747))/(\$100 – \$75 (0.5403)) = 1.18
B/C_B = (\$12 (5.747))/(\$80 – \$50 (0.5403)) = 1.30
B/C_C = (\$9.7 (5.747))/(\$60 – \$50 (0.5403)) = 1.69
B/C_d = (\$12.2 (5.747))/\$50 = 1.40

All alternatives have B/C > 1. Proceed with Δ analysis.

Incremental Analysis

C – D Increment

	C – D
Δ Cost	$10
Δ Uniform Annual Benefit	–$2.5

Δ Salvage Value $50

The apparent confusion may be cleared up by a detailed examination of the cash flows:

Year	Cash Flow C	Cash Flow D	Cash Flow C- D
0	–$60	–$50	–$10
1–7	+$9.7	+$12.2	–$2.5
8	+$9.7 +$50	+$12.2	+$47.5

B/C ratio = ($47.5 (P/F, 8%, 8)/($10 + $2.5 (P/A, 8%, 7))
= ($47.5 (0.5403)/($10 + $2.5 (5.206)
= 1.11

The C – D increment is desirable. Reject D.

B – C Increment

	B – C
Δ Cost	$20.0
Δ Uniform Annual Benefit	$2.3
Δ Salvage Value	$0

B/C ratio = ($2.3 (0.5403)/$20
= 0.062

Reject B.

A – C Increment

	A – C
Δ Cost	$40.0
Δ Uniform Annual Benefit	$2.5
Δ Salvage Value	$25.0

B/C ratio = ($2.5 (0.5403)/($40 – $25 (0.5403))
= 0.051

Reject A.

Conclusion: Select C.

9-35

	A	B	C	D	E
Cost	$100	$200	$300	$400	$500
UAB	$37	$69	$83	$126	$150
PW of Benefits = UAB (P/A, 15%, 5)	$124	$231.3	$278.2	$422.4	$502.8
B/C Ratio	1.24	1.16	0.93	1.06	1.01

We can eliminate alternative C since its B/C ratio is less than 1 and that of the other alternatives are greater than one.

	B – A	D – B	E – B
Δ Cost	$100	$200	$300
Δ UAB	$32	$57	$81
PW of Benefits	$107.3	$191.1	$271.5
ΔB/ΔC	1.07	0.96	0.91
Decision	Reject A.	Reject D.	Reject E.

Conclusion: Select B.

9-36

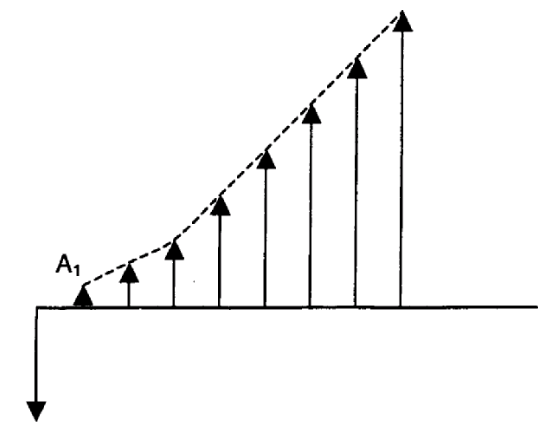
Ordered Alternatives	B	C	D	A	E
Cost	$100	$125	$150	$200	$225
UAB	$25	$42	$52	$68	$68
PW of Benefits = UAB (P/A, 15%, 5)	$83.8	$141	$174	$218	$228
B/C Ratio	0.84	1.13	1.16	1.09	1.01

By inspection one can see that A, with its smaller cost and identical benefits, is preferred to E in all situations, hence E may be immediately rejected. Based on the B/C ratio for the remaining four alternatives, three exceed 1.0 and only B is less than 1.0. On this basis B may be rejected. That leaves A, C, and D for incremental B/C analysis.

	D – C	A – D
Δ Cost	$25	$50
Δ Benefits	$10	$16
PW of Benefits	$10 (P/A, 15%, 5) = $10 (3.352)	$16 (P/A, 15%, 5) = $16 (3.352)
ΔB/ΔC	$10 (3.352)/$25 = 1.34	$16 (3.352)/$50 = 1.07
Decision	Reject C.	Reject D.

Therefore, select investment A.

9-37



Geometric gradient at a 10% uniform rate.

A_1 = $10,000	i = 10%	g = 1%	n = 8 yrs

Where i = g: $P = A_1 n (1 + i)^{-1}$

B/C = PW of Benefits/PW of Cost
= [$10,000 (8) $(1 + 0.10)^{-1}$]/$50,000
= <u>1.45</u>

9-38

(a) A: PW Benefit = 2.1M (P/A, 9%, 15) = (2.1M) (8.061) = 16.928M
PW Cost = 6.9M + 1.2M + 0.75M (P/A, 9%, 15) = 14.146M
$\frac{PW\ Benefit}{PW\ Cost} = \frac{16.928M}{14.146M} = 1.197$ > 1, so do nothing is eliminated.

B: PW Benefit = 2.6M (P/A, 9%, 15) = 20.959M
PW Cost = 9.9M + 2.1M + 0.825 (P/A, 9%, 15) = 18.650M
$\frac{PW\ Benefit}{PW\ Cost} = \frac{20.959M}{18.650M} = 1.124$

B – A: $\frac{20.959M - 16.928M}{18.650M - 14.146M} = 0.895$ < 1, so choose the lower-cost alternative A.

(b) There is no salvage, so same as part a.

(c) A: PW = –1.2M + 2.1M (P/A, 9%, 15) = 15.728M
PC = 6.9M + 0.75M (P/A, 9%, 15) = 12.946M
$\frac{B}{C} = \frac{15.728M}{12.946M} = 1.215$ > 1, so do nothing is eliminated.

B: PW Benefit = –2.1M + 2.6M (P/A, 9%, 15) = 18.859M
PW Cost = 9.9M + 0.825M (P/A, 9%, 15) = 16.550M
$\frac{B}{C} = \frac{18.859M}{16.550M} = 1.140$

B – A: $\frac{18.859M - 15.728M}{16.550M - 12.946M} = 0.869$ < 1, so choose the lower-cost alternative A.

(d) A: PW (years 1 to 15) = (2.1M – 0.75M) (P/A, 9%, 15) = 10.882M
PC = 6.9M + 1.2M = 8.1M
$PW\ index = \frac{10.882M}{8.1M} = 1.343$ > 1, so do nothing is eliminated.

B: PW (years 1 to 15) = (2.6M – 0.825M) (P/A, 9%, 15) = 14.308M
PC = 9.9M + 2.1M = 12.0M
$PW\ index = \frac{14.308M}{12.0M} = 1.192$

B – A: $\frac{14.308M - 10.882M}{12.0M - 8.1M} = 0.878$ < 1, so choose the lower-cost alternative A.

(e) Yes, they are all consistent. The largest ratio is the present worth index as would be expected since only initial costs show up in the denominator.

9-39

(a) A: PW Benefit = 2.1M (P/A, 8%, 20) = (2.1M) (9.818) = 20.618M
PW Cost = 8.8M + 0.8M + 0.95M (P/A, 8%, 20) = 18.927M
$\frac{PW\ Benefit}{PW\ Cost} = \frac{20.618M}{18.927M} = 1.089$ > 1, so do nothing is eliminated.

B: PW Benefit = 3.1M (P/A, 8%, 20) = 30.436M
PW Cost = 10.4M + 0 + 1.7M (P/A, 8%, 20) = 27.091M
$\frac{PW\ Benefit}{PW\ Cost} = \frac{30.436M}{27.091M} = 1.123$

B – A: $\frac{30.436M - 20.618M}{27.091M - 18.927M} = 1.203$ > 1, so choose the higher-cost alternative B.

(b) There is no salvage, so same as part a.

(c) A: PW Benefit = –0.8M + 2.1M (P/A, 8%, 20) = 19.818M
PW Cost = 8.8M + 0.95M (P/A, 8%, 20) = 18.127M
$\frac{B}{C} = \frac{19.818M}{18.127M} = 1.093$ > 1, so do nothing is eliminated.

B: PW Benefit = 0 + 3.1M (P/A, 8%, 20) = 30.436M
PW Cost = 10.4M + 1.7M (P/A, 8%, 20) = 27.091M
$\frac{B}{C} = \frac{30.436M}{27.091M} = 1.123$

B – A: $\frac{30.436M - 19.818M}{27.091M - 18.127M} = 1.185$ > 1, so choose the higher-cost alternative B.

(d) A: PW (years 1 to 20) = (2.1M – 0.95M) (P/A, 8%, 20) = 11.291M
PC = 8.8M + 0.8M = 9.6M
$PW\ index = \frac{11.291M}{9.6M} = 1.176$ > 1, so do nothing is eliminated.

B: PW (years 1 to 20) = (3.1M – 1.7M) (P/A, 8%, 20) = 13.745M
PC = 10.4M + 0 = 10.4M
$PW\ index = \frac{13.745M}{10.4M} = 1.322$

B – A: $\frac{13.745M - 11.291M}{10.4M - 9.6M} = 3.068$ > 1, so choose the higher-cost alternative B.

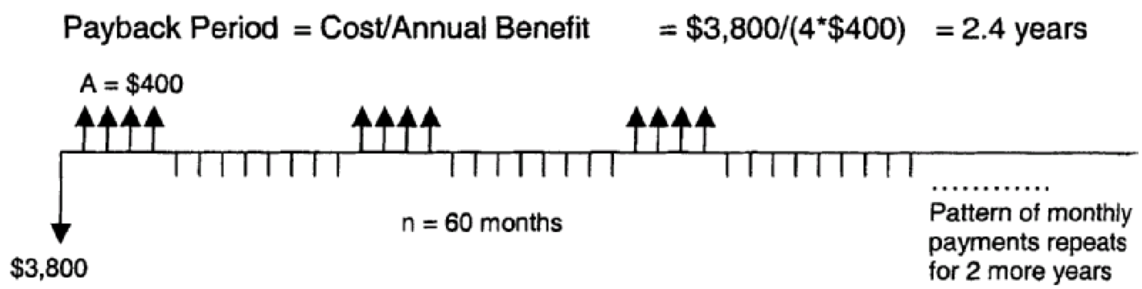
(e) Yes, they are all consistent. The largest ratio is the present worth index as would be expected since only initial costs show up in the denominator.

9-40

Investment = \$67,000
Annual Benefit = \$26,000/yr for 2 years
Payback Period = \$67,000/\$26,000 = 2.6 years
Do not buy because total benefits (2 yr) (\$26,000/yr) < Cost as seen by the payback period be greater than 2 years.

9-41

Payback Period = Cost/Annual Benefit = \$3,800/(4*\$400) = 2.4 years



\$3,800 = \$400 (P/A, i%, 4) + \$400 (P/A, i%, 4) (P/F, i%, 12) +
\$400 (P/A, i%, 4) (P/F, i%, 24) + \$400 (P/A, i%, 4) (P/F, i%, 36) +
\$400 (P/A, i%, 4) (P/F, i%, 48)

\$3,800 = \$400 (P/A, i%, 4) [1 + (P/F, i%, 12) + (P/F, i%, 24) + (P/F, i%, 36) +
(P/F, i%, 48)]

Try i = 3%
P(3%) = \$400 (3.717) [1 + 0.7014 + 0.4919 + 0.3450 + 0.2420] = \$1,486.80 [2.7803]
= \$4,134 so i is too low

Try i = 4%
P(4%) = \$400 (3.630) [1 + 0.6246 + 0.3901 + 0.2437 + 0.1522]
= \$1,452 [2.4106]
= \$3,500 so i is too high

Try i = 3.5%
P(3.5%) = \$400 (3.673) [1 + 0.6618 + 0.4380 + 0.2898 + 0.1918]
= \$1,469.20 [2.5814]
= \$3,793

So i = 3.5% per month
Nominal Rate of Return = 12 (3.5%) = 42%

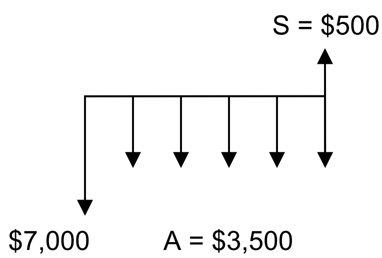
9-42

Year	Costs	Benefit	Benefit - Costs
0	$1,400.00		–$1,400.00
1	$500.00		–$1,900.00
2	$300.00	$400.00	–$1,800.00
3		$300.00	–$1,500.00
4		$300.00	–$1,200.00
5		$300.00	–$900.00
6		$300.00	–$600.00
7		$300.00	–$300.00
8		$300.00	$0.00

Costs = Benefits at end of year 8
Therefore, payback period = 8 years.

9-43

Lease: A = $5,000/yr
Purchase:



(a) Payback Period
Cost = $7,000
Benefit = $1,500/yr + $500 at any time
Payback = ($7,000 – $500)/$1,500 = 4.3 years

(b) Benefit-Cost Ratio
B/C = EUAB/EUAC
= [$1,500 + $500 (A/F, 10%, 6)]/[$7,000 (A/P, 10%, 6)]
= [$1,500 + $500 (0.1296)]/[$7,000 (0.2296)]
= 0.97

9-44

(a) **Payback Periods**

	Alternative A		Alternative B	
Period	Cash Flow	Sum CF	Cash Flow	Sum CF
−2	−$30	−$30	−$30	−$30
–1	–$100	–$130	–$100	–$130
0	–$70	–$200	–$70	–$200
1	$40	–$160	$32.5	–$167.5
2	$40	–$120	$32.5	–$135
3	$40	–$80	$32.5	–$102.5
4	$40	–$40	$32.5	–$70
5	$40	$0	$32.5	–$37.5
6	$40	$40	$32.5	–$5
7	$40	$80	$32.5	$27.5

Payback_A = 5.0 years
Payback_B = 7 years (based on end of year cash flows)

(b) **Equivalent Investment Cost**
= $30 (F/P, 10%, 2) + $100 (F/P, 10%, 1) + $70
= $30 (1.210) + $100 (1.100) + $70
= <u>$216.3 million</u>

(c) **Equivalent Uniform Annual Worth** = EUAB − EUAC
EUAW_A = $40 – $216.3 (A/P, 10%, 10) = $4.81 million
EUAW_B = $32.5 – $216.3 (A/P, 10%, 20) = $7.08 million

<u>Since the EUAW for the Alternative B is higher, this alternative should be selected.</u> Alternative A may be considered if the investor is very short of cash and the short payback period is of importance to him.

9-45

(a)

	Increment B – A
Δ Cost	$300
ΔUAB	$50

Incremental Payback = Cost/UAB = $300/$50 = 6 years

(b) ΔB/ΔC = [$50 (P/A, 12%, 8)]/$300 = 0.83
<u>Reject B and select A.</u>

9-46

Part (a)

Year	Conventional	Solar	Solar − Conventional	Net Investment
0	–$200	–$1,400	–$1,200	–$1,200
1-4	–$230/yr	–$60/yr	+$170/yr	–$520
4		–$180	–$180	–$700
5-8	–$230/yr	–$60/yr	+$170/yr	–$20
8		–$180	−$180	–$200
9-12	–$230/yr	–$60/yr	+$170/yr	+$480 ← Payback
12		–$180	–$180	+$300

Payback = 8 yrs + $200/$170 = 9.18 yrs

9-47

(a) **Net Future Worth**

NFW_A = $18.8 (F/A, 10%, 5) – $75 (F/P, 10%, 5) = –$6.05
NFW_B = $13.9 (F/A, 10%, 5) – $50 (F/P, 10%, 5) = +$4.31 ←
NFW_C = $4.5 (F/A, 10%, 5) – $15 (F/P, 10%, 5) = +$3.31
NFW_D = $23.8 (F/A, 10%, 5) – $90 (F/P, 10%, 5) = +$0.31
Select B.

(b) **Incremental B/C Ratio Analysis**

	C	B	A	D
Cost	$15.0	$50.0	$75.0	$90.0
UAB	$4.5	$13.9	$18.8	$23.8
Computed Uniform Annual Cost (UAC)	$3.96 = $15 (A/P,10%,5)	$13.19	$19.78	$23.74
B/C Ratio	1.14	1.05	0.95	1.00
Decision	Ok	Ok	Reject	Ok

	B – C	D – B
Δ UAB	$9.40	$9.90
Δ UAC	$9.23	$10.55
ΔB/ΔC	1.02	0.94
Decision	Reject C.	Reject D.

Conclusion: Select B.

(c) **Payback Period**
Payback_A = \$75/\$18.8 = 4.0
Payback_B = \$50/\$13.9 = 3.6
Payback_C = \$15/\$4.5 = 3.3 ←
Payback_D = \$90/\$23.8 = 3.8
To minimize Payback, select C.

9-48

	A	B	C
Cost	\$50	\$150	\$110
Annual Benefit	\$28.8	\$39.6	\$39.6
Useful Life	2 yr	6 yr	4 yr

(a) Solve by Future Worth analysis. In future worth analysis there must be a common future time for all calculations. In this case, 12 years hence is a practical future time.

NFW_A = \$28.8 (F/A, 12%, 12) – \$50 (A/P, 12%, 2) (F/A, 12%, 12)
= \$28.8 (24.133) – \$50 (0.5917) (24.133)
= –\$18.94

NFW_B = \$39.6 (F/A, 12%, 12) – \$150 (F/P, 12%, 6) – \$150 (F/P, 12%, 12)
= \$39.6 (24.133) – \$150 [1.974 + 3.896]
= +\$75.17

NFW_C = \$39.6 (F/A, 12%, 12) – \$110 (F/P, 12%, 4) – \$110 (F/P, 12%, 8) − \$110 (F/P, 12%, 12)
= \$39.6 (24.133) – \$110 [1.574 + 2.476 + 3.896]
= +\$81.61

Choose Alternative C because it maximizes Future Worth.

(b) Solve by Benefit–Cost ratio analysis
With neither input nor output fixed, incremental analysis is required.

Alternative C – Alternative A

Year	Alt. C	Alt. A	C- A
0	–\$110	–\$50	–\$60
1	+\$39.6	+\$28.8	+\$10.8
2	+\$39.6	+\$28.8 −\$50	+\$60.8
3	+\$39.6	+\$28.8	+\$10.8
4	+\$39.6	+\$28.8	+\$10.8

Four years is a suitable analysis period for Alternatives C and A.

For the increment C – A:
PW of Cost = $60
PW of Benefits = $10.8 (P/A, 12%, 4) + $50 (P/F, 12%, 2)
= $10.8 (3.037) + $50 (0.7972)
= $72.66
ΔB/ΔC = PW of Benefits/PW of Cost = $72.66/$60> 1
The increment of investment is acceptable and therefore Alternative C is preferred over Alternative A.

Increment B – C

Year	Alt. B	Alt. C	B- C
0	–$150	–$110	–$40
1-4	+$39.6	+$39.6	$0
4	$0	–$110	+$110
5-6	+$39.6	+$39.6	$0
6	–$150	$0	–$150
7-8	+$39.6	+$39.6	$0
8	$0	–$110	+$110
9- 12	+$39.6	+$39.6	$0

Twelve years is a suitable analysis period for Alternatives B and C.

For the increment B – C
Ignoring the potential difficulties signaled by 3 sign changes in the B – C cash flow:
PW of Cost = $40 + $150 (P/F, 12%, 6)
= $40 + $150 (0.5066)
= $115.99

PW of Benefits = $110 (P/F, 12%, 4) + $110 (P/F, 12%, 8)
= $110 (0.6355) + $110 (0.4039)
= $114.33

ΔB/ΔC = PW of Benefits/PW of Cost = $114.33/$115.99 < 1

The increment is undesirable and therefore Alternative C is preferred over Alternative B.

Alternative Analysis of the Increment B – C
An examination of the B- C cash flow suggests there is an external investment of money at the end of Year 4. Using an external interest rate (say 12%) the +$110 at Year 4 becomes +$110 (F/P, 12%, 2) = $110 (1.254) = $137.94 at the end of Year 6.

The altered cash flow becomes:

Year	B – C
0	–$40
1-6	$0
6	–$150 +$137.94 = –$12.06
7-8	$0
8	+$110

For the altered B – C cash flow:
PW of Cost = $40 + $12.06 (P/F, 12%, 6)
= $40 + $12.06 (0.5066)
= $46.11

PW of Benefits = $110 (P/F, 12%, 8)
= $110 (0.4039)
= $44.43

ΔB/ΔC = PW of Benefits/PW of Cost = $44.43/$46.11 < 1
The increment is undesirable and therefore Alternative C is preferred to Alternative B.

Solutions for part (b): <u>Choose Alternative C.</u>

(c) Payback Period
Alternative A: Payback = $50/$28.8 = 1.74 yr
Alternative B: Payback = $150/$39.6 = 3.79 yr
Alternative C: Payback = $110/$39.6 = 2.78 yr
To minimize the Payback Period, <u>choose Alternative A.</u>

(d) Payback period is the time required to recover the investment ignoring the time value of money. Here we have three alternatives that have rates of return varying from 10% to 16.4%. Thus each generates uniform annual benefits in excess of the cost, during the life of the alternative. From this it must follow that the alternative with a 2-year life has a payback period less than 2 years. The alternative with a 4-year life has a payback period less than 4 years, and the alternative with a 6-year life has a payback period less than 6 years.

Thus we see that the shorter-lived asset automatically has an advantage over longer-lived alternatives in a situation like this. While Alternative A takes the shortest amount of time to recover its investment, Alternative C is best for long-term economic efficiency.

9-49

(a) B/C of Alt. x = [$25 (P/A, 10%, 4)]/$100 = 0.79

(b) X: Payback period = $100/$25 = 4 years
Y: Payback period = $50/$16 = 3.1 years
Z: Payback period = $50/$21 = 2.4 years
Based on payback Alternative Z is the best.

(c) No computations are really necessary. The problem may be solved by inspection.
Alternative x has a 0% rate of return (Total benefits = cost).
Alternative z dominates Alternative y. (Both cost $50, but Alternative z yields more benefits).
Alternative z has a positive rate of return (actually 24.5%) and is obviously the best of the three mutually exclusive alternatives.
Choose Alternative z.

9-50

(a) **Payback Period**
Payback $_A$ = 4 + $150/$350 = Year 4.4
Payback$_B$ = Year 4
Payback$_C$ = 5 + $100/$200 Year 5.5
For shortest payback, choose Alternative B.

(b) **Net Future Worth**
NFW$_A$ = $200 (F/A, 12%, 5) + [$50 (P/G, 12%, 5) – $400] (F/P, 12%, 5) – $500 (F/P, 12%, 6)
= $200 (6.353) + [$50 (6.397) – $400] (1.762) – $500 (1.974)
= +$142.38

NFW$_B$ = $350 (F/A, 12%, 5) + [–$50 (P/G, 12%, 5) – $300] (F/P, 12%, 5) – $600 (F/P, 12%, 6)
= $350 (6.353) + [–$50 (6.397) – $300] (1.762) – $600 (1.974)
= –$53.03

NFW$_C$ = $200 (F/A, 12%, 5) – $900 (F/P, 12%, 6)
= $200 (6.353) – $900 (1.974)
= –$506.00

To maximize NFW, choose Alternative A.

9-51

(a) $Payback_A$ = 4 years
$Payback_B$ = 2.6 years
$Payback_C$ = 2 years

To minimize payback, choose C.

(b) B/C Ratios:
B/C_A = ($100 (P/A, 10%, 6) + $100 (P/F, 10%, 1)/$500
= 1.05
B/C_B = ($125 (P/A, 10%, 5) + $75 (P/F, 10%, 1)/$400
= 1.36
B/C_C = ($100 (P/A, 10%, 4) + $100 (P/F, 10%, 1)/$300
= 1.36

Incremental Analysis
B – C Increment

Year	B – C
0	–$100
1	$0
2	+$25
3	+$25
4	+$25
5	+$125

$\Delta B/\Delta C_{B\text{-}C}$ = ($25 (P/A, 10%, 3)(P/F, 10%, 1) + $125 (P/F, 10%, 5))/$100
= 1.34
This is a desirable increment. Reject C.

A – B Increment

Year	A- B
0	–$100
1	$0
2	–$25
3	–$25
4	–$25
5	+$100

By inspection we see that $\Delta B/\Delta C < 1$
$\Delta B/\Delta C_{A\text{-}B}$ = ($100 (P/F, 10%, 6))/($100 + $25 (P/A, 10%, 3) (P/F, 10%, 1))
= 0.36
Reject A.

Conclusion: Select B.

9-52

(a) **Future Worth Analysis at 6%**

NFW_E = \$20 (F/A, 6%, 6) – \$90 (F/P, 6%, 6) = +\$11.79
NFW_F = \$35 (F/A, 6%, 4) (F/P, 6%, 2) – \$110 (F/P, 6%, 6) = +\$16.02*
NFW_G = [\$10 (P/G, 6%, 6) – \$100] (F/P, 6%, 6) = +\$20.70 →
NFW_H = \$180 – \$120 (F/P, 6%, 6) = +\$9.72
To maximize NFW, select G.

(b) **Future Worth Analysis at 15%**

NFW_E = \$20 (F/A, 15%, 6) – \$90 (F/P, 15%, 6) = –\$33.09
NFW_F = [\$35 (P/A, 15%, 4) – \$110] (F/P, 15%, 6) = –\$23.30* →
NFW_G = [\$10 (P/G, 15%, 6) – \$100] (F/P, 15%, 6) = –\$47.72
NFW_H = \$180 – \$120 (F/P, 15%, 6) = –\$97.56
* Note: Two different equations that might be used.

To maximize NFW, select F.

(c) **Payback Period**

$Payback_E$ = \$90/\$20 = 4.5 yr
$Payback_F$ = \$110/\$35 = 3.1 yr →
$Payback_G$ = 5 yr
$Payback_H$ = 5.7 yr

To minimize payback period, select F.

(d) B/C_G = PW of Benefits/PW of Cost = [\$10 (P/G, 7%, 6)]/\$100 = 1.10

9-53

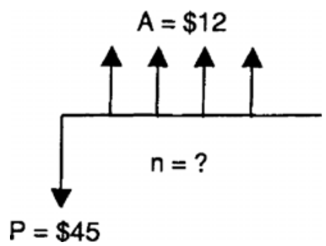
$EUAC_{AMERICAN}$ =(\$8,900 – \$1,700) (A/P, 8%, 3)+ \$1,700 (0.08) + 12,000(0.09)
= \$4,010

$EUAC_{FIASCO}$ =(\$8,000 − x) (A/P, 8%, 3) + x (0.08) + 12,000 (0.08)
= \$3,104 − 0.3880x + 0.08x + \$960

Set $EUAC_{AMERICAN}$ = $EUAC_{FIASCO}$
\$4,010 = \$4,064 − 0.308X

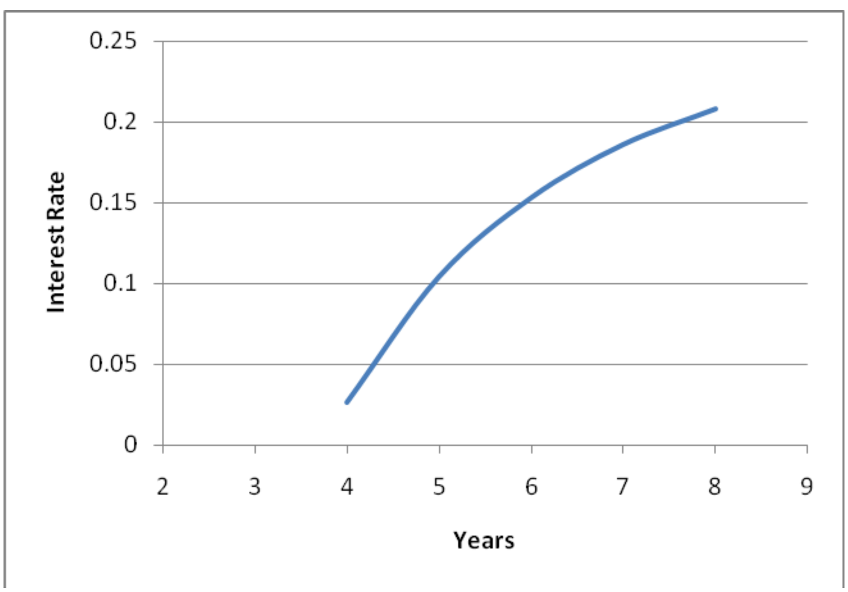
Minimum Fiasco Resale Value x = \$54/0.308 = \$175

9-54



$45 = $12 (P/A, i%, n)
(P/A, i%, n) = $45/$12 = 3.75

n	i%
4	2.6%
5	10.4%
6	15.3%
7	18.6%
8	20.8%
∞	A/P = $12/$45 = 26.7%



(b) Using the equation for P/A (inside cover of book). Insert 12% and solve for "n". For a 12% rate of return, the useful life must be 5.28 years.

(c) When n = ∞, capitalized cost P = A/i so, i = A/P = $12/$45 = 0.2666.
Rate of return = 26.7%

9-55

$(EUAB - EUAC)_A$ = \$230 – \$800 (A/P, 12%, 5) = +\$8.08

Set $(EUAB - EUAC)_B$ = +\$8.08 and solve for x.

$(EUAB - EUAC)_B$ = \$230 – \$1,000 (A/P, 12%, x) = +\$8.08

(A/P, 12%, x) = [\$230 – \$8.08]/\$1,000 = 0.2219
From the 12% compound interest table, <u>x = 6.9 yr.</u>

9-56

NPW_A = \$40 (P/A, 12%, 6) + \$100 (P/F, 12%, 6) – \$150
= +\$65.10
Set NPW_B = NPW_A
= \$65 (P/A, 12%, 6) + \$200 (P/F, 12%, 6) − x
= +\$65.10
\$368.54 − x = +\$65.10

<u>x = 303.44</u>

9-57

NPW Solution
NPW_A = \$75/0.10 – \$500 = +\$250
NPW_B = \$75 (P/A, 10%, n) – \$300 = +\$250
(P/A, 10%, n) = \$550/\$75 = 7.33
From the 10% table, <u>n = 13.9 yr.</u>

9-58

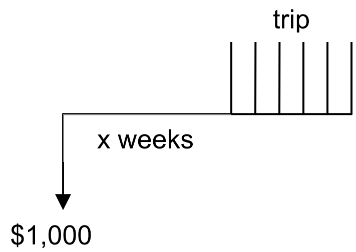
The key to solving this part of the problem is selecting a suitable analysis method. The Present Worth method requires common analysis period, which is virtually impossible for this problem. The problem is easy to solve by Annual Cash Flow Analysis.

$EUAC_{conventional\text{-}20\ yrs}$ = \$200 (A/P, 10%, 20) + \$230 = \$253.50
$EUAC_{solar\text{-}n\ yrs}$ = \$1,400 (A/P, 10%, n) + \$60

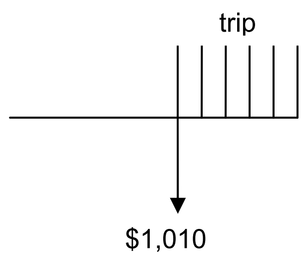
For equal EUAC:
(A/P, 10%, n) = [\$253.50 – \$60]/\$1,400 = 0.1382
From the interest tables, <u>n = 13.5 years.</u>

9-59

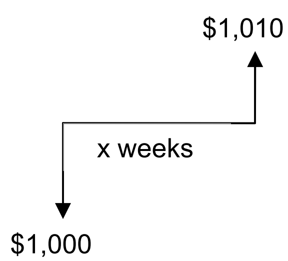
Alternative 1: Buy by May 31st



Alternative 2: Buy just before trip (1% service charge)



Difference between alternatives



i = ¼% per week
$1,000 = $1,010 (P/F, ¼%, x weeks)
(P/F, ¼%, x) = 0.9901
<u>x = 4 weeks</u>

9-60

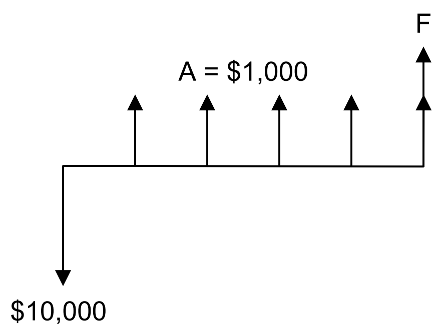
Untreated: EUAC = $10.50 (A/P, 10%, 10) = $10.50 (0.1627)
= $1.71

Treated: EUAC = ($10.50 + treatment) (A/P, 10%, 15)
= $1.38 + 0.1315 (treatment)

Set $EUAC_{UNTREATED}$ = $EUAC_{UNTREATED}$
$1.71 = $1.38 + 0.1315 (treatment)
Treatment = ($1.71 – $1.38)/0.1315 = $2.51

So, <u>up to $2.51 could be paid for post treatment.</u>

9-61



F = $10,000 (F/P, 10%, 5) – $1,000 (F/A, 10%, 5)
= $10,000 (1.611) – $1,000 (6.105)
= <u>$10,005</u>

9-62

Year	Cash Flow
0	–x
1	+$8,400
2	+$8,400
3	+$8,400
4	+$8,400
5	+$8,400
6	+$8,400
7	+$8,400
8	+$8,400
9	+$8,400
10	+$8,400
11	+$8,400
12	+$8,400
	+$80,000

Where x = maximum purchase price,
x = ($14,400 – $6,000) (P/A, 7%, 12) + $80,000 (P/F, 7%, 12)
= $8,400 (7.943) + $80,000 (0.4440)
= <u>$102,241</u>

9-63

Have three options for a tax-free annuity. (Note: \$359.60 x 12 = \$4,315.20 for option B and \$513.80 x 12 = \$6,165.60 for option C. Also, option C pays off the full 10 years regardless of year of death.)

Year	A	B	C
0	\$30,976	0	0
1	0	\$4,315.2	\$6,165.6
.	.	.	.
10	0	4,315.2	6,165.6
11	0	4,315.2	0
.	.	.	.
Death–1	0	4,315.2	0
Death	0	0	0

(a) If i = 6% develop a choice table for lives from 5 to 30 years. Calculate EUAB for each option. The C1 column is based strictly on economic equivalency. The C2 column ignores economic equivalency (except past year 10) and is from the viewpoint that the recipient needs the money to live on and the money that his heirs get after he dies does him no good!

EUAB(A) = 30976 (A/P, 6%, n)
EUAB(B) = 4315.20
EUAB(C) = 6165.6 (P/A, 6%, 10) (A/P, 6%, n)
(for all of C1 but past 10 years for C2)

Year	A	B	C1	C2
5	\$7,354	\$4,315.2	\$10,773	\$6,165.6
6	6,301	4,315.2	9,230	6,165.6
7	5,548	4,315.2	8,127	6,165.6
10	4,210	4,315.2	6,165.6	6,165.6
15	3,191	4,315.2	4,674	4,674
17	2,955	4,315.2	4,329	4,329
20	2,701	4,315.2	3,957	3,957
25	2,422	4,315.2	3,549	3,549
30	2,249	4,315.2	3,295	3,295

Choice Table:

5 yr ≤ C1 ≤ 17 yr	OR	5 yr ≤ A ≤ 6 yr
18 yr ≤ B ≤ 30 yr		7 yr ≤ C2 ≤ 17 yr
		18 yr ≤ B ≤ 30 yr

(b) If i = 10% develop a choice table for lives from 5 to 30 years. Same considerations as above.
EUAB(A) = 30976 (A/P, 10%, n)
EUAB(B) = 4315.20
EUAB(C) = 6165.6 (P/A, 10%, 10) (A/P, 10%, n) (for all of C1 but past 10 years for C2)

Year	A	B	C1	C2
5	$8,171	$4315.2	$9,995	$6165.6
7	6362	4315.2	7782	6165.6
8	5805	4315.2	7100	6165.6
10	5040	4315.2	6165.6	6165.6
15	4073	4315.2	4982	4982
20	3640	4315.2	4452	4452
22	3531	4315.2	4319	4319
25	3414	4315.2	4175	4175
30	3287	4315.2	4020	4020

Choice Table:

5 yr ≤ C1 ≤ 22 yr	OR	5 yr ≤ A ≤ 7 yr
23 yr ≤ B ≤ 30 yr		8 yr ≤ C2 ≤ 22 yr
		23 yr ≤ B ≤ 30 yr

(c) As interest rate goes up, the choice table extends further out into the future for each of the choices.

9-64

Since both motors have the same annual maintenance cost, it may be ignored in the computations. Here, however, we will include it.
Assuming a 20 year life for each motor.

Graybar
$EUAC_G$ = $7,000 (A/P, 10%, 20) + $300 + [[(200 hp) (0.746 kw/hp) ($0.072/kwhr)]/0.89 eff]
= $7,000 (0.1175) + $300 + 12.07 hr
= $1,122.50 + $12.07/hr

Blueball
$EUAC_B$ = $6,000 (A/P, 10%, 20) + $300 + [[(200 hp) (0.746 kw/hp) ($0.072/kwhr)]/0.85 eff]
= $6,000 (0.1175) + $300 + 12.64 hr
= $1,005 + 12.64 hr

Set $EUAC_B = EUAC_G$
$1,005 + 12.64 hr = $1,122.50 + $12.07/hr

The minimum number of hours the graybar, with its smaller power cost, must be used is: (12.64 − 12.07) hr = $1,122.50 − $1,005
hr = $117.50/$0.57 = <u>206 hours</u>

9-65

The difference between the alternatives is that Plan A requires $20,000 extra now and Plan B requires $40,000 extra n years hence.
At breakeven:
$20,000 = $40,000 (P/F, 8%, n)
(P/F, 8%, n) = 0.5
From the 8% interest table, <u>n = 9 years.</u>

9-66

The annual cost of the untreated part:
$350 (A/P, 10%, 6) = $350 (0.2296) = $80.36

The annual cost of the treated part must be at least this low so:
$80.36 = $500 (A/P, 10%, n)
(A/P, 10%, n) = $80.36/$500 = 0.1607

So n = 10 yr + (1) [(0.1627 − 0.1607)/(0.1627 − 0.1540)] = <u>10.2 years</u>

9-67

(a) PW of $Cost_A$ = PW of $Cost_B$
$55,000 + $16,200 (P/A, 10%, n) = $75,000 + $12,450 (P/A, 10%, n)
(P/A, 10%, n) = ($75,000 − $55,000)/($16,200 − $12,450) = 5.33

From the 10% interest table, (P/A, 10%, 8) = 5.335 so the machines are equivalent at 8 years.

(b) At 0% interest, from (a):
(P/A, 0%, n) = 5.33
Since (P/A, 0%, n) = n, the machines are equivalent at 5⅓ years.

9-68

(a) **Payback Period**
At first glance, payback would appear to be
$5,240/$1,000 = 5.24 years

However, based on end-of-year benefits, as specified in the problem, the correct answer is
Payback = 6 years

(b) **Breakeven Point (in years)**
Here interest is used in the computations.
Using continuous compounding allows us to solve directly
$P = A[(e^{rn} - 1)/(e^{rn} (e^{r} - 1)]$

P = $5,240	A = $1,000	R = 0.10	n = ?

$\$5{,}240 = \$1{,}000[(e^{0.10n} - 1)/(e^{0.10n} - 1)]$
$= \$1{,}000\ [(e^{0.10n} - 1)/(0.1052\ e^{10n})]$
$[e^{0.10n} - 1] = 5.24\ [0.1052\ e^{0.10n}]$
$e^{0.10n}\ [1 - 0.5511] = 1$
$e^{0.10n} = 1/(1 - 0.5511) = 2.23$
Solving, n = 8 years.

For annual compounding: $P = A\ [(1+i)^n - 1/(i(1+i)^n)]$
$\$5{,}240 = \$1{,}000[(1.1^n - 1)/(0.1*(1.1^n))]$
$0.5240 = (1.1^n - 1)/(1.1^n)$
Iterating we get at n=7.8 0.5240 = 0.5245. So n=8 years is the closest whole year

(c) Both (a) and (b) are “correct.” Since the breakeven analysis takes all eight years of benefits into account, as well as the interest rate, it is a better measure of long -term economic efficiency.

9-69

(a) First solve for the future unknown, F:
F = (12,000 – 1,000*(P/A,10%,5))/(P/F,10%,5)
Then code into Excel:
F = (12,000 + PV(0.10,5,1000))/(1+ 0.10)^–5
Result:
F = $13,221.02
For the remaining cases just change the appropriate variables in the part (a) Excel equation.

(b) Here change 12,000 back to 10,000 and 1,000 to 925.
F = $10,457.88

(c) Here change 12,000 back to 10,000 and 5 to 7.
F = $10,000.00

(d) Here change 1,000 to 925 and 5 to 7.
F = $14,608.97

9-70

(a) First solve for unknown by setting PWC(A) = PWC(B):
X = (8,000 + 12,000*0.075*(P/A,8%,3) – 8,900 – 12,000*0.09*(P/A,8%,3) + 1700*(P/F,8%,3))/(P/F,8%,3)
Then code into Excel:

X = (8000 – PV(0.08,3,12000*0.075) – 8,900 + PV(0.08,3,12,000*0.09) + 1,700*(1+ 0.08)^–3)/(1+ 0.08)^–3

Result: X = –$18.09 Since negative no breakeven price exists. You can give the Fiasco away!

For the remaining cases just change the appropriate variables in the part (a) Excel equation.

(b) Here change 0.075 back to 0.08 and 12,000 to 9,000.
X = $274.08

(c) Here change 0.075 back to 0.08 and 8% to 6%.
X = $246.05

(d) Here change 12,000 to 9,000 and 8% to 6%.
X = $198.30

9-71

	A	B
Cost	$800	$1,000
UAB	230	230
Useful Life (yrs)	5	X
MARR	12%	12%

(a) B's first cost is changed to $1,200.
For A the EUAB = 230 – 800*(A/P,12%,5). Program into Excel:
EUAB(A) – EUAC(A) = 230 + PMT(0.12,5,800) = $8.07

Next program EUAB(B) into Excel then calculate over several years to obtain above value.

Δ = EUAB(B) – EUAC(B) = 230 + PMT(0.12,X,1000) where X is the cell containing the number of years.

Years	Δ
8	($11.56)
9	4.79
9.23	8.01
10	17.62
11	27.90

Very close to 9.2 years.

(b) B’s annual benefit changed to $280

Years	Δ
4	($49.23)
5	2.59
5.14	8.20
6	36.77
7	60.88

Very close to 5.1 years.

(c) MARR is changed to 10%
EUAB(A) – EUAC(A) = $18.96

Years	Δ
5	($33.80)
6	0.39
6.74	19.01
7	24.59
8	42.56

Very close to 6.7 years.

(d) All three changes in (**a**), (**b**) and (**c**) are made.
EUAB(A) – EUAC(A) = $18.96

Years	Δ
5	($36.56)
6	4.47
6.46	18.98
7	33.51
8	55.07

Very close to 6.5 years.

9-72

	Untreated	Treated
Cost	$350	$500
Useful Life (yrs)	6	N
MARR	10%	10%

(a) Treated's installed cost is changed to $600.
EUAC (untreated) = 350*(A/P,10%,6). Program into Excel.
EUAC(untreated) = PMT(10%,6,350) = $80.36
Next program EUAC(treated) into Excel and calculate over several years to obtain $80.36.

EUAC(treated) = 600*(A/P,10%,N) = –PMT(0.10,N,600), where N is the unknown number of years.

Years	EUAC(treated)
13	$84.47
14	81.45
14.4	80.37
15	78.88
16	76.69

Very close to 14.4 years.

(b) Untreated's expected lifetime is changed to 4 years.
EUAC(untreated) = PMT(10%,4,350) = $110.41
EUAC(treated) = 500*(A/P,10%,N) = –PMT(0.10,N,500).

Years	EUAC(treated)
5	$131.90
6	114.80
6.33	110.37
7	102.70
8	93.72

Very close to 6.3 years

(c) MARR is changed to 12%.
EUAC(untreated) = PMT(12%,6,350) = $85.13
EUAC(treated) = 500*(A/P,12%,N) = –PMT(0.12,N,500).

Years	EUAC(treated)
9	$93.84
10	88.49
10.77	85.11
11	84.21
12	80.72

Very close to 10.8 years.

(d) All three changes in (a), (b) and (c) are made.
EUAC(untreated) = PMT(12%,4,350) = $115.23
EUAC(treated) = 600*(A/P,12%,N) = –PMT(0.12,N,600).

Years	EUAC(treated)
7	$131.47
8	120.78
8.65	115.24
9	112.61
10	106.19

Very close to 8.65 years.

Chapter 10: Uncertainty in Future Events

10-1

(a) Some reasons why a pole might be removed from useful service:
1. The pole has deteriorated and can no longer perform its function of safely supporting the telephone lines
2. The telephone lines are removed from the pole and put underground. The poles, no longer being needed, are removed.
3. Poles are destroyed by damage from fire, automobiles, etc.
4. The street is widened and the pole no longer is in a suitable street location.
5. The pole is where someone wants to construct a driveway.

(b) Telephone poles face varying weather and soil conditions, hence there may be large variations in their useful lives. Typical values for Pacific Telephone Co. in California are:
 Optimistic Life: 59 years
 Most Likely Life: 28 years
 Pessimistic Life: 2.5 years

Recognizing there is a mortality dispersion it would be possible, but impractical, ‘ to define optimistic life as the point where the last one from a large group of telephone poles is removed (for Pacific Telephone this would be 83.5 years). This is <u>not</u> the accepted practice. Instead, the optimum life is where only a small percentage (often 5%) of the group remains in service. Similarly, pessimistic life is when, say, 5% of the original group of poles have been removed from the group.

10-2

If 10,000 miles per year, then fuel cost = oil/tires/repair = $990/year, and salvage value = 8,000 − 5*10,000*.08 = 9,000 − 4,000 = 5,000

$EUAC_{10,000}$ = 9,000(*A/P,*8%,5) + 2*990 − 5,000(*A/F,*8%,5)
= 9,000*.2505 + 1,980 − 5,000*. 1705
= 2,254.5 + 1,980 − 852.5 = $3,382

Increasing annual mileage to 15,000 is a 50% increase so it increases operating costs by 50%. The salvage value drops by 5*5,000*.08 = 2,000.

$EUAC_{15,000}$ = 9,000(*A/P,*8%,5) + $2^{*}1.5^{*}990$ − 3,000(*A/F,* 8%,5)
= $9,000^{*}.2505 + 1.5^{*}1,980 - 3,000^{*}. 1,705$
= 2,254.5 + 2,970 − 511.5 = $4,713

Decreasing annual mileage to 5,000 is a 50% decrease so it decreases operating costs by 50%. The salvage value increases by $5^{*}5,000^{*}.08$ = 2,000

$EUAC_{5,000}$ = 9,000(*A/P,*8%,5) + $2^{*}.5^{*}990$ − 7,000(*A/F,*8%,5)
= $9,000^{*}.2505 + .5^{*}1980 - 7,000^{*}. 1,705$
= 2,254.5 + 990 − 1,193.5 = $2,051

10-3

Mean Life= (12 + 4 x 5 + 4)/6 = 6 years
PW of Cost = PW of Benefits
$80,000 = $20,000 (P/A, i%, 6)
Rate of Return is between 12% and 15%
Rate of Return <EQ> 13%

10-4

Since the pessimistic and optimistic answers are symmetric about the most likely value of 10,000, the weighted average is 10,000 miles. If 10,000 miles per year, then fuel cost = oil/tires/repair = $990/year, and salvage value = 8,000 − $5^{*}10,000^{*}.08$ = 9,000 − 4,000 = 5,000

$EUAC_{10,000}$ = 9,000(*A/P,*8%,5) + $2^{*}990$ − 5,000(*A/F,*8%,5)
= $9,000^{*}.2505 + 1,980 - 5,000^{*}. 1705$
= 2,254.5 + 1,980 − 852.5 = $3,382

10-5

(a) PW(A) = –25,0000 + 20,000 (P/A, 6%, 30) = –250,000 + (20,000) (13.765)
= $25,300
PW(B) = –250,000 + 15,000 (13.765) = –$43,525
PW(C) = –250,000 + 8,000 (13.765) = –$139,880

(b) Mean Annual Savings = $\frac{20000+(4)(15000)+8000}{6}$ = $14,667

PW(MAS) = –250000 + 14667 (13.765) = –$48,109

(c) No, because the pessimistic estimate was \$2000 further below the most likely than the most optimistic was above the most likely.

10-6

There are six ways to rolls a 7:1 & 6, 2 & 5, 3 & 4, 4 & 3, 5 & 2, 6 & 1
There are two ways to roll an 11: 5 & 6 or 6 & 5
Probability of rolling a 7 or 11 = (6 + 2)/36 = <u>8/36</u>

10-7

Since the *P* values must sum to 1: *P*(20%) = 1 − 2/10 − 3/10 = .5
E(*i*) = .2(10%) + .3(15%) + .5(20%) = 16.5%

10-8

State of Nature	Completion Time	Probability
Sunny and Hot	250 days	0.2
In Between Weather	300 days	0.5 = 1 − 0.2 − 0.3
Cool and Damp	350 days	0.3

E(days) = .20(250) + .5(300) + .3(350) = 305 days

10-9

If you have another accident or a violation this year, which has a .2 probability, it is assumed to occur near the end of the year so that it affects insurance rates for years 1–3. A violation in year 1 affects the rates in years 2 and 3 only if there was no additional violation in this year, which is P(none in 0).P(occur in 1) = .8· .2 = .16. So the total probability of higher rates for year 2 is .2 + .16 or .36. This also equals 1 − P(no violation in 0 or 1) = 1 − $.8^2$.

For year 3, the result can be found as P(higher in year 2) + P(not higher in year 2). P(viol. in year 2) = .36 + .64·.2 = .488. This also equals either 1 − P(no violation in 0 to 2) = 1 − $.8^3$.

Rates for Year	0	1	2	3
P(\$600)	0	.2	.36	.488

10-10

Al's Score was x + (5/20) s = x + 0.25 s
Bill's Score was x + (2/4) s = x + 0.50 x
Therefore, Bill ranked higher in his class.

10-11

(a)

First Cost	P	Net Revenue	P	Joint Probability
$300,000	0.2	$70,000	0.3	0.06
400,000	0.5	70,000	0.3	0.15
600,000	0.3	70,000	0.3	0.09—pessimistic
300,000	0.2	90,000	0.5	0.10
400,000	0.5	90,000	0.5	0.25—most likely
600,000	0.3	90,000	0.5	0.15
300,000	0.2	100,000	0.2	0.04—optimistic
400,000	0.5	100,000	0.2	0.10
600,000	0.3	100,000	0.2	0.06
			Total =	1.00

(b) optimistic: PW = –300,000 + 100,000 (P/A, 12%, 10)
= –300,000 + 100,000 (5.650) = $265,000
most likely: PW = –400,000 + 90,000 (5.650) = $108,500
pessimistic: PW = –600,000 + 70,000 (5.650) = –$204,500

10-12

(a)

Savings per Year	P	Useful Life (yr)	P	Joint Probability
$18,000	0.2	12	1/6	0.033
20,000	0.7	12	2/3	0.117
22,000	0.1	12	1/6	0.017–optimistic
18,000	0.2	5	1/6	0.133
20,000	0.7	5	2/3	0.467–most likely
22,000	0.1	5	1/6	0.067
18,000	0.2	4	1/6	0.033–pessimistic
20,000	0.7	4	2/3	0.117
22,000	0.1	4	1/6	0.017
			Total =	1.001 (rounding error)

(b) optimistic: NPW = 0 = –81,000 + 22,000 (P/A, ROR, 12), so
(P/A, ROR, 12) = 3.682 and interpolating

$$\text{ROR} = 25\% + (5\%)\left[\frac{3.725-3.682}{3.725-3.190}\right] = 25.4\%$$

most likely: NPW = 0 = –81,000 + 20,000 (P/A, ROR, 5), so
(P/A, ROR, 5) = 3.682 and interpolating

$$\text{ROR} = 7\% + (1\%)\left[\frac{4.100-4.05}{4.100-3.993}\right] = 7.47\%$$

pessimistic: NPW = 0 = –81000 + 18000 (P/A, ROR, 4), so
(P/A, ROR, 4) = 4.500 and ROR = –4.55% (Excel)

10-13

(a)

Savings per Year	P	Useful Life (yr)	P	Joint Probability
$15,000	0.3	3	0.6	0.18–pessimistic
30,000	0.5	3	0.6	0.30–most likely
45,000	0.2	3	0.6	0.12
15,000	0.3	5	0.4	0.12
30,000	0.5	5	0.4	0.20
45,000	0.2	5	0.4	0.08–optimistic
			Total =	1.00

(b) optimistic: PW = –80,000 + 45,000 (P/A, 9%, 5) = –80,000 + 45,000 (3.890)
= $95,050
most likely: PW = –80000 + 30,000 (P/A, 9%, 3) = –80,000 + 30,000 (2.531)
= –$4,070
pessimistic: PW = –80,000 + 15,000 (P/A, 9%, 3) = –80,000 + 15,000 (2.531)
= –$42,035

10-14

Since the *P*s must sum to 1: *P*(30K) = 1 − .2 − .3 = .5
E(savings) = .3(20K) + .5(30K) + .2(40K) = $29K

10-15

		Instructor A		Instructor B	
Grade		Grade Distribution	Expected Grade Point	Grade Distribution	Expected Grade Point
A	4.0	0.10	0.40	0.15	0.60
B	3.0	0.15	0.45	0.15	0.45
C	2.0	0.45	0.90	0.30	0.60
D	1.0	0.15	0.15	0.20	0.20
F	0	0.15	0	0.20	0
Sum		1.00	1.90	1.00	1.85

To minimize the Expected Grade Point, <u>choose instructor A.</u>

10-16

P(20%) = 10 – P(10%) – P(15%) = 10 – 2 – 3 = 5
EV(discount rate) = (10%) (0.2) + (15%) (0.30) + (20%) (0.50) = 16.5%

10-17

P(otherwise) = 100% – 20% – 30% = 50%
EV(completion date) = 250 (0.2) + 300 (0.5) + 350 (0.3) = 305 days

10-18

Expected outcome = \$2,000 (0.3) + \$1,500 (0.1) + \$1,000 (0.2) + \$500 (0.3) + \$0 (0.1)
= \$1,100

10-19

The sum of probabilities for all possible outcomes is one.
An inspection of the Regular Season situation reveals that the sum of the probabilities for the outcomes enumerated is 0.95. Thus one outcome (win less than three games), with probability 0.05, has not been tabulated. This is not a faulty problem statement. The student is expected to observe this difficulty.

Similarly, the complete probabilities concerning a post-season Bowl Game are:
Probability of playing = 0.10
Probability of not playing = 0.90

Expected Net Income for the team
= (0.05 + 0.10 + 0.15 + 0.20) (\$250,000) + (0.15 + 0.15 + 0.10) (\$400,000) + (0.07 + 0.03) (\$600,000) + (0.10) (\$100,000)
= 0.50 (\$250,000) + 0.40 (\$400,000) + 0.10 (\$600,000) + 0.10 (\$100,000) + 0.90 (\$0)
= \$355.00

10-20

Determine the different ways of throwing an 8 with a pair of dice.

Die 1	Die 2
2	6
3	5
4	4
5	3
6	2

The five ways of throwing an 8 have equal probability of 0.20.
The probability of winning is 0.20
The probability of losing is 0.80
The outcome of a \$1 bet = 0.20 (\$4) + 0.80 (\$0) = \$0.80
This means a \$0.20 loss.

10-21

Expected number of wins in 100 attempts = 100/38 = 2.6316
Results of a win = 35 × \$5 + \$5 bet return = \$180.00
Expected winnings = \$180.00 (2.6313) = \$473.69
Expected loss = \$500.00 − \$473.69 = \$26.31

10-22

(a) EV(EUAC) = (2,051) (0.3) + (3,382) (0.5) + (4713) (0.2) = \$3,249

(b) EV(number of miles) = (5,000) (0.3) + (10,000) (0.5) + (15,000) (0.2) = 9,500

fuel cost = $\left(\frac{9500}{10000}\right)(990)$ = \$940.5 and 9,500 x 5 = 47,500 miles in 5 years

EV(EUAC) = 9,000 (A/P, 8%, 5)+(2)(940.5)–[9,000 – (47,500)(0.08)] (A/F,8%,5)
= 2,254.4 + 1881 – 886.6 = \$3,249

(c) They match! If you work algebraically from the equation in part (a), you would eventually be able to factor out the EV(number of miles) formula out everywhere miles appears in the original equation.

10-23

(a) 4 years: EUAC –EUAB = 0 = 80000 (A/P, ROR, 4) – 20000
(A/P, ROR, 4) = 0.25 and ROR = 0 by inspection.

5 years: (A/P, ROR, 5) = 0.25 and interpolating

$$\text{ROR} = 7\% + (1\%)\left[\frac{0.2439-0.25}{0.2439-0.2505}\right] = 7.92\%$$

12 years: (A/P, ROR, 12) = 0.25 and interpolating

$$\text{ROR} = 20\% + (5\%)\left[\frac{0.2253-0.25}{0.2253-0.2684}\right] = 22.87\%$$

EV(ROR) = (0) (0.05) + (7.92%) (0.80) + (22.87%) (0.15) = 9.77%

(b) EV(life) = (40) (0.05) + (5) (0.80) + (12) (0.15) = 6
(A/P, ROR, 6) = 0.25 and interpolating

$$\text{ROR} = 12\% + (3\%)\left[\frac{0.2432-0.25}{0.2432-0.2642}\right] = 12.97\%$$

(c) No, the equation for Find a Given P is nonlinear with respect to the n parameter.

10-24

(a) EV(PW) = (–139,880) (0.3) + (–43,525) (0.5) + (25,300) (0.2) = –$58,667

(b) EV(annual savings) = (8,000) (0.3) + (15,000)(0.5) + (20,000) (0.2) = $13,900
PW = –250,000 + 13,900 (P/A, 6%, 30) = –250,000 + (13,000) (13.765) = –$58,667

(c) Yes, if you work algebraically from the equation in part (a), you will be able to factor out the EV(annual savings) equation where annual savings occurs in the original equation.

10-25

(a) EV(annual savings) = 20,000 (0.3) + 30,000 (0.5) + 40,000 (0.2) = $29,000
EUAW = EUAB – EUAC = 29,000 – 150,000 (A/P, 8%, 10)
= 29,000 – (150,000) (0.1490)
= $6,650

(b) EUAW(pessimistic) = 20,000 – 150,000 (0.1490) = –$2,350
EUAW(most likely) = 30,000 – 150,000 (0.1490) = $7,650
EUAW(optimistic) = 40,000 – 150,000 (0.1490) = $17,650
EV(EUAW) = –2,350 (0.3) + 7,650 (0.5) + 17,650 (0.2) = $6,650

(c) Yes, if you work algebraically from the equation in part (a), you will be able to factor out the EV(EUAW) equation where annual savings occurs in the original equation.

10-26

Height above Roadway	Annual Probability of Flood Damage	× Damage	= Expected Annual Damage
2 m	0.333	$300,000	= $100,000
2.5 m	0.125	$300,000	= $37,500
3 m	0.04	$300,000	= $12,000
3.5 m	0.02	$300,000	= $6,000
4 m	0.01	$300,000	= $3,000

Height above Roadway	Initial Cost	x (A/P, 12%, 50)	= EUAC of Embankment	Expected Annual Damage	Total Expected Annual Cost
2 m	$100,000	0.1204	= $12,040	$100,000	$112,040
2.5 m	$165,000	0.1204	= $19,870	$37,500	$53,370
3 m	$300,000	0.1204	= $36,120	$12,000	$48,120 ↔
3.5 m	$400,000	0.1204	= $48,160	$6,000	$54,160
4 m	$550,000	0.1204	= $66,220	$3,000	$69,220

Select 3 meter embankment to minimize total Expected Annual Cost.

10-27

$E(PW_{extra\ costs})$ = .2*600(*P/F,*8%, 1) +.36*600(*P/F,*8%,2) +.488*600(*P/F,*8%,3)
= .2*600*.9259 +.36*600*.8573 +.488*600*.7938 = $528.7

10-28

(a) PW = –(First Cost) + (Net Revenue)*(P/A, 12%, 10)

PW = –300K + 70K (5.650) = $95,500
PW = –300K + 90K (5.650) = $208,500
PW = –300K + 100K (5.650) = $265,000
PW = –400K + 70K (5.650) = –$4,500
PW = –400K + 90K (5.650) = $108,500
PW = –400K + 100K (5.650) = $165,000
PW = –500K + 70K (5.650) = –$204,500
PW = –500K + 90K (5.650) = –$91,500
PW = –500K + 100K (5.650) = –$35,000

E(PW) = (0.2)(0.3)$95,500 + (0.2)(0.5)$208.500 + (0.2)(0.2)$265,000 +
(0.5)(0.3)(–$4,500) + (0.5)(0.5)$108.500 + (0.5)(0.2)$165,000 +
(0.3)(0.3)(–$204,000) + (0.3)(0.5)(–$91.500) + (0.3)(0.2)(–$35,000)
= $45,900

(b)
E(first cost) = 300,000(.2) + 400,000(.5) + 600,000(.3) = $440K

E(net revenue)= 70,000(.3) + 90,000(.5) + 100,000(.2) = $86K

E(PW) = −440K + 86K(*P/A,* 12%, 10) = $45.9K, do the project

(c) Yes the expected costs are the same. If you look at the calculations you will see that they are essentially identical.

10-29

Use the data from Problem 10-12.

A									
	Savings per year = 18,000*(0.2) + 20,000*(0.7) + 22,000(0.1) = $19,800								
	Useful Life (years) = 12*(1/6) + 5*(2/3) + 4*(1/6) = 6								
	PW = 0 = –81000 + 19,800(P/A,ROR,6);				ROR = IRR(B6:H6) = 12.18%				
	–81000	19800	19800	19800	19800	19800	19800		
b									
	—81000	—81000	—81000	—81000	—81000	—81000	—81000	—81000	—81000
	18000	20000	22000	18000	20000	22000	18000	20000	22000
	18000	20000	22000	18000	20000	22000	18000	20000	22000
	18000	20000	22000	18000	20000	22000	18000	20000	22000
	18000	20000	22000	18000	20000	22000	18000	20000	22000
	18000	20000	22000	18000	20000	22000	–4.55%	–0.50%	3.40%
	18000	20000	22000	3.62%	7.46%	11.15%			
	18000	20000	22000						
	18000	20000	22000		Example: ROR = IRR(F8:F13,0.1)				
	18000	20000	22000			= 7.46%			
	18000	20000	22000						
	18000	20000	22000						
	18000	20000	22000						
	19.64%	22.54%	25.36%						
	EV(ROR) = 0.1964*0.033 + 0.2254*0.117 + 0.2536*0.017 + 0.0362*0.133 + 0.0746*0.467 + 0.1115*0.067 – 0.0455*0.033 – 0.0050*0.117 + 0.034*0.017 = 0.082779 or 8.28%								
	No, the answers do not match because the interest formulas are nonlinear.								

Note: In (a) the Excel function IRR uses row entries, while in (b) IRR uses column entries.

10-30

Use the data from Problem 10-13.

(a)	Savings per year = 15000(0.3) + 30000*(0.5) + 45000*(0.2) = $28,500					
	Useful life = 3*(0.6) + 5*(0.4) = 3.8 years					
	PW = –80,000 + 28500*(P/A,9%,3.8) = –80,000 – PV(0.09,3.8,28500) = $8,432					
(b)	Savings/yr	P	Useful Life (yr)	P	Joint Prob.	PW
	$15,000	0.3	3	0.6	0.18	($42,031)
	30,000	0.5	3	0.6	0.30	($4,061)
	45,000	0.2	3	0.6	0.12	$33,908
	15,000	0.3	5	0.4	0.12	($21,655)
	30,000	0.5	5	0.4	0.20	$36,690
	45,000	0.2	5	0.4	0.08	$95,034
				Total =	1.00	
	EV(PW) = –42031*(0.18) – 4,061*(0.30) + 33,908*(0.12) – 21,655*(0.12) + 36,690*(0.20) + 95,034*(0.08) = $7,627					
(c)	No, the useful life enters in a nonlinear way, i.e., as an exponent.					

10-31

Leave the Valve as it is
Expected PW of Cost = 0.60 ($10,000) + 0.50 ($20,000) + 0.40 ($30,000)
= $28,000

Repair the Valve
Expected PW of Cost = $10,000 repair + 0.40 ($10,000) + 0.30 ($20,000) + 0.20 ($30,000)
= $26,000

Replace the Valve
Expected PW of Cost = $20,000 replacement + 0.30 ($10,000) + 0.20 ($20,000) + 0.10 ($30,000)
= $30,000

To minimize Expected PW of Cost, repair the valve.

10-32

Do Nothing
EUAC = Expected Annual Damage
= 0.20 ($10,000) + 0.10 ($25,000) = $4,500

$15,000 Building Alteration
Expected Annual Damage = 0.10 ($10,000) = $1,000
Annual Floodproofing Cost = $15,000 (A/P, 15%, 15) = $2,565
EUAC = $3,565

$20,000 Building Alteration
Expected Annual Damage = $0
Annual Floodproofing Cost = $20,000 (A/P, 15%, 15) = $3,420
EUAC = $3,420

To minimize expected EUAC, recommend $20,000 building alteration.

10-33

Since $250,000 of dam repairs must be done in all alternatives, this $250,000 can be included or ignored in the analysis. Here it is ignored. (Remember, only the differences between alternatives are relevant.)

	Flood	Probability of Damage in Any year = 1/yr Flood	Downstream Damage	Spillway Damage
	25 yr	0.04	$50,000	
	50 yr	0.02	$200,000	
For 10 yr:	100 yr	0.01	$1,000,000	$250,000
Thereafter:	100 yr	0.01	$2,000,000	$250,000

Alternative I: Repair existing dam but make no other alterations
Spillway damage: Probability that spillway capacity equaled or exceeded in any year is 0.02. Damage if spillway capacity exceed: $250,000

Expected Annual Cost of Spillway Damage = $250,000 (0.02)
= $5,000

Downstream Damage during next 10 years:

Flood	Probability that Flow* Will Be Equaled or Exceeded	Damage	Δ Damage over More Frequent Flood	Annual Cost of Flood Risk
25 yr	0.04	$50,000	$50,000	$2,000
50 yr	0.02	$200,000	$150,000	$3,000
100 yr	0.01	$1,000,000	$800,000	$8,000

Next 10 year expected annual cost of downstream damage = $13,000

Downstream Damage after 10 years: Following the same logic as above, Expected annual cost of downstream damage
= $2,000 + $3,000 + 0.1 ($2,000,000 − $200,000)
= $23,000

Present Worth of Expected Spillway and Downstream Damage
PW = $5,000 (P/A, 7%, 50) + $13,000 (P/A, 7%, 10) + $23,000 (P/A, 7%, 40) (P/F, 7%, 10)
= $5,000 (13.801) + $13,000 (7,024) + $23,000 (13.332) (0.5083)
= $316,180

Equivalent Uniform Annual Cost
Annual Cost = $316,180 (A/P, 7%, 50)
= $316,180 (0.0725)
= <u>$22,920</u>

* An N-year flood will be equaled or exceed at an average interval of N years.

Alternative II: Repair the dam and redesign the spillway
Additional cost to redesign/reconstruct the spillway = $250,000
PW to Reconstruct Spillway and Expected Downstream Damage Downstream Damage—same as alternative 1

PW = $250,000 + $13,000 (P/A, 7%, 10) + $23,000 (P/A, 7%, 40) (P/F, 7%, 10)
= $250,000 + $13,000 (7.024) + $23,000 (13.332) (0.5083)
= $497,180

EUAC = $497,180 (A/P, 7%, 50)
= $497,180 (0.0725)
= <u>$36,050</u>

Alternative III: Repair the dam and build flood control dam upstream
Cost of flood control dam = $1,000,000
EUAC = $1,000,000 (A/P, 7%, 50)
= $1,000,000 (0.7225)
= <u>$72,500</u>

Note: One must be careful not to confuse the frequency of a flood and when it might be expected to occur. The occurrence of a 100-year flood this year is no guarantee that it won't happen again next year. In any 50-year period, for example, there are 4 chances in 10 that a 100-year flood (or greater) will occur.

Conclusion: Since we are dealing with conditions of risk, it is not possible to make an absolute statement concerning which alternative will result in the least cost to the community. Using a probabilistic approach, however, Alternative I is most likely to result in the least equivalent uniform annual cost.

10-34

If the savings are only \$15K per year, spending \$50K for 3 more years would not make sense. For the two or three shift situations, the table from 10-30 can be modified for 3 extra years, and to include the \$50K at the end of 3 or 5 years. For example, the first and second rows' PWs are unchanged. The third row's PW = −80K + 15K(*P/A,*9%,6) − 50K(*P/F,*9%,3).

Savings/yr	*P*	Life	*P*	*P*	PW	*P*·PW
15,000	.3	3	.6	0.18	−42,031	−7,566
15,000	.3	5	.4	0.12	−21,655	−2,599
30,000	.5	6	.6	0.30	15,968	4,791
30,000	.5	8	.4	0.20	53,548	10,710
45,000	.2	6	.6	0.12	83,257	9,991
45,000	.2	8	.4	0.08	136,57 0	10,926
				Expected Values		26,252

The option of extending the life is not used for single shift operations, but it increases the expected PW by 26,252 − 7,627 = \$18,625.

10-35

(a) Expected fire loss in any year = 0.010 (\$10,000) + 0.003 (\$40,000) +
0.001 (\$200,000)
= \$420.00

(b) The engineer buys the fire insurance because
1. a catastrophic loss is an unacceptable risk
or 2. he has a loan on the home and fire insurance is required by the lender.

10-36

P	.3	.5	.2	*E*(*x*)
PW	\$6,570	\$8,590	\$9,730	\$8,212
PW^2	43,164,900	73,788,100	94,672,900	68,778,100

$\sigma_{PW} = (68{,}778{,}100 - 8{,}212^2)^{1/2} = \$1{,}158$

10-37

$PW_1 = -25{,}000 + 7{,}000(P/A,\ 12\%,4) = -\$3{,}739$
$PW_2 = -25{,}000 + 8{,}500(P/A,\ 12\%,4) = \$817$
$PW_3 = -25{,}000 + 9{,}500(P/A,12\%,4) = \$3{,}855$

From the table the E(PW) = \$361.9
$\sigma_{PW} = (8{,}918{,}228 - 361.9^2)^{1/2} = \$2964$

P	.3	.4	.3	*E*(*x*)
Annual Savings	\$7,000	\$8,500	\$9,500	\$8,350
PW	−3,739	817	3,855	361.87
PW^2	13,976,7 90	668,256	14,859,6 28	8,918,228

10-38

The \$35K is a sunk cost and should be ignored.
a. E(PW) = \$5,951
b. P(PW < 0) = .3 and σ = \$65,686.

State	Bad	OK	Great		
Probability	.3	.5	.2		
Net Revenue	\$−15,000	\$15,000	\$20,000		
Life (yr)	5	5	10		
PW	−86,862	26,862	92,891	\$5,951	E_{PW}
PW^2 · Prob	2,263,491,770	360,778,191	1,725,760,288	\$65,686	σ_{PW}

10-39

(a) The $35K is still a sunk cost and should be ignored. Note: P(PW < 0) = .3 and *N* = 1 used for PW_{bad} since termination allowed here. This improves the E_{PW} by 18, in years 2−5, which equals .3 (1/1.1)* 15,000(*P/A*,. 1,4).

(b) The P(loss) is unchanged at .3. However, the standard deviation improves by 65,686 − 47,957 = $17,709.

State	Bad	OK	Great		
Probability918 − 5951 = $12,967. This also equals the E(PW) of the avoided negative net revenue	.3	.5	.2		
Net Revenue	$−15,000	$15,000	$20,000		
Life (yr)	1	5	10		
PW	−43,636	26,862	92,891	$18,918	E_{PW}
PW^2 · Prob	571,239,669	360,778,191	1,725,760,288	$47,957	σ_{PW}

10-40

To calculate the risk, it is necessary to state the outcomes based on the year in which the next accident or violation occurred.

Year Of 2nd Offense	**0**	**1**	**2**	**OK**	
Extra $600 in years	1−3	2−3	3	none	
P	.2	.16	.128	.512	*E*(*x*)
PW	$−1546	$−991	$−476	$0	$−529
PW^2	2,390,914	981,492	226,861	0	664,260

$\sigma_{PW} = (664{,}260 - 529^2)^{1/2} = \$620.0$

10-41

For example, the first row's PW = −300K + 70K(*P/A,* 12%, 10)

First Cost	*P*	Net Revenue	*P*	*P*	PW	*P*·PW	*P*·PW^2
−300	.2	70	.3	0.06	95.5	5.73	547
−300	.2	90	.5	0.10	208.5	20.85	4,347
−300	.2	100	.2	0.04	265.0	10.60	2,809
−400	.5	70	.3	0.15	−4.5	−0.68	3
−400	.5	90	.5	0.25	108.5	27.13	2,943
−400	.5	100	.2	0.10	165.0	16.50	2,723
−600	.3	70	.3	0.09	−204.5	−18.41	3,764
−600	.3	90	.5	0.15	−91.5	−13.73	1,256
−600	.3	100	.2	0.06	−35.0	−2.10	74
				Expected Values	45.90	18,468	

Risk can be measured using the P(loss), range, or the standard deviation of the PWs.
P(loss) = .15 + .09 + .15 + .06 = .45
The range is −204.5K to $265K.
The standard deviation is $\sigma_{PW} = \sqrt{(18{,}468 - 45.90^2)}$ = $127.9K.

10-42

(a) The probability of a negative PW is .18 + .12 + .3 = .6

Savings/yr	*P*	Life	*P*	*P*	PW	*P*·PW	*P*·PW^2
15,000	.3	3	.6	0.18	−42,031	−7,566	317,982,538
15,000	.3	5	.4	0.12	−21,655	−2,599	56,273,884
30,000	.5	3	.6	0.30	−4,061	−1,218	4,947,906
30,000	.5	5	.4	0.20	36,690	7,338	269,224,438
45,000	.2	3	.6	0.12	33,908	4,069	137,972,411
45,000	.2	5	.4	0.08	95,034	7,603	722,521,558
				Expected Values	7,627	1,508,922,738	

Risk can also be measured using the standard deviation of the PWs. The standard deviation is $\sigma_{PW} = \sqrt{(1{,}508{,}922{,}738 - 7{,}627^2)}$ = $38,089.

(b) Extending the life for 2 & 3 shift operations, reduces the probability of a negative PW by .3 to .3.

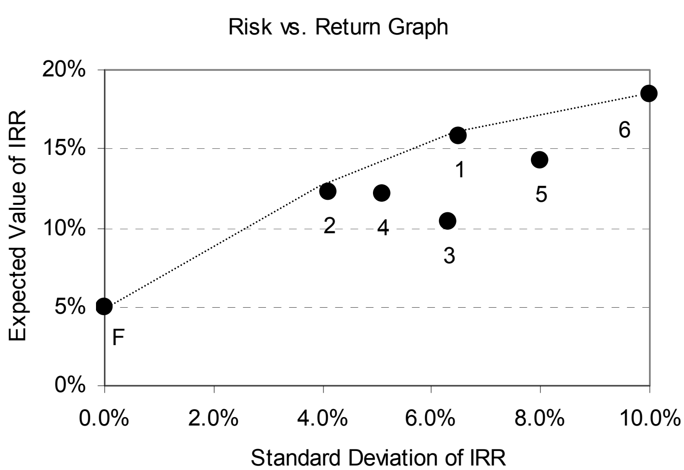
Savings/yr	*P*	Life	*P*	*P*	PW	*P*·PW	*P*·PW^2
15,000	.3	3	.6	0.18	−42,031	−7,566	317,982,538
15,000	.3	5	.4	0.12	−21,655	−2,599	56,273,884
30,000	.5	6	.6	0.30	15,968	4,791	76,496,783
30,000	.5	8	.4	0.20	53,548	10,710	573,477,749
45,000	.2	6	.6	0.12	83,257	9,991	831,810,614
45,000	.2	8	.4	0.08	136,570	10,926	1,492,115,547
				Expected Values	26,252	3,348,157,118	

Risk can also be measured using the standard deviation of the PWs. The standard deviation is increased by $13,477. This illustrates why standard deviation alone is not the best measure of risk. Extending the life makes the project more attractive, and increases the spread of the possible values. The standard deviation is higher, but the P(loss) has dropped by half.

$\sigma_{PW} = \sqrt{(3,348,157,118 - 26,252^2)} = \$51,565$

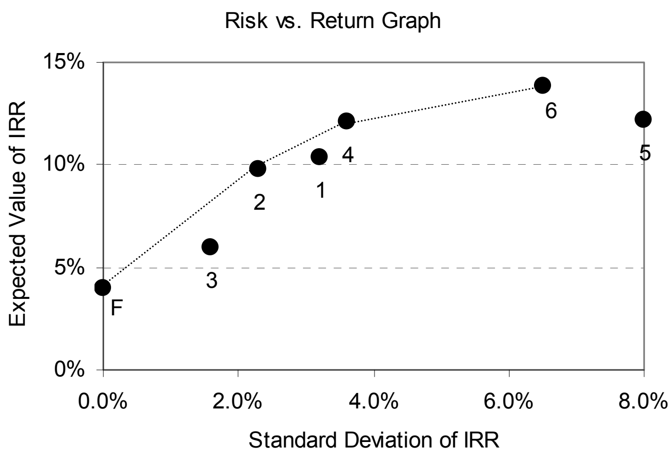
10-43

Project	IRR	Std.Dev.
1	15.8%	6.5%
2	12.3%	4.1%
3	10.4%	6.3%
4	12.1%	5.1%
5	14.2%	8.0%
6	18.5%	10.0%
F	5.0%	0.0%



10-44

Project	IRR	Std.Dev.
1	10.4%	3.2%
2	9.8%	2.3%
3	6.0%	1.6%
4	12.1%	3.6%
5	12.2%	8.0%
6	13.8%	6.5%
F	4.0%	0.0%



10-45

	First Cost	$25,000	i = 7%
	Life	Annual Benefit	
Min	7	$4,400	Mean
Max	10	1,000	Std Dev
Iteration			PW
1	7	4,587	($277)
2	9	2,964	(5,688)
3	7	4,726	467
4	10	4,477	6,443
5	9	4,248	2,678
6	7	4,781	767
7	8	4,919	4,372
8	7	3,867	(4,159)
9	7	4,414	(1,211)
10	8	4,446	1,551
11	10	4,236	4,754
12	9	4,526	4,485
13	10	3,797	1,666
14	7	3,350	(6,945)
15	10	5,631	14,551
16	10	3,449	(774)
17	9	2,624	(7,905)
18	10	4,727	8,203
19	8	5,750	9,333
20	7	5,806	6,289
21	9	4,046	1,358
22	10	5,606	14,372
23	9	4,927	7,097
24	9	6,168	15,185
25	8	4,445	1,545
Mean	8.56	$4,501	$3,126
Std Dev	1.19	882	6,232
Life = 7 + INT(4*RAND())			
Annual Benefit = NORMINV(RAND(), 4,400,1,000)			
PW = –25000 – PV(0.07,Life,Annual Benefit)			
Mean = AVERAGE(E7:E31)			
Std Dev = STDEV(E7:E31)			

10-46

Annual benefit $55,000			
Annual operating cost		$10,000	i = 8%
	Life	First Cost	
Min	3	$150,000	Mean
Max	7	50,000	Std Dev
Iteration			PW
1	5	$72,782	$106,890
2	6	175,896	$32,133
3	5	163,094	$16,578
4	3	239,955	($123,986)
5	4	159,658	($10,612)
6	7	121,566	$112,721
7	7	160,090	$74,197
8	5	151,896	$27,776
9	7	86,637	$147,650
10	7	63,750	$170,537
11	5	112,293	$67,379
12	7	141,319	$92,968
13	7	141,505	$92,781
14	6	169,007	$39,023
15	3	38,376	$77,593
16	3	106,321	$9,648
17	4	111,970	$37,076
18	3	149,845	($33,875)
19	4	174,423	($25,377)
20	4	110,218	$38,828
21	7	237,986	($3,699)
22	5	109,253	$70,419
23	3	175,610	($59,640)
24	4	177,276	($28,230)
25	5	185,633	($5,961)
Mean	5.04	$141,454	$36,913
Std Dev	1.51	48,959	66,922
Life = 3 + INT(5*RAND())			
Annual Benefit = NORMINV(RAND(),150000,50000)			
PW = –First Cost – PV(0.08,Life,55000–10000)			
Mean = AVERAGE(E8:E32)			
Std Dev = STDEV(E8:E32)			

Chapter 11: Depreciation

11-1

Year	SOYD	DDB
1	\$2,400	\$3,333
2	\$2,000	\$2,222
3	\$1,600	\$1,482
4	\$1,200	\$988
5	\$800	\$375*
6	\$400	\$0
Sum	\$8,400	\$8,400

*Computed \$658 must be reduced to \$375 to avoid depreciating the asset below its salvage value.

11-2

DDB Schedule is:

Year n	d(n) = (2/n)[P − sum d(n)]	DDB Depreciation
1	(2/6) (\$1,000,000 − \$0)	= \$333,333
2	(2/6) (\$1,000,000 − \$333,333)	= \$222,222
3	(2/6) (\$1,000,000 − \$555,555)	= \$148,148
4	(2/6) (\$1,000,000 − \$703,703)	= \$98,766
5	(2/6) (\$1,000,000 − \$802,469)	= \$65,844
6	See below	= \$56,687

If switch DDB to SL for year 5:
SL = (\$1,000,000 − \$802,469 − \$75,000)/2 = \$61,266
Do not switch.

If switch DDB to SL for year 6:
SL = (\$1,000,000 − \$868,313 − \$75,000)/1 = \$56,687
Do switch.

Sum-of-Years Digits Schedule is:
SOYD in N = [(Remain. useful life at begin. of yr.)/[(N/2)(N +1)]] (P − S)
1st Year: SOYD = (6/21) ($1 mil − $75,000) = $264,286
2nd Year: = (5/21) ($1 mil − $75,000) = $220,238
3rd Year: = (4/21) ($1 mil − $75,000) = $176,190
4th Year: = (3/21) ($1 mil − $75,000) = $132,143
5rd Year: = (2/21) ($1 mil − $75,000) = $ 88,095
6th Year: = (1/21) ($1 mil − $75,000) = $ 44,048

Question: Which method is preferred?
Answer: It depends, on the MARR%, i% used by the firm (individual)

As an example:

If i% is	PW of DDB is	PW of SOYD is	Preferred is
0%	$925,000	$925,000	Equal, same
2%	$881,211	$877,801	DDB
10%	$738,331	$724,468	DDB
25%	$561,631	$537,130	DDB

Thus, if MARR% is > 0%, DDB is best. One can also see this by inspection of the depreciation schedules above.

11-3

DDB Depreciation

Year		DDB Depreciation
1	(2/5) ($16,000 − $0)	= $6,400
2	(2/5) ($16,000 − $6,400)	= $3,840
3	(2/5) ($16,000 − $10,240)	= $2,304
4	(2/5) ($16,000 − $13,926)	= $830
Sum		$14,756

Converting to Straight Line Depreciation

If Switch for Year	Beginning of Year Book Value	Remaining Life	SL = (Book – Salvage)/Remaining Life	Decision
2	$9,600	4 yr	$2,400	Do not switch
3	$5,760	3 yr	$1,920	Do not switch
4	$3,456	2 yr	$1,728	Switch to SL
5	$2,074	1 yr	$2,074	

Resulting Depreciation Schedule:

Year	DDB with Conversion to Straight Line
1	$6,400
2	$3,840
3	$2,304
4	$1,728
5	$1,728
Sum	$16,000

11-4

P = $12,000	S = $600	N = 4 years

(a) **Straight Line Depreciation**
SL depreciation in each year = (P – S)/N = ($12,000 − $600)/4
= $2,850

(b) **Sum-of-Years Digits Depreciation**
SOYD in yr. N = [(Remain. useful life at begin. of yr)/[(N/2)(N +1)]] (P – S)
1st Year: SOYD = (4/10) ($12,000 − $600) = $4,560
2nd Year: = (3/10) ($12,000 − $600) = $3,420
3rd Year: = (2/10) ($12,000 − $600) = $2,280
4th Year: = (1/10) ($12,000 − $600) = $1,140
Sum = $11,400

(c) **Double Declining Balance Depreciation**
DDB in any year = 2/N (Book Value)
1st Year: DDB = (2/4) ($12,000 − $0) = $6,000
2nd Year: = (2/4) ($12,000 − $6,000) = $3,000
3rd Year: = (2/4) ($12,000 − $9,000) = $1,500
4th Year: = (2/4) ($12,000 − $10,500) = $750
Sum = $11,250

(d) The special handling devices fall into the 3-year MACRS class life from Table 11-2. The percentages from Table 11-3 are multiplied by the initial cost of $12,000 and the asset is depreciated to a book value of 0. In year 4 the $600 salvage value is recaptured depreciation. If the MACRS rule of ½ year in year of disposal is applied, then the last year's depreciation is $444.60; and the recaptured depreciation equals the salvage value minus the final book value of $600 − $444.60 = $155.40.

Year	MACRS %	Depreciation
1	33.33%	$3,999.60
2	44.45%	$5,334.00
3	14.81%	$1,777.20
4	7.41%	$889.20

11-5

The computations for the first three methods (SL, DB, and SOYD) are similar to Problem 11-4.

(d) **Accelerated Cost Recovery System (MACRS)**
Read the appropriate percentages from the 7-year class personal property table.

Year	Percentage	Year	Percentage
1	14.29	5	8.93
2	24.49	6	8.92
3	17.49	7	8.93
4	12.49	8	4.46

Computed MACRS depreciation:

Year		MACRS	Year		MACRS
1	14.29%($50,000)	= $7,145	5	8.93% ($50,000)	= $4,465
2	24.49% ($50,000)	= $12,245	6	8.92% ($50,000)	= $4,460
3	17.49% ($50,000)	= $8,745	7	8.93% ($50,000)	= $4,465
4	12.49% ($50,000)	= $6,245	8	4.46% ($50,000)	= $2,230

Sum = $50,000

Summary of Methods

Year	SL	DDB	SOYD	MACRS
1	$5,000	$10,000	$9,091	$7,145
2	$5,000	$8,000	$8,182	$12,245
3	$5,000	$6,400	$7,273	$8,745
4	$5,000	$5,120	$6,364	$6,245
5	$5,000	$4,096	$5,455	$4,465
6	$5,000	$3,277	$4,545	$4,460
7	$5,000	$2,621	$3,636	$4,465
8	$5,000	$2,097	$2,727	$2,230
9	$5,000	$1,678	$1,818	$0
10	$5,000	$1,342	$909	$0

11-6

(a)

Year	SL	SOYD		DDB
1	$15,200	$25,333	(2/5) ($76,000 − $0)	= $30,400
2	$15,200	$20,267	(2/5) ($76,000 − $30,400)	= $18,240
3	$15,200	$15,200	(2/5) ($76,000 − $48,640)	= $10,944
4	$15,200	$10,133	(2/5) ($76,000 − $59,584)	= $6,566
5	$15,200	$5,067	(2/5) ($76,000 − $66,150)	= $3,940
Sum	$76,000	$76,000		$70,090

(b) By looking at the data in Part (a), some students may jump to the conclusion that one should switch from DDB to Straight Line depreciation at the beginning of Year 3. This mistaken view is based on the fact that in the table above the Straight Line depreciation for Year 3 is $15,2000, while the DDB depreciation is only $10,944. This is not a correct analysis of the situation.

This may be illustrated by computing the Straight Line depreciation for Year 3, if DDB depreciation had been used in the prior years.

With DDB depreciation for the first two years, the book value at the beginning of Year 3 = $76,000 − $30,400 − $18,240 = $27,360.

SL depreciation for subsequent years = ($27,360 − $0)/3 = $9,120.
Thus, the choice for Year 3 is to use DDB = $10,944 or SL = $9,120.
One would naturally choose to continue with DDB depreciation.

For subsequent years:

If Switch for Year	Beginning of Yr Book Value	Remaining Life	SL = (Book – Salvage)/Remaining Life
4	$16,416	2 yrs	$8,208
5	$9,850	1 yr	$9,850

When SL is compared to DDB in Part (a), it is apparent that the switch should take place at the beginning of Year 4. The resulting depreciation schedule is:

Year	DDB with Conversion to Straight Line
1	$30,400
2	$18,240
3	$10,944
4	$8,208
5	$8,208
Sum	$76,000

11-7

(a) Straight Line
SL depreciation in any year = ($45,000 − $0)/5 = $9,000

(b) SOYD
Sum = (n/2) (n +1) = (5/2) (5) = 15
Depreciation in Year 1 = (5/15) ($45,000 − $0) = $15,000
Gradient = (1/15) ($45,000 − $0) = −$3,000

(c) DDB

Year		DDB
1	(2/5) ($45,000 − $0)	= $18,000
2	(2/5) ($45,000 − $18,000)	= $10,800
3	(2/5) ($45,000 − $28,800)	= $6,480
4	(2/5) ($45,000 − $35,280)	= $3,888
5	(2/5) ($45,000 − $39,168)	= $2,333

(d) MACRS
Depreciation Percentages: 20%, 32%, 19.20%, 11.52%, 11.52%, 5.76%
Summary of Depreciation Schedules

Year	**SL**	**DDB**	**SOYD**	**MACRS**
1	$9,000	$18,000	$15,000	$9,000
2	$9,000	$10,800	$12,000	$14,400
3	$9,000	$6,480	$9,000	$8,640
4	$9,000	$3,888	$6,000	$5,184
5	$9,000	$2,333	$3,000	$5,184
6				$2,592
Sum	$45,000	$41,501	$45,000	$45,000

11-8

Year	SL	SOYD	DDB	UOP*	MACRS	Year
1	$1,060	$1,767	$2,600	$707	$1,300	1
2	$1,060	$1,413	$1,560	$1,178	$2,080	2
3	$1,060	$1,060	$936		$1,248	3
4	$1,060	$707	$204		$749	4
5	$1,060	$353	$0		$749	5
6					$374	6
Sum	$5,300	$5,300	$5,300		$6,500	

*Total Lifetime Production = 225,000 tons
UOP (year 1) = (3000/22,500)($6,500 – $1,200) = 706.7

11-9

MACRS Depreciation

Year		MACRS
1	20.00%($1.5 × 10^6)	= $300,000
2	32.00%($1.5 × 10^6)	= $480,000
3	19.20%($1.5 × 10^6)	= $288,000
4	11.52%($1.5 × 10^6)	= $172,800
5	11.52%($1.5 × 10^6)	= $172,800
6	5.76%($1.5 × 10^6)	= $86,400
Sum		= $1,500,000

11-10

From the description in Table 11-2 this is a MACRS 7-Year Property.

Useful Life	MACRS Depreciation for Year *t*	Sum of Depreciation	Book Value at the End of Year *t*
1	$10,000 (0.1429) = $1,429	$1,429	$10,000 − $1,429 = $8,571
2	$10,000 (0.2449) = $2,449	$3,878	$10,000 − $3,878 = $6,122
3	$10,000 (0.1749) = $1,749	$5,627	$10,000 − $5,627 = $4,373
4	$10,000 (0.1249) = $1,249	$6,876	$10,000 − $6,876 = $3,122
5	$10,000 (0.0893) = $893	$7,769	$10,000 − $7,769 = $2,231
6	$10,000 (0.0892) = $892	$8,661	$10,000 − $8,661 = $1,339
7	$10,000 (0.0893) = $893	$9,554	$10,000 − $9,554 = $446
8	$10,000 (0.0446) = $446	$10,000	$10,000 − $10,000 = $0
9	$0	$10,000	$10,000 − $10,000 = $0
10	$0	$10,000	$10,000 − $10,000 = $0

11-11

ADR Class Life = 4 year, thus from Table 11-2 it is a MACRS 3-Year Property.

Useful Life	MACRS Depreciation for Year t	Sum of Depreciation	Book Value at the End of Year t
(t)	(d)	∑ d	BV = B − ∑ d
1	$75,000 (0.3333) = $24,997.50	$24,997.50	$75,000 − $24,997.50 = $50,002.50
2	$75,000 (0.4445) = $33,337.50	$58,335	$75,000 − $58,335 = $16,665
3	$75,000 (0.1481) = $11,107.50	$69,442.50	$75,000 − $69,442.50 = $5,557.50
4	$75,000 (0.0741) = $5,557.50	$75,000	$75,000 − $75,000 = $0

11-12

Year	Possible UOP	SL	SOYD	150% DB	DDB	MACRS
1	$35	$27	$45	$43.50	$58.00	$29.00
2	$20	$27	$36	$30.45	$34.80	$46.40
3	$30	$27	$27	$21.32	$20.88	$27.84
4	$30	$27	$18	$14.92	$12.53	$16.70
5	$20	$27	$9	$10.44	$7.52	$16.70
6						$8.36
	$132	$135	$135	$120.63	$133.73	$145.00
Ans.	B		A	E	D	C

Based on Cost = $145 and Salvage Value = $10.

11-13

The depreciation schedules are:
A: Sum of the Years Digits
B: MACRS
C: Double Declining Balance
D: Straight line
E: Units of Production

11-14

The depreciation schedules are:
A: Sum-of-Years digits
B: 150% Declining Balance
C: MACRS
D: SL
It is very helpful, as a first step, to compute the sum of the depreciation schedule.

11-15

(a) **SOYD Depreciation**
N = 8
SUM = (N/2) (N +1) = 36
1st Year SOYD Depreciation = (8/36) ($600,000 − $60,000)
= $120,000
Subsequent years are a declining gradient:
G = (1/36) ($600,000 − $60,000)
= $15,000

Year	SOYD Depreciation
1	$120,000
2	$105,000
3	$90,000
4	$75,000
5	$60,000
6	$45,000
7	$30,000
8	$15,000
Sum	$540,000

(b) **Unit of Production (UOP) Depreciation**
Depreciation/hour = $540,000/21,600 hours = $25/hr

Year	Utilization hrs/yr	UOP Depreciation
1	6,000	$150,000
2	4,000	$100,000
3	4,000	$100,000
4	1,600	$40,000
5	800	$20,000
6	800	$20,000
7	2,200	$55,000
8	2,200	$55,000
Sum		$540,000

11-16

	A	B	C	D	E	F	G
1	100,000	First cost					
2	20,000	Salvage	(not used in MACRS calculations)				
3	5	Life					
4	200%	Factor					
5	10%	interest rate					
6							
7	Period	Depreciation					
8	1	$20,000	=VDB($A$1,0,$A$3,MAX(0,A8-1.5),MIN($A$3,A8-0.5),$A$4)				
9	2	$32,000	or (cost, salvage, life, max(0, *t*-1.5), min (life, *t*-.5), factor)				
10	3	$19,200					
11	4	$11,520					
12	5	$11,520					
13	6	$5,760					
14		$100,000	= Sum				
15		$77,326	=NPV(A5,B8:B13)				
16							
17							

11-17

	A	B	C	D	E	F	G	H
1	100,000	First cost						
2	10,000	Salvage	(not used in MACRS calculations)					
3	6	Life						
4	200%	Factor						
5	7	Class Life	(MACRS class life not expected life)					
6								
7		Straight line		Sum-of-years digits		MACRS		
8	Period	Depr	BookValue	Depr	BookValue	Depr	BookValue	
9	1	$15,000	85,000	$25,714	74,286	$14,286	85,714	
10	2	$15,000	70,000	$21,429	52,857	$24,490	61,224	
11	3	$15,000	55,000	$17,143	35,714	$17,493	43,732	
12	4	$15,000	40,000	$12,857	22,857	$12,495	31,237	
13	5	$15,000	25,000	$8,571	14,286	$8,925	22,312	
14	6	$15,000	10,000	$4,286	10,000	$8,925	13,387	
15	7					$8,925	4,462	
16	8		=SYD($A$1,$A$2,$A$3,A14)			$4,462	0	
17								
18		=SLN($A$1,$A$2,$A$3)						
19								
20								
21		=VDB($A$1,0,$A$5,MAX(0,A14-1.5),MIN($A$5,A14-0.5),$A$4)						
22		or (cost, salvage, life, max(0, *t*-1.5), min (life, *t*-.5), factor)						
23								

11-18

	A	B	C	D	E	F	G
1	1,750,000	First cost					
2	200,000	Salvage	(not used in MACRS calculations)				
3	7	Life	(MACRS class life not expected life)				
4	200%	Factor					
5							
6	Period	Depreciation					
7	1	$250,000	=VDB($A$1,0,$A$3,MAX(0,A7-1.5),MIN($A$3,A7-0.5),$A$4)				
8	2	$428,571	or (cost, salvage, life, max(0, *t*-1.5), min (life, *t*-.5), factor)				
9	3	$306,122					
10	4	$218,659					
11	5	$156,185					
12	6	$156,185					
13	7	$156,185					
14	8	$78,092					
15		$1,750,000	= Sum				
16							

11-19

Year	Unit of Product Depreciation	Sum of Depreciation	Book Value
0			$110,000
1	$6,500	$6,500	103,500
2	7,500	14,000	96,000
3	8,500	22,500	87,500
4	9,500	32,000	78,000
5	10,500	42,500	67,500
6	11,500	54,000	56,000
7	12,500	66,500	43,500
8	13,500	80,000	30,000
9	14,500	94,500	15,500
10	15,500	110,000	0

11-20

Production Total = 1,500,000					
Cost Basis = $65,000 – $5000 = $60,000					
Unit Depreciation Rate = $60,000/1.5 million = $0.04					
Year	Production	Sum of Production	Depreciation	Sum of Depreciation	Book Value
0					$65,000
1	140,000	140,000	$5,600	$5,600	59,400
2	140,000	280,000	5,600	$11,200	53,800
3	400,000	680,000	16,000	$27,200	37,800
4	400,000	1,080,000	16,000	$43,200	21,800
5	140,000	1,220,000	5,600	$48,800	16,200
6	140,000	1,360,000	5,600	$54,400	10,600
7	140,000	1,500,000	5,600	$60,000	5,000
Depreciation = Production * Unit Depreciation Rate					
Book Value = Initial Cost – Sum of Depreciation					
Note that the machine is depreciated down to its salvage value.					

11-21

Since the building is a leasehold improvement, which reverts to the landowner at the end of the lease, it may be depreciated over the period of the lease. Below MACRS is based on straight line depreciation using the mid-month convention and 15 years.

Recovery Year		MACRS
1	(11.5 mo/12 mo) ($250,000/15 yr)	= $15,972
2−15	($250,000/15 yr)	= $16,666
16	(0.5 mo/12 mo) ($250,000/15 yr)	= $704*

* Calculation gives $694 but increase to $704 for a total depreciation of $250,000.

Year	**MACRS Depreciation**	**PW at Yr 0 at 10%**	**SOYD Depreciation**	**PW at Yr 0 at 10%**
1	$15,972	$14,520	$31,250	$28,409
2	$16,666	$13,773	$29,167	$24,104
3	$16,666	$12,521	$27,083	$20,347
4	$16,666	$11,383	$25,000	$17,075
5	$16,666	$10,348	$22,917	$14,229
6	$16,666	$9,408	$20,833	$11,760
7	$16,666	$8,553	$18,750	$9,622
8	$16,666	$7,775	$16,667	$7,775
9	$16,666	$7,068	$14,583	$6,185
10	$16,666	$6,425	$12,500	$4,819
11	$16,666	$5,841	$10,417	$3,651
12	$16,666	$5,310	$8,333	$2,655
13	$16,666	$4,828	$6,250	$1,811
14	$16,666	$4,388	$4,167	$1,097
15	$16,666	$3,990	$2,083	$499
16	$704	$153		
Sum	$250,000	$126,284	$250,000	$154,038

To maximize PW, choose SOYD depreciation.

11-22

(a) Straight Line Method

Year	SL Depreciation	PW of Depreciation
1	($100,000 − $20,000)/5 = $16,000	$14,546
2	$16,000	$13,222
3	$16,000	$12,021
4	$16,000	$10,928
5	$16,000	$9,934
Sum	$80,000	$60,651

(b) Double Declining Balance Method

Year	Depreciation	DDB Depreciation	PW of Depreciation
1	(2/5) ($100,000 − $0)	= $40,000	$36,364
2	(2/5) ($100,000 − $40,000)	= $24,000	$19,834
3	(2/5) ($100,000 − $64,000)	= $14,400	$10,819
4	(2/5) ($100,000 − $78,400)	= $1,600*	$1,093
5	(2/5) ($100,000 − $87,040)	= $0	$0
Sum		= $80,000	$68,110

* DDB depreciation must stop when it reaches salvage value.

(c) MACRS Method

Year	Depreciation	MACRS Depreciation	PW of Depreciation
1	20.00% ($100,000)	= $20,000	$18,182
2	32.00% ($100,000)	= $32,000	$26,445
3	19.20% ($100,000)	= $19,200	$14,425
4	11.52% ($100,000)	= $11,520	$7,868
5	11.52% ($100,000)	= $11,520	$7,153
6	5.76% ($100,000)	= $5,760	$3,252
Sum		= $100,000	$77,325

Conclusion: Select the depreciation method that has the largest PW of depreciation. <u>Choose MACRS.</u>

11-23

SOYD Depreciation

Sum = (5/2) (5 + 1) = 15

Year		SOYD	PW at Yr 0 at 8%
1	(5/15) ($120,000)	= $40,000	$37,036
2	(4/15) ($120,000)	= $32,000	$27,434
3	(3/15) ($120,000)	= $24,000	$19,051
4	(2/15) ($120,000)	= $16,000	$11,760
5	(1/15) ($120,000)	= $8,000	$5,445
Sum		= $120,000	=$100,726

Unit of Production Depreciation

Year		UOP	PW at Yr 0 at 8%
1	($15,000/$40,000) ($120,000)	= $45,000	$41,666
2	($11,000/$40,000) ($120,000)	= $33,000	$28,291
3	($4,000/$40,000) ($120,000)	= $12,000	$9,526
4	($6,000/$40,000) ($120,000)	= $18,000	$13,230
5	($4,000/$40,000) ($120,000)	= $12,000	$8,167
Sum		= $120,000	= $100,880

To maximize PW at Year 0, <u>choose UOP depreciation.</u>

11-24

	A	B	C	D	E
1	1,000	First cost			
2	50	Salvage	(not used in MACRS calculations)		
3	5	Life			
4	10%	interest rate			
5					
6		Straight line		Sum-of-years digits	
7	Period	Depr	BookValue	Depr	BookValue
8	1	$190	810	$317	683
9	2	$190	620	$253	430
10	3	$190	430	$190	240
11	4	$190	240	$127	113
12	5	$190	50	$63	50
13	NetPW(10%)	$720		$766	
14					
15	Thus, SOYD is better than straight line, since the PW of				
16	the depreciation deduction for SOYD is $46 higher.				
17					
18					

11-25

	A	B	C	D	E	F
1	12,000	First cost				
2	400	Salvage	(not used in MACRS calculations)			
3	5	Life				
4	200%	Factor				
5	7%	interest rate				
6						
7		Declining balance		Sum-of-years digits		
8	Period	Depr	BookValue	Depr	BookValue	
9	1	$4,800	7,200	$3,867	8,133	
10	2	$2,880	4,320	$3,093	5,040	
11	3	$1,728	2,592	$2,320	2,720	
12	4	$1,037	1,555	$1,547	1,173	
13	5	$622	933	$773	400	
14	NetPW(7%)	$9,647		$9,941		
15						
16	Without switch to straight-line, declining balance BV5 is $533 high					
17	Thus, SOYD is better than DB without switching since					
18	depreciation deduction for SOYD has a higher present worth.					
19						

11-26

(a) DDB

Year		DDB
1	(2/4) ($10,000 − $0)	= $5,000
2	(2/4) ($10,000 − $5,000)	= $2,500

2nd-year depreciation = <u>$2,500</u>

(b) SOYD
2nd-year SOYD = (3/10) ($10,000 − $0) = <u>$3,000</u>

(c) MACRS
Special tools with 4-year life are in the 3-year property class.
2nd-year MACRS = 44.45% ($10,000) = <u>$4,445</u>

11-27

DDB with conversion to Straight Line depreciation
One-half-year depreciation in first and last years

Year		DDB
1 (1/2 yr)	(1/2) (2/5) ($100 − $0)	= $20.00
2	(2/5) ($100 − $20.00)	= $32.00
3	(2/5) ($100 − $52.00)	= $19.20
4	(2/5) ($100 − $71.20)	= $11.52

Check for conversion to SL in Year 4:
SL = ($100 − $71.20)/2.5 = $11.52
Yes− Convert to SL.

Year		SL
5		$11.52
6	½(SL)	$5.76
Sum		= $100.00

These computed values are the same as Table 11-3.

11-28

Year		DDB
1 (1/2)	(1/2) (2/10) (Cost − $0)	= 10.00%
2	(2/10) (Cost − 0.10 Cost)	= 18.00%
3	(2/10) (Cost − 0.28 Cost)	= 14.40%
4	(2/10) (Cost − 0.424 Cost)	= 11.52%
5	(2/10) (Cost − 0.5392 Cost)	= 9.22%
6	(2/10) (Cost − 0.6314 Cost)	= 7.37%
7	(2/10) (Cost − 0.7051 Cost)	= 5.89%

Or, if switch to Straight Line:

Year		SL
7	(1.000 − 0.7051)/4.5	= 6.55%

Since SL depreciation > DDB depreciation, use SL for year 7 and subsequent years 8, 9, 10.

Year		SL
11	[(1.00 − 0.7051)/4.5] (1/2)	= 3.28%

Check to see if the total depreciation equals 100%.
The MACRS depreciation for 10-year personal property is:

Year	MACRS
1	10.00%
2	18.00%
3	14.40%
4	11.52%
5	9.22%
6	7.37%
7	6.55%
8	6.55%
9	6.56%
10	6.55%
11	3.28%

This is the same as Table 11-3.

11-29

(1) Use Table 11-1 to find the MACRS GDS Property Class for each asset:
  (a) MACRS 5-year Property
  (b) MACRS 7-year Property
  (c) MACRS 15-year Property

(2) Depreciation in year 3, using Table 11-3 values
  (a) $17,000(0.1920) = $3,264
  (b) $30,000(0.1749) = $5,247
  (c) $130,000(0.0855) = $1,111.50

(3) Book Value = Cost Basis − Sum of Depreciation Charges
  (a) $17,000 − $17,000 (0.200 + 0.3200 + 0.1920) = $4,896
  (b) $30,000 − $30,000 (0.1429 + 0.2449 + 0.1749) = $13,119
  (c) $130,000 − $130,000 (0.0500 + 0.0950 + 0.855) = $100,035

11-30

Year		MACRS	
1	1.177% ($600,000)	= $7,062	5½ months
2–4	2.564% ($600,000)	= $15,384	
5	1.391% ($600,000)	= $8,346	6½ months

Note that Year 1 and Year 5 do not equal $15,384. This is caused by rounding in the MACRS table.

11-31

A hotel is nonresidential real property with a 39-year useful life. Using Table 11-4, with the midmonth convention, the MACRS depreciation is:

  Calendar Year 1 (purchased in June*)
  1.391% x $850,000 = $11,823.50

  Calendar Years 2 & 3
  2.564% x $850,000 = $21,794.00

  Calendar Year 4 (sold in June*)
  1.177% x $850,000 = $10,004.50

* The mid-month convention means we assume June 15th for the property placed in service in June. Thus there are 6½ months (June 15 to December 31) of depreciation for the first calendar year. In the fourth calendar year the June sale is taken as June 15th also. This time there would be just 5½ months (January 1 to June 15) of depreciation.

11-32

Computers are in the 5-year property class. Year 1 will be double declining balances, with the computer assumed to be put in service February 15th (the mid-quarter).

For full year:
DDB = (2/5) ($70,000) = $28,000

For the mid-first quarter installation:
MACRS depreciation = (10.5 months/12 months) ($28,000)
= $24,500

11-33

(a) $EUAC_I$= (P − S) (A/P, i%, n) + Si + Annual operating cost
= ($80,000 − $20,000) (A/P, 10%, 20) + $20,000 (0.10) + $18,000
= $60,000 (0.1175) + $2,000 + $18,000
= $27,050

$EUAC_{II}$ = ($100,000 − $25,000) (A/P, 10%, 25) + $25,000 (0.10) + $20,000 − $5,000 (P/A, 10%, 10) (A/P, 10%, 25)
= $75,000 (0.1102) + $2,500 + $20,000 − $5,000 (6.145) (0.1102)
= $27,380

To minimize EUAC, select Machine II.

(b) Capitalized Cost of Machine I = PW of an infinite life = EUAC/i
In part (a), EUAC = $27,050, so:
Capitalized Cost = $27,050/0.10 = $270,500

(c) Fund to replace Machine I
Required future sum F = $80,000 − $20,000 = $60,000
Annual Deposit A = $60,000 (A/P, 10%, 20)
= $60,000 (0.0175) = $1,050

(d)

Year	Cash Flow
0	−$80,000
1−	+$28,000
20	−$18,000
20	+$20,000

$80,000 = ($28,000 − $18,000) (P/A, i%, 20) + $20,000 (P/F, i%, 20)
Solve by trial and error:

Try i = 10%
($10,000) (8.514) + $20,000 (0.1486) = $88,112 ≠ $80,000

Try i = 12%
($10,000) (7.469) + $20,000 (0.1037) = $76,764 ≠$80,000
Rate of Return = 10% + (2%) [($88,112 − $80,000)/($88,112 − $76,764)]
= <u>11.4%</u>

(e) SOYD depreciation
Book value of Machine I after two periods
Depreciation charge in any year = (Remaining useful life at beginning of yr/SOYD for total useful life)(P − S)

Sum of years digits = (n/2) (n + 1) = 20/2 (20 + 1) = 210
1st-Year depreciation = (20/210) ($80,000 − $20,000) = $5,714
2nd-Year depreciation = (19/210) ($80,000 − $20,000) = $5,429
Sum = $11,143
Book value = Cost – Depreciation to date
= $80,000 − $11,143
= <u>$68,857</u>

(f) DDB Depreciation
Book value of Machine II after three years
Depreciation charge in any year
= (2/n) (P – Depreciation charge to date)

1st-Year Depreciation = (2/25) ($100,000 − $0) = $8,000
2nd-Year Depreciation = (2/25) ($100,000 − $8,000) = $7,360
3rd-Year Depreciation = (2/25) ($100,000 − $15,360) = $6,771
Sum = $22,131
Book Value = Cost – Depreciation to date
= $100,000 − $22,131
= <u>$77,869</u>

(g) MACRS Depreciation (7-Year Class)
Machine II Third year

From Table 11-3 read: 17.49%

MACRS Depreciation = 0.1749 ($100,000) = $17,490

11-34

Students will develop different answers to distinguish between capital gains and ordinary gains. Capital gains are the amount realized at disposal above the original cost basis; ordinary gains refer to gains above book value but below cost basis, also referred to as recaptured depreciation. It is important to distinguish between these because they are taxed at different rates and thus affect the size of cash flows after taxes. Capital gains are almost never seen for depreciated business assets, but are often part of individual tax situations involving disposal of homes, investment property, stocks, jewelry, or other collectibles that appreciate in value.

11-35

This is a MACRS 3-Year Property with:
B= $20,000
t = 2
MV_2 = $14,000
BV_2 = $20,000 − $20,000 (0.3333 + (0.4445/2) = $8,889

$MV_2 - BV_2$ = $14,000 − $8,889 = $5,111, which is >0, thus there is $5,111 in deprecation recapture.

11-36

(1) Using MACRS GDS Depreciation
8-Year ADR is a 5-Year MACRS Property
B = $50,000
BV_3 = $50,000 − $50,000 (0.2000 + 0.3200 + (0.1920/2)) = $19,200

(a) MV3 − BV_3 = $15,000 − $19,200 = −$4,200
Thus there is a $4,200 loss.

(b) $MV_3 - BV_3$ = $25,000 − $19,200= $5,800
Thus there is a $5,800 depreciation to be recaptured.

(c) Because $MV_3 > B$ there is a capital gain. We divide the overall difference of $MV_3 - BV_3$ as

$MV_3 - B$ = \$60,000 − \$50,000 = \$10,000 capital gain
$B - BV_3$ = \$50,000 − \$19,200 = \$30,800 in recaptured depr.
\$40,800 is the total difference between $MV_3 - BV_3$
\$40,800 = \$60,000 − \$19,200

(2) Using Straight line Depreciation

B = \$50,000
n = 8 years
S = \$10,000
d_{SL} = (\$50,000 − \$10,000)/8 = \$5,000
BV_3 = \$50,000 − 3(\$5,000) = \$35,000

(a) $MV_3 - BV_3$ = \$15,000 − \$35,000 = −\$20,000
Thus there is a \$20,000 loss.

(b) $MV_3 - BV_3$ = \$25,000 − \$35,000 = −\$10,000
Thus there is a \$10,000 loss.

(c) Because $MV_3 > B$ there is a capital gain. We divide the overall difference of $MV_3 - BV_3$ as

$MV_3 - B$ = \$60,000 − \$50,000 = \$10,000 capital gain
$B - BV_3$ = \$50,000 − \$35,000 = \$15,000 in recaptured depr.
\$25,000 is the total difference between $MV_3 - BV_3$
\$25,000 = \$60,000 − \$35,000

11-37

At disposal we are interested in capital gains, depreciation recapture, or ordinary losses. We need to know the market value and book value at the time of disposal.

Market Value at year 5 = \$90,000
Book Value at year 5 = \$150,000 – [(150,000 – 30,000)/8] (5) = \$75,000
Depreciation Recapture (Ordinary Gain) = Market Value – Book Value
= \$90,000 – \$75,000 = \$15,000

We would include as part of the after tax cash flow at disposal (year 5) an amount that reflects the taxes owed on this ordinary gain. The size would be (\$15,000) (ordinary tax rate of the firm).

11-38

At disposal we look at Market Value at year 3 and Book Value at year 3. For all three cases the Market Value is = (0.60) (50,000) = $30,000 .

(a) SOYD = (8(8+1)/2 = 36 with salvage = $2,000
depreciation (year 1) = [(8-1+1)/36] (50,000 – 2,000) = $10,667
depreciation (year 2) = [(8-2+1)/36] (50,000 – 2,000) = $9,333
depreciation (year 3) = [(8-3+1)/36] (50,000 – 2,000) = $8,000

Book Value (year 3) = 50,000 – (10667 +9333 + 8,000) = $22,000
Gain/Loss = Market Value – Book Value = 30,000 – 22,000 = $8,000 (ordinary gain)

(b) Straight-line depreciation = (50,000 – 2,000)/8 = $6,000 each year
Book Value (year 3) = 50,000 – (3) (6,000) = $32,000
Gain/Loss = 30,000 – 32,000 = –$2,000 (ordinary loss)

(c) MACRS GDS depreciation; classified as a 7-year property.
Book Value (year3) = 50,000 – 50,000(0.1429 + 0.2449 + 0.1749/2) = $26,237.5
Gain/Loss = 30,000 – 26237.5 = $3,762.5 (ordinary gain)
Note that a half year of depreciation has been taken in the disposal year

11-39

Gross income from sand and gravel
$\$0.65/m^3$ $(45,000\ m^4)$ = $29,250
To engineering student − $2,500
Taxable Income inc. depletion = $26,750
Percentage depletion = 5% ($29,250) = $1,462.50

Therefore, <u>allowable depletion is $1,462.50.</u>

11-40

Mr. Salt's cost of depletion = $45,000 (1,000 Bbl/15,000Bbl) = $3,000
Percentage depletion = 15% ($12,000) = $1,800

The % depletion value is limited to 50% of taxable income before depletion
Taxable Income = 50% ($12,000 − $3,000) = $4,500 (so no constraint)

So we select the maximum between ($1,800 and $3,000)
Therefore, <u>allowable depletion = $3,000.00.</u>

11-41

(a) This is a cost depletion problem. We first calculate the depletion rate:
Depletion Rate = $450,000/150 = $3,000 per million board feet harvested.

Year	Depletion Allowance
1	(3000)(42) = $126,000
2	(3000)(45) = 135,000
3	(3000)(35) = 105,000
	$366,000

(b) After year 3 a total of $366,000 has been depleted. Based on the new estimate (180 million board feet), a new depletion rate must be computed.

New Cost Basis = $450,000 – 366,000 = $84,000
New board feet available for harvest = 180 – (42 + 45 + 35) = 58 million
New depletion rate = $84,000 / 58 = $1448.28 per million board feet harvested.

11-42

Allowable depreciation per hour of operation = (45,000 – 5,000) / 10,000 = $4 per hour. After 4,000 hours of operation the book value is:
Book Value = 45,000 – (4) (4,000) = 45,000 – 16,000 = $29,000

11-43

Unit depletion cost = $600 million / 10 million metric tons = $60 per metric ton
(A metric ton = 1,000 kg = 2,204.6 lb.) Annual production is 350,000 metric tons/year:

(1) Cost depletion method*
Annual depletion allowance = (60 $/ton) (350,000 tons/year) = $21 million

(2) Percentage depletion method*

(a) Gross income from sales (annual)
Nickel = (350,000 tons/yr)(0.02 tons nickel/ton ore)(2204.6 lb/ton)($3.75/lb)
= $57.871 million
Copper = (350,000 tons/yr)(0.04 tons copper/ton ore)
(2204.6 lb/ton)($0.65/lb)
= $20.062 million

(b) Percentage allowance amount (from Table 11-6)
Nickel = ($57.871 million) (22%) = $12.732 million
Copper = ($20.062 million) (15%) = $3.009 million

(c) Maximum annual depletion allowance (50% of net taxable income)
N = (350,000 ton/yr)(0.02 tons nickel/ton ore)(2204.6 lb/ton)
($3.75–0.50/lb)(0.50)
= ($50.155 million) (0.50) = $25.077 million
C = (350,000 ton/yr)(0.04 tons copper/ton ore)(2204.6 lb/ton)
($0.65–0.50)/lb)(0.50)
= ($4.630 million) (0.50) = $2.315 million

(d) Annual depletion allowance final. Here we compare the percentage allowance to the maximum depletion amount.

Nickel: Since the percentage allowance is less than 50% of taxable income, the full amount can be depleted. Thus, annual depletion is $12.732 million.
Copper: Since the percentage allowance is greater than 50% of taxable income we must use the maximum limit. Thus, annual depletion is $2.315 million.

(e) Combined Nickel and Copper depletion
The combined annual depletion = $12.732M + $2.315M = $15.047 million.

(3) Compare and decide
Answer: Since the cost depletion method provides a larger annual depletion allowance, we use that method. Annual depletion is set to $21 million.

* The cost depletion method considers only the cost basis and mineral resource quantity, not the value of the minerals mined. The percentage depletion method does incorporate the value of those resources.

Chapter 12: Income Taxes

12-1

(a) Adjusted Gross Income – Itemized or Standard Deduction – Exemptions = Taxable Income. Since the person's itemized deductions exceed their standard deduction ($5350), the itemized deductions will be used.

$70,000 − $6,000 − $3,400 = $60,600 taxable income
Federal Taxes = $4,386.50 + ($60,600 − $31,850) (0.25) = <u>$11,574</u>

(b) Adjusted Gross Income = $70,000 + $16,000
= $86,000 − $6,000 − $3,400
= $76,600 taxable income

Federal Taxes = $4,386.50 + ($76,600 − $31,850) (0.25)
= <u>$15,574</u>

Since AGI is in same tax bracket, the tax rate is still 25%, so we could have computed the tax increase directly as: $16,000(0.25) = $4,000 more taxes.

New Federal Tax = $11,574 + <u>$4,000 = </u>$15,574

12-2

	John	Mary	Joint
Adjusted Gross Income	$65,000	$75,000	$140,000
Exemptions	−$3,400	−$3,400	−$6,800
Deductions	−$7,200	−$5,350	−$10,200
Taxable Income	$54,400	$66,250	$123,000

Federal Taxes as Individuals
John = $4,386.50 + ($54,400 − $31,850) (0.25) = $10,024
Mary = $4,386.50 + ($66,250 − $31,850) (0.25) = $12,986.50
Total = $23,010.50

Federal Taxes Filed as Joint Income
John + Mary = $8,772.50 + ($123,000− $63,700)(0.25) = $23,597.50
Penalty for Joint Filing = $23,597.50− $23,010.50= $587

12-3

Without the extra work:	Taxable Income = $1,800		Tax = $180
	After-Tax Income = $1,620		
With the extra work:	Taxable Income = $3,400		Tax = $340
	After-Tax Income = $3,060		

Additional Income = $3,060 − $1,620 = $1,440
or Additional Income = $1,600 − ($1,600) (0.10) = $1,440

12-4

Adjusted Gross Income = $75,000
Exemptions = −(2) ($3,400) = −$6,800
Deductions = −$10,700
Taxable Income = $57,500
Federal Taxes = $1,565 + ($57,500 − $15,650) (0.15)
= $7,842.50

12-5

Proprietorship
Exemption = $3,400
Standard Deduction = $5,350
Taxable Income = $100,000 − $3,400 − $5,350 = $91,250
Tax = $15,698.75 + ($91,250 − $77,100) (0.28) = $19,660.75

Corporation
Jane's Tax on $40,000
Exemption = $3,400
Standard Deduction = $5,350
Taxable Income = $40,000 − $3,400 − $5,350= $31,250
Tax = $782.50 + 0.15 ($31,250– $7,825) = $4,296.25

Corporate Tax on $60,000 taxable income:
Tax = $7,500 + 0.25 ($60,000 – $50,000) = $10,000

Total Tax = $4,296.25+ $10,000 = $14,296.25

12-6

Let x = number of months that Valerie will work in the year
Adjusted Gross Income = $70,000 + $2,000x
Exemptions = −(2) ($3,400) = $6,800
Deductions = –$10,700
Taxable Income = $70,000 + $2,000x − $6,800 − $10,700
= $52,500 + $2,000x
Tax = $8,772.50 + (0.25) (Taxable income above $63,700)

Breakeven point is:
$11,500 = $8,772.50 + (0.25) ($52,500 + $2,000x − $63,700)
= $8,772.50 + (0.25) (–$11,200 + $2,000x)
= $8,772.50 – $2,800 + $500x
$11,500 = $5,972.50 + $500x
Thus, <u>x = 11 months.</u>

12-7

Combined incremental tax rate
= Δ State tax rate + (Δ Fed tax rate) (1 − Δ State tax rate)
= 0.093 + (0.28) (1 − 0.093) = 0.3470 = <u>34.7%</u>

12-8

Taxable Income = Adjustable Gross Income − Allowable Deductions
= ($500,000 − $300,000) − $30,000
= <u>$170,000</u>

Tax Bill = 0.15 ($50,000) + 0.25 ($25,000) + 0.34 ($25,000) + 0.39 ($70,000)
− tax credits
= $49,550 − 8,000
= <u>$41,550</u>

12-9

Generally all depreciation methods allocated the cost of the equipment (less salvage value) over some assigned useful life. While the depreciation charges in any year may be different for different methods, the <u>sum</u> of the depreciation charges will be the same. This will affect the amount of taxes paid in any year, but with a stable income tax rate, <u>the total taxes paid will be the same.</u> (The difference is not the amount of the taxes, but their timing.)

12-10

Let i_a = annual effective after-tax cost of capital.

Sole Brother. is paying
((100%) /(100% − 3%)) − 1 = 0.030928 = 3.0928%
for use of the money for
45 − 5 = 40 days.

Another way to think to picture this:
You order $100 in shoes. If you pay early you will pay only $97.
If you do not pay early, then you have paid:
($100 –$97)/$97 = 1.030928 or 3.0928% more for the order to make use of the money for the remaining 40 day period.

Number of 40-day periods in 1 year = 365/40 = 9.125
$i_a = [1 + (0.030928)(1 - 0.4)]^{9.125} - 1 = 0.1827$
= 18.27%

12-11

State Tax = 9.6% ($150,000) = $14,400

Federal Taxable Income = $150,000 − $14,400 = $135,600

Federal Tax = $22,250 + 0.39 ($135,600 − $100,000) = $36,134

Total State + Federal Tax = $50,534

Combined incremental state and federal income tax rate:
0.096 + 0.39 (1 − 0.096) = 0.4486 = <u>44.86%</u>

12-12

A = $5,000 (A/P, 15%, 4) = $5,000 (0.3503) = $1,751.50

	n = 0	1	2	3	4
Loan Balance	$5,000				
Interest Payment		$750.00	$599.80	$427.02	$228.35
Principal Payment		$1,001.50	$1,151.70	$1,324.48	$1,523.15
Loan Balance		$3,998.50	$2,846.80	$1,522.32	$0
Sum of Payments		$1,751.50	$1,751.50	$1,751.50	$1,751.50
Additional "Point" Interest		$75.00	$75.00	$75.00	$75.00
BTCF	+$4,700	−$1,751.50	−$1,751.50	−$1,751.50	−$1,751.50
Tax Benefit-Interest Deduction					
Interest		$825.00	$674.80	$502.02	$303.35
Tax Saving (Interest x 0.40*)		+$330.00	+$269.92	+$200.80	+$121.34
ATCF	+$4,700	−$1,421.50	−$1,481.58	−$1,550.70	−$1,630.16

*Assume that the corporate tax rate is 40%.

Solving the After-Tax Cash Flow, the after-tax interest rate is 10.9%.

12-13

(a) Bonds plus Loan

Year	Before-Tax Cash Flow	Taxable Income	Income Taxes	After-Tax Cash Flow
0	−$75,000 +$50,000			−$25,000
1-5	+$5,000 −$5,000	$0	$0	$0
5	+$100,000 −$50,000	$25,000* capital gain	−$5,000	+$45,000

* Taxed at 20%, the capital gain rate.

After-Tax Rate of Return
$25,000 = $45,00 (P/F, i%, 5)
(P/F, i%, 5) = 0.5556, thus the Rate of Return = 12.47%

Note: The Tax Reform Act of 1986 permits interest paid on loans to finance investments to continue to be deductible, but only up to the taxpayer's investment income.

(b) Bonds but no loan

Year	Before-Tax Cash Flow	Taxable Income	Income Taxes	After-Tax Cash Flow
0	−$75,000			−$75,000
1-5	+$5,000	$5,000	−$2,500	$2,500
5	+$100,000	$25,000* capital gain	−$5,000	+$95,000

* Taxed at 20%

After-Tax Rate of Return
$75,000 = $2,500 (P/A, i%, 5) + $95,000 (P/F, i%, 5)
Try i = 7%, $2,500 (4.100) + $95,000 (0.7130) = $77,985
Try i = 8%, $2,500 (3.993) + $95,000 (0.6806) = $74,639

Using linear interpolation, Rate of Return = 7.9%

12-14

Income = $800/month
Expenses = $600/year
Net = $800/month(12 months/year) – $600 = $9,000/year

SOYD Depreciation
N = 20 SUM = (N/2) (N + 1) = 210
1st-Year Depreciation = (20/210) ($93,000 − $9,000) = $8,000
Declining Gradient = (1/210) ($93,000 − $9,000) = $400

Year	Before-Tax Cash Flow	SOYD Depr.	Taxable Income	Income Taxes at 38%	After-Tax Cash Flow
0	−$93,000				−$93,000
1	+$9,000	$8,000	$1,000	−$380	+$8,620
2	+$9,000	$7,600	$1,400	−$532	+$8,468
3	+$9,000	$7,200	$1,800	−$684	+$8,316
.	.	.	.	.	.
.	.	.	.	.	G = −$152

Using assumption (a)

Year	Before-Tax Cash Flow	SOYD Depr.	Taxable Income	Income Taxes at 38%	After-Tax Cash Flow
20	+$9,000	$400	$8,600	−$3,268	+$5,732
	+$9,000 Lot + Bldg.		$0		+$9,000

Using assumption (b): Assume building value is at least $84,000

Year	Before-Tax Cash Flow	SOYD Depr.	Taxable Income	Income Taxes at 38%	After-Tax Cash Flow
20	+$9,000	$400	$8,600	−$3,268	+$5,732
	+$100,000 Lot + Bldg.		$91,000*	−$33,320**	+$66,680

* Capital Gain = Selling Price – Cost = $100,000 − $93,000 = $7,000
* Recaptured Depreciation = Cost – Book Value = $84,000
** Capital Gain taxed at 20% → Tax = 0.2 ($7,000) = $1,400
** Recaptured depreciation taxed at 38% → Tax = 0.38 ($84,000) = $31,920
Total Tax = $33,320

After-Tax Rate of Return, based on assumption (a)
PW of Benefits – PW of Cost = 0
$8,620(P/A, i%, 20) − $152(P/G, i%, 20) + $9,000(P/F, i%, 20) − $93,000 = 0

Try i = 4½%
$8,620(13.008) − $152(104.78)+$9,000(0.4146) − $93,000 = +$6,934

Try i = 6%
$8,620(11.470) − $152(87.23)+$9,000(0.3118) − $93,000= −$4,581
i^* = 4½% + (1½%) [$6,934/($4,581 + $6,934)] = 5.4%

After-Tax Rate of Return, based on assumption (b)
PW of Benefits – PW of Cost = 0
$8,620(P/A, i%, 20) − $152(P/G, i%, 20) + $66,680(P/F, i%, 20) − $93,000 = 0

Try i = 7%
$8,620(10.594) − $152(77.509)+$66,680(0.2584) − $93,000 = +$3,769

Try i = 8%
$8,620(9.818) − $152(69.090)+$66,680(0.2145) − $93,000 = −$4,568
i^* = 7% + (1%) [$3,769/($4,568 + $3,769)] = 7.45%

12-15

SOYD Depreciation
N = 8 SUM = (N/2)(N + 1) = 36
1st-Year Depreciation = (8/36) ($120,000 − $12,000) = $24,000
Annual Decline = (1/36) ($120,000 − $12,000) = $3,000

Year	Before-Tax Cash Flow	SOYD Depr.	Taxable Income	Income Taxes at 46%	After-Tax Cash Flow
0	−$120,000				−$120,000
1	+$29,000	$24,000	$5,000	−$2,300	+$26,700
2	+$26,000	$21,000	$5,000	−$2,300	+$23,700
3	+$23,000	$18,000	$5,000	−$2,300	+$20,700
4	+$20,000	$15,000	$5,000	−$2,300	+$17,700
5	+$17,000	$12,000	$5,000	−$2,300	+$14,700
6	+$14,000	$9,000	$5,000	−$2,300	+$11,700
7	+$11,000	$6,000	$5,000	−$2,300	+$8,700
8	+$8,000	$3,000	$5,000	−$2,300	+$5,700
	+$12,000		$0	$0	+$12,000
Sum		$108,000			

Will the firm obtain a 6% after tax rate of return?
PW of Cost = PW of Benefits
$120,000 =$26,700(P/A, i%, 8)−$3,000(P/G, i%, 8)+$12,000(P/F, i%, 8)

At i = 6%
PW of Benefits = $26,700(6.210)−$3,000(19.841)+$12,000(0.6274)
= $113,813 < PW of Cost

Therefore, the firm will not obtain a 6% after-tax rate of return.
Further calculations show actual rate of return to be approximately 4.5%.

12-16

Year	Before-Tax Cash Flow	SOYD Depr.	Taxable Income	Income Taxes at 20%	After-Tax Cash Flow
0	−$50,000				−$50,000
1	+$20,000	$15,000	$5,000	−$1,000	+$19,000
2	+$17,000	$12,000	$5,000	−$1,000	+$16,000
3	+$14,000	$9,000	$5,000	−$1,000	+$13,000
4	+$11,000	$6,000	$5,000	−$1,000	+$10,000
5	+$8,000	$3,000	$5,000	−$1,000	+$7,000
	+$5,000 (salvage val.)		$0	$0	+$5,000
Sum		$45,000			

PW of Benefits – PW of Cost = 0
$19,000 (P/A, i%, 5) − $3,000 (P/G, i%, 5) + $5,000 (P/F, i%, 5) − $50,000 = 0

Try i = 15%
$19,000 (3.352) − $3,000 (5.775) + $5,000 (0.4972) − $50,000 = −$1,151

Try i = 12%
$19,000 (3.605) − $3,000 (6.397) + $5,000 (0.5674) − $50,000 = +$2,141

Using linear interpolation, find that i = 14%. (Actual 13.6%)

12-17

Year	Before-Tax Cash Flow	SL Deprec.	Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	−$20,000				−$20,000
1− 8	+$5,000	$2,500	$2,500	−$1,000	+$4,000
Sum		$20,000			

(a) Before Tax Rate of Return
$20,000 = $5,000 (P/A, i%, 8)
(P/A, i%, 8) = $20,000/$5,000 = 4
i^* = 18.6%

(b) After Tax Rate of Return
$20,000 = $4,000 (P/A, i%, 8)

(P/A, i%, 8) = $20,000/$4,000 = 5
i^* = 11.8%

(c)

Year	Before-Tax Cash Flow	SL Depr.	Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	−$20,000				−$20,000
1− 8	+$5,000	$1,000	$4,000	−$1,600	+$3,400
9− 20	$0	$1,000	−$1,000	+$400	+$400
Sum		$20,000	$20,000	−$8,000	

Note that the changed depreciable life does not change Total Depreciation, Total Taxable Income, or Total Income Taxes. It does change the timing of these items.

After-Tax Rate of Return
PW of Benefits – PW of Cost = 0
$400 (P/A, i%, 20) + $3,000 (P/A, i%, 8) − $20,000 = 0

Try i = 9%
$400 (9.129) + $3,000 (5.535) − $20,000 = +$256.60

Try i = 10%
$400 (8.514) + $3,000 (5.335) − $20,000 = −$589.40
Using linear interpolation, i^* = 9.3%.

12-18

Year	Before-Tax Cash Flow	DDB Depr.	Taxable Income	Income Taxes at 34%	After-Tax Cash Flow	NPW at 10%
0	−$1,000				−$1,000	−$1,000
1	+$500	$400	$100	−$34	+$466	$423.6
2	+$340	$240	$100	−$34	+$306	$252.9
3	+$244	$144	$100	−$34	+$210	$157.8
4	+$100	$86.4	$13.6	−$4.6	+$95.4	$65.2
5	+$100 +$125	$4.6*	$95.4	−$32.4	+$192.6	$119.6
Sum		$875				+$19.1

* Reduced to $4.60 so book value not less than salvage value.

At 10%, NPW = +$19.1
Thus the rate of return exceeds 10%. (Calculator solution is 10.94%)
The project should be undertaken.

12-19

Double Declining Balance with Conversion to Straight Line

Year		Depreciation
1	(2/10) ($100,000 − $0)	= $20,000
2	(2/10) ($100,000 − $20,000)	= $16,000
3	(2/10) ($100,000 − $36,000)	= $12,800
4	(2/10) ($100,000 − $48,800)	= $10,240
5	(2/10) ($100,000 − $59,040)	= $8,192

There is no switch to straight line in the first five years.

Year	BTCF	DDB/SL Depr.	Taxable Income	Income Taxes 34%	ATCF
0	−$100,000				−$100,000
1	$30,000	$20,000	$10,000	−$3,400	$26,600
2	$30,000	$16,000	$14,000	−$4,760	$25,240
3	$30,000	$12,800	$17,200	−$5,848	$24,152
4	$30,000	$10,240	$19,760	−$6,718	$23,282
5	$30,000	$8,192	$21,808	−$7,415	$22,585
	$35,000 (SL)		$2,232*	−$759	$34,241

* Depreciation Recapture = $35,000 − ($100,000 − $67,232) = $2,232
After-Tax Rate of Return = 14.9%

12-20

SOYD Depreciation
Sum = (n/2) (n + 1) = (5/2) 6 = 15
Year 1 Depreciation = (5/15) ($120,000 − $0) = $40,000
Gradient = (−1/15) ($120,000 − $0) = −$8,000

Year	BTCF	SOYD Depr.	Taxable Income	Income Taxes 34%	ATCF
0	−$120,000				−$120,000
1	$32,000	$40,000	−$8,000	+$2,720	+$34,720
2	$32,000	$32,000	$0	$0	+$32,000
3	$32,000	$24,000	$8,000	−$2,720	+$29,280
4	$32,000	$16,000	$16,000	−$5,440	+$26,560
5	$32,000	$8,000	$24,000	−$8,160	+$50,240
	$40,000		$40,000	−$13,600	
Sum		$120,000			

After-Tax Rate of Return: Try i = 12%

NPW = \$34,720 (P/A, 12%, 4) − \$2,720 (P/G, 12%, 4)
+ \$50,240 (P/F, 12%, 5) − \$120,000
= \$105,445 − \$11,225 + \$28,506 −\$120,000
= +\$2,726 (Calculator solution: ROR = 12.88%)

Therefore, investment was satisfactory.

12-21

Year	BTCF	DDB Depr.	Taxable Income	Income Taxes at 46%	ATCF
0	−\$100,000				−\$100,000
1	\$30,000	\$50,000	−\$20,000	+\$9,200	\$39,200
2	\$30,000	\$25,000	\$5,000	−\$2,300	\$27,700
3	\$35,000	\$12,500	\$22,500	−\$10,350	\$24,650
4	\$40,000	\$6,250	\$33,750	−\$15,525	\$24,475
5	\$10,000	\$0	\$10,000	−\$4,600	\$5,400
6	\$10,000 \$6,250 (S)	\$0	\$10,000 \$0*	−\$4,600	\$11,650
Sum		\$93,750			

* Sold for Book Value.

After-Tax Rate of Return = <u>11.6%</u>

12-22

\$25,240*
Loan Payment

Year	BTCF	DDB Depr.	Principal	Interest	Taxable Income	Income Taxes at 46%	ATCF
0	−\$20,000						−\$20,000
1	\$30,000	\$50,000	\$17,240	\$8,000	−\$28,000	+\$12,880	\$17,640
2	\$30,000	\$25,000	\$18,964	\$6,276	−\$1,276	+\$587	\$5,347
3	\$35,000	\$12,500	\$20,860	\$4,380	\$18,120	−\$8,335	\$1,425
4	\$40,000	\$6,250	\$22,936†	\$2,294	\$31,456	−\$14,470	\$300
5	\$10,000	\$0			\$10,000	−\$4,600	\$5,400
6	\$10,000 \$6,250 (S)	\$0			\$10,000 \$0	−\$4,600	\$11,650
Sum		\$93,750	\$80,000		\$0		

* Loan payment = $80,000 (A/P, 10%, 4) = $25,240
† $10 adjustment. If the loan payment had been exactly computed, it is $25,237.66
** Taxable Income = BTCF – DDB Depreciation – Interest Payment

(a) After-Tax Rate of Return = 34.3%

(b) The purchase of the special tools for $20,000 cash plus an $80,000 loan represents a leveraged situation.

Under the tax laws all the interest paid is deductible when computing taxable income, so the after-tax cost of the loan is not 10%, but 5.4%. The resulting rate of return on the $20,000 cash is therefore much higher in this situation.

Note, however, that the investment now is not just $20,000, but really $20,000 plus the obligation to repay the $80,000 loan.

12-23

Year	BTCF	SOYD Depr.	Taxable Income	34% Income Tax	ATCF	PW at 15%
0	−$108,000 −$25,000				−$133,000	−$133,000
1	$24,000	$24,000	$0	$0	$24,000	$20,870
2	$24,000	$21,000	$3,000	−$1,020	$22,980	$17,376
3	$24,000	$18,000	$6,000	−$2,040	$21,960	$14,439
4	$24,000	$15,000	$9,000	−$3,060	$20,940	$11,973
5	$24,000	$12,000	$12,000	−$4,080	$19,920	$9,904
6	$24,000	$9,000	$15,000	−$5,100	$18,900	$8,171
7	$24,000	$6,000	$18,000	−$6,120	$17,880	$6,722
8	$24,000 $25,000	$3,000	$21,000 $25,000	−$15,640	$33,360	$10,905
Sum		$108,000				−$32,640

SOYD Depr.
Sum = (n/2) (n+1) = (8/2) (9) = 36
1st Year Depreciation = (8/36) ($108,000 − $0) = $24,000
Gradient = (−1/36) ($108,000 − $0) = −$3,000

<u>NPW at 15% is negative. Therefore the project should not be undertaken.</u>
(Calculator solution: i = 7.14%)

12-24

Depreciation Schedule

Year		DDB	If we convert to SL	Convert to SL?
1	(2/6) ($12,000 − $0)	= $4,000		
2	(2/6) ($12,000 − $4,000)	= $2,667		
3	(2/6) ($12,000 − $6,667)	= $1,778		
4	(2/6) ($12,000 − $8,445)	= $1,185	($12,000 − $8,445 − $700)/3 = $952	Do not convert.
5	(2/6) ($12,000 − $9,630)	= $790	($12,000 − $9,630 − $700)/2 = $835	Do convert.
6			$835	

Year	BTCF	DDB w/ conv. to SL	Taxable Income	Income Taxes at 34%	ATCF
0	−$12,000				−$12,000
1	$1,727	$4,000	−$2,273	+$773	$2,500
2	$2,414	$2,667	−$253	+$86	$2,500
3	$2,872	$1,778	$1,094	−$372	$2,500
4	$3,177	$1,185	$1,992	−$677	$2,500
5	$3,358	$835	$2,523	−$858	$2,500
6	$1,997 $1,000	$835	$1,162 $300	−$395 −$102	$2,500

Annual Cash Flow Analysis:
EUAC = $12,000 (A/P, 9%, 6) = $12,000 (0.2229) = $2,675
EUAB = $2,500
<u>Since EUAC > EUAB, the investment is not desirable.</u>

12-25

(a) Payback = $500,000/(12,000,000 x ($0.05 − $0.03)) = 2.08 years

(b) After-Tax Payback:

Year	BTCF	SL Depr.	Taxable Income	Income Taxes at 40%	ATCF
0	−$500,000				−$500,000
1−5	$240,000	$100,000	$140,000	−$56,000	$184,000

After-Tax Payback = $500,000/$184,000 = 2.72 years
After-Tax Rate of Return: $500,000 = $184,000 (P/A, i%, 5)
(P/A, i%, 5)= $500,000/$184,000 = 2.7174
Rate of Return = <u>24.5%</u>

12-26

SOYD Depreciation: (n/2) (n + 1) = (7/2) (8) = 28
1^{st} Year Depreciation = (7/28) ($14,000 − $0) = $3,500
Gradient = −(1/28) ($14,000 − $0) = −$500

Year	BTCF	SOYD Depr.	Taxable Income	Income Taxes at 47%	ATCF
0	−$14,000				−$14,000
1	+$3,600	$3,500	$100	−$47	$3,553
2	+$3,600	$3,000	$600	−$282	$3,318
3	+$3,600	$2,500	$1,100	−$517	$3,083
4	+$3,600	$2,000	$1,600	−$752	$2,848
5	+$3,600	$1,500	$2,100	−$987	$2,613
6	+$3,600	$1,000	$2,600	−$1,222	$2,378
7	+$3,600	$500	$3,100	−$1,457	$2,143
		= $14,000		Δ = $235	Δ = −$235

Solve for rate of return:
NPW = PW of Benefits − PW of Costs = 0
$3,553 (P/A, i%, 7) − $235 (P/G, i%, 7) − $14,000 = $0

Try i = 10%, NPW = $3,553 (4.868) − $235 (12.763) − $14,000 = +$296.7
Try i = 12%, NPW = $3,553 (4.564) − $235 (11.644) − $14,000 = −$520.4

After-Tax Rate of Return = 10% + (2%) ($296.7/($296.7 − $520.4)
= <u>10.73%</u>

12-27

GIVEN: First Cost = $18,600
Annual Cost = $16,000
Salvage Value = $3,600
Depreciation = S/L with n = 10, S = $3,600
Savings/bag = $0.030
Cartons/year = 200,000
Savings bag/carton = 105.5 bags – 102 bags = 3.5
Annual Savings = ($0.03/bag)(3.5 bag/carton)(200,000 cartons) = $21,000
Annual Benefit = $21,000 – $16,000 = $5,000

After-Tax Cash Flows Table

Year	Before-Tax Cash Flow	SL Depr.	Taxable Income	Income Taxes	After-Tax Cash Flow
0	−$18,600			+$1,860*	−$16,740
1−10	$5,000	$1,500**	$3,500	−$1,750	$3,250
10	$3,600		$0***	$0	$3,600

* SL Depreciation = ($18,000 − $3,600)/10 = $1,500/year
** 10% ITC on $18,600 = 0.10 ($18,600) = $1,860
*** Depreciation Recapture = $3,600 − [$18,600 − 10 ($1,500)] = $0

(a) PW = −$16,740 + $3,250 (P/A, 20%, 10) + $3,600 (P/F, 20%, 10)
= <u>−$2,535</u>

(b) Set PW = 0 at i^* and solve for i^*:
$0 = −$16,740 + $3,250 (P/A, i^*, 10) + $3,600 (P/F, i^*, 10)
by trial-and-error method, <u>i^* = 15.7% per year.</u>

(d) After tax payback period = $16,740/$3,250 = 5.2 years

12-28

For this problem the AT PW at 10% is −11,028, thus the project would not be acceptable.

Year	BTCF	MACRS Depr*	Taxable Income	Income Taxes**	ATCF	AT-PW
0	−82,000	(bld)				
0	−30,000	(land)			−112,000	−112,000
1	9,000	2,018	6,982	−1,885	7,115	6,468
2	9,000	2,012	6,988	−1,887	7,113	5,879
3	9,000	2,012	6,988	−1,887	7,113	5,344
4	9,000	2,012	6,988	−1,887	7,113	4,858
5	9,000	2,012	6,988	−1,887	7,113	4,417
5	125,000		23,066***	5,813***	119187	74,006
					TOTAL =	−11,028
*	MACRS Deprec (39 years)		Year	Depreciation		
			1	2.461%(82,000) = 2,018		
			2–5	2.564%(82,000) = 2,102		
**	27% on ordinary income, up to (114,650−63,900)= 50,750 more income					
**	27% on depreciation recapture and losses, 20% on capital gains					
***	MV at year 5 = 125,000 = 95,000(building) + 30,000(land)					
Capital Gain = 95,000 − 82,000 = 13,000						
Tax @ 20% = (13,000)(.20) = 2600						
Depr. Recapture = MV − BV = 82,000 − 71,934 = 10,066						
MV = 82,000 (in non-capital gain dollars)						
BV = 82,000 − (2018+(4)2012) = 71,934						
Tax @ 27% = (10,066)(.27) = 2718						
Total Taxes = 2600 + 2718 = 5318						

After tax present worth at 10% is –$11,028. A loss.

12-29

MACRS Depreciation (27½ yrs)

Year		MACRS Depreciation
1 (11½ months)	3.485% ($90,000)	= $3,136
2–4	3.636% ($90,000)	= $3,276
5 (1/2 month)	0.152% ($90,000)	= $137
Sum		= $13,089

Book value of house and lot after four years = $99,700 − $13,089 = $86,611

Year	Before-Tax Cash Flow	MACRS Depr.	Taxable Income	Income Taxes at 24%	After-Tax Cash Flow
0	−$99,700				−$99,700
1	+$5,500	$3,136	$2,364	−$567	+$4,933
2	+$6,000	$3,276	$2,724	−$654	+$5,346
3	+$6,000	$3,276	$2,724	−$654	+$5,346
4	+$6,000	$3,276	$2,724	−$654	+$5,346**
5	+$500 +$105,000	$137	$363 $18,264*	−$87 −$4,171	+$100,829**

* MV on Building = ($105,000 − $9,700) = $95,300
Capital Gain = $95,300 − $90,000 = $5,300 taxed at 20%
Book Value = $90,000 − $13,101 = $77,036
Depreciation Recapture = $90,000 − $77,036 = $12,964 taxed at 24%
Total Tax = $5,300 (0.20) + $12,964 (0.24) = $4,171
The year 4-year to 5 timing is a little confusing due to the MACRS "mid-month" convention.

***All assumed to be Year 4 because the sales receipts are closer to the end of Year 4 than to the end of Year 5.

PW of Benefits = PW of Cost
$4,933 (P/F, i%, 1) + $5,346 (P/A, i%, 3) (P/F, i%, 1)
+ $100,829 (P/F, i%, 4) = $99,700

Calculator solution: After-Tax Rate of Return = <u>5.60%</u>

12-30

Year	Before-Tax Cash Flow	5-yr class MACRS Depr.	Taxable Income	Income Taxes at 34%	After-Tax Cash Flow
0	−$50,000				−$50,000
1	$2,000	$10,000	−$8,000	+$2,720	+$4,720
2	$8,000	$16,000	−$8,000	+$2,720	+$10,720
3	$17,600	$9,600	$8,000	−$2,720	+$14,880
4	$13,760	$5,760	$8,000	−$2,720	+$11,040
5	$5,760	$5,760	$0	$0	+$5,760
6	$2,880	$2,800	$0	$0	+$2,880
Sum	$0	$50,000	$0	$0	$0

(a) The sum of the After-Tax Cash Flows equals zero, indicating that the After-Tax Rate of Return is 0%.

(b) Similarly, the Before-Tax Rate of Return equals 0%.

12-31

Year	BTCF	MACRS Depr.	Taxable Income	Income Taxes at 34%	After-Tax Cash Flow	Cumulative ATCF
0	− $100,000				−$100,000	−$100,000
1	$35,000	$20,000	$15,000	−$5,100	$29,900	−$70,100
2	$35,000	$32,000	$3,000	−$1,020	$33,980	−$36,120
3	$35,000	$19,200	$15,800	−$5,372	$29,628	−$6,492
4	$35,000	$11,520	$23,480	−$7,983	$27,017	+$20,525
5	$35,000	$11,520	$23,480	−$7,983	$27,017	+$47,542
6	$35,000	$5,760	$29,240	−$9,942	$25,058	+$72,600
Sum		$100,000				

After-Tax Payback Period = 3 years + ($6,942/($6,942 + $20,525)
= <u>3.24 years</u>

12-32

Year	Before-Tax Cash Flow	MACRS Depr.	Taxable Income	Income Taxes at 34%	After-Tax Cash Flow
0	−$400				−$400
1	$200*	$133	$67	−$23	+$177
2	$200	$178	$22	−$7	+$193
3	$200	$59	$141	−$48	+$152
4	$200	$30	$170	−$58	+$142

* For 2-year payback, annual benefits must be ½($400) = $200

(a) Before-Tax Rate of Return
$400 = $200 (P/A, i%, 4)
(P/A, i%, 4) = 2
Before-Tax Rate of Return = <u>34.9%</u>

(b) After-Tax Rate of Return
$400 = $177 (P/F, i%, 1) + $193 (P/F, i%, 2) + $152 (P/F, i%, 3)
+ $142 (P/F, i%, 4)
After-Tax Rate of Return = <u>25.2%</u>

12-33

Just need the first 4 years of the 5 year property. Early disposal means they will only get one-half year of depreciation in year 4.

Year		Depreciation
1	20.00% ($14,000)	= $2,800
2	32.00% ($14,000)	= $4,480
3	19.20% ($14,000)	= $2,688
4	11.52% ($14,000)(0.5)	= $806

Year	Before-Tax Cash Flow	MACRS Depr.	Taxable Income	Income Taxes 45%	After-Tax Cash Flow
0	−$14,000				−$14,000
1	$5,000	$2,800	$2,200	−$990	+$4,010
2	$5,000	$4,480	$520	−$234	+$4,766
3	$5,000	$2,688	$2,312	−$1,040	+$3,960
4	$5,000 $3,000	$806	$4,194 –$226*	−$1,887 $102	+$6,215

* End of Yr 4
Book Value = $14,000 − $10,774 = $3,226
Depreciation Recapture = $3,000 − $3,226 = –$226
After-Tax Rate of Return = 12.5%

12-34

Year	BTCF	Bldg. MACRS Depr.	Machinery MACRS Depr.	Taxable Income	Income Taxes at 34%	ATCF
0	−$400,000					−$400,000
1	$17,500	$1,070	$30,000	−$13,570	+$4,614	+$22,114
2	$70,000	$5,128	$48,000	$16,872	−$5,736	+$64,264
3	$70,000	$5,128	$28,800	$36,072	−$12,264	+$57,736
4	$70,000	$5,128	$17,280	$47,592	−$16,181	+$53,819
5	$70,000	$5,128	$17,280	$47,592	−$16,181	+$53,819
6	$52,500 $328,000	$4,066	$4,320	$44,114 $99,328**	−$14,999 −$33,772	+$37,501 +$294,228
Sum		$25,648	$145,680			

Year 1 BTCF arises from the asset being placed in service on Oct 1.
BTCF (year 1) = $70,000 * (3 months service/12 months) = $17,500
BTCF (year 6) = $70,000 * (9 months service/12 months) = $52,500
End of 5 years (in Year 6) Book Value = $400,000 − $25,648 − $145,680 = $228,672
** Depreciation Recapture = $328,000 − $228,672 = $99,328

* MACRS Depreciation:

Year	Building	Machinery
1	0.535%	20.00%
2	2.564%	32.00%
3	2.564%	19.20%
4	2.564%	11.52%
5	2.564%	11.52%
6	2.033	2.88%

Early disposal results in one-half of depreciation in year 6.

After-Tax Rate of Return = <u>8.40%</u>
<u>The project fails to meet the corporation's criterion.</u>

12-35

Year	BTCF	MACRS Depr.	Taxable Income	Income Taxes at 34%	ATCF
0	−$55,000				−$55,000
1	$10,000	$5,500	$4,500	−$1,530	$8,470
2	$10,000	$9,900	$100	−$34	$9,966
3	$10,000	$7,920	$2,080	−$707	$9,293
4	$10,000	$6,336	$3,664	−$1,246	$8,754
5	$10,000	$5,071	$4,929	−$1,676	$8,324
6	$10,000 $35,000	$2,027*	$7,973 $16,754**	−$2,711 −$5,696	$7,289 $29,304
Sum		$36,754			

*Assumes the small vessel is 10 year MACRS property with one-half year of depreciation in year 6 for early disposal.
** End of Year 6 Book Value = $55,000 − $36,754 = $18,246

Depreciation Recapture = $35,000 − $18,246 = $16,754
<u>After-Tax Rate of Return = 9.86%</u>

12-36

Year	BTCF	MACRS Deprec.	Taxable Income	Income Taxes at 34%	After-Tax Cash Flow	PW at 10%
0	−$1,800,000				−$180,000	−$180,000
1	$450,000	$360,000	$9,000	−$30,600	$419,400	$381,277
2	$450,000	$576,000	−$126,000	+$42,840	$492,840	$407,283
3	$450,000	$345,600	$104,400	−$35,496	$414,504	$311,417
4	$450,000	$207,360	$242,640	−$82,498	$367,502	$251,004
5	$450,000	$207,360	$242,640	−$82,498	$367,502	$228,182
6	$450,000	$103,680	$346,320	−$117,749	$332,251	$187,556
7	$450,000	$0	$450,000	−$153,000	$297,000	$152,420
8	$450,000	$0	$450,000	−$153,000	$297,000	$138,550

NPW(10%) = +$257,689
After-Tax Rate of Return = <u>14.2%</u>
<u>Therefore, the investment is satisfactory.</u>

12-37

MACRS Depreciation: Special tools- plastic products = 3-year property class

Year		MACRS Depr.
1 (1/2 yr. dep)	33.33% ($300,000)	= $99,990
2	44.45% ($300,000)	= $133,350
3	14.81% ($300,000)	= $44,430
4	7.41% ($300,000)	= $22,230
Sum		= $300,000

Yr	BTCF	MACRS Depr.	Taxable Income	Inc. Taxes at 39%	After-Tax Cash Flow	Unrec. Investment	PW at 12%
0	−$300,000				−$300,000	$300,000	−$300,000
1	$150,000	$99,990	$50,010	−$19,504	$130,496	$169,504	$116,520
2	$150,000	$133,350	$16,650	−$6,493	$143,507	$25,997	$114,404
3	$150,000	$44,430	$105,570	−$41,172	$108,828	$0	$77,464
4	$150,000	$22,230	$127,770	−$49,830	$100,170		$63,658
5	$150,000	$0	$150,000	−$58,500	$91,500		$51,917
						Sum =	+$123,963

(a) After-Tax Payback
- assuming end-of-period benefits = 3 yr
- assuming benefits throughout year = 2.24 yr

(b) This is a desirable investment (PW at 12% > 0; After-Tax ROR = 29%)

12-38

(a)

Year	Gross Income	Expense	BTCF	MACRS Depr.	Δ Taxable Income	40% Δ Tax	ATCF
0			−$10,000				−$10,000
1	$2,000	$200	$1,800	$1,429	$371	−$148	$1,652
2	$2,200	$400	$1,800	$2,449	−$649	+$260	$2,060
3	$2,400	$600	$1,800	$1,749	$51	−$20	$1,780
4	$2,600	$800	$1,800	$1,249	$551	−$220	$1,580
5	$2,800	$1,000	$1,800	$893	$907	−$363	$1,437
6	$3,000	$1,200	$1,800	$892	$908	−$363	$1,437
7	$3,200	$1,400	$1,800	$893	$907	−$363	$1,437
8	$3,400	$1,600	$1,800	$446	$1,354	−$542	$1,258
9	$3,600	$1,800	$1,800	$0	$1,800	−$720	$1,080
10	$3,800	$2,000	$1,800	$0	$1,800	−$720	$1,080

(b) Solving the ATCF for the rate of return

Try i = 8%

NPW = −\$10,000 − \$1,652 (P/F, 8%, 1) + \$2,060 (P/F, 8%, 2)
+ \$1,780 (P/F, 8%, 3) + \$1,580 (P/F, 8%, 4)
+ \$1,437 [(P/F, 8%, 5) + (P/F, 8%, 6) + (P/F, 8%, 7)]
+ \$1,258 (P/F, 8%, 8) + \$1,080 [(P/F, 8%, 9) + (P/F, 8%, 10)]
= +\$312

Interest rate too low.

Try i = 9%

NPW = −\$10,000 − \$1,652 (P/F, 9%, 1) + \$2,060 (P/F, 9%, 2)
+ \$1,780 (P/F, 9%, 3) + \$1,580 (P/F, 9%, 4)
+ \$1,437 [(P/F, 9%, 5) + (P/F, 9%, 6) + (P/F, 9%, 7)]
+ \$1,258 (P/F, 9%, 8) + \$1,080 [(P/F, 9%, 9) + (P/F, 9%, 10)]
= −\$95

Interest rate too high.

Rate of Return = 8% + (1%) [\$312/(\$312 + \$95)] = 8.8%

(c) End at year 5 so last two rows in table for part a would be:

Year	BTCF	MACRS Depr.	Δ Taxable Income	40% Δ Tax	ATCF
5	\$1,800	\$447	\$1,354	−\$541	\$1,259
5	\$7,000		\$4,323	−\$1,729	\$5,271

End of Year 5 Book Value = \$10,000 − \$7,323 = \$2,677
Recaptured Depreciation = \$7,000 − \$2,677 = \$4,323

Compute NPW at 9% for sale of gas generator at the end of Year 5.
NPW = −\$14

At 9%, the 10-year life gives NPW = −\$96 and the sale of the generator at the end of 5 years give a NPW = −\$14. The two situations are almost identical, but the sale of the generator at the end of 5 years is slightly preferred.

12-39

Year	BTCF Truck	MACRS Depr.	Taxable Income	Income Taxes at 40%	ATCF
0	−$25,000				−$25,000
1	$8,000	$8,332	−$332	+$133	$8,133
2	$8,000	$11,113	−$3,113	+$1,245	$9,245
3	$8,000	$3,702	$4,298	−$1,719	$6,281
4	$8,000 $5,000	$1,853	$6,147 $5,000*	−$2,459 −$2,000	$5,542 $3,000

* Recaptured Depreciation

Compute the NPW of the ATCF at 10%
NPW = −$25,000 + $8,133 (P/F, 10%, 1) + $9,245 (P/F, 10%, 2)
+ $6,281 (P/A, 10%, 3) + ($5,542 + $3000) (P/F, 10%, 4)
= +$587

Since NPW is positive at 10%, the after-tax rate of return exceeds 10% (the after-tax rate of return is about 11.1%). So, Granny should buy the churn.

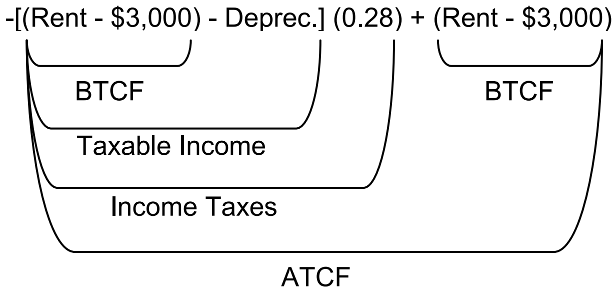
12-40

MACRS Depreciation
(a) 1st Recovery Year: 11½ months depreciation = 3.485% ($60,000) = $2,091
2−4 Recovery Year: 3.636% ($60,000) = $2,182
5th Recovery Year: 11½ months depreciation = $3.485% ($60,000) = $2,091
Total MACRS depreciation = $10,728

(b) Market Value year 5 = $80,000 − $10,000= $70,000
Cap Gain on House = $70,000 − $60,000 = <u>$10,000</u>
Tax on Cap Gain = 0.15 ($10,000) = $1,500
Recaptured Depreciation on House = $10,728
Tax on Recapture Depreciation = 0.28 ($10,728) = $3,004
Total Tax on Disposal = $1,500 + $3,004 = $4,504
ATCF at disposal = $80,000 − $4,504 = <u>$75,496</u>

(c) −$70,000 + A (P/A, 15%, 5) + $75,496 (P/F, 15%, 5) = $0
−$70,000 + A (3.352) + $75,496 (0.4972) = $0
−$70,000 + 3.352A + $37,537 = $0
<u>A = $9,685 (This would be the ATCF for uniformyears 1-5)</u>

(d)



Years 1 & 5

–(Rent − $3,000 − $2,091) (0.28) + (Rent − $3,000) = ATCF = $9,685
− 0.28(Rent) + $1,425 + Rent − $3,000 = $9,685
0.72 (Rent) = $11,260
<u>Rent = $15,638</u>

Years 2–4

–(Rent − $3,000 − $2,182) (0.28) + (Rent − $3,000) = ATCF = $9,743
− 0.28(Rent) + $1,451 + Rent − $3,000 = $9,685
0.72 (Rent) = $11,234
<u>Rent = $15,603</u>

Shown in table format:

Year	Income (Rent)	Expense	BTCF	MACRS Depr.	Taxable Income	28% Income Tax	ATCF
0			($70,000)				($70,000)
1	$15,638	$3,000	$12,638	$2,091	$10,547	$2,953	$9,685
2	$15,603	$3,000	$12,603	$2,182	$10,421	$2,918	$9,685
3	$15,603	$3,000	$12,603	$2,182	$10,421	$2,918	$9,685
4	$15,603	$3,000	$12,603	$2,182	$10,421	$2,918	$9,685
5	$15,638	$3,000	$12,638	$2,091	$10,547	$2,953	$9,685
5			$80,000	$10,728		$4,504	$75,496

12-41

Year	Before-Tax Cash Flow	MACRS Depr.*	Taxable Income	Income Tax (45%)	After-Tax Cash Flow	PW (12%)
0	−$20,000				−$20,000	−$20,000
1	$8,000	$4,000	$4,000	−$1,800	$6,200	$5,536
2	$8,000	$6,400	$1,600	−$720	$7,280	$5,804
3	$8,000 $10,000	$1,920	$6,080 $4,240**	−$2,736 −$1,044	$5,264 $8,956	$3,747 $6,375

Net Present Worth = $1,462

* Computers are in the 5-year property class.

** Book Value = $7,680

Recaptured Depreciation = $10,000 − $7,690 = $2,320

12-42

MACRS Depreciation: Pickup in 5-Year Property class

Year		MACRS Depr.
1	20% ($14,000)	= $2,800
2	32% ($14,000)	= $4,480
3	19.2% ($14,000)	= $2,688
4	11.52% ($14,000)(0.5)	= $806

(a)

Year	BTCF Truck	BTCF Loan	MACRS Depr.	Taxable Income	Income Taxes at 45%	After-Tax Cash Flow
0	−$14,000	+$10,000				−$4,000
1	$5,000	−$2,500 − $1,000	$2,800	$1,200	−$540	+$960
2	$5,000	−$2,500 − $750	$4,480	−$230	+$103	+$1,853
3	$5,000	−$2,500 − $500	$2,688	$1,812	−$815	$1,185
4	$5,000 $3,000	−$2,500 − $250	$806	$3,944 –$226*	−$1,775 $102	$3,577
Sum			$10,774			

Book Value = $14,000 − $10,774 = $3,226

* Depreciation Recapture = $3,000 − $3,226 = –$226

Computed After-Tax Rate of Return = <u>25.3%</u>

(b) This problem illustrates the leverage that a loan can produce. The cash investment is greatly reduced. Since the truck rate of return (12.5% in Problem 12-33) exceeds the loan interest rate (10%), combining the two increased the overall rate of return.

Two items worth noting:
1. The truck and the loan are independent decisions and probably should be examined separately.
2. There is increased risk when investments are leveraged.

12-43

Year	Before-Tax Cash Flow	Depr.	Δ Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	− x − $5,500*		−$3,000	+$1,200	− x − $4,300
1	+$7,000		$7,000	−$2,800	+$4,200
2	+$7,000		$7,000	−$2,800	+$4,200
…	…		…	…	…
…	…		…	…	…
9	+$7,000		$7,000	−$2,800	+$4,200
10	+$7,000		$7,000	−$2,800	+$4,200
	+x + $2,500	$0	$0		+x + $2,500

*$2,500 for initial cost to demolish + $3,000 expense to prep lot.

Where x = maximum purchase price for old building and lot.

PW of benefits – PW of cost = 0

$4,200 (P/A, i%, 10) + (+x + $2,500) (P/F, i%, 10) − x − $4,300 = 0

At the desired i = 15%:

$4,200 (5.019) + (+x + $2,500) (0.2472) − x − $4,300 = 0

$21,080 + 0.2472x + $618 − x − $4,300 = 0

x = ($21,080 + $618 − $4,300)/0.7528 = <u>$23,100</u>

12-44

Year	Before-Tax Cash Flow	Depr.	Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	−P				−P
1	+\$87,500 − 0.065P	0.0667P	+\$87,500 − 0.1317P	−\$35,000 + 0.0527 P	+\$52,500 − 0.0123 P
2	…	….	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
15	+\$87,500 − 0.065P	0.0667P	+\$87,500 − 0.1317 P	−\$35,000 + 0.0527 P	+\$52,500 − 0.0123 P

Wages = (\$14,000/operator)(5 operators) = \$70,000
Benefits = 25% of wage = (0.25) (\$70,000) = \$17,500
P = maximum expenditure for new equipment.
Property taxes and maintenance = 0.025P + 0.04P = 0.065P
Solve the after-tax cash flow for P
PW of Cost = PW of Benefits
P = (\$52,500 − 0.0123P) (P/A, 8%, 15)
= (\$52,500 − 0.0123P) (8.559)
= \$449,348 − 0.1053 P
= \$449,348/1.1053 = <u>\$406,500</u>

12-45

Let x = number of days/year that the trucks are used.
Annual Benefit of truck ownership = (\$83 − \$35)x − \$1,100= \$48x − \$1,100

Year	Before-Tax Cash Flow	Depr.	Taxable Income	Income Taxes at 40%	After-Tax Cash Flow
0	−\$13,000				−\$13,000
1	\$48x–\$1,100	\$1,429	\$48x−\$2,529	−\$24x+\$1,264	\$24x+\$164
2	…	….	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
7	\$48x−\$1,100+\$3,000	\$1,429	\$48x−\$2,529 \$0	−\$24x+\$ 1,264 \$0	\$24x+\$164 \$3,000

Set PW of Cost = PW of Benefits
\$13,000 = (\$24x + \$164) (P/A, 10%, 7) + \$3,000 (P/F, 10%, 7)
= (\$24x + \$164) (4.868) + \$3,000 (0.5132)
= \$116.8x + \$798 + \$1,540
x = (\$13,000 − \$798 − \$1,540)/116.8= <u>91.5 days</u>

Alternate Analysis
An alternate approach is to compute the after-tax cash flow of owning the truck. From this the after-tax EUAC may be calculated (= $2,189 + $17.5x).

In a separate calculation the after-tax EUAC of hiring a truck is determined (= $41.5x). By equating the EUAC for the alternatives we get:

$2,189 + $17.5 x = $41.5 x
x = 91.2 which is approximately equal to 91.5 days.

12-46

SOYD Depreciation
N = 5 SUM = (N/2) (N + 1) = (5/2) (6) = 15
1st year depreciation = (5/15) ($20,000 − $5,000)= $5,000
Annual decline = (1/15) ($20,000 − $5,000)= $1,000

Year	Before-Tax Cash Flow	Deprec.	Taxable Income	Income Taxes at 50%	After-Tax Cash Flow
0	−$20,000				−$20,000
1	+A	$5,000	A − $5,000	−0.5A + $2,500	0.5A + $2,500
2	+A	$4,000	A − $4,000	−0.5A + $2,000	0.5A + $2,000
3	+A	$3,000	A − $3,000	−0.5A + $1,500	0.5A + $1,500
4	+A	$2,000	A − $2,000	−0.5A + $1,000	0.5A + $1,000
5	+A + $5,000	$1,000	A − $1,000 $0	−0.5A + $500 + $0	0.5A + $500 +$5,000

A = Before-Tax Annual Benefit
After-Tax Cash flow computation:

$20,000 = (0.5A + $2,500) (P/A, 8%, 5) −$500 (P/G, 8%, 5) + $5,000 (P/F, 8%, 5)
= (0.5A + $2,500) (3.993) − $500 (7.372) + $5,000 (0.6806)

A = ($20,000 − $9,983 + $3,686 − $3,403)/1.9965 = $5,159
<u>Required Before-Tax Annual Benefit = $5,159</u>

12-47

Year	Before-Tax Cash Flow	SL Depr.	Taxable Income	Income Taxes at 28%	After-Tax Cash Flow
0	–$155,000				−$155,000
1− 10	+$12,000	$4,000	$8,000	−$2,240*	+$9,760
10	+x		(x − $155,000)*	− 0.20 (x − $155,000)	0.8x + $31,000
Sum		$40,000**		−0.28 ($40,000)	–$11,200

* Portion of capital gain taxed at 20%
** Depreciation recapture taxed at 28%

$ATCF_{10}$ = x − 0.20x + (0.20) ($155,000) − (0.28) ($40,000)
Year 10 Disposal = 0.80x + $19,800

Set PW = 0 and solve for x:
PW = −$155,000 + $9,760(P/A,10%,10) + (0.80x + $31,000 – $11,200)(P/F,10%,10)
= −$155,000 + $9,760 (6.145) + (0.80x + $19,800) (0.3855)
= −$155,000 + $59,975 + 0.3084x + $7,633
= –$87,392 + 0.3084x = $0
So <u>x = $283,372</u>

12-48

This problem is similar to 12-44

Year	Before-Tax Cash Flow	SOYD Depr.*	Taxable Income	Income Taxes at 50%	After-Tax Cash Flow
0	− P				− P
1	$110,000	(6/21) P	$110,000 − (6/21) P	−($55,000 − (3/21) P)	+$55,000 + (3/21) P
2	$110,000	(5/21) P	$110,000 − (5/21) P	−($55,000 − (2.5/21) P)	+$55,000 + (2.5/21) P
3	$110,000	(4/21) P	$110,000 − (4/21) P	−($55,000 − (2/21) P)	+$55,000 + (2/21) P
4	$110,000	(3/21) P	$110,000 − (3/21) P	−($55,000 − (1.5/21) P)	+$55,000 + (1.5/21) P
5	$110,000	(2/21) P	$110,000 − (2/21) P	−($55,000 − (1/21) P)	+$55,000 + (1/21) P
6	$110,000	(1/21) P	$110,000 − (1/21) P	−($55,000 − (0.5/21) P)	+$55,000 + (0.5/21) P

* Sum = (N/2) (N + 1) = (6/2) (7) = 21
Annual Benefit = ($32,000/operator)(4 operators) – $18,000 = $110,000
Benefits = 25% of wage = (0.25) ($70,000) = $17,500
Write an equation for the After-Tax Cash Flow:

P = ($55,000 + (3/21) P) (P/A, 15%, 6) − (0.5/21) (P/G, 15%, 6)
= ($55,000 + (3/21) P) (3.784) − (0.5/21) (7.937)
= $208,120 + 0.5406 P − 0.1889 P

= \$208,120/0.6483 = \$321,024

12-49

Let X = number of days car used per year. Automobiles are in the MACRS 5-year property class.

Year	BTCF	MACRS Depreciation	Taxable Income	Income Taxes at 28%	ATCF
0	−$14,500				−$14,500
1	$80X − $1,000 − $50X = $30X − $1,000	0.20 ($14,500) = $2,900	$30X − $3,900	−$8.4X + $1,092	$21.6X + $92
2	$30X − $1,500	0.32 ($14,500) = $4,640	$30X − $6,140	−$8.4X + $1,719	$21.6X + $219
3	$30X − $2,000 + $5,000*	0.192 ($14,500) = $2,784	$30X − $4,784 + $824	−$8.4X + $1,109	$21.6X + $4,109

* Salvage value of which $824 is subject to tax on recaptured depreciation.

NPW = −$14,500 + $21.6X (P/A, 12%, 3) + $92 (P/F, 12%, 1)
+ $219 (P/F, 12%, 2) + $4,109 (P/F, 12%, 3) = 0
= −$14,500 + $21.6X (2.402) + $92 (0.8929) + $219 (0.7972) + $4,109 (0.7118) = 0

X = <u>218 days</u>

12-50

NOTE: All yield benefits are in thousands of dollars.

Year	0	1	2	3	4
Purchase/Sale	− P				0.2 P
Benefits		$10	$15	$20	$20
Depreciation/ Book Value		0.20 P	0.32 P	0.192 P	0.115 P
Taxable Income		$10 − 0.20 P	$15 − 0.32 P	$20 − 0.192 P	$20 −0.115 P + 0.027P*
40% Income Taxes		−$4 + 0.08 P	−$6 + 0.128 P	−$8 + 0.077 P	−$8 + 0.046 P - 0.0108P
ATCF		$6 + 0.08 P	$9 + 0.128 P	$12 + 0.077 P	$12 + 0.2352 P
(P/F, 10%, n)		0.9091	0.8264	0.7513	0.6830
Discounted NPW	− P	$5.4546 + 0.073 P	$7.4376 + 0.106 P	$9.01256 + 0.058 P	$8.1960 + 0.161 P

*BV (end of year 4) = P – (0.2P + 0.32P + 0.192P + 0.115P) = P – 0.827P = 0.173P
Sell at end of year 4 for 0.2P, so recapture some depreciation = 0.0.027P.
Tax = 0.4*0.027P = 0.0108P

NPW = $27,625 − 0.602 P = $0
P = $30,104/0.602 = <u>$50,007 maximum purchase price.</u>

12-51

Ann Arbor Municipal Bonds

Year	Before-Tax Cash Flow	Taxable Income	Income Taxes at 20%	After-Tax Cash Flow
0	−$800			−$800
1−15	$60			$60
15	$1,000	$200	−$40	$960

Using Excel's = RATE (15,60, −800, 960) the answer is <u>8.22%.</u>

Southern Coal Corporation Bonds

Year	Before-Tax Cash Flow	Taxable Income	Income Taxes at 28%	After-Tax Cash Flow
0	−$1,000			−$1,000
1− 20	$100	$100	−$28	$72
1− 20	$1,000			$1,000

By inspection the answer is <u>7.2%.</u>

12-52

We must use incremental analysis to solve this problem. Placing the alternatives in ascending order of initial investment we get: C - A - B. Doing nothing is not an alternative.

The depreciation charges for each alternative using MACRS 3-year property

Year	Alt A		Alt B		Alt C
1	$4,666		$5,999		$3,333
2	$6,223		$8,001		$4,445
3	$2,073		$2,666		$1,481
4	$1,037		$1,334		$741

Since each alternative is fully depreciated to its salvage value, there is no cash flow associated with depreciation recapture, capital gain, or loss.

Alternative A - Alternative C

Year	Before-Tax Cash Flow	Depreciation	Taxable Income	Income Taxes 45%	After-Tax Cash Flow
0	–4,000				–4,000
1	2,500	1,333	1,167	–525	1,975
2	2,500	1,778	722	–325	2,175
3	2,500	592	1,908	–858	1,642
4	2,500	296	2,204	–992	1,508
5	2,500	–	2,500	–1,125	1,375

Incremental ROR = 36.2% which is > MARR so go with Alternative A (reject C)

Alternative B - Alternative A

Year	Before-Tax Cash Flow	Depreciation	Taxable Income	Income Taxes 45%	After-Tax Cash Flow
0	–4,000				–4,000
1	1,500	1,333	167	–75	1,425
2	1,500	1,778	–278	125	1,625
3	1,500	592	908	–408	1,092
4	1,500	296	1,204	–542	958
5	1,500	–	1,500	–675	825

Incremental ROR = 16.7 which is < MARR so go with lower cost Alternative A (reject B)

Therefore, of the three alternatives, we would select Alternative A.

12-53

This multiple alternative after-tax rate of return problem is one where neither input nor output is fixed. From the given data, compute the after-tax cash flow for each alternative. From this the after-tax rate of return may be computed.

Alternative B

Year	Before-Tax Cash Flow	Depreciation	Taxable Income	Income Taxes at 20%	After-Tax Cash Flow
0	−$25				−$25
1− 5	+$7.5	$5	$2.5	−$0.5	+$7

Rate of Return = 12.4%

Alternative C

Year	Before-Tax Cash Flow	Depreciation	Taxable Income	Income Taxes at 20%	After-Tax Cash Flow
0	−$10				−$10
1− 5	+$3	$2	$1.0	−$0.2	+$2.8

Rate of Return = 12.4%

Alternative D

Year	Before-Tax Cash Flow	Depreciation	Taxable Income	Income Taxes at 20%	After-Tax Cash Flow
0	−$5				−$5
1− 5	+$1.7	$1	$0.7	−$0.14	+$1.56

Rate of Return = 16.9%

Alternative E

Year	Before-Tax Cash Flow	Depreciation	Taxable Income	Income Taxes at 20%	After-Tax Cash Flow
0	−$15				−$15
1− 5	+$5	$3	$2.0	−$0.4	+$4.6

Rate of Return = 16.2%

Alternative F

Year	Before-Tax Cash Flow	Depreciation	Taxable Income	Income Taxes at 20%	After-Tax Cash Flow
0	−$30				−$30
1− 5	+$8.7	$6	$2.7	−$0.54	+$8.16

Rate of Return = 11.2%

Alternatives B through F each meet the 10% after-tax rate of return criterion. Therefore, the do-nothing and A alternatives may be discarded. Proceed with incremental analysis, starting with the least cost remaining alternative.

The order of least cost is: D ($5), C ($10), E ($15), B ($25), F ($30)

Alternative C– Alternative D

Year	After-Tax Cash Flow
0	−$5
1− 5	+$1.24

Incremental Rate of Return = 7.6%. Reject Alternative C.

Alternative E– Alternative D

Year	After-Tax Cash Flow
0	−$10
1− 5	+$3.04

Incremental Rate of Return = 15.8%. Reject Alternative D.

Alternative B– Alternative E

Year	After-Tax Cash Flow
0	−$10
1− 5	+$2.40

Incremental Rate of Return = 6.4%. Reject Alternative B.

Alternative F–Alternative E

Year	After-Tax Cash Flow
0	−$15
1− 5	+$3.56

Incremental Rate of Return = 6%. Reject Alternative F.

Conclusion: Choose Alternative E.

12-54

Alternative 1

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	−$10,000				−$10,000
1−10	$4,500	$1,000	$3,500	−$1,190	$3,310
11−20	$0	$0			$0

($7M company is in the 34% tax bracket)

Year	ATCF	PW(10%)	EUAB–EUAC (over 20 years)	FW(10%)
0	−$10,000	−$10,000	−$1,175	−$67,280
1−10	$3,310	+$20,338	+$2,390	+$136,832
11−20	$0	$0		$0
Sum		+$10,338	+$1,215	+$69,552

Alternative 2

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	−$20,000				−$20,000
1−10	$4,500	$2,000	$2,500	−$850	$3,650
11−20	$4,500	$0	$4,500	−$1,530	$2,970

Year	ATCF	PW(10%)	EUAB–EUAC (over 20 years)	FW(10%)
0	−$20,000	−$20,000	−$2,350	−$134,560
1−10	$3,650	+$22,427	+$3,462	+$150,876
11−20	$2,970	+$7,036		+$47,339
Sum		+$9,463	+$1,112	+$63,655

Increment 2- 1 After-Tax Cash Flow

Year	Alt. 1	Alt. 2	Alt. 2 − Alt. 1
0	−$10,000	−$20,000	−$10,000
1−10	$3,310	$3,650	+$340
11−20	$0	$2,970	+$2,970
Rate of Return	30.9%	16.9%	9.2%
B/C Ratio	$2,390/$1,175 =2.03	1.47	0.91*

(a) To maximize NPW, choose Alternative 1 with a total present worth of $10,338.
(b) To maximize (EUAB – EUAC), choose Alternative 1 with (EUAB – EUAC) = $1,215.
(c) Based on the rate of return of 9.2% from investing in Alt. 2 instead of 1, note that the increment is unacceptable. Choose Alternative 1.
(d) To maximize Net Future Worth, choose Alternative 1.

(e) Because the 2− 1 increment has a B/C ratio less than 1, reject the increment and select Alternative 1.
* B/C = ($3,462 – $2,390)/($2,350 – $1,175) = 0.91

12-55

Alternative A

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	−$3,000				−$3,000
1	$1,000	$1,000	$0	$0	$1,000
2	$1,000	$800	$200	−$68	$932
3	$1,000	$600	$400	−$136	$864
4	$1,000	$400	$600	−$204	$796
5	$1,000	$200	$800	−$272	$728

*Company is in 34% tax bracket given income between $5M–$10M

Alternative B

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	−$5,000				−$5,000
1	$1,000	$1,000	$0	$0	$1,000
2	$1,200	$1,000	$200	−$68	$1,132
3	$1,400	$1,000	$400	−$136	$1,264
4	$2,600	$1,000	$1,600	−$544	$2,056
5	$2,800	$1,000	$1,800	−$612	$2,188

Alternative B–Alternative A

Year	B- A ATCF	PW at MARR of 10%
0	−$2,000	−$2,000
1	$0	$0
2	$200	$165
3	$400	$301
4	$1,260	$861
5	$1,460	$907
Sum		$234

The NPW of B − A is greater than zero so we are making more than 10% on the increment of investment of Alternative B over A. (Actual rate is 13%). <u>Choose B.</u>

12-56

Alternative A

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	−$11,000				−$11,000
1	$3,000	$3,000	$0	$0	$3,000
2	$3,000	$3,000	$0	$0	$3,000
3	$3,000	$3,000	$0	$0	$3,000
4	$3,000	$0	$3,000	−$1,020	$1,980
5	$3,000 $2,000	$0	$3,000 $0	−$1,020	$3,980

NPW(12%) = −$278

Alternative B

Year	BTCF	SL Dep.	TI	34% Inc.Tax	ATCF
0	−$33,000				−$33,000
1	$9,000	$12,000	−$3,000	+$1,020	$10,020
2	$9,000	$9,000	$0	$0	$9,000
3	$9,000	$6,000	$3,000	−$1,020	$7,980
4	$9,000	$3,000	$6,000	−$2,040	$6,960
5	$9,000 $5,000	$0	$9,000 $2,000	−$3,060 −$680	$10,260

NPW(12%) = −$954
Neither A nor B meet the 12% criterion. By NPW one can see that A is the better of the two undesirable alternatives.

<u>Select Alternative A.</u>

12-57

Purchase Option

Year	BTCF	MACRS Dep.	TI	40% Inc.Tax	ATCF	PW Of Cost at 12%
1 (6 mo-2006)	−$5,838	$7,660	−$7,660*	+$3,064	−$2,774	−$2,477
2 (2007)	−$11,676	$4,900	−$4,900	+$1,960	−$9,716	−$7,746
3 (2008)	−$11,676	$2,950	−$2,950	+$1,180	−$10,496	−$7,471
4 (6 mo-2009)	$0	$888	−$888	+$355	+$355	$226
4	+$15,200		$2,410**	−$964	+$14,236	$9,047
				Net Present Worth = −$8,421		

* BTCF finance expenses are not tax deductible because interest is 0%. In this solution we have eliminated mileage and insurance costs because these are the same for both alternatives.
** Book Value = Cost – Depreciation = $29,188 − $16,398= $12,790

Capital loss on disposal = $15,200 − $12,790= $2,410

Lease Option

Year	BTCF	TI	40% Inc.Tax	ATCF	PW Of Cost
1 (6 mo-2006)	−$4,464	−$4,464	+$1,786	−$2,678	$2,391
2 (2007)	−$4,428	−$4,428	+$1,771	−$2,657	$2,118
3 (2008)	−$4,428	−$4,428	+$1,771	−$2,657	$1,891
4 (6 mo – 2009)	−$2,214	−$2,214	+$886	−$1,328	$844
			Net Present Worth = -$7,244		

<u>The lease option, with the smaller PW of Cost, is preferred.</u>

12-58

(a) **Purchase**

SL Depreciation = (1st Cost – Salvage Value)/n
= ($1,000,000 − $400,000)/10
= $60,000

Annual Taxes = ($800,000 − $200,000 – $60,000) (0.40) = $216,000

ATCF = ($800,000 − $200,000 – $216,000)
= $384,000/year + Year-0 Purchase + Year-10 Resale

EUAB – EUAC = $384,000 + $400,000 (A/F, 10%, 10) − $1,000,000 (A/P, 10%, 10)
= $384,000 + $400,000 (0.0627) − $1,000,000 (0.1627)
= +$246,380

Lease
Annual Taxes = ($800,000 − $200,000 − $200,000) (0.40) = $160,000
EUAB – EUAC = $800,000 − $200,000 − $200,000 − $160,000 = +$240,000

<u>Purchasing the plant is preferred.</u>

(b) **Breakeven Rate of Return**
Set $(EUAB – EUAC)_{purchase}$ = $(EUAB – EUAC)_{lease}$ = 0
Solve for the breakeven rate of return.
$384,000 + $400,000 (A/F, 10%, 10) − $1,000,000 (A/P, 10%, 10)−
$240,000 = $0
From part (a), we know that at 10%, $(EUAB – EUAC)_{purchase}$ –
$(EUAB – EUAC)_{lease}$ = $246,380 − $240,000 = +$6,380

Tryi = 12%
$384,000 + $400,000 (0.0570) − $1,000,000 (0.1770) − $240,000
= −$10,200
So breakeven rate of return
= 10% + (2%) [$6,830/($6,830 + $10,200)] = <u>10.8%</u>

12-59

PURCHASE OPTION						
Year	BTCF	MACRS Depr*	Taxable Income	Income Taxes (40%)	ATCF	AT-PW @18%
0	–$95,000				–$95,000	–$95,000
1	–7,500	$13,575	–$21,075	$8,430	930	788
2	–7,500	23,265	–30,765	12,306	4,806	3,452
3	–7,500	16,615	–24,115	9,646	2,146	1,306
3	25,000		–16,545**	6,618	31,618	<u>19,242</u>
					PW Costs = $70,212	
*MACRS GDS Depreciation (7-year property)						
** Gain/Loss = MV(yr) – BV(yr- 3) = 25,000 – 41,545 = –$16,545						
LEASE OPTION						
0	–$45,000		–$45,000	$18,000	–$27,000	
1	–45,000		–45,000	18,000	–27,000	
2	–45,000		–45,000	18,000	–27,000	
		PW Costs = 27,000 + 27000 (P/A, 18%, 2) = $69,282				

To minimize PW of Costs, one should choose the LEASE OPTION.

12-60

Since the projects all have the same useful life we will use an AT-Present Worth analysis to compare and select.

Alternative I					0.28		0.15
		MACRS					
Year	BTCF	Depr. *	DEPR	TI	IT	ATCF	PW
0	–9,000					–9,000	–9,000
1	–25	0.20000	1,800	–1,825	511	486	423
2	–25	0.32000	2,880	–2,905	813.4	788	596
3	–25	0.19200	1,728	–1,753	490.84	466	306
4	–25	0.11520	1,037	–1,062	297.304	272	156
5	–25	0.11520	1,037	–1,062	297.304	272	135
6	–25	0.05750	518	–543	151.9	127	55
7	–25			–25	7	–18	–7
8	–25			–25	7	–18	–6
9	–25			–25	7	–18	–5
10	–25			–25	7	–18	–4
10	2,250			2,250	–630	1,620	400
					PW=		($6,950.79)
Alt. II					0.28		0.15
		MACRS					
Yr	BTCF	Depr *	DEPR	TI	IT	ATCF	PW
0	–8,000					–8,000	–8,000
1	–200	0.20000	1,600	–1,800	504	304	264
2	–200	0.32000	2,560	–2,760	772.8	573	433
3	–200	0.19200	1,536	–1,736	486.08	286	188
4	–200	0.11520	922	–1,122	314.048	114	65
5	–200	0.11520	922	–1,122	314.048	114	57
6	–200	0.05750	460	–660	184.8	–15	–7
7	–200			–200	56	–144	–54
8	–200			–200	56	–144	–47
9	–200			–200	56	–144	–41
10	–200			–200	56	–144	–36
10	2,000			2,000	–560	1,440	356
					PW =		($6,820.88)

Alternative III					0.28		0.15
		MACRS					
Year	BTCF	Depr *	DEPR	TI	IT	ATCF	PW
0	–7,500					–7,500	–7,500
1	–300	0.20000	1,500	–1,800	504	204	177
2	–300	0.32000	2,400	–2,700	756	456	345
3	–300	0.19200	1,440	–1,740	487.2	187	123
4	–300	0.11520	864	–1,164	325.92	26	15
5	–300	0.11520	864	–1,164	325.92	26	13
6	–300	0.05750	431	–731	204.75	–95	–41
7	–300			–300	84	–216	–81
8	–300			–300	84	–216	–71
9	–300			–300	84	–216	–61
10	–300			–300	84	–216	–53
10	1,875			1,875	–525	1,350	334
					PW=		($6,801.10)

Alternative IV					0.28		0.15
		MACRS					
Year	BTCF	Depr *	DEPR	TI	IT	ATCF	PW
0	–6,200					–6,200	–6,200
1	–600	0.20000	1,240	–1,840	515.2	–85	–74
2	–600	0.32000	1,984	–2,584	723.52	124	93
3	–600	0.19200	1,190	–1,790	501.312	–99	–65
4	–600	0.11520	714	–1,314	367.9872	–232	–133
5	–600	0.11520	714	–1,314	367.9872	–232	–115
6	–600	0.05750	357	–957	267.82	–332	–144
7	–600			–600	168	–432	–162
8	–600			–600	168	–432	–141
9	–600			–600	168	–432	–123
10	–600			–600	168	–432	276
10	1,550			1,550	–434	1,116	
					PW =		($6,894.20)

Thus Padre Pio should select Alternative III to minimize Total PW of Costs for this equipment.

Chapter 13: Replacement Analysis

13-1

For the Replacement Analysis Decision Map, the appropriate analysis method is a function of the cash flows and assumptions made regarding the defender and challenger assets. Thus, the answer would be the last it depends on the data and the assumptions

13-2

The replacement decision is a function of both the defender and the challenger. The statement is false.

13-3

The book value of the equipment describes past actions or a *sunk cost* situation. The answer is the last it should be ignored in this *before-tax* analysis.

13-4

The value to use is the present market value of the defender equipment. (The book indicates that trade-in value may be purposely inflated as a selling strategy, hence it may or may not represent market value.)

13-5

Looking at Figure 13-1: For this problem marginal cost data is available, and is not strictly increasing. This would lead to the use of Replacement Analysis Technique #2. In this case we compute the minimum cost life of the defender and compare the EUAC at that life against the EUAC of the best available challenger. We chose the options with the smallest EUAC.

13-6

$50,000	first cost
20%	amount market value declines each year
$3,500	operating and maintenance cost first year
$2,000	amount O&M increases each year
9% MARR	

Year	Cost	Salvage Value	PW	EUAC
0	$50,000			
1	3,500	$40,000	−$16,514	$18,000
2	5,500	32,000	−30,906	17,569
3	7,500	25,600	−43,864	17,329
4	9,500	20,480	−55,853	17,240
5	11,500	16,384	−67,187	17,273
6	13,500	13,107	−78,070	17,403
EUAC minimum = $17,240				
Economic life = 4 years.				

13-7

$10,000	first cost
20%	amount market value declines each year
$0	repair cost first year since on warranty
$600	amount repair increases each year
15%	MARR

Year	Cost	Salvage Value	PW	EUAC
0	$10,000			
1	0	$8,000	−$3,043	$3,500
2	600	6,400	−5,614	3,453
3	1,200	5,120	−7,876	3,450
4	1,800	4,096	−9,930	3,478
5	2,400	3,277	−11,836	3,531
6	3,000	2,621	−13,629	3,601
EUAC minimum = $3,450				
Economic life is 3 years.				

13-8

$20,000 first cost
15% amount market value declines each year
$700 amount “image cost” increases each year
10% MARR

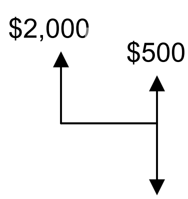
Year	Cost	Salvage Value	PW	EUAC
0	$20,000			
1	700	$17,000	−$5,182	$5,700
2	1,400	14,450	−9,851	5,676
3	2,100	12,283	−14,143	5,687
4	2,800	10,440	−18,153	5,727
5	3,500	8,874	−21,947	5,789
6	4,200	7,543	−25,570	5,871
EUAC minimum = $5,676				
Economic life is 2 years.				

13-9

EUAC of Capital Recovery
In this situation P = S = $15,000
So EUAC of Capital Recovery = $15,000 (0.15) = $2,250 for all useful lives.

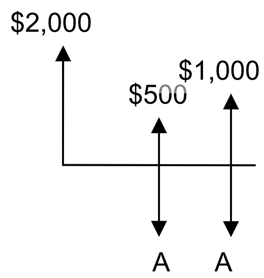
EUAC of Maintenance

For a **1-year useful life**



EUAC = $2,000 (1 + 0.15)^1 + $500 = $2,800

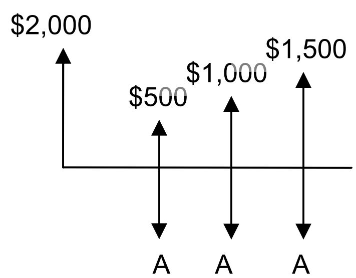
For a **2-year useful life**



$FW_{yr\,2}$ = \$2,000 (F/P, 15%, 2) + \$500 (F/P, 15%, 1) + \$1,000
= \$4,220

EUAC = A = \$1,963

For a **3-year useful life**

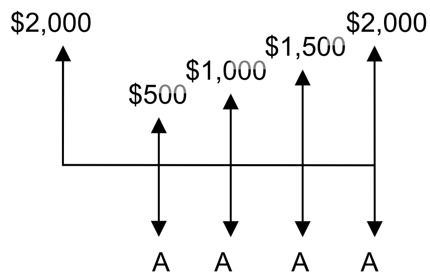


$FW_{yr\,3}$ = \$2,000 (F/P, 15%, 3) + \$500 (F/P, 15%, 2) + \$1,000 (F/P, 15%, 1) + 1,500
= \$6,353

A = \$6,353 (A/F, 15%, 3) = \$1,829

EUAC = A = \$1,829

For a 4-year useful life



$FW_{yr\ 4}$ = \$2,000 (F/P, 15%, 4) + \$500 (P/G, 15%, 5) (F/P, 15%, 5)
= \$9,305

A = \$9,305 (A/F, 15%, 4) = \$1,864

EUAC = A = \$1,864

Alternate computation of maintenance in any year N:
$EUAC_N$ = A = \$2,000 (A/P, 15%, N) + \$500 + \$500 (A/G, 15%, N)

(a) Total EUAC = \$2,250 + EUAC of Maintenance
Therefore, to minimize Total EUAC, choose the alternative with minimum EUAC of maintenance.

<u>Economical life = 3 years</u>

(b) The stainless steel tank will always be compared with the best available replacement (the challenger). If the challenger is superior, then the defender tank probably will be replaced.

It will cost a substantial amount of money to remove the existing tank from the plant, sell it to someone else, and then buy and install another one. As a practical matter, it seems unlikely that this will be economical.

13-10

Year	Salvage Value	Maintenance	Year	Salvage Value	Maintenance
0	P = $10,000		4	$4,500	$600
1	$3,000	$300	5	$5,000	$1,200
2	$3,500	$300	6	$5,500	$2,400
3	$4,000	$300	7	$6,000	$4,800

EUAC of Maintenance

$EUAC_1$ = $EUAC_2$ = $EUAC_3$ = $300
$EUAC_4$ = $300 + $300 (A/F, 15%, 4) = $360
$EUAC_5$ = $300 + [$300 (F/P, 15%, 1) + $900] (A/F, 15%, 5) = $485
$EUAC_6$ = $300 + [$300 (F/P, 15%, 2) + $900 (F/P, 15%, 1) + $2,100] (A/F, 15%, 6)
= $703
$EUAC_7$ = $300 + [$300 (F/P, 15%, 3) + $900 (F/P, 15%, 2) + $2,100 (F/P, 15%, 1) + $4,500] (A/F, 15%, 7)
= $1,074

EUAC of Installed Cost

Yr.	(P − S) (A/P, i%, n) + (S) (i)	= EUAC of Installed Cost
1	($10,000 − $3,000) (A/P, 15%, 1) + $3,000 (0.15)	= $8,500
2	($10,000 − $3,500) (A/P, 15%, 2) + $3,500 (0.15)	= $4,523
3	($10,000 − $4,000) (A/P, 15%, 3) + $4,000 (0.15)	= $3,228
4	($10,000 − $4,500) (A/P, 15%, 4) + $4,500 (0.15)	= $2,602
5	($10,000 − $5,000) (A/P, 15%, 5) + $5,000 (0.15)	= $2,242
6	($10,000 − $5,500) (A/P, 15%, 6) + $5,500 (0.15)	= $2,014
7	($10,000 − $6,000) (A/P, 15%, 7) + $6,000 (0.15)	= $1,862

Year	EUAC of Installed Cost +	EUAC of Maintenance	= Total EUAC
1	$8,500	$300	= $8,800
2	$4,523	$300	= $4,823
3	$3,228	$300	= $3,528
4	$2,602	$360	= $2,962
5	$2,242	$485	= $2,727
6	$2,014	$703	= $2,717 ←
7	$1,862	$1,074	= $2,936

The Economical Life is 6 years because this life has the smallest total EUAC.

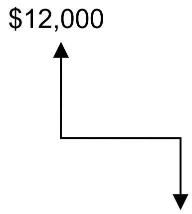
13-11

With no resale value, and maintenance costs that are expected to be higher in the future, EUAC would be a minimum for one year. (This is such a common situation that the early versions of the MAPI replacement analysis model were based on a one year remaining life for the defender.) The answer is one year.

13-12

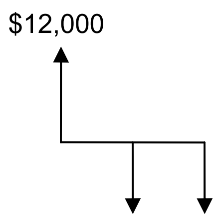
For various lives, determine the EUAC for the challenger assuming it is retired at the end of the period. The best useful life will be the one whose EUAC is a minimum.

Useful Life—1 year



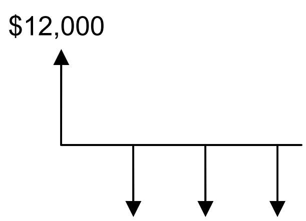
EUAC = $12,000 (A/P, 10%, 1) = $13,200

Useful Life—2 years



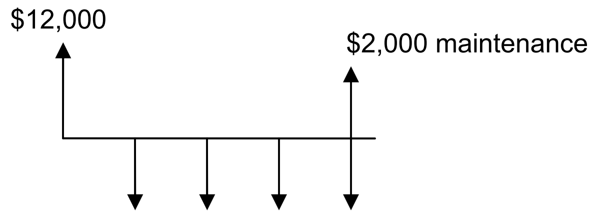
EUAC = $12,000 (A/P, 10%, 2) = $6,914

Useful Life—3 years



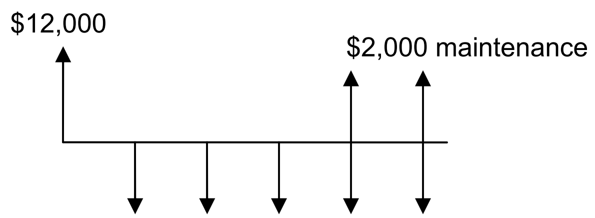
EUAC = $12,000 (A/P, 10%, 3) = $4,825

Useful Life—4 years



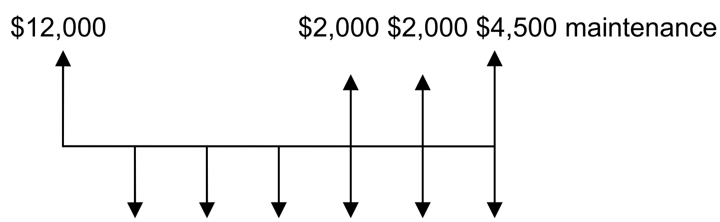
EUAC = $12,000 (A/P, 10%, 4) + $2,000 (A/F, 10%, 4)
= $12,000 (0.3155) + $2,000 (0.2155)
= $4,217

Useful Life—5 years



EUAC = $12,000 (A/P, 10%, 5) + [$2,000 (1 + (F/P, 10%, 1)] (A/F, 10%, 5)
= $12,000 (0.2638) + [$2,000 (1 + (1.100)] (0.1638)
= $3,854

Useful Life—6 years



EUAC = [$12,000 (F/P, 10%, 6) + $2,000 (F/A, 10%, 3) + $2,500](A/F, 10%, 6)
= [$12,000 (1.772) + $2,000 (3.310) + $2,500](0.1296)
= $3,938

Summary

Useful Life	EUAC
1 yr	$13,200
2 yr	$6,914
3 yr	$4,825
4 yr	$4,217
5 yr	$3,854 ← Best Useful Life is 5 years
6 yr	$3,938

13-13

First Cost = \$1,050,000
Salvage Value = \$225,000
Maintenance & Operating Cost = \$235,000
Maintenance & Operating Gradient = \$75,000

MARR = 10%

EUAB – EAUC = \$1,050,000 (A/P, 10%, n) + \$225,000 (A/F, 10%, n) − \$235,000 − \$75,000 (A/G, 10%, n)

Try n = 4 years:

EUAB – EAUC = \$331,275 + \$48,488 − \$235,000 − \$103,575 = −\$621,362

Try n = 5 years:

EUAB – EUAC = −\$276,990 + \$36,855 − \$235,000 − \$135,750 = −\$610,885

Try n = 6 years:

EUAB – EUAC = −\$241,080 + \$29,160 − \$235,000 − \$166,800 = −\$613,720

Thus, year 5 has the minimum EUAB – EUAC, hence <u>the most economic life is 5 years.</u>

13-14

For this problem we have marginal cost data for the defender, so we will check to see if these data are strictly increasing.

Defender

Current Market Value = $25,000 $(0.90)^5$ = $14,762

Year	Time Line	Market Value (n)	Loss in MV (n)	Annual Costs (n)	Lost Interest in (n)	Total Marg. Cost
0		$25,000				
1	−5	$22,500	$2,500	$1,250	$2,000	$5,750
2	−4	$20,250	$2,250	$1,750	$1,800	$5,800
3	−3	$18,225	$2,025	$2,250	$1,620	$5,895
4	−2	$16,403	$1,823	$2,750	$1,458	$6,031
5	−1	$14,762	$1,640	$3,250	$1,312	$6,202
6	1	$13,286	$1,476	$3,750	$1,181	$6,407
7	2	$11,957	$1,329	$4,250	$1,063	$6,641
8	3	$10,762	$1,196	$4,750	$957	$6,902
9	4	$9,686	$1,076	$5,250	$861	$7,187
10	5	$8,717	$969	$5,750	$775	$7,493

We see that these data are strictly increasing from the Time Line of today → onward (year 6 of the original life). Thus we use Replacement Analysis Technique #1 and compare the marginal cost data of the defender against the min. EUAC of the challenger. Let’s find the Challenger’s min. EUAC at its 5-year life.

Challenger
Challenger’s minimum cost life is given at 5 years in the problem.
EUAC = $27,900 (A/P, 8%, 5) = $6,989

From this we would recommend that we <u>keep the Defender for three more years</u> and then replace it with the Challenger. This is because after three years the marginal costs of the Defender become greater than the min. EUAC of the Challenger.

13-15

For this problem we have marginal cost data for the defender, so we will check to see if these data are strictly increasing.

Defender:

Current Market Value = $25,000 (0.70)^5 = $4,202

Year	Time Line	Market Value (n)	Loss in MV (n)	Annual Costs (n)	Lost Interest in (n)	Total Marg. Cost
0		$25,000				
1	−5	$17,500	$7,500	$3,000	$2,000	$12,500
2	−4	$12,250	$5,250	$3,300	$1,400	$9,950
3	−3	$8,575	$3,675	$3,630	$980	$8,285
4	−2	$6,003	$2,573	$3,993	$686	$7,252
5	−1	$4,202	$1,801	$4,392	$480	$6,673
6	1	$2,941	$1,261	$4,832	$336	$6,428
7	2	$2,059	$882	$5,315	$235	$6,432
8	3	$1,441	$618	$5,846	$165	$6,629
9	4	$1,009	$532	$6,431	$115	$6,978
10	5	$706	$303	$7,074	$81	$7,457

Again here the marginal costs of the Defender are strictly increasing from the Time Line of today → onward (year 6 of the original life). Thus, we use Replacement Analysis Technique #1 and compare the marginal cost data of the defender against the min. EUAC of the challenger.

From the previous problem the Challenger's minimum EUAC at its 5-year life is EUAC = $27,900 (A/P, 8%, 5) = <u>$6,989</u>

From this we would recommend that we <u>keep the Defender for four more years</u> and then replace it with the Challenger. This is because after four years the marginal costs of the Defender become greater than the minimum EUAC of the Challenger.

13-16

Yr.	Time Line	Salvage	Oper.	Insurance	Maint.	Lost Interest	Lost MV	Total Marg. Cost
1	−5	$80,000	$16,000	$17,000	$5,000	$31,250	$45,000	$114,250
2	−4	$78,000	$20,000	$16,000	$10,000	$20,000	$2,000	$68,000
3	−3	$76,000	$24,000	$15,000	$15,000	$19,500	$2,000	$75,500
4	−2	$74,000	$28,000	$14,000	$20,000	$19,000	$2,000	$83,000
5	−1	$72,000	$32,000	$13,000	$25,000	$18,500	$2,000	$90,500
6	1	$70,000	$36,000	$12,000	$30,000	$18,000	$2,000	$98,000
7	2	$68,000	$40,000	$11,000	$35,000	$17,500	$2,000	$105,500
8	3	$66,000	$44,000	$10,000	$40,000	$17,000	$2,000	$113,000
9	4	$64,000	$48,000	$10,000	$45,000	$16,500	$2,000	$121,500
10	5	$62,000	$52,000	$10,000	$50,000	$16,000	$2,000	$130,000

(a) Total marginal cost for this previously implemented asset is given above.

(b) In looking at the table above one can see that the marginal cost data of the defender is strictly increasing over the next five-year period. Thus the Replacement Decision Analysis Map would suggest that we use Replacement Analysis Technique #1. We compare the defender marginal cost data against the challenger's minimum EUAC.

We would keep the defender asset for two more years and then replace it with the new automated shearing equipment. After two years the MC (def) > minimum EUAC (chal): $113,000 > $110,000.

13-17

(a) The minimum cost life is where the EUAC of ownership is minimized for the number of years held. Since the costs are increasing, the minimum cost would occur at 1 year for the defender, where EUAC = $3,000.

(b) The minimum cost life of the challenger is 3 years where the EUAC = $3,300.

(c) Using Replacement Analysis Technique #1: Given these costs for the defender and challenger we should keep the defender 2 more years.

13-18

A tabulation of the decline in resale value plus the maintenance is needed to solve the problem.

Age	Value of Car	Decline in Value for the Year	Maintenance for the Year	Sum of Decline in Value + Maintenance
New	$11,200			
1 yr	$8,400	$2,800	$50	$2,850
2	$6,300	$2,100	$150	$2,250
3	$4,725	$1,575	$180	$1,755
4	$4,016	$709	$200	$909
5	$3,414	$602	$300	$902
6	$2,902	$512	$390	$902
7	$2,466	$536	$500	$936

From the table it appears that minimum cost would result from buying a 3-year-old car and keeping it for three years.

13-19

The EUAC of installed cost will decline as the service life increases. The EUAC of maintenance is constant. Thus total EUAC is declining over time. Answer: <u>For minimum EUAC, keep the bottling machine indefinitely.</u>

13-20

In this case we first compute the total marginal costs of the defender asset. From Figure 13-1 the marginal cost data are available, and it is not strictly increasing (see Total MC column in the table below). Thus, we use Replacement Analysis Technique #2, comparing minimum EUAC defender against minimum EUAC of challenger. In the table below, the minimum EUAC is at year 5 for the old paver (five years from today), the value is $59,703. We compare this value to the minimum EUAC for the challenger of $62,000. Thus, we recommend keeping the defender for at least one more year and reviewing the data for changes.

MARR%	20%							
First Cost	120,000							
Year	Oper.	Maint.	MV in	Lost MV	Lost Int.	Total MC	NPW	EUAC
(n)	Cost	Cost	(n)	(n)	(n)	(n)	(1→n)	(1→n)
1	15000	9000	85000	35000	24000	83000	$69,166.67	$83,000.00
2	15000	10000	65000	20000	7000	52000	$105,277.78	$68,909.09
3	17000	12000	50000	15000	4000	48000	$133,055.56	$63,164.84
4	20000	18000	40000	10000	3000	51000	$157,650.46	$60,898.66
5	25000	20000	35000	5000	2000	52000	$178,548.10	$59,702.86
6	30000	25000	30000	5000	1000	61000	$198,976.87	$59,833.49
7	35000	30000	25000	5000	1000	71000	$218,791.67	$60,698.04

13-21

(a) 1 year: EUAC = 2500(A/F, 10%, 1) = $2,500
2 years: EUAC = [2500(P/F,10%,1) + 2400(P/F,10%,2)](A/P,10%,2) = $2,452
3 years: EUAC = [2500(P/F,10%,1) + 2400(P/F,10%,2) + 2300(P/F,10%,3)](A/P,10%,3) = $2,406
4 years: EUAC = [2500(P/F,10%,1) + 2400(P/F,10%,2) + 2300(P/F,10%,3) + 2550(P/F,10%,4)](A/P,10%,4) = $2,437
The minimum cost life is where the EUAC of ownership is minimized for the number of years held. This would occur at 3 years for the defender where the EUAC = $2,406.

(b) The minimum cost life of the challenger is 4 years where the EUAC = $2,600.

(c) Using Replacement Analysis Technique #2: we compare the minimum EUAC of the defender ($2,406) vs. minimum EUAC of the challenger ($2,600). Thus we keep the defender. Assuming that the defender and challenger costs do not change in the near future we should keep the defender for at least three years and then reevaluate the costs with challengers at that time.

13-22

Here we use Replacement Analysis Technique #3. Because the remaining life of the defender and the life of the challenger are both 10 years, we can use either the “opportunity cost” or “cash flow” approach to setting the first cost of each option (keep defender or replace with challenger). Let’s show each solution:

Opportunity Cost Approach
EUAC (def) = 4 ($600) (A/P, 25%, 10) = <u>$672</u>
EUAC (chal) = $5,000 (A/P, 25%, 10) − $10,000 (0.075) = <u>$650</u>

Cash Flow Cost Approach
EUAC (def) = $0.00
EUAC (chal) = ($5,000 − 4*$600) (A/P, 25%, 10) − $10,000 (0.075) = −$22

In either case we recommend that the new high-efficiency machine be implemented today.

13-23

Before-Tax Analysis

Year	New Machine BTCF	Existing Machine BTCF	New Machine Rather than Existing Machine BTCF
0	−$3,700	−$1,000	−$2,700
1	+$900	$0	+$900
2	+$900	$0	+$900
3	+$900	$0	+$900
4	+$900	$0	+$900

Compute Rate of Return
PW of Cost = PW of Benefit
$2,700 = $900 (P/A, i%, 4)
(P/A, i%, 4) = $2,700/$900 = 3.0
Rate of return = <u>12.6%</u>

13-24

Find: $NPW_{OVERHAUL}$ and $NPW_{REPLACE}$
Note: All costs which occur before today are *sunk costs* and are irrelevant.

$NPW_{OVERHAUL}$ = −$1,800 − $800 (P/A, 5%, 2)
= −$1,800 − $800 (1.859) = −$3,287

$NPW_{REPLACE}$ = +$1,500 − ($2,500 + $300) (P/A, 5%, 2)
= +$1,500 − $2,800 (1.859) = −$3,705

Since the PW of Cost of the overhaul is less than the PW of Cost of the replacement car, the decision is to overhaul the 1988 auto.

13-25

Alternative I: Retire the 4 old machines and buy 6 new machines.

Initial Cost:	6 new machines at $32,000 each	$192,000
	Training Program at 6 x $700	+$4,200
	Total	= $196,200

Savings:	Annual Labor Saving	$12,000
	Less Maintenance	$3,600
	Total	=$8,400

Compute Equivalent Uniform Annual Cost (EUAC)

Initial Cost: $196,000 (A/P, 9%, 8) = $196,000 (0.1807)	= $35,453
Less Salvage Value: (6 x $750) (A/F, 9%, 8)= $4,500 (0.0907)	= −$408
Less Net Annual Benefit:	= −$8,400
EUAC	= $26,645

Alternative II: Keep 4 old machines and buy 3 new ones
Initial Cost: Value of 4 old machines 4 x $2,000 $8,000
3 new machines at $32,000 each $96,000
Training Program at 3 x $700 $2,100
Total = $106,100
Annual Maintenance= 4 old x $1,500 + 3 new x $600 = $7,800 per year
Salvage Value 8 years hence= 4 old x $500 + 3 new x $750 = $4,250
Compute Equivalent Uniform Annual Cost (EUAC)
Initial Cost: $106,100 (A/P, 9%, 8) = $106,100 (0.1807) = $19,172
Less Salvage Value: ($4,250) (A/F, 9%, 8) = $4,250 (0.0907) = −$385
Add Annual Maintenance: = +$7,800
EUAC = $26,587
Decision: Choose Alternative II with its slightly lower EUAC.

13-26

In a before-tax computation the data about depreciation are unneeded.
No marginal cost data is available, so use replacement technique #3.

Defender: Compute EUAC over its remaining life of 10 years
P = $10,000 + $35,000 = $45,000 (outsider's perspective)
EUAC = $45,000 (A/P,15%,10) – $10,000 (A/F,15%,10) = $8,457.50

Challenger: Computer EUAC at minimum cost life.
Note: We do not have MV data for years 1 to 9, so can't compute EUAC for years 1 to 9, so we will assume minimum cost life is at 10 years.
EUAC = $85,000(A/P, 15%,10) – $15,000(A/F, 15%,10) – $7,000 = $9,201

Decision: Keep the defender (recondition the old tank car).

13-27

(a) Expected good performance, productivity, energy efficiency, safety, long service life. Retraining in operation and maintenance may be required. High comfort of operation. High purchase price. May not be immediately available. Sales taxes to be paid. Can be depreciated. Supplier warranty and spare parts backup available.

(b) All as in (a) except for lower price and probably faster delivery.

(c) All as in (a) except for still lower cost, lost production during the rebuild period, and that the rebuild costs can be expensed, at least partially. No sales tax applies.

(d) Performance and productivity may not be as good as in option (c). Retraining in operation and maintenance is not required. Production will be lost during the rebuilding period. Cost may be substantially lower than in previous options. The rebuild costs can be expensed. No sales tax applies.

(e) Performance, productivity, service life, energy efficiency, safety, reliability may be significantly lower than in the other options. Retraining in operation and maintenance may be required if the new unit is different from the previous one. Cost may be only 20–50% of the new equipment. Immediate delivery is a possibility. The sales tax applies. Equipment can be depreciated.

13-28

No marginal cost data so we will use technique #3.
From the facts stated, we see that if the old forklift is retained the EUAC is minimum for a one year useful life. The problem says the challenger economic life is 10 years. (Using the data provided, this fact could be verified, but that is not part of the problem.)

Annual Cash-Flow Analysis:
Keep Old Forklift Another Year

Year	BTCF	Depr.	Taxable Income	40% Income Taxes	ATCF
0	$0				$0
1	–$400	$0	−$400	+$160	−$240

EUAC for one more year with old forklift = $240

Buy New Forklift

Year	BTCF	SL Depr.	Taxable Income	40% Income Taxes	ATCF
0	−$6,500				−$6,500
1−10	−$50	$650	−$700	+$280	+$230

EUAC = $6,500 (A/P, 8%, 10) − $230
= $6,500 (0.1490) − $230
= $738.50

Decision: <u>Choose the alternative with the minimum EUAC. Keep the old forklift another year.</u>

13-29

No marginal cost data so we will use technique #3.
Book value of Machine A now = Cost − Depreciation to date
= $54,000 − (9/12) ($54,000 − $0)
= $13,500

Recaptured Deprec. If sold now = $30,000 − $13,500
= $16,500

Machine A annual depreciation = (P − S)/n = ($54,000 − $0)/12 = $4,500
Machine B annual depreciation = (P − S)/n = ($42,000 − $0)/12 = $3,500

Alternate 1: Keep A for 12 more years

Year	BTCF	SL Depr.	Taxable Income	40% Income Taxes	ATCF
0	−$30,000*		−$16,500	+$6,600	−$23,400
1	$0	$4,500	−$4,500	+$1,800	+$1,800
2	$0	$4,500	−$4,500	+$1,800	+$1,800
3	$0	$4,500	−$4,500	+$1,800	+$1,800
4- 12	$0	$0	$0	$0	$0

* If A were sold, the Year 0 entries would be:

Year	BTCF	SL Deprec.	Taxable Income	40% Income Taxes	ATCF
0	+$30,000		$16,500	+$6,600	+$23,400

($9,000 is a sunk cost)
If A is kept, the entries are just the reverse.
After-Tax Annual Cost
= [$23,400 − $1,800 (P/A, 10%, 4)] (A/P, 10%, 12)
= [$23,400 − $1,800 (2.487)] (0.1468)
= <u>$2,778</u>

The cash flow in year 0 reflects the loss of income after Recaptured Depreciation tax from <u>not</u> selling Machine A. This is the preferred way to handle the current market value of the “defender.”

Alternate 2: Buy Machine B

Year	BTCF	SL Depr.	Taxable Income	40% Income Taxes	ATCF
0	−$42,000				−$42,000
1- 12	+$2,500	$3,500	−$1,000	+$400	+$2,900

Since the ATCF values for years 1 through 12 are positive, it will be best to hold the machine as long as possible.
After-Tax Annual Cost
= $42,000 (A/P, 10%, 12) − $2,900
= $42,000 (0.1468) − $2,900
= <u>$3,266</u>

Choose the alternative with the smaller annual cost. <u>Keep Machine A.</u>

13-30

Here we use the Opportunity Cost Approach for finding the first costs.

(a) **Problem as given**
Defender: SL Depreciation = ($50,000 − $15,000)/10
= $3,500 per year
MV today = $30,000

	Year	BTCF	Depr.	TI	IT	ATCF
Sell	0	$30,000		$4,500*	−$2,025	$27,975
Keep	0	−$30,000		−$4,500	+$2,025	−$27,975

* TI = Taxable Inc. = Recaptured Depreciation
= $30,000 − [$50,000 − 7 ($3,500)]
= $4,500

(b) Defender Market Value = $25,500
Defender: SL Depr = ($50,000 − $15,000)/10 = $3,500 per year
MV (today) = $25,500

	Year	BTCF	Depr.	TI	IT	ATCF
Sell	0	$25,500		$0*	$0	$25,500
Keep	0	−$25,500		$0	$0	−$25,500

* Recaptured Depreciation = $25,500 − [$50,000 − 7 ($3,500)] = $0

Challenger

Year	BTCF	Depr.	TI	IT	ATCF
0	−$85,000			+$8,500	−$76,500

(c) Defender Market Value = $18,000
Defender: SL Depreciation = ($50,000 − $15,000)/10 = $3,500 per year
MV (today) = $18,000

	Year	BTCF	Depr.	TI	IT	ATCF
Sell	0	$18,000		−$7,500*	+$3,375	$21,375
Keep	0	−$18,000		+$7,500	−$3,375	−$21,375

* Loss = $18,000 − [$50,000 − 7 ($3,500)] = −$7,500

Challenger

Year	BTCF	Depr.	TI	IT	ATCF
0	−$85,000			+$8,500	−$76,500

13-31

(a) The defender was implemented six years ago with a cost basis (1st cost) of $5,000. The estimated salvage value for tax purposes was $1,000 and the straight line depreciation method was used.

SL Depreciation = (B − S)/n
$500 = ($5,000 − $1,000)/n
n = 8 years

(b) The ATCFs for defender and challenger are as follows:

Defender: - 3 year remaining life
- depreciated over 8 years (six in the past)
- $2,500 expensed at time 0
- present MV = $1,000
- MV in 3 years = $500

	Year	BTCF	Depr.	TI	IT	ATCF
(sell)	0	$1,000	−$1,000*		+$350	$1,350
(keep)	0	−$1,000	$1,000		−$350	−$1,350
	0	−$2,500	−$2,500		+$875	−$1,625
	1	−$600	$500	−$1,100	+$385	−$215
	2	−$750	$500	−$1,250	+$438	−$312
	3	−$900	−	−$900	+$315	−$585
	3	$500		−$500**	+$175	$675

* TI = Taxable Inc. = Recaptured Depreciation
= $1,000 − ($5,000 − 6 ($500)]
= −$1,000

** IT = Income Tax = $500 − ($5,000 − 8($500)]
= −$500

Challenger: - 6-year useful life
- MACRS depreciation w/ 7-yr class life
- MV at 6 years = $1,000

Year	BTCF	Depr.	TI	IT	ATCF
0	−$10,000				−$10,000
1	−$100	$1,429	−$1,529	$535	$435
2	−$150	$2,449	−$2,599	$910	$760
3	−$200	$1,749	−$1,949	$682	$482
4	−$250	$1,249	−$1,499	$525	$275
5	−$300	$893	−$1,193	$418	$118
6	−$350	$446*	−$796	$279	−$71
6	$1,000		−$785**	$275	$1,275

* MACRS depreciation = 0.50 ($10,000) (0.0892)
** Recovered Depr.= $1,000 − [$10,000 (0.0446 + 0.0893 + 0.0446)] = −$785
Here we use Replacement Analysis Technique #3
AW (def) = [−$1,350 − $1,625 − $215 (P/F, 18%, 1) − $312 (P/F, 18%, 2) − ($585 − $675) (P/F, 18%, 3)] (A/P, 18%, 3)
= $1,530

AW (chal) = [−$10,000 + $435 (P/F, 18%, 1) + $760 (P/F, 18%, 2) +$482 (P/F, 18%, 3)+$275 (P/F, 18%, 4)+$118 (P/F, 18%, 5) + (−$71 + $1,275) (P/F, 18%, 6)] (A/P, 18%, 6)
= $2,331

Choose the defender

The assumptions that are made here are the Repeatability Replacement Assumptions: the same challenger will always be available at the current cost, and there is an indefinite need of this asset for operations.

13-32

The problem, with a 7-year analysis period, may be solved in a variety of ways. A first step is to compute an after-tax cash flow for each alternative.

Alternative A

Year	BTCF	Depr.	Taxable Income	40% Income Taxes	ATCF
0	−$44,000		−$44,000	+$17,600	−$26,400
1− 7	$0		$0		$0

Alternative B
This alternative is less desirable than Alternative D and may be immediately rejected.

Alternative C

Year	BTCF	SOYD Depr.	Taxable Income	40% Income Taxes	ATCF
0	−$56,000				−$56,000
1	$12,000	$14,000	−$2,000	+$800	+$12,800
2	$12,000	$12,000	$0	$0	+$12,000
3	$12,000	$10,000	$2,000	−$800	+$11,200
4	$12,000	$8,000	$4,000	−$1,600	+$10,400
5	$12,000	$6,000	$6,000	−$2,400	+$9,600
6	$12,000	$4,000	$8,000	−$3,200	+$8,800
7	$12,000	$2,000	$10,000	−$4,000	+$8,000

Alternative D

Year	BTCF	Depr.	Taxable Income	40% Income Taxes	ATCF
0	−$49,000				−$49,000
1−7	$7,000	$7,000	$0	$0	+$7,000

Alternative E (Do Nothing)

Year	BTCF	Depr.	Taxable Income	40% Income Taxes	ATCF
0	$0				$0
1-7	−$8,000	$0	−$8,000	+$3,200	−$4,800

A NPW solution is probably easiest to compute:
NPW_A = −$26,400
NPW_C = −$56,000 + $12,800 (P/A, 10%, 7) − $800 (P/G, 10%, 7)
= −$56,000 + $12,800 (4.868) − $800 (12.763)
= −$3,900
NPW_D = −$49,000 + $7,000 (P/A, 10%, 7)
= −$49,000 + $7,000 (4.868)
= −$14,924
NPW_E = −$4,800 (P/A, 10%, 7)
= −$4,800 (4.868)
= −$23,366
Choose the solution that maximizes NPW. Choose Alternative C.

Rate of Return Solution
Alternative A rather than Alternative E (Do nothing)

Year	Alt. A ATCF	Alt. E ATCF	(A – E) ATCF
0	−$26,400	$0	−$26,400
1-7	$0	−$4,800	+$4,800

ΔROR = 6.4%
Reject Alternative A.

Alternative D rather than Alternative E

Year	Alt. D ATCF	Alt. E ATCF	(D – E) ATCF
0	−$49,000	$0	−$49,000
1-7	+$7,000	−$4,800	+$11,800

ΔROR = 12.8%
Reject Alternative E.

Alternative C rather than Alternative D

Year	Alt. C ATCF	Alt. D ATCF	(C – D) ATCF
0	−$56,000	−$49,000	−$7,000
1	+$12,800	$7,000	$5,800
2	+$12,000	$7,000	$5,000
3	+$11,200	$7,000	$4,200
4	+$10,400	$7,000	$3,400
5	+$9,600	$7,000	$2,600
6	+$8,800	$7,000	$1,800
7	+$8,000	$7,000	$1,000

$7,000 = $5,800 (P/A, i%, 7) − $800 (P/G, i%, 7)

ΔROR > 60%
(Calculator Solution: ΔROR = 65.9%)
Reject D.

Conclusion: Choose Alternative C.

13-33

(a) SONAR
SOYD = (8/2) (9) = 36
ΔD/yr = (1/36) ($18,000 − $3,600) = $400

	Original Year j	SOYD Depr.	Book Value	
	1	$3,200	$14,800	
	2	$2,800	$12,000	
	3	$2,400	$9,600	
	4	$2,000	$7,600	
Now →	5	$1,600	$6,000	← BV_5
	6	$1,200	$4,800	
	7	$800	$4,000	
	8	$400	$3,600	← BV_8

Orig. Year	Analysis Year	BTCF	SOYD Depr.	Δ Tax Income	Δ Tax	ATCF
5	0	−$7,000		−$1,000*	+$400	−$6,600
6	1		$1,200	−$1,200	+$480	+$480
7	2		$800	−$800	+$320	+$320
8	3	$1,600	$400	−$400 $2,000**	+$160 +$800	+$2,560

* Foregone recaptured depreciation is $7,000 − BV_5 = $1,000
** Loss is $1,600 − BV_8 = −$2,000

(b) SHSS

Year	BTCF	MACRS Depr.	Δ Tax Income	Δ Tax	ATCF
0	−$10,000				−$10,000
1	$500	$2,000	−$1,500	+$600	$1,100
2	$500	$3,200	−$2,700	+$1,080	$1,580
3	$500 $4,000	$960[1]	−$460 $160*	+$184 −$68	$684 $3,936

[1]Half year convention in year of disposal.

* Recaptured Depreciation $= \$4{,}000 - BV_3$
$= \$4{,}000 - (\$10{,}000 - \$2{,}000 - \$3{,}200 - \$960)$
$= \$160$

(c) Difference between Alternatives

Year	$\Delta ATCF = ATCF_{SHSS} - ATCF_{Sonar}$
0	−$3,400
1	+$620
2	+$1,260
3	+$2,060

(d) Compute the NPW on the difference between alternative at 20%

$NPW_{\Delta} = -\$3{,}400 + \$620\,(P/F, 20\%, 1) + \$1{,}260\,(P/F, 20\%, 2) + \$2{,}060\,(P/F, 20\%, 3)$
$= -\$3{,}400 + \$620\,(0.8333) + \$1{,}260\,(0.6944) + \$2{,}060\,(0.5787)$
$= -\$816.29$

Since NPW is negative, the incremental rate of return < 20%.
Stay with the sonar device.

13-34

After-Tax Analysis

New Machine

Year	BTCF	SOYD Depr.	Taxable Income	40% Income Taxes	ATCF
0	−$3,700				−$3,705
1	+$900	$1,480	−$580	+$232	+$1,132
2	+$900	$1,110	−$210	+$84	+$984
3	+$900	$740	+$160	−$64	+$836
4	+$900	$370	$530	−$212	+$688

SOYD Depr.
Sum = (4/2) (5) = 10
1st Year SOYD = (4/10) ($3,700 − $0)
Annual Decline = (1/10) ($3,700 − $0) −$370

Existing Machine

Year	BTCF	SL Depr.	Taxable Income	40% Income Taxes	ATCF
0	−$1,000		$1,000*	−$200**	−$1,200
1	$0	$500	−$500	−$200	+$200
2	$0	$500	−$500	+$200	+$200
3	$0	$500	−$500	+$200	+$200
4	$0	$500	−$500	+$200	+$200

* Long-term capital loss foregone by keeping machine:
$2,000 Book Value − $1,000 Selling Price = $1,000 Capital Loss

** The $1,000 long-term capital loss foregone would have offset $1,000 of long-term capital gains elsewhere in the firm. The result is a tax saving of 20% ($1,000) = $200 is foregone.

New Machine rather than Existing Machine

Year	New Tool ATCF	Existing Tool ATCF	New-Existing ATCF	PW AT 5%	PW AT 6%
0	−$3,705	−$1,200	−$2,500	−$2,500	−$2,500
1	+$1,132	+$200	$932	$888	$879
2	+$984	+$200	$784	$711	$698
3	+$836	+$200	$636	$549	$534
4	+$688	+$200	$488	$400	$387
			Sum	= +$50	−$2

Δ After-Tax rate of return = <u>5.96%</u>

13-35

	A	B	C	D	E	F	G	H	I
1	15,000	First Cost			Initial Depreciation				
2	10,000	Initial salvage			Year	MACRS	Book Val.		
3	-1,000	Salvage gradient			0		15,000		
4	1,000	Initial O&M			1	3000	12,000		
5	1,000	O&M gradient			2	4800	7,200		
6	0.45	Tax rate			3	2880	4,320		
7	5	Life			4	1728	2,592		
8	0.3	Interest rate			5	1728	864		
9					6	864	0		
10	**Year**	**Capital Cost**	**MACRS**	**Book Value**	**AT Salvage**	**O&M Cash Flow**	**Taxable Income**	**PW Sum O&M Tax**	**EAC**
11	0	-15,000		4320					
12	1	10,000	1728	2592	6,666	-1,000	-2,728	175	12,606
13	2	9,000	1728	864	5,339	-2,000	-3,728	-16	8,712
14	3	8,000	864	0	4,400	-3,000	-3,864	-590	7,481
15	4	7,000			3,850	-4,000	-4,000	-1,360	6,930
16	5	6,000			3,300	-5,000	-5,000	-2,101	6,656
17	6	5,000			2,750	-6,000	-6,000	-2,784	6,514
18	7	4,000			2,200	-7,000	-7,000	-3,398	6,441
19	8	3,000			1,650	-8,000	-8,000	-3,937	6,406
20	9	2,000			1,100	-9,000	-9,000	-4,404	6,393
21	10	1,000			550	-10,000	-10,000	-4,803	**6,393**
22	11	0			0	-11,000	-11,000	-5,141	6,399
23			=B21+($A$6)*(D21-B21)			=F21-C21			
24									
25		=NPV($A$8,$F$12:F21)-$A$6*NPV($A$8,$G$12:G21)							
26									
27						=PMT($A$8,A21,H21-$A$1,E21)			
28					=PMT(interest,year,Pwsum+firstcost,Atsalv)				
29	**10 years is min EAC**								

Solution from time of purchase:

L30

	A	B	C	D	E	F	G	H	I
1	15,000	First Cost			Initial Depreciation				
2	10,000	Initial salvage			Year	MACRS	Book Val.		
3	-1,000	Salvage gradient			0		15,000		
4	1,000	Initial O&M			1	3000	12,000		
5	1,000	O&M gradient			2	4800	7,200		
6	0.45	Tax rate			3	2880	4,320		
7	5	Life			4	1728	2,592		
8	0.3	Interest rate			5	1728	864		
9					6	864	0		
10	**Year**	**Capital Cost**	**MACRS**	**Book Value**	**AT Salvage**	**O&M Cash Flow**	**Taxable Income**	**PW Sum O&M Tax**	**EAC**
11	0	-15,000		15,000					
12	1	10,000	3000	12,000	10,900	-1,000	-4,000	615	7,800
13	2	9,000	4800	7,200	8,190	-2,000	-6,800	1,243	6,548
14	3	8,000	2880	4,320	6,344	-3,000	-5,880	1,081	6,074
15	4	7,000	1728	2,592	5,016	-4,000	-5,728	583	5,844
16	5	6,000	1728	864	3,689	-5,000	-6,728	52	5,729
17	6	5,000	864	0	2,750	-6,000	-6,864	-551	5,669
18	7	4,000			2,200	-7,000	-7,000	-1,164	5,644
19	8	3,000			1,650	-8,000	-8,000	-1,704	**5,642**
20	9	2,000			1,100	-9,000	-9,000	-2,171	5,653
21	10	1,000			550	-10,000	-10,000	-2,570	5,670
22			=B21+($A$6)*(D21-B21)			=F21-C21			
23									
24		=NPV($A$8,$F$12:F21)-$A$6*NPV($A$8,$G$12:G21)							
25									
26						=PMT($A$8,A21,H21-$A$1,E21)			
27					=PMT(interest,year,Pwsum+firstcost,Atsalv)				
28	**8 years is min EAC**								

13-36

	A	B	C	D	E	F	G	H	I	J	K	L
1	125,000	First cost										
2	80,000	Initial salvage										
3	-2000	Salvage gradient										
4		Initial O&M		replaced by columns F, G, & H								
5		O&M gradient										
6	0.35	Tax rate						yrs 9 & 10 = 10,000				
7	7	class life										
8	0.25	Interest rate			initial	16000	5000	17000				
9					gradient	4000	5000	-1000				
10	year	Capital Cost	MACRS	Book Value	AT salvage	Oper.	Maint.	Insur-ance	O&M cash	Taxable Income	PW Sum	EAC
11	0	-125,000		125,000								
12	1	80,000	17857	107143	89500	16000	5000	17000	-38000	-55857	-14760	85200
13	2	78,000	30612	76531	77486	20000	10000	16000	-46000	-76612	-27039	71144
14	3	76,000	21866	54665	68533	24000	15000	15000	-54000	-75866	-41092	67112
15	4	74,000	15618	39046	61766	28000	20000	14000	-62000	-77618	-55360	65659
16	5	72,000	11156	27890	56562	32000	25000	13000	-70000	-81156	-68989	65243
17	6	70,000	11156	16734	51357	36000	30000	12000	-78000	-89156	-81257	65322
18	7	68,000	11156	5578	46152	40000	35000	11000	-86000	-97156	-92161	65635
19	8	66,000	5578	0	42900	44000	40000	10000	-94000	-99578	-102084	66054
20	9	64,000		0	41600	48000	45000	10000	-103000	-103000	-111070	66554
21	10	62,000		0	40300	52000	50000	10000	-112000	-112000	-118887	67094
22												
23			=B21+($A$6)*(D21-B21)					=I21-C21				
24												
25				=NPV($A$8,$I$12:I21)-$A$6*NPV($A$8,$J$12:J21)								
26												
27								=PMT($A$8,A21,K21+$B$11,E21)				
28	**5 years is the minimum EAC**						=PMT(interest,year,PWsum+firstcost,ATsalv)					
29												

Chapter 14: Inflation and Price Change

14-1

During times of inflation, the purchasing power of a monetary unit is reduced. In this way the currency itself is less valuable on a per unit basis. In the U.S.A., what this means is that during inflationary times our dollars have less purchasing power, and thus we can purchase less products, goods and services with the same $1, $10, or $100 dollar bill as we did in the past.

14-2

Actual dollars are the cash dollars that we use to make transactions in our economy. These are the dollars that we carry around in our wallets and purses, and have in our savings accounts. Real dollars represent dollars that do not carry with them the effects of inflation, these are sometimes called "inflation free" dollars. Real dollars are expressed as of purchasing power base, such as Year-2000-based-dollars.

The inflation rate captures the loss in purchasing power of money in a percentage rate form. The real interest rate captures the growth of purchasing power, it does not include the effects of inflation is sometimes called the "inflation free" interest rate. The market interest rate, also called the combined rate, combines the inflation and real rates into a single rate.

14-3

There are a number of mechanisms that cause prices to rise. In the chapter the authors talk about how *money supply, exchange rates, cost-push,* and *demand pull* effects can contribute to inflation.

14-4

Yes. Dollars and interest rates are used in engineering economic analyses to evaluate projects. As such, the purchasing power of dollars, and the effects of inflation on interest rates, are important.

The important principle in considering effects of inflation is not to mix-and-match dollars and interest rates that include, or do not include, the effect of inflation. A constant dollar analysis uses real dollars and a real interest rate, a then-current (or actual) dollar analysis uses actual dollars and a market interest rate. In much of this book actual dollars (cash flows) are used along with a market interest rate to evaluate projects — this is an example of the later type of analysis.

14-5

The stable price assumption is really the same as analyzing a problem in Year 0 dollars, where all the costs and benefits change at the same rate. Allowable depreciation charges are based on the original equipment cost and do not increase. Thus the stable price assumption may be suitable in some before-tax computations, but is not satisfactory where depreciation affects the income tax computations.

14-6

F = P (F/P, *f*%, 10 yr) = \$10 (F/P, 7%, 10) = \$10 (1.967) = \$19.67

14-7

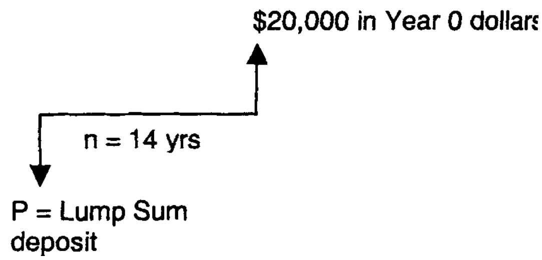
$i_{equivalent} = i'_{inflation\ corrected} + f\% + (i'_{inflation\ corrected})\ (f\%)$
In this problem: $i_{equivaient} = 5\%$
$f\% = +2\%$
$i'_{inflation\ corrected}$ = unknown
$0.05 = i'_{inflation\ corrected} + 0.02 + (i'_{inflation\ corrected})\ (0.02)$
$i'_{inflation\ corrected} = (0.05 - 0.02)/(1 + 0.02) = 0.02941 = 2.941\%$

That this is correct may be proved by the year-by-year computations.

Year	Cash Flow	$(1 + f)^{-n}$ (P/F, f%, n)	Cash Flow in Year 0 dollars	PW at 2.941%
0	−$1,000	0	−$1,000.00	−$1,000.00
1	+$50	0.9804	+$49.02	+$47.62
2	+$50	0.9612	+$48.06	+$45.35
3	+$50	0.9423	+$47.12	+$43.20
4	+$50	0.9238	+$46.19	+$41.13
5	+$50	0.9057	+$45.29	+$39.18
6	+$50	0.8880	+$44.40	+$37.31
7	+$50	0.8706	+$43.53	+$35.54
8	+$50	0.8535	+$42.68	+$33.85
9	+$50	0.8368	+$41.84	+$32.23
10	+$50	0.8203	+$41.02	+$30.70
11	+$50	0.8043	+$40.22	+$29.24
12	+$50	0.7885	+$39.43	+$27.85
13	+$50	0.7730	+$38.65	+$26.52
14	+$50	0.7579	+$37.90	+$25.26
15	+$50	0.7430	+$37.15	+$24.05
16	+$50	0.7284	+$36.42	+$22.90
17	+$50	0.7142	+$35.71	+$21.82
18	+$50	0.7002	+$35.01	+$20.78
19	+$50	0.6864	+$34.32	+$19.79
20	+$1,000	0.6730	+$706.65	+$395.76
				+$0.08

Therefore, $i_{inflation\ corrected} = 2.94\%$.

14-8



Actual Dollars 14 years hence = $\$20{,}000\ (1 + f\%)^{n}$
$= \$20{,}000\ (1 + 0.08)^{14}$
$= \$58{,}744$

At 5% interest:
$P = F\ (1 + i)^{-n} = \$58{,}744\ (1 + 0.05)^{-14} = \$29{,}670$

Since the inflation rate (8%) exceeds the interest rate (5%), the money is annual losing purchasing power.
<u>Deposit $29,670.</u>

14-9

$(1 + f)^{5} = 1.50$
$(1 + f) = 1.50^{1/5} = 1.0845$
$f = 0.845 = 8.45\%$

14-10

To buy $1 worth of goods today will require:
$F = P\ (F/P, f\%, n)$ n years hence.
$F = \$1\ (1 + 0.05)^{5} = \$1.47$ 5 years hence.

For the subsequent 5 years the amount required will increase to:
$\$1.47\ (F/P, f\%, n) = \$1.47\ (1 + 0.06)^{5} = \$1.97$

Thus for the ten year period $1 must be increased to $1.97. The average price change per year is:
$(\$1.97 - \$1.00)/10 \text{ yrs} = 9.7\%$ per year

14-11

f = 0.06
i' = 0.10
i = 0.10 + 0.06 + (0.10) (0.06) = 16.6%

14-12

Number of dollars required five years hence to have the buying power of one dollar today = \$1 (F/P, 7%, 5) = \$1.403

Number of cruzados required five years hence to have the buying power of 15 cruzados today = 15 (F/P, 25%, 5) = 45.78 cruzados.

Combining: \$1.403 = 45.78 cruzados
\$1 = 32.6 cruzados (Brazil uses cruzados.)

14-13

Price increase = $(1 + 0.12)^8$ = 2.476 x present price
Therefore, required fuel rating = 10 x 2.476 = 24.76 km/liter

14-14

P= 1.00 F = 1.80 n=10 f=?
1.80 = 1.00 (F/P, f%, 10)
(F/P, f%, 10) = 1.80
From tables, f is slightly greater than 6%. (f = 6.05% exactly).

14-15

i = i' + f + (i')(f)
0.15 = i' + 0.12 + 0.12(i')
1.12 i' = 0.03
i' = 0.03/1.12 = 0.027 = 2.7%

14-16

Compute an equivalent i:

$i_{equivalent}$ = i' + *f* + (i') (*f*)
= 0.05 + 0.06 + (0.05) (0.06)
= 0.113
= 11.3%

Compute the PW of Benefits of the annuity:

PW of Benefits = \$2,500 (P/A, 11.3%, 10)
= \$2,500 $[((1.113)^{10} - 1)/(0.113\ (1.113)^{10})]$
= \$14,540

Since the cost is \$15,000, the benefits are less than the cost computed at a 5% real rate of return. Thus the actual real rate of return is less than 5% and the annuity should not be purchased.

14-17

$1 = 0.20\ (1.06)^n$
log (1/0.20) = n log (1.06)
n = <u>27.62 years</u>

14-18

Use \$97,000 $(1 + 1\%)^n$, where f% = 7% and n = 15
\$97,000 $(1 + 0.07)^{15}$ = \$97,000 (F/P, 7%, 15)
= \$97,000 (2.759)
= \$268,000

If there is 7% inflation per year, a \$97,000 house today is equivalent to \$268,000 15 years hence. But will one have "profited" from the inflation?

Whether one will profit from owning the house depends somewhat on an examination of the alternate use of the money. Only the differences between alternatives are relevant. If the alternate is a 5% savings account, neglecting income taxes, the profit from owning the house, rather than the savings account, would be: \$268,000 − \$97,000 (F/P, 5%, 15) = \$66,300.

On the other hand, compared to an alternative investment at 7%, the profit is \$0. And if the alternative investment is at 9% there is a loss. If "profit" means an enrichment, or being better off, then multiplying the price of everything does no enrich one in real terms.

14-19

See the table below for (a) through (e)

Year	Average Price	Inflation for Year
5 years ago	165,000.0	(a) = 1.2%
4 years ago	167,000.0	(b) = 3.0%
3 years ago	172,000.0	(c) = 4.7%
2 years ago	180,000.0	(d) = 1.7%
last year	183,000.0	(e) = 3.8%
This year	190,000.0	(f) see below

One could predict the inflation (appreciation) in the home prices this year using a number of approaches. One simple rule might involve using the average of the last 5 years inflation rates. This rate would be (1.2 + 3 + 4.7 + 1.7 + 3.8)/5 = 2.9%.

14-20

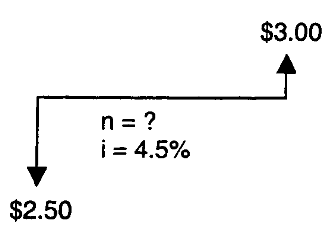
(a) Here 10 years has 12 x 10 = 120 months.
$18,000 (F/P, i, 120) = 30000$, so, $(F/P, i, 120) = (1 + i)^{120} = 1.667$
Solving for i yields 0.004266. Thus, $f_m = 0.4266\%$.

(b) $f = (1 + f_m)^{12} - 1 = (1.004266)^{12} - 1 = 0.05241$ or $f = 5.241\%$.

(c) $F = 30,000 (F/P, 5.241\%, 10) = 30,000 (1 + 0.05241)^{10} = \$50,000$

14-21

Compute equivalent interest/3 mo. = x
$i_{eff} = (1 + x)^n - 1$
$0.1925 = (1 + x)^4 - 1$
$(1 + x) = 1.1925^{0.25} = 1.045$
<u>x = 0.045 = 4.5%/3 mo.</u>



\$2.50 = \$3.00 (P/F, 4.5%, n)
(P/F, 4.5%, n) = \$2.50/\$3.00 = 0.833
n is slightly greater than 4.
So purchase pads of paper- one for immediate use plus 4 extra pads.

14-22

(a) R today \$ in year 15= \$10,000 (P/F, i_r%, 15)
i_r = (0.15 − 0.08)/1.08 = 6.5%
R today \$ in year 15 = \$10,000 $(1.065)^{15}$ = \$25,718

(b) i_c = 15%*f* = 8%
F = \$10,000 $(1.15)^{15}$ = \$81,371

14-23

Year	Cost to City (Year 0 \$)	Benefits to City	Description of Benefits
0	−\$50,000		
1−10	−\$5,000/yr	+ A	Fixed annual sum in then-current dollars
10		+\$50,000	In then-current dollars

i = i' + *f* + i'*f*
= 0.03 + 0.07 + 0.03 (0.07)
= 0.1021 = 10.21%

PW of Cost = PW of Benefits
\$50,000 + \$5,000 (P/A, 3%, 10) = A(P/A, 10.21 %, 10) +\$50,000 (P/F, 10.21%, 10)
\$50,000 + \$5,000 (8.530) = A (6.0895*) + \$50,000 (0.3783*)
\$92,650 = 6.0895A + \$18,915

A = (\$92,650 − \$18,915)/6.0895
= \$12,109

* Computed on hand calculator

14-24

No Inflation Situation

Alternative A: PW of Cost = \$6,000

Alternative B: PW of Cost = \$4,500 + \$2,500 (P/F, 8%, 8)
= \$4,500 + \$2,500 (0.5403)
= \$5,851

Alternative C: PW of Cost = \$2,500 + \$2,500 (P/F, 8%, 4) + \$2,500 (P/F, 8%, 8)
= \$2,500 (1 + 0.7350 + 0.5403)
= \$5,688

To minimize PW of Cost, <u>choose Alternative C.</u>

For *f* = +5% (Inflation)

Alternative A: PW of Cost = \$6,000

Alternative B: PW of Cost = \$4,500 + \$2,500 (F/P, 5%, 8) (P/F, 8%, 8)
= \$4,500 + \$2,500 $(1 + i\%)^8$ (P/F, 8%, 8)
= \$4,500 + \$2,500 (1.477) (0.5403)
= \$6,495

Alternative C: PW of Cost = \$2,500 + \$2,500 (F/P, 5%, 4) (P/F, 8%, 4) +
\$2,500 (F/P, 5%, 8) (P/F, 8%, 8)
= \$2,500 + \$2,500 (1.216) (0.7350) + \$2,500 (1.477)
(0.5403)
= \$6,729

To minimize PW of Cost in year 0 dollars, <u>choose Alternative A.</u>
This problem illustrates the fact that the prospect of future inflation encourages current expenditures to be able to avoid higher future expenditures.

14-25

Cash Flow:

Year	\$500 Kit	\$900 Kit
0	−\$500	−\$900
5	−\$500	\$0

(a) $PW_{\$500\ kit}$ = \$500 + \$500 (P/F, 10%, 5) = \$810
$PW_{\$900\ kit}$ = \$900
To minimize PW of Cost, <u>choose \$500 kit.</u>

(b) Replacement cost of \$500 kit, five years hence
= \$500 (F/P, 7%, 5) = \$701.5
$PW_{\$500\ kit}$ = \$500 + \$701.5 (P/F, 10%, 5) = \$935.60
$PW_{\$900\ kit}$ = \$900

To minimize PW of Cost, <u>choose \$900 kit.</u>

14-26

If one assumes the 5-year hence cost of the Filterco unit is:
$7,000 (F/P, 8%, 5) = $10,283
in Actual Dollars <u>and $7,000 in Year 0 dollars,</u> the year 0 $ cash flows are:

Year	Filterco	Duro	Duro – Filterco
0	−$7,000	−$10,000	−$3,000
5	−$7,000	$0	+$7,000

ΔROR = 18.5%
Therefore, <u>buy Filterco.</u>

14-27

Month	BTCF
0	$0
1− 36	−$1,000
36	+$40,365

$1,000 (F/A, i%, 36 mo) = $40,365
(F/A, i%, 36) = 40.365

Performing linear interpolation:

(F/A, i%, 36))	*i*
41.153	¾%
39.336	½%

i = 0.50% + 0.25% [(40.365 − 39.336)/(41.153 − 39.336)]
= 0.6416% per month
Equivalent annual interest rate
i per year = $(1 + 0.006416)^{12} − 1$ = 0.080 = 8%

So, we know that i = 8% and *f* = 8%. Find i'.
i = i' + *f* + (i') (*f*)
0.08 = i' + 0.08 + (i') (0.08)
i' = 0%

Thus, <u>Before-Tax Rate of Return = 0%</u>

14-28

(a) F = \$2,500 $(1.10)^{50}$= <u>\$293,477</u> in A\$ today

(b) R\$ today in (−50) purchasing power = \$293,477 (P/F, 4%, 50)
= <u>\$41,296</u>

14-29

(a) PW = \$2,000 (P/A, i_c, 8)
$i_{combined}$ = i_{real} + f + (i_{real}) (*f*) = 0.03 + 0.05 + (0.03) (0.05)
= <u>0.0815</u>
PW = \$2,000 (P/A, 8.15%, 9) = <u>\$11,428</u>

(b) PW = \$2,000 (P/A, 3%, 8) = <u>\$14,040</u>

14-30

Find PW of each plan over the next 5-year period.
$i_r = (i_c - f)/(1 + f)$ = (0.08 − 0.06)/1.06 = 1.19%

PW(A) = \$50,000 (P/A, 11.5%, 5) = \$236,359
PW(B) = \$45,000 (P/A, 8%, 5) + \$2,500 (P/G, 8%, 5) = \$198,115
PW(C) = \$65,000 (P/A, 1.19, 5) (P/F, 6%, 5) = \$229,612

<u>Here we choose Company A's salary to maximize PW.</u>

14-31

The Consumer Price Index (CPI) is a composite price index that is managed by the US Department of Labor Statistics. It measures the historical cost of a bundle of "consumer goods" over time. The goods included in this index are those commonly purchased by consumers in the US economy (e.g. food, clothing, entertainment, housing, etc.).

Composite indexes measure a collection of items that are related. The CPI and Producers Price Index (PPI) are examples of composite indexes. The PPI measures the cost to produce goods and services by companies in our economy (items in the PPI include materials, wages, overhead, etc.). Commodity specific indexes track the costs of specific and individual items, such as a labor cost index, a material cost index, a "football ticket" index, etc.

Both commodity specific and composite indexes can be used in engineering economic analyses. Their use depends on how the index is being used to measure (or predict) cash flows. If, in the analysis, we are interested in estimating the labor costs of a new production process, we would use a specific labor cost commodity index to develop the estimate. Much along the same lines, if we wanted to know the cost of treated lumber 5 years from today, we might use a commodity index that tracks costs of treated lumber. In the absence of commodity indexes, or in cases where we are more interested in capturing aggregate effects of inflation (such as with the CPI or PPI) one would use a composite index to incorporate/estimate how purchasing power is affected.

14-32

EAT(today) = \$330 (F/P, 12%, 10) = <u>\$1,025</u>

14-33

(a) Overall LCI change = [(250 − 100)/100] x 100% = 150%
(b) Overall LCI change = [(415 − 250)/250] x 100% = 66.8%
(c) Overall LCI change = [(650 − 417)/417] x 100% = 31.9%

14-34

(a) LCI(1970) = 100
LCI(1979) = 250
$n = 9$
$i^* = ?$
$i^* = (250/100)^{(1/9)} - 1 = 10.7\%$

(b) LCI(1980) = 250
LCI(1989) = 417
$n = 9$
$i^* = ?$
$i^* = (417/250)^{(1/9)} - 1 = 5.85\%$

(c) LCI(1990) = 417
LCI(1999) = 550
$n = 9$
$i^* = ?$
$i^* = (550/417)^{(1/9)} - 1 = 3.12\%$

14-35

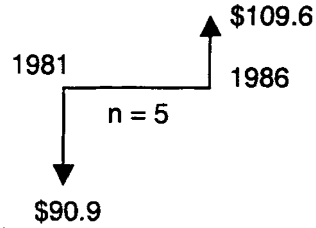
(a) CPI (1973) = 44.4
CPI (1982) = 96.5
$n = 9$
$i^* = ?$
$i^* = (96.5/44.4)^{(1/9)} - 1 = \underline{9.0\%}$

(b) CPI (1980) = 82.4
CPI (1989) = 124.0
$n = 9$
$i^* = ?$
$i^* = (124.0/82.4)^{(1/9)} - 1 = \underline{4.6\%}$

(c) CPI (1985) = 107.6
CPI (2002) = 179.9
$n = 17$
$i^* = ?$
$i^* = (179.9/107.6)^{(1/17)} - 1 = \underline{3.1\%}$

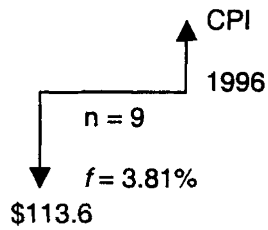
14-36

(a)



$109.6 = $90.9 (F/P, f%, 5)
(F/P, f%, 5) = $109.6/$90.9 = 1.2057
f% = 3.81%

(b)



CPI_{1996} = $113.6 (F/P, 3.81%, 9)
= $113.6 $(1 + 0.0381)^9$ = $159.0

14-37

From Table 14-2 we have P = 156.9 and F= 201.6.
The number of years = 2006 – 1996 = 10. Thus,
$F = P (1 + i)^{10}$, so, $201.6 = 156.9 (1 + i)^{10}$ giving
$i = 10^{0.010887} - 1 = 0.025385$ or 2.5385%.

To find the number of years it will take for the purchasing power of today's dollars to equal 1/5 of their present value, set
1 = 0.2 (F/P, 2.5385%, n)
$5 = (1 + 0.025385)^n$
$n = \frac{\log 5}{\log 1.025385} = 64.20$ years.

14-38

(a) Unknown quantities are calculated as follows:
a. % change = [($100 − $89)/$89] x 100% = 12.36%
b. PSI = 100 (1.04) = 104
c. % change = ($107 − $104)/$104 = 2.88%
d. % change = ($116 − $107)/$107 = 8.41%
e. PSI = 116 (1.0517) = 122

(b) The base year is 1993. This is the year of which the index has a value of 100.

(c) i. PSI (1991) = 82
PSI (1995) = 107
h = 4 years
$i^* = ?$
$i^* = (107/82)^{0.25} - 1 =$ <u>6.88%</u>

ii. PSI (1992) = 89
PSI (1998) = 132
n = 6 years
$i^* = ?$
$i^* = (132/89)^{(1/6)} - 1 =$ <u>6.79%</u>

14-39

(a)

Year	Brick Cost	CBI
1970	2.10	442
1998	X	618

x/2.10 = 618/442
x = $2.94
Total Material Cost = 800 x $2.94 = <u>$2,350</u>

(b) Here we need *f*% of brick cost
CBI(1970) = 442
CBI(1998) = 618
n = 18
$i^* = ?$
$i^* = (618/442)^{(1/18)} - 1 = 1.9\%$

We assume the past average inflation rate continues for 10 more years.
Brick Unit Cost in 2008 = 2.94 (F/P, 1.9%, 10) = $3.54
Total Material Cost = 800 x $3.54 = <u>$2.833</u>

14-40

Actual Dollars: F = \$10,000 (F/P, 10%, 15) = <u>\$41,770</u>
Real Dollars:

Year	Inflation
1−5	3%
6−10	5%
11−15	8%

R\$ in today's base = \$41,770 (P/F, 8%, 5) (P/F, 5%, 5) (P/F, 3%, 5)
= \$18,968

Thus, the real growth in purchasing power has been:
$\$18,968 = \$10,000\,(1 + i^*)^{15}$
<u>i^* = 4.36%</u>

14-41

To minimize purchase price Mary Clare should select the vehicle from company X.

Car	Current Price	Inflation	Future Price
X	27500	4.0%	30933.8
Y	30000	1.5%	31370.4
Z	25000	8.0%	31492.8
		Min =	30933.8

14-42

$F_{YEAR\ 5}$ = \$100 (F/A, 12/4 = 3%, 5 x 4 = 20) = <u>\$2,687</u>
$F_{YEAR\ 10}$ = \$2,687 (F/P, 4%, 20) + \$100 (F/A, 4%, 20) = <u>\$8,865</u>
$F_{YEAR\ 15\ (TODAY)}$ = \$8,865 (F/P, 2%, 20) + \$100 (F/A, 2%, 20) = <u>\$15,603</u>

14-43

(a) To pay off the loan Andrew will need to write a check for $ 18,116

Year	**Amt Due Begin yr**	**Loan Rate**	**Amt Due End yr**
1	15,000	5.0%	15,750.0
2	15,750.0	6.5%	16,773.8
3	16,773.8	8.0%	18,115.7
		Due =	18,115.7

(b) Payment (year 0 $) = $18,115.70 (P/F, 4%, 3) = $16,122.97

14-44

Year	Cost 1	Cost 2	Cost 3	Cost 4	TOTAL	PW−TOTAL
1	$4,500	$7,000	$10,000	$8,500	$30,000	$24,000
2	$4,613	$7,700	$10,650	$8,288	$31,250	$20,000
3	$4,728	$8,470	$11,342	$8,080	$32,620	$16,702
4	$4,846	$9,317	$12,079	$7,878	$34,121	$13,976
5	$4,967	$10,249	$12,865	$7,681	$35,762	$11,718
6	$5,091	$11,274	$13,701	$7,489	$37,555	$9,845
7	$5,219	$12,401	$14,591	$7,302	$39,513	$8,286
8	$5,349	$13,641	$15,540	$7,120	$41,649	$6,988
9	$5,483	$15,005	$16,550	$6,942	$43,979	$5,903
10	$5,620	$16,506	$17,626	$6,768	$46,519	$4,995

PW = −$60,000 − ($24,000 + $20,000 + $16,702 + … +$4,995) +
$15,000 (P/F, 25%, 10)
= <u>$180,802</u>

14-45

Item	Year 1	Year 2	Year 3
Structural	$125,160	$129,165	$137,690
Roofing	$14,280	$14,637	$15,076
Heat etc.	$35,560	$36,306	$37,614
Insulating	$9,522	$10,093	$10,850
Labor	$89,250	$93,266	$97,463
Total	$273,772	$283,467	$298,693

(a) $89,250; $93,266; $97,463

(b) PW = $9,522 (P/F, 25%, 1) + $10,093 (P/F, 25%, 2) + $10,850 (P/F, 25%, 3)
= <u>$19,632</u>

(c) FW = ($9,522 + $89,250) (F/P, 25%, 2) + ($10,093 + $93,266) (F/P, 25%, 1) +
($10,850 + $97,463) = <u>$391,843</u>

(d) PW = $273,772 (P/F, 25%, 1) + $283,467 (P/F, 25%, 2) +
$298,693 (P/F, 25%, 3) = <u>$553,367</u>

14-46

The total cost of the bike 10 years from today would be $2,770

	Current		**Future**
Item	**Cost**	**Inflation**	**Cost**
Frame	800	2.0%	975.2
Wheels	350	10.0%	907.8
Gearing	200	5.0%	325.8
Braking	150	3.0%	201.6
Saddle	70	2.5%	89.6
Finishes	125	8.0%	269.9
Sum =	1695	**Sum** =	2769.8

14-47

Let x = selling price
Then long-term capital gain = x− $18,000
Tax = 0.15 (x − $18,000)
After-Tax cash flow in year 10 = x − 0.15 (x − $18,000) = 0.85x + $2,700

Year	ATCF	Multiply by	Year 0 $ ATCF
0	−$18,000	1	−$18,000
10	+0.85x + $2,700	1.06^{-10}	0.4743x + $1,508

For a 10% rate of return:
$18,000 = (0.4746x + $1,508) (P/F, 10%, 10)
= 0.1830 x + $581
x = $95,186

Alternate Solution using an equivalent interest rate

$i_{equiv} = i' + f + (i')(f) = 0.10 + 0.06 + (0.10)(0.06) = 0.166$
So $\$18,000 (1 + 0.166)^{10} = 0.85x + \$2,700$
$83,610 = 0.85x + $2,700
Selling price of the lot = x = ($83,610 − $2,700)/0.85 = $95,188

14-48

Depreciation charges that a firm makes in its accounting records allow a profitable firm to have that amount of money available for replacement equipment without any deduction for income taxes.

If the money available from depreciation charges is inadequate to purchase needed replacement equipment, then the firm may need also to use after-tax profit for this purpose.

Depreciation charges produce a tax-free source of money; profit has been subjected to income taxes. Thus substantial inflation forces a firm to increasingly finance replacement equipment out of (costly) after-tax profit.

14-49

Year	BTCF	TI	42% Income Taxes	ATCF	Multiply by	Year 0 $ ATCF
0	−$10,000			−$10,000	1	−$10,000
1	$1,200	$1,200	−$504	$696	1.07^{-1}	$650
2	$1,200	$1,200	−$504	$696	1.07^{-2}	$608
3	$1,200	$1,200	−$504	$696	1.07^{-3}	$568
4	$1,200	$1,200	−$504	$696	1.07^{-4}	$531
5	$1,200 $10,000	$1,200	−$504	$10,696	1.07^{-5}	$7,626
					Sum	−$17

(a) **Before-Tax Rate of Return ignoring inflation**

Since the $10,000 principal is returned unchanged,
i = A/P= $1,200/$10,000 = 12%

If this is not observed, then the rate of return may be computed by conventional means.
$10,000 = $1,200 (P/A, i%, 5) + $10,000 (P/F, i%, 5)
Rate of Return = <u>12%</u>

(b) **After-Tax Rate of Return ignoring inflation**

Solved in the same manner as Part (a):
i = A/P= $696/$10,000 = 6.96%

(c) **After-Tax Rate of Return after accounting for inflation**

An examination of the Year 0 dollars after-tax cash flow shows the algebraic sum of the cash flow is −$17. Stated in Year 0 dollars, the total receipts are less than the cost, hence there is <u>no positive rate of return.</u>

14-50

Now:
Taxable Income = \$60,000
Income Taxes = \$1,565.00 + 0.15 (\$60,000 − \$15,650) = \$8,217.50
After-Tax Income = \$60,000 − \$8,218= \$51,782

Twenty Years Hence: To have some buying power, need:
After-Tax Income = \$51,782 $(1.07)^{20}$ = \$200,380
= Taxable Income – Income Taxes
Income Taxes = \$43,830.5 + 0.33 (Taxable Income − \$195,850)
Taxable Income = After-Tax Income + Income Taxes
= \$200,380 + \$43,830.5 + 0.33 (TI − \$195,850)
= \$244,210.5 + 0.33 (TI) − \$64,630.5
0.67TI = \$179,580

<u>Taxable Income = \$268,030</u>

14-51

(a) F = P(F/P, 5%, 20) = 28000 $(1+0.05)^{20}$ = (28000) (2.6533) = \$74,292

(b) P = F(P/F, 3%, 20) = 74292 $(1 + 0.03)^{-20}$ = (74292) (0.55368) = \$41,134

(c) The question seems ambiguous with respect to timing. It will be assumed here that the calculation made in part a was made in August and then the child starts college in August, 20 years later when the first tuition payment is due at the inflated value. It will also be assumed that the tuition still increases by 5% each year. Next, we have i = 7% and an income tax rate = 40%. The effective rate is (1 – 0.4) (0.07) = 0.042 . Another way to view this is to calculate the amount of money that you would earn in one year minus income taxes (where x is the original amount): x + 0.07x – (0.4) (0.07x) = x + 0.07x – 0.028x = x + 0.042x

PW = Tuition (P/F, 4.20%, n) = Tuition $(1 + 0.042)^{-n}$

Year	Tuition	Present Worth
20	\$74,292	\$32,628
21	78,007	32,878
22	81,907	33,131
23	86,002	<u>33,385</u>
	Total = \$132,022	

14-52

(a) Before-tax asset increases by 6% of the previous year's after-tax asset.

Year	Before-Tax Total Asset	Taxable Income	20% Income Tax	After-Tax Asset
0	$2,500.00			$2,500.00
1	2,650.00	$150.00	$30.00	2,620.00
2	2,777.20	157.20	31.44	2,745.76

Next, P = F (P/F, 4%, 2) = 2745.76 $(1 + 0.04)^{-2}$ = $2,538.61
Extra year 0 dollars = $2,538.61 − $2,500.00 = $38.61

(b) Same calculation with no taxes.

Year	Total Asset
0	$2,500.00
1	2,650.00
2	2,809.00

P = (2,809.00) (1 + 0.04)−2 = $2,597.08
Extra year 0 dollars = $2,597.08 − $2,500.00 = $97.08

14-53

(a)

Year	BTCF	SL Depr.	TI	34% Income Taxes	ATCF
0	−$85,000				−$85,000
1	$8,000	$1,500	$6,500	−$2,210	$5,790
2	$8,000	$1,500	$6,500	−$2,210	$5,790
3	$8,000	$1,500	$6,500	−$2,210	$5,790
4	$8,000	$1,500	$6,500	−$2,210	$5,790
5	$8,000 $77,500	$1,500	$6,500 $0	−$2,210	$83,290
Sum		$7,500			

SL Depreciation = ($67,500 − $0)/45 = $1,500
Book Value at end of 5 years = $85,000 − 5 ($1,500) = $77,500
After-Tax Rate of Return = <u>5.2%</u>

(b)

Year	BTCF	SL Depr.	TI	34% Income Taxes	Actual Dollars ATCF
0	−$85,000				−$85,000
1	$8,560	$1,500	$7,060	−$2,400	$6,160
2	$9,159	$1,500	$7,659	−$2,604	$6,555
3	$9,800	$1,500	$8,300	−$2,822	$6,978
4	$10,486	$1,500	$8,986	−$3,055	$7,431
5	$11,220 $136,935*	$1,500	$9,720	−$3,305 −$16,242**	$131,913
Sum		$7,500			

* Selling Price = $85,000 (F/P, 10%, 5) = $85,000 (1.611) = $136,935
** On disposal, there are capital gains and depreciation recapture

Capital Gain	= $136,935 − $85,00	= $51,935
Tax on Capital Gain	= (20%) ($51,935)	= <u>$10,387</u>
Recaptured Depr.	= $85,000 − $77,500	= $7,500
Tax on Recap. Depr.	= (34%)($7,500)	= <u>$2,550</u>
Total Tax on Disposal	= $10,387 + $2,550	= <u>$12,937</u>

<u>After-Tax IRR = 14.9%</u>

After-Tax Rate of Return in Year 0 Dollars

Year	Actual Dollars ATCF	Multiply by	Year 0 $ ATCF
0	−$85,000	1	−$85,000
1	$6,160	1.07^{-1}	$5,757
2	$6,555	1.07^{-2}	$5,725
3	$6,978	1.07^{-3}	$5,696
4	$7,431	1.07^{-4}	$5,669
5	$131,913	1.07^{-5}	$94,052
Sum			

In year 0 dollars, After-Tax Rate of Return = <u>7.4%</u>

14-54

Yr	BTCF	MACRS Depr.	TI	40% Income Taxes	Actual Dollars ATCF	Conv. Factor	Yr 0 $ ATCF	PW at 12%
0	−$10,000				−$10,000	1	−$10,000	−$10,000
1	$2,000	$2,000	$0	$0	$2,000	1.07^{-1}	$1,869	$1,669
2	$3,000	$3,200	−$200	+$80	$3,080	1.07^{-2}	$2,690	$2,145
3	$4,000	$1,920	$2,080	−$832	$3,168	1.07^{-3}	$2,586	$1,841
4	$5,000	$1,152	$3,848	−$1,539	$3,461	1.07^{-4}	$2,640	$1,678
5	$6,000	$1,152	$4,848	−$1,939	$4,061	1.07^{-5}	$2,895	$1,643
6	$7,000	$576	$6,424	−$2,570	$4,430	1.07^{-6}	$2,952	$1,496
7	$8,000	$0	$8,000	−$3,200	$4,800	1.07^{-7}	$2,989	$1,352
							NPW	−$1,824

Thus the Year 0 $ After-Tax Rate of Return is greater than 12% (actually 17.17%). Therefore, the purchase is justified.

14-55

Alternative I: Continue to Rent the Duplex Home
Compute the Present Worth of renting and utility costs in Year 0 dollars.
Assuming end-of-year payments, the Year 1 payment is
= ($750 + $139) (12) = $7,068

The equivalent Year 0 payment in Year 0 dollars is
$7,068 $(1 + 0.05)^{-1}$ = $6,713.40

Compute an equivalent i
$i_{equivalent} = i' + f + (i')(f)$
where i' = interest rate without inflation = 15.5%
f = inflation rate = 5%

$i_{equivalent}$ = 0.155 + 0.05 + (0.155) (0.05)
= 0.21275
= 21.275%

PW of 10 years of rent plus utilities:
= $6,731.40 (P/A, 21.275%, 10)
= $6,731.40 $[(1 + 0.21275)^{(10-1)})/(0.21275 (1 + 0.21275)^{10})]$
= $6,731.40 (4.9246)
= $33,149

An Alternative computation, but a lot more work:
Compute the PW of the 10 years of inflation adjusted rent plus utilities using 15.5% interest.

$PW_{year\ 0}$ = 12[\$589 $(1 + 0.155)^{-1}$ + \$619 $(1 + 0.155)^{-2}$ + … + \$914 $(1 + 0.155)^{-10}$]
= 12 (\$2,762.44)
= \$33,149

Alternative II: Buying a House
\$3,750 down payment plus about \$750 in closing costs for a cash requirement of \$4,500.
Mortgage interest rate per month = 8%/12 = 0.667%
n = 30 years x 12 = 360 payments

Monthly Payment: A = (\$75,000 − \$3,750) (A/P, 0.667%, 360)
= \$71,250 [(0.00667 $(1.00667)^{360}$)/($(1.00667)^{360}$ − 1)]
= −\$523.00

Mortgage Balance After the 10-year Comparison Period:
A' = \$523 (P/A, 0.667%, 240)
= \$523 [($(1.00667)^{240}$ − 1)/(0.00667 $(1.00667)^{240}$)]
= \$62,504

Thus:
\$523 x 12 x 10 = \$62,760 total payments
\$71,250 − \$62,504 = \$8,746 principal repayments (12.28% of loan)
= \$54,014 interest payments

The couple is in the 30% marginal income tax bracket. Assuming sufficient other deductions, and the interest averages 87.72% of the loan payment, their monthly tax saving will be \$523 (0.8772) (0.30) = \$138/month

The after-tax cost of the mortage = \$523 − \$138 = \$385

Sale of the property at 6% appreciation per year in year 10:

F = \$75,000 $(1.06)^{10}$	= \$134,314
Less 5% commission	= −\$6,716
Less mortgage balance	= −\$62,504
Net Income from the sale	= <u>\$65,094</u>

Assuming no capital gain tax is imposed, the Present Worth of Cost is
PW = \$4,500 [down payment + closing costs in constant dollars]
+ \$385 x 12 (P/A, 15.5%, 10) [actual dollar mortgage]
+ \$160 x 12 (P/A, 10%, 10) [constant dollar utilities]
+ \$50 x 12 (P/A, 10%, 10) [constant dollar insurance & maintenance]
− \$65,094 (P/F, 15.5%, 10) [actual dollar net income from sale]

PW = \$4,500 + \$385 x 12 (4.9246) + \$160 x 12(6.145)
+ \$50 x 12 (6.145)− \$65,094 (0.2367)
= \$27,329

The PW of Cost of owning the house for 1 year = \$27,329 in Year 0 dollars.
Thus \$33,149 > \$27,329 and so buying a house is the more attractive alternative.

14-56

Year	BTCF	MACRS Depr.*	TI	35% Income Taxes	Actual Dollars ATCF
0	−\$150,000				−\$150,000
1	\$15,750	\$3,607	\$12,143	−\$4,250	\$11,500
2	\$15,750	\$3,763	\$11,987	−\$4,195	\$11,555
3	\$15,750	\$3,763	\$11,987	−\$4,195	\$11,555
4	\$15,750	\$3,763	\$11,987	−\$4,195	\$11,555
5	\$14,438 \$150,000	\$3,607	\$10,831 \$18,503	−\$3,791 −\$6476**	\$154,170
Sum		\$18,503			

* See table below.
** A 35% tax rate is used on the Recaptured Depreciation:
(\$18,503)(0.35) = \$6476.

After-Tax Rate of Return is 6.84%

MACRS Depreciation
Home = \$150,000 − \$46,500 = \$103,500

Year		MACRS Depreciation
1	3.485% (\$103,500)	= \$3,607
2	3.636% (\$103,500)	= \$3,763
3	3.636% (\$103,500)	= \$3,763
4	3.636% (\$103,500)	= \$3,763
5	3.485% (\$103,500)	= \$3,607

14-57

Year	Actual $ BTCF	Mkt. Val. Of Property + 12%/yr	MACRS Deprec	TI	35% Income Taxes	Actual $ ATCF
0	−$150,000	$150,000				−$150,000
1	$12,000	$168,000	$3,607	$8,393	−$2,938	$9,062
2	$13,440	$188,160	$3,763	$9,677	−$3,387	$10,053
3	$15,053	$210,739	$3,763	$11,290	−$3,951	$11,101
4	$16,859	$236,028	$3,763	$13,096	−$4,584	$12,275
5†	$17,309* $261,991*	$261,991*	$3,607	$13,702	−$4,796 −$28,874	$245,630
Sum			$18,503			

(see table footnotes below)

† Note that is a slight error assuming the 11 month values occur at end of Year 5.

* Assume 11 months rents and 11 month increase in Year 5 market value.

* Selling Price = $261,991

** On disposal, there are capital gains and depreciation recapture

Capital Gain = $261,991 − $150,000 = $111,991

Tax on Capital Gain = (20%) ($111,991) = $22,398

Recaptured Depreciation = $150,000 − ($150,000−$18,503) = $18,503

Tax on Recaptured Depreciation = (35%)($18,503) = $6,474

Total Tax on Disposal = $22,398 + $6,476 = $28,874

(a) After Tax IRR = 15.3%

After-Tax Rate of Return in Year 0 Dollars

Year	Actual Dollars ATCF	Multiply by	Year 0 $ ATCF
0	−$150,000	1	$−150,000
1	$9,062	1.1^{-1}	$8,239
2	$10,053	1.1^{-2}	$8,308
3	$11,102	1.1^{-3}	$8,341
4	$12,275	1.1^{-4}	$8,384
5	$245,630	1.1^{-5}	$152,517

In Year 0 dollars, After-Tax Rate of Return = 4.8%

Alternate Solution

$i = i' + f + (i')(f)$

$0.153 = i' + 0.10 + 0.10i'$

$i' = 0.053/1.10 = 0.048$ = 4,8%

14-58

Alternative A

Year	Cash Flow in Year 0 $	Cash Flow in Actual $	SL Depr.	TI	25% Income Tax	ATCF in Actual $	ATCF in Year 0 $
0	−$420	−$420				−$420	−$420
1	$200	$210	$140	$70	−$17.5	$192.5	$183.3
2	$200	$220.5	$140	$80.5	−$20.1	$200.4	$181.8
3	$200	$231.5	$140	$91.5	−$22.9	$208.6	$180.2

Alternative B

Year	Cash Flow in Year 0 $	Cash Flow in Actual $	SL Depr.	TI	25% Income Tax	ATCF in Actual $	ATCF in Year 0 $
0	−$300	−$300				−$300	−$300
1	$150	$157.5	$100	$57.5	−$14.4	$143.1	$136.3
2	$150	$165.4	$100	$65.4	−$16.4	$149.0	$135.1
3	$150	$173.6	$100	$73.6	−$18.4	$155.2	$134.1

Quick Approximation of Rates of Return:

Alternative A:
$420 = $182 (P/A, i%, 3)
(P/A, i%, 3) = $420/$182 = 2.31

12% < ROR < 15%
(Actual ROR = 14.3%)

Alternative B:
$300 = $135 (P/A, i%, 3)
(P/A, i%, 3) = $300/$135 = 2.22

15% < ROR < 18%
(Actual ROR = 16.8%)

Incremental ROR Analysis for A – B

Year	A	B	A – B
0	−$420	−$300	−$120
1	$183.3	$136.3	$47
2	$181.8	$135.1	$46.7
3	$180.2	$134.1	$46.1

Try i = 7%
NPW = −$120 + $47 (P/F, 7%, 1) + $46.7 (P/F, 7%, 2)
+ $46.1 (P/F, 7%, 3)
= +$2.3
So the rate of return for the increment A – B is greater than 7% (actually 8.1%).

Choose the higher cost alternative — choose Alternative A.

Chapter 15: Selection of a Minimum Attractive Rate of Return

15-1

The interest rates on these securities vary greatly over time, making it impossible to predict rates. Three factors that distinguish the three securities:

	Bond Income	Bond Duration	Bond Safety
U.S. Treasury Bond	Taxable	5 years	Safest
Municipal Bond	Not Taxable	20 years	Safe
Corporate Bond	Taxable	20 years	Less Safe

The importance of the nontaxable income feature usually makes the municipal bond the one with the lowest interest rate. Next higher, with its safety and shorter duration, is the Treasury bond. The corporate bond generally will have the highest interest rate.

15-2

Cost of capital = [125K(0.08) + 75K(0.12)] / [125K + 75K] = 0.095 or 9.5%.

15-3

Cost of capital = [725K(0.07) + 600K(0.11) + 1200K(0.11)] / [725K + 600K + 1200K]
= 0.0985 or 9.85%.

15-4

Cost of capital = [12M(0.15) + 5M(0.07) + 8M(0.06)] / [12M + 5M + 8M]
= 0.1052 or 10.52%.

15-5

Cost of capital = [22M(0.18) + 9M(0.08) + 14M(0.04)] / [22M + 9M + 14M]
= 0.1164 or 11.64%.

15-6

Investment opportunities may include:
1. Deposit of the money in a Bank.
2. Purchase of common stock, U.S. Treasury bonds, or corporate bonds.
3. Investment in a new business, or an existing business.
4. And so on.

Assuming the student has a single investment in which more than $2,000 could be invested, the MARR equals the projected rate of return for the investment.

15-7

Venture capital syndicates typically invest money in situations with a substantial amount of risk. The process of identifying and selecting investments is a time-consuming (and hence costly) process. The group would therefore only make a venture capital investment where (they think) the rate of return will be high- probably 25% or more.

15-8

CPI – U US City Average (All Items) = 2.0% since August 2006 (August 2007 number) obtained 10/12/2007 from U.S. Dept. of Labor at www.bls.gov/cpi/#overview.

Two-year CD rate = 4.73% average (Indiana) obtained 10/12/2007 from www.bankrate.com.

$i' = \frac{i-f}{1+f} = \frac{0.0473-0.020}{1+0.020} = 0.026765$, so $i' = 2.68\%$.

15-9

CPI – U US City Average (All Items) = 2.0% since August 2006 (August 2007 number) obtained 10/12/2007 from U.S. Department of Labor at www.bls.gov/cpi/#overview.

48-month new car loan overnight average (Indiana) = 6.73% obtained 10/12/2007 from www.Bankrate.com.

$i' = \frac{i-f}{1+f} = \frac{0.0673-0.020}{1+0.020} = 0.046373$, so $i' = 4.64\%$.

15-10

For the inflation rate over the last 10 years between 1996 and 2006 use the CPI from Table 14-2 (see Problem 14-37): 201.6 = 156.9 $(1 + f)^{10}$ yielding f = 2.539%.

The "Dow" average over the same period has changed from 6000 (Oct 1996) to 11,700 (Oct. 2006) obtained from www.djindexes.com and www.smallinvestors.com.

Thus,
11,700 = 6,000 (F/P, i, 10) = 6,000 $(1 + i)^{10}$, yielding i = 0.06906 or 6.906%.

Finally, $i' = \frac{i-f}{1+f} = \frac{0.06906 - 0.02539}{1+0.02539} = 0.04259$ or 4.26%.

15-11

For the inflation rate over the last 10 years between 1996 and 2006 use the CPI from Table 14-2 (see Problem 14-37): 201.6 = 156.9 $(1 + f)^{10}$,yielding f = 2.539%.

The NASDAQ average over the same period has changed from 1000 (Oct. 1996) to 2,300 (Oct. 2006) obtained from www.money.cnn.com.

Thus,
2,300 = 1,000 (F/P, i, 10) = 1,000 $(1 + i)^{10}$,yielding i = 0.08686 or 8.686%.

Finally, $i' = \frac{i-f}{1+f} = \frac{0.08686 - 0.02539}{1+0.02539} = 0.05995$ or 6.00%.

15-12

The IRR for each project is calculated using the Excel function = RATE (life, annual benefit, first cost, salvage value), and then the table is sorted with IRR as the key. Projects A and B are the top two projects, which fully utilize the $100,000 capital budget. The opportunity cost of capital is 12.0% if based on the first project rejected.

Project	IRR	First Cost	Annual Benefits	Life	Salvage Value
A	13.15%	$50,000	$13,500	5 yr	$5,000
B	12.41%	$50,000	$9,000	10 yr	$0
D	11.99%	$50,000	$9,575	8 yr	$6,000
C	10.66%	$50,000	$13,250	5 yr	$1,000

15-13

The IRR for each project is calculated using the Excel function = RATE (3, annual benefit, first cost) since N = 3 for all projects. Then the table is sorted with IRR as the key. Do projects 3, 1 and 7 with a budget of $70,000. The opportunity cost of capital is 26.0% if based on the first project rejected.

Project	IRR	Cumulative First Cost	First Cost	Annual Benefit
3	36.31%	$10,000	$10,000	$6,000
1	29.92%	$30,000	$20,000	$11,000
7	26.67%	$70,000	$40,000	$21,000
5	26.01%	$95,000	$25,000	$13,000
4	20.71%	$100,000	$5,000	$2,400
2	18.91%	$130,000	$30,000	$14,000
6	18.91%	$145,000	$15,000	$7,000

15-14

The IRR for each project is calculated using the Excel function = RATE (life, annual benefit, first cost, salvage value), and then the table is sorted with IRR as the key. With a budget of $500,000, the opportunity cost of capital is 19.36% if based on the first project rejected. Projects 3, 1, 4, and 6 should be done.

Project	IRR	Cumulative First Cost	First Cost	Annual Benefit	Life (years)
3	28.65%	$100,000	$100,000	$40,000	5
1	24.01%	$300,000	$200,000	$50,000	15
4	21.41%	$350,000	$50,000	$12,500	10
6	20.85%	$500,000	$150,000	$32,000	20
2	19.36%	$800,000	$300,000	$70,000	10
7	16.99%	$1,200,000	$400,000	$125,000	5
5	15.24%	$1,450,000	$250,000	$75,000	5

15-15

The IRR for each project is calculated using the Excel function = Rate (life, annual benefit, first cost), and then the table is sorted with IRR as the key. The top 6 projects required $260K in capital funding, and the opportunity cost of capital based on the first rejected project is 8.0%.

Project	IRR	Cumulative First Cost	First Cost	Annual Benefit	Life (years)
E	15.00%	$40,000	$40,000	$11,933	5
H	13.44%	$100,000	$60,000	$12,692	8
C	12.00%	$130,000	$30,000	$9,878	4
G	10.97%	$165,000	$35,000	$6,794	8
I	10.00%	$240,000	$75,000	$14,058	8
B	9.00%	$260,000	$20,000	$6,173	4
D	8.00%	$285,000	$25,000	$6,261	5
A	7.01%	$300,000	$15,000	$4,429	4
F	5.00$	$350,000	$50,000	$11,550	5

15-16

The IRR for each project is calculated using the Excel function = RATE (life, annual benefit, first cost, salvage value), and then the table is sorted with IRR as the key. With a budget of $100,000, the top 5 projects should be done (6, 5, 4, 1, and 7). The opportunity cost of capital based on the first rejected project is 16.41%.

Project	IRR	First Cost	Annual Benefits	Life (years)	Salvage Value
6	26.16%	$20,000	$5,800	10	$0
5	22.50%	$20,000	$4,500	25	−$20,000
4	21.25%	$20,000	$4,500	15	$0
1	19.43%	$20,000	$4,000	20	$0
7	19.26%	$20,000	$4,000	15	$10,000
3	16.41%	$20,000	$3,300	30	$10,000
2	16.00%	$20,000	$3,200	20	$20,000

15-17

Assume Project B in Problem 15-12 is a “new product in a new market.” From Table 15-1 the interest rate that should be used for this project is 16%.

	First Cost	Annual Benefit	Life (years)	Salvage
Project B	$50,000	$9,000	10	0

NPW = 0 = –50,000 + 9,000 (P/A, i, 10), so, (P/A, i, 10) = 5.5556 and interpolating

i = 12% + (3%) $\left[\frac{5.650-5.5556}{5.650-5.019}\right]$ = 12.45% < 16%, so, the project should NOT be done.

15-18

Assume Project E in Problem 15-15 is a “new product in an existing market.” From Table 15-1 assuming “existing” = “normal” the interest rate that should be used for this project is 10%.

	First Cost	Annual Benefit	Life (years)	Salvage
Project E	$40,000	$11,933	5	0

NPW = 0 = –40000 + 11933 (P/A, i, 5), so, (P/A, i, 5) = 3.352 and from tables
i = 15% (exact) > 10%, so, the project should be done.

15-19

Assume Project 1 in Problem 15-16 is a “new product in an foreign market.” From Table 15-1 the interest rate that should be used for this project is 20%.

	First Cost	Annual Benefit	Life (years)	Salvage
Project 1	$20,000	$4,000	20	0

NPW = 0 = –20,000 + 4000 (P/A, i, 20), so, (P/A, i, 20) = 5.000 and interpolating

i = 18% + (2%) $\left[\frac{5.353-5.000}{5.353-4.870}\right]$ = 19.46% < 20%, so, the project should NOT be done.

15-20

(a) With no budget constraint, do all projects except Project #4.
Cost = $115,000

(b) Ranking the 9 projects by NPW/Cost

Project	Cost	Uniform Benefit	NPW at 12%	NPW/Cost
1	$5	$1.03	$0.82	0.16
2	$15	$3.22	$3.19	0.21
3	$10	$1.77	$0	0
5	$5	$1.19	$1.72	0.34
6	$20	$3.83	$1.64	0.08
7	$5	$1.00	$0.65	0.13
8	$20	$3.69	$0.85	0.04
9	$5	$1.15	$1.50	0.30
10	$10	$2.23	$2.60	0.26

Projects ranked in order of desirability

Project	Cost	NPW at 12%	NPW/Cost	Cumulative Cost
5	$5	$1.72	0.34	$5
9	$5	$1.50	0.30	$10
10	$10	$2.60	0.26	$20
2	$15	$3.19	0.21	$35
1	$5	$0.82	0.16	$40
7	$5	$0.65	0.13	$45
6	$20	$1.64	0.08	$65
8	$20	$0.85	0.04	$85
3	$10	$0	0	$95

(c) At $55,000 we have more money than needed for the first six projects ($45,000), but not enough for the first seven projects ($65,000). This is the "lumpiness" problem. There may be a better solution than simply taking the first six projects, with total NPW equal to 10.48. There is in this problem. By trial and error we see that if we forego Projects 1 and 7, we have ample money to fund Project 6. For this set of projects, ∑ NPW = 10.65.

To maximize NPW the proper set of projects for $55,000 capital budget is:

<u>Projects 5, 9, 10, 2, and 6</u>

15-21

(a) Approve all projects except D.

(b) Ranking Computations for NPW/Cost

Project	Cost	Uniform Benefit	NPW at 14%	NPW/Cost
A	$10	$2.98	$0.23	0.023
B	$15	$5.58	$4.16	0.277
C	$5	$1.53	$0.25	0.050
D	$20	$5.55	−$0.95	−0.048
E	$15	$4.37	$0	0
F	$30	$9.81	$3.68	0.123
G	$25	$7.81	$1.81	0.072
H	$10	$3.49	$1.98	0.198
I	$5	$1.67	$0.73	0.146
J	$10	$3.20	$0.99	0.099

Ranking:

Project	Cost	NPW/Cost	Cumulative Cost
B	$15	0.277	$15
H	$10	0.198	$25
I	$5	0.146	$30
F	$30	0.123	$60
J	$10	0.099	$70
G	$25	0.072	$95
C	$5	0.050	$100
A	$10	0.023	$110
E	$15	0	$125
D	$20	−0.048	$145

(c) Budget = $85,000
The first five projects (B, H, I, F, and J) equal $70,000. There is not enough money to add G, but there is enough to add C and A. Alternately, one could delete J and add G. So two possible selections are:

B H I F G NPW(14%) = $28.36
B H I F J C A NPW(14%) = $28.26

For $85,000, maximize NPW.
<u>Choose: B, H, I, F, and G.</u>

15-22

(a) Select projects, given MARR = 10%. Incremental analysis is required.

Project		Δ Cost	Δ Uniform Annual Benefit	Δ Rate of Return	Conclusion
1	Alt. 1A − Alt. 1C	$15	$2.22	7.8%	Reject 1A
	Alt. 1B − Alt. 1C	$40	$7.59	13.7%	Reject 1C Select 1B
2	Alt. 2B − Alt. 2A	$15	$2.57	11.2%	Reject 2A Select 2B
3	Alt. 3A − Alt. 3B	$15	$3.41	18.6%	Reject 3B Select 3A
4		$10	$1.70	11%	Select 4

Conclusion: Select Projects 1B, 2B, 3A, and 4.

(b) Rank separable increments of investment by rate of return

Alternative	Cost or Δ Cost	Δ Rate of Return	For Budget of $100,000
1C	$10	20%	
3A	$25	18%	3A $25
2A	$20	16%	2A $20
1B − 1C	$40	13.7%	1B $50
2B − 2A	$15	11.2%	−
4	$10	11%	−
			Σ = $95

* The original choice of 1C is overruled by the acceptable increment of choosing 1B instead of 1C.

Conclusion: Select Projects 3A, 2A, and 1B.

(c) The cutoff rate of return equals the cost of the best project foregone. Project 1B, with a Rate of Return of 13.7% is accepted and Project 2B with a Rate of Return of 11.2% is rejected. Therefore the cutoff rate of return is actually 11.2%, but could be considered as midway between 13.7% and 11.2% (12%).

(d) Compute NPW/Cost at i = 12% for the various alternatives

Project	Cost	Uniform Benefit	NPW	NPW/Cost
1A	$25	$4.61	$1.05	0.04
1B	$50	$9.96	$6.28	0.13
1C	$10	$2.39	$3.50	0.35
2A	$20	$4.14	$3.39	0.17
2B	$35	$6.71	$2.91	0.08
3A	$25	$5.56	$6.42	0.26
3B	$10	$2.15	$2.15	0.21
4	$10	$1.70	−$0.39	−0.03

Project Ranking

Project	Cost	NPW/Cost
1C	$10	0.35
3A	$25	0.26
3B	$10	0.21
2A	$20	0.17
1B	$50	0.13
2B	$35	0.08
1A	$25	0.04
4	$10	−0.03

(e) For a budget of $100 x 10^3, select:
3A($25) + 2A($20) + 1B ($50) thus ∑ = $95

15-23

(a) Cost to maximize total ohs — no budget limitation Select the most appropriate gift for each of the seven people

Recipient	Gift	Oh Rating	Cost
Father	Shirt	5	$20
Mother	Camera	5	$30
Sister	Sweater	5	$24
Brother	Camera	5	$30
Aunt	Candy	5	$20
Uncle	Sweater	4	$24
Cousin	Shirt	4	$20
Total			$168

Cost of Best Gifts = <u>$168</u>

(b) This problem differs from those described in the book where a project may be rejected by selecting the do-nothing alternative. Here, each person must be provided a gift. Thus while we can move the gift money around to maximize "ohs", we cannot eliminate a gift. This constraint destroys the validity of the NPW-p (PW of Cost) or Ohs – P (Cost) technique.

The best solution is to simplify the problem as much as possible and then to proceed with incremental analysis. The number of alternatives may be reduced by observing that since the goal is to maximize "ohs," for any recipient one should not pay more than necessary for a given number of "ohs," or more dollars for less "ohs."

For example, for Mother the seven feasible alternatives (the three O-oh alternatives are not feasible) are:

Alternative	Cost	Ohs
1	$20	4
4	$20	3
5	$24	4
6	$30	5
8	$16	3
9	$18	4
10	$16	2

Careful examination shows that for five ohs, one must pay $30, for four ohs, $18, and $16 for three ohs. The other three and four oh alternatives cost more, and the two alternative costs the same as the three oh alternatives.

Thus for Mother the three dominant alternatives are:

Alternative	Cost	Ohs
6	$30	5
9	$18	4
10	$16	2

All other alternatives are either infeasible or inferior.

If the situation is examined for each of the gift recipients, we obtain:

Ohs	Father Cost	Father Δ Cost /oh	Mother Cost	Mother Δ Cost /oh	Sister Cost	Sister Δ Cost /oh	Brother Cost	Brother Δ Cost /oh
5	$20		$30		$24		$30	
		$4		$12		$8		$14
4	$16		$18		$16		$16	
		$4		$2				
3	$12		$16					
						$3.3		$1.3
2								
1					$6		$12	

Ohs	Aunt Cost	Aunt Δ Cost /oh	Uncle Cost	Uncle Δ Cost /oh	Cousin Cost	Cousin Δ Cost /oh
5	$20					
		$2				
4	$18		$24		$20	
		$2		$8		$4
3	$16		$16		$16	
		$1.3				$4
2				$2	$12	$4.6
		$5				$6
1	$6		$12		$6	

In part (a) we found that the most appropriate gifts cost $168. This table confirms that the gifts with the largest oh for each person cost $20 + $30 + $24 + $30 + $20 +$24 + $20 = $168. (This can be found by reading across the top of the table on the previous page.)

For a budget limited to $112 we must forego increments of Cost/Oh that consume excessive dollars. The best saving available is to go from a five-oh to a four-oh gift for Brother, thereby savings $14. This makes the cost of the seven gifts = $168 − $14 = $154. Further adjustments are required, first on Mother, then Sister, then Father, and finally a further adjustment of Sister. The selected gifts are:

Recipient	Gift	Ohs	Cost
Father	Shirt	5	$20
Mother	Book	4	$18
Sister	Magazine	4	$16
Brother	Magazine	4	$16
Aunt	Candy	5	$20
Uncle	Necktie	3	$16
Cousin	Calendar	1	$6
Total		26	$112

(c) For a budget of $90 the process described above must be continued. The selected gifts are:

Recipient	Gift	Ohs	Cost
Father	Cigars	3	$12
Mother	Book	4	$18
Sister	Magazine	4	$16
Brother	Magazine	4	$16
Aunt	Calendar	1	$6
Uncle	Necktie	3	$16
Cousin	Calendar	1	$6
Total		20	$90

15-24

The solution will follow the approach of Example 17-5. The first step is to compute the rate of return for each increment of investment.

Project A1—no investment

Project A2 (A2–A1)

Year	Cash Flow	PW at 20%
0	−$500,000 (keep land)	−$500,000
1- 20	+$98,700	+$480,669
20	+$750,000	$15,000
Total		+$244

Therefore, Rate of Return ≈ 20%.

Project A3 (A3–A1)
Expected Annual Rental Income
= 0.1 ($1,000,000) + 0.3 ($1,100,000) + 0.4 ($1,200,000) + 0.2 ($1,900,000)
= $1,290,000

Year	Cash Flow	PW at 18%
0	−$5,000,000	−$5,000,000
1- 2	$0	$0
3- 20	+$1,290,000	+$4,885,200
20	+$3,000,000	+$109,000
Total		−$5,300

Therefore, Rate of Return ≈ 18%.

Project A3–Project A2

Year	Project A3	Project A2	A3–A2
0	−$5,000,000	−$500,000	−$4,500,000
1	$0	+$98,700	−$98,700
2	$0	+$98,700	−$98,700
3- 20	+$1,290,000	+$98,700	+$1,191,300
20	+$3,000,000	+$750,000	+$2,250,000

Year	A3–A2	PW at 15%	PW at 18%
0	−$4,500,000	−$4,500,000	−$4,500,000
1	−$98,700	−$85,830	−$83,650
2	−$98,700	−$74,630	−$70,890
3- 20	+$1,191,300	+$5,519,290	+$4,511,450
20	+$2,250,000	+$137,480	+$82,120
Total		+$996,310	−$60,970

Δ Rate of Return ≈ 17.7% (HP−12C Answer = 17.8%)

Project B
Rate of Return = $i_{eff} = e^r - 1 = e^{0.1375} - 1 = 0.1474 = 14.74\%$

Project C

Year	Cash Flow	PW at 18%
0	−$2,000,000	−$2,000,000
1- 10	+$500,000	+$1,785,500
10	+$2,000,000	+$214,800
Total		+$300

Actually the rate of return is exactly $500,000/$2,000,000 = 25%.

Project D
Rate of Return = 16%

Project E
$i_{eff} = (1 + 0.1406/12)^{12} - 1 = 15.00\%$

Project F

Year	Cash Flow	PW at 18%
0	−$2,000,000	−$2,000,000
1	+$1,000,000	+$847,500
2	+$1,604,800	+$1,152,600
Total		+$100

Rate of Return = 18%

Rank order of increments of investment by rate of return

Project	Increment	Rate of Return
C	$2,000,000	25%
A2	$500,000	20%
F	$2,000,000	18%
A3- A2	$4,500,000	17.7%
D	$500,000	16%
E	Any amount > $100,000	15%
B	Not stated	14.7%

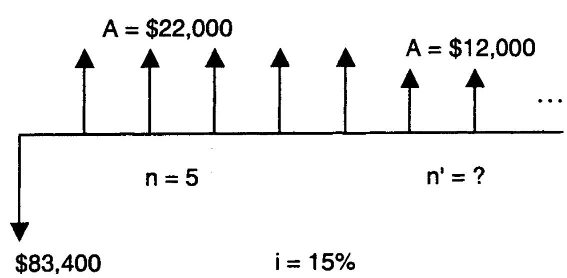
Note that $500,000 value of Project A land is included.

(a) Budget = $4 million (or $4.5 million including Project A land)
Go down the project list until the budget is exhausted
Choose Project C, A2, and F.
MARR= Cutoff rate of Return = Opportunity cost ≈ 17.7%–18%

(b) Budget = $9 million (or $9.5 million including Project A land)
Again, go down the project list until the budget is exhausted.
Choose Projects C, F, A3, D.
Note that this would become a lumpiness problem at a capital budget of $5 million (or many other amounts).

15-25

Project 1: Liquid Storage Tank
Saving at 0.1 cent per kg of soap:
First five years = $0.001 x 22,000 x 1,000 = $22,000
Subsequent years = $0.001 x 12,000 x 1,000 = $12,000
How long must the tank remain in service to produce a 15% rate of return?



$83,400 = $22,000 (P/A, 15%, 5) + $12,000 (P/A, 15%, n’) (P/F, 15%, 5)
= $22,000 (3.352) + $12,000 (P/A, 15%, n’) (0.4972)

(P/A, 15%, n’) = 1.619

n’ = 2 years (beyond the 5-year contract)

Thus the storage tank will have a 15% rate of return for a useful life of 7 years. This appears to be far less than the actual useful life of Raleigh's tank.
Install the Liquid Storage Tank.

Project 2: Another sulfonation unit
There is no alternative available, so the project must be undertaken to provide the necessary plant capacity.
Install Solfonation Unit.

Project 3: Packaging department expansion
Cost = $150,000
Salvage value at tend of 5 years = $42,000
Annual saving in wage premium = $35,000

Rate of Return:
$150,000 − $42,000 (P/F, i%, 5) = $35,000 (P/A, i%, 5)

Try i = 12%
$150,000 − $42,000 (0.5674) = $35,000 (3.605)
$126,169 = $126,175
The rate of return is 12%.
Reject the packaging department expansion and plan on two-shift operation.

Projects 4 & 5: New warehouse or leased warehouse
Cash Flow

Year	Leased Warehouse	New Warehouse	New Rather than Leased
0	$0	−$225,000	−$225,000
1	−$49,000	−$5,000	+$44,000
2	−$49,000	−$5,000	+$44,000
3	−$49,000	−$5,000	+$44,000
4	−$49,000	−$5,000	+$44,000
5	−$49,000	−$5,000 + $200,000	+$244,000

Compute the rate of return on the difference between the alternatives.
$225,000 = $44,000 (P/A, i%, 5) + $200,000 (P/F, i%, 5)

Try i = 18%
$225,000 = $44,000 (3.127) + $200,000 (0.4371)
= $225,008

The incremental rate of return is 18%.
Build the new warehouse.

15-26

Project	Cost (P)	Annual Benefit (A)	(A/P, i%, 10)	ROR
1A	$5,000	$1,192.50	0.2385	20%
1B- 1A	$5,000	$800.50	0.1601	9.6%
2A	$15,000	$3,337.50	0.2225	18%
2B- 2A	$10,000	$1,087.50	0.1088	1.6%

(a) 1A

(b) 8%

(c) 1B and 2A

Chapter 16: Economic Analysis in the Public Sector

16-1

Public decision making involves the use of public money and resources to fund public projects. Often there are those who are advocating for particular projects, those who oppose projects, those who will be immediately affected by such project, and those who may be affected in the future. There are those who represent their own stated interests, and those who are representing others' interests. Thus the "multi-actor" aspect of the phrase refers to the varied and wide group of "stakeholders" who are involved with, affected by, or place some concern on the decision process.

16-2

Public decision making is focused on *promoting the general welfare* of the aggregate public. There is an explicit recognition in promoting the good of the whole, in some cases, that individual's goals must be subordinate (e.g., eminent domain). Private decision making, on the other hand, is generally focused on increasing stakeholder wealth or investment. This is not to say that private decision making is entirely focuses on financials, clearly private decision making focuses on nonmonetary issues. However, the goal and objective of the enterprise is economic survival and growth, and thus the primary objective is financial in nature (for without financial success all other objectives are moot is the firm dissolves).

16-3

The general suggestion is that the viewpoint should be at least as broad as those who pay the costs and/or receive the benefits. This approach balances local decisions, which may suboptimize decision making if not taken. Example 16-1 describes this dilemma for a municipal project funded partly by federal money (50%). In this example, it still made sense to approve the project from the municipality's viewpoint but not the federal government, after the benefit estimate was revised.

16-4

This phrase refers to the fact that most benefits are confined locally for government investments. As the authors state, "Other than investments in defense and social programs, most benefits provided by government are realized at the local or regional levels." This is true for projects funded with full or partial government money. The conflict arises when some regions, states, municipalities perceive that they are consistently passed over for projects that would benefit their region, state, municipality. Powerful members in Congress and state legislatures with key committee/subcommittee appointments can influence government spending in their districts. Politics have an effect in this regard. However, many projects, including the U.S. parks system, the interstate highway, and others reach many beyond even regional levels.

16-5

This is a list of potential costs, benefits, and disbenefits for a nuclear power plant.

Costs	Benefits	Disbenefits
Land Acquisition	Environment	Fission product material to contend with forever
Site Preparation	- No greenhouse gas	Not in my backyard
Cooling System	- No leakage	Risk of reactor
- Reservoir dams	- No combustion	- Real
- Reservoir cooling	Jobs & Economy	- Psychological
Construction	- At enrichment plants	Loss to economy
- Reactor vessel/core	- At power plant	- Coal
- Balance of plant	- Increase tax base	- Electric
- Spent fuel storage	Increase Demand	
- Water cleaning	- Uranium plants	

16-6

Students will have a variety of answers. They may include the following:

Costs	Benefits	Disbenefits
Buy property (land acquisition) New signage and traffic lights Construction -Breakup and removal of old concrete -Earth moving, grading -New cement/blacktop -Lane-line painting -Labor, management -Utility relocation -Surveying -Design work	Increased traffic flow Increased safety Increase in jobs Economic increase	Traffic disruption during construction Increase in noise and dust during construction Land acquisition -Loss of two gas stations -Loss of bank -Loss of church

From the city's viewpoint, virtually all the items listed in columns two and three must be included in the evaluation of the project since it is receiving virtually all of the benefits (and disbenefits) and, also, they should consider their share of the costs (30%).

The state is receiving very little benefit except "some through traffic"; thus, they probably should use increased traffic flow, traffic disruption during construction, and their share of all costs (70%).

The state's viewpoint should be used to evaluate the project since it is providing the largest share of the funding.

16-7

Overpass Cost = $1,800,000	Salvage Value = $100,000	n = 30	i = 6%

Benefits to Public
Time Saving for 1000 vehicles per day
400 trucks x (2 min/60 min/hr) x ($18/hr) = $240 per day
600 others x (2 min/60 min/hr) x ($5/hr) = $100 per day
Total = $340 per day

Benefits to the State
Saving in accident investigation costs = $6,000 per year

Combined Benefits
Benefits to the Public + Benefits to the State
= $340/day (365 days) + $6,000 = $130,100 per year

Benefits to the Railroad
Saving in crossing guard expense = $48,000 per year
Saving in accident case expense = $60,000 per year
Total = $108,000 per year

Should the overpass be built?

Benefit–Cost Ratio Analysis
Annual Cost (EUAC) = $1,700,000 (A/P, 6%, 30) + $100,000 (0.06)
= $1,700,000 (0.0726) + $6,000
= $129,420

Annual Benefit (EUAB) = $130,100 + $108,000
= $238,100

B/C = EUAB/EUAC = $238,100/$129,420 = 1.84

With a B/C ratio > 1, the project is economically justified.

Allocation of the $1,800,000 cost
The railroad should contribute to the project in proportion to the benefits received.

PW of Cost = $1,800,000 − $100,000 (P/F, 6%, 30)
= $1,800,000 − $100,000 (0.1741)
= <u>$1,782,590</u>

The railroad portion would be
($108,000/$238,100) ($1,782,590) = <u>$808,567</u>

The State portion would be
($130,100/$238,100) ($1,782,590) + $100,000 (P/F, 6%, 30)
= ($130,100/$238,100) ($1,782,590) + $100,000 (0.1741)
= <u>$991,433</u>

Note that $808,567 + $991,433 = $1,800,000
While this problem is a simplified representation of the situation, it illustrates a realistic statement of benefits and an economic analysis solution to the allocation of costs.

16-8

Students will pull elements from the discussion of this topic in the textbook. In the text the concepts discussed include (1) No Time Value of Money, (2) Cost of Capital, and (3) Opportunity Cost. The Recommended Concept is to select the largest of the cost of capital, the government opportunity cost, or the taxpayer opportunity cost.

16-9

Based on B/C ratios and its capital budget, projects B, C, and E should be funded (all of the $600,000 budgeted is used up). The government's opportunity cost is then 19% based on project D which is the best opportunity foregone. This decision method did not work well because two of the three projects not funded had higher rates-of-return than two of the funded projects.

16-10

Based on B/C ratios and its capital budget, projects B, D, E, and F should be funded (all of the budgeted $9M is used up efficiently). The government's opportunity cost is then 12% based on project G which is the best opportunity foregone. In this decision all of the projects with the lowest rate-of-return are not funded. The method worked well.

16-11

Since the interest rate specified in OMB A94 is "what a taxpayer could have received," it is a nominal interest rate and, thus, used with actual (inflated) dollars. This answer seems to be consistent with Appendix C to circular No. A94, which specifies nominal and real interest rates. The real rates are lower.

16-12

It will be assumed that the bond is purchased for its face value. The number of pay periods is 20 x 2 = 40.

Thus, 10,000 = 400 (P/A, i , 40) + 10,000 (P/F, i, 40) and at i = 4% one obtains an equality. So, $i_a = (1+ 0.04)^2 - 1 = 0.0816$ or 8.16%.

The effective interest rate earned by the cash flow is established at purchase and will not change as time passes as long as the original purchaser keeps the bond until maturity. Thus, for this case the only allowance for inflation would be the original offer made by the purchaser who would need to build expected inflation into his offer. However, usually these types of bonds can be bought and sold on the open market. The actual value of the bond's cash flow will rise and fall in relationship to market conditions one of which is inflation.

The municipality's cost of capital will actually be approximately 2 percentage points smaller than the cash flow calculated above since the Federal Government subsidizes the bond by not taxing it, so here 6.2%. An offer for the bond would need to be adjusted higher in order to represent an effective interest rate of 6.2%, otherwise the municipality most likely won't sell it to you.

16-13

Benefit–Cost Ratio = PW of Benefits/PW of Cost
= [$20,000 (P/A, 7%, 9) (P/F, 7%, 1)]/[$100,000 + $150,000 (P/F, 7%, 1)]
= [$20,000 (6.515) (0.9346)]/[$100,000 + $150,000 (0.9346)]
= <u>0.51</u>

16-14

(a) Conventional B/C Ratio
= [PW (Benefits − Disabilities)]/[PW (1st Cost + Annual Cost)]
= [($500,000−$25,000) (P/A, 10%, 35)]/[($1,200,000 + $125,000) (P/A, 10%, 35)]
= 1.9

(b) Modified B/C Ratio
= [PW (Benefits − Disbenefits − Cost)]/[PW (1st Cost)]
= [($500,000 − $25,000 − $125,000) (P/A, 10%, 35)]/$1,200,000
= 2.8

16-15

Using the Conventional B/C Ratio
(i) Using PW B/C Ratio = 1.90 (as above)

(ii) Using AW B/C Ratio = (\$500,000 − \$25,000)/[\$1,200,000 (A/P, 10%, 35) + \$125,000]
= 1.90

(iii) Using FW B/C Ratio = [(\$500,000 − \$25,000) (F/A, 10%, 35)]/[\$1,200,000 (F/P, 10%, 35) + \$125,000 (F/A, 10%, 35)]
= 1.90

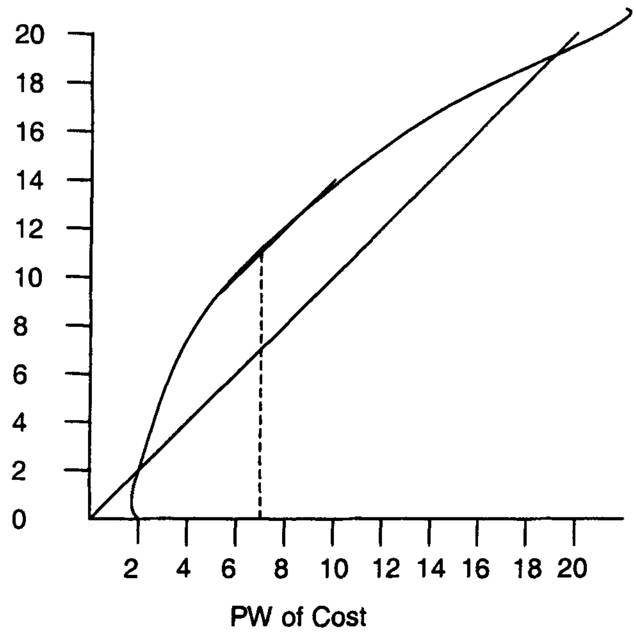
16-16

The *conventional* benefit–cost ratio has net benefits to the users in the numerator and cost to the sponsor in the denominator. The *modified* B–C ratio takes the project operating and maintenance costs paid by the sponsor, and subtracts these from the net benefits to the users. This quantity is all in the numerator. These leaves only the projects initial costs in the denominator.

The *conventional* and *modified* versions of the B–C ratio use different algebra/math to calculate the ratio, but the resulting recommendation will always be the same. That is, for any problem, **both** ratios will be either greater than or less than 1.0 at the same time.

16-17

The problem requires the student to use calculus. The text points out in Example 8-9 (of Chapter 8) that one definition of the point where ΔB = ΔC is that of the slope of the benefits curve equals the slope of the NPW = 0 line.



Values for the graph:

PW of Cost (x)	PW of Benefits (y)
2	0
4	6.6
6	9.4
8	11.5
10	13.3
12	14.8
16	17.6
20	19.9

Let x = PW of Cost and y = PW of Benefits

$y^2 - 22x + 44 = 0$ or $y = (22x - 44)^{1/2}$

$dy/dx = ½ (22x - 44)^{(-1/2)} (22) = 1$
(Note that the slope of the NPW = 0 line is 1)

$22x - 44 = [(1/2)(22)]^2$

$x = (11^2 + 44)/22 =$ <u>7.5 = optimum PW of cost</u>

16-18

Since we have a 40-year analysis period, the problem could be solved by any of the exact analysis techniques. Here the problem specifies a present worth analysis. The annual cost solution, with a 10% interest rate, is presented in Problem 6-44.

Gravity Plan
PW of Cost = \$2,800,000 + \$10,000 (P/A, 8%, 40)
= \$2,800,000 + \$10,000 (11.925) = \$2,919,250

Pumping Plan
PW of Cost = \$1,400,000 + \$200,000 (P/F, 8%, 10)
+ (\$25,000 + \$50,000) (P/A, 8%, 40)
+ \$50,000 (P/A, 8%, 30) (P/F, 8%, 10)
= \$1,400,000 + \$200,000 (0.4632)
+ (\$25,000 + \$50,000) (11.925)
+ \$50,000 (11.258) (0.4632)
= \$2,647,700

To minimize PW of Cost, choose pumping plan.

16-19

Annual Travel Volume = (2,500) (365) = 912,500 cars/year

The High Road
1st Cost = \$200,000 (35) = \$7,000,000
Annual Benefits = 0.015 (\$912,500) 35) = \$479,063
Annual O & M Cost = \$2,000 (35) = \$70,000

The Low Road
1st Cost = \$450,000 (10) = \$4,500,000
Annual Benefits = 0.045 (\$912,500) (10) = \$410,625
Annual O & M Cost = \$10,000 (10) = \$100,000

These are two mutually exclusive alternatives, and we use an incremental analysis process.

Rank Order based on denominator = Low Road, High Road

	Do Nothing vs.Low	Low vs. High
Δ 1st Cost	\$4,500,000	\$2,500,000
Δ Annual Benefits	\$410,625	\$68,438
Δ Annual O & M Costs	\$100,000	−\$30,000
Δ B / Δ C	1.07[a]	0.61[b]
Justified?	Yes	No

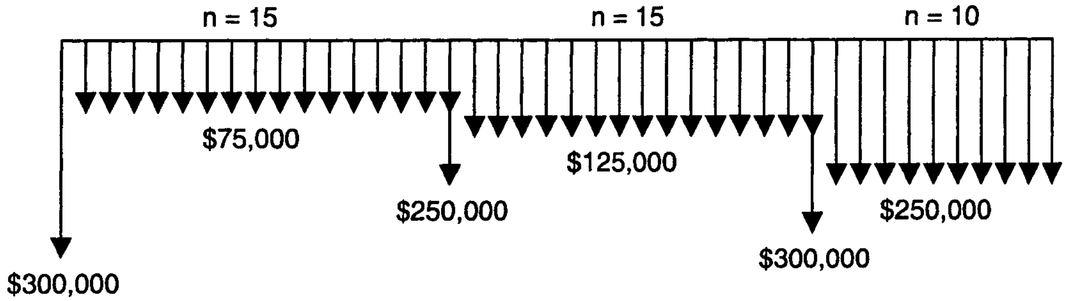
<u>Recommend investing in the Low road, it is the last justified increment.</u>

[a] [(\$410,625 − \$100,000) (\$15,456)]/\$4,500,000 = 1.07

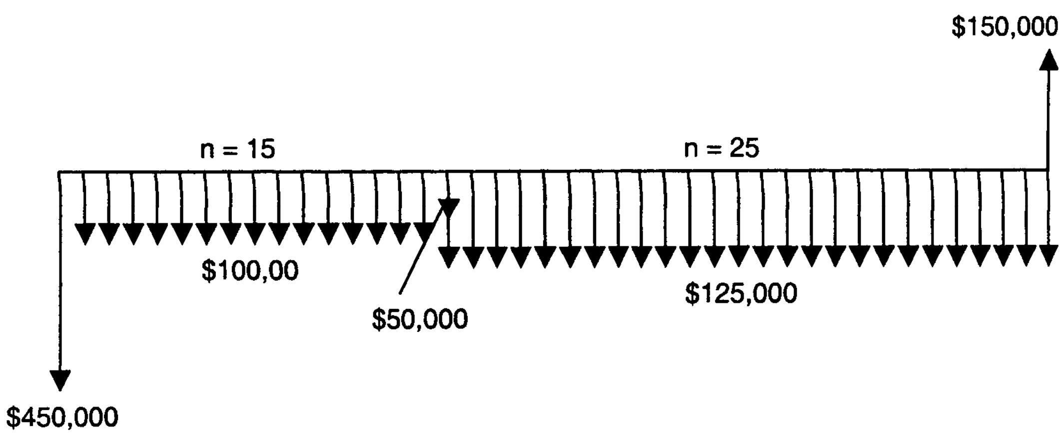
[b] [(\$68,438 + \$30,000) (\$15,456)]/\$2,500,000 = 0.61

16-20

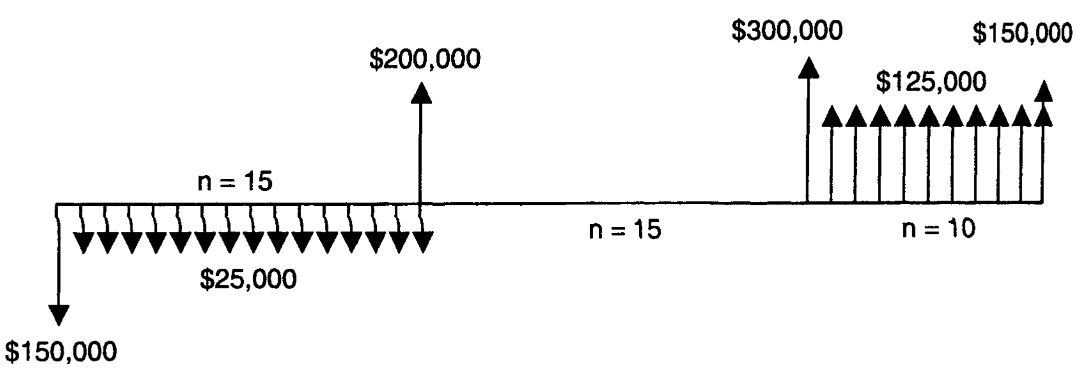
Plan A



Plan B



Differences between Alternatives A and B



3.5

An examination of the differences between the alternatives will allow us to quickly determine which plan is preferred.

Explore increment Plan B- Plan A:

PW(Benefits) = $200,000(P/F,7%,15) + $150,000(P/F,7%,40) +
[$300,000 + $125,000(P/A,7%,10)] (P/F,7%,30)
PW(Benefits) = $200,000(0.3624) + $150,000(0.0668) +
[$300,000 + $125,000(7.024)](0.1314) = $237,289

PW(Costs) = $150,000 + $25,000 (P/A,7%,15)
PW(Costs) = $150,000 + $25,000 (9.108) = $377,700

PW(Benefits)/PW(Costs) = $237,289/$377,700 = 0.63 < 1
So we select Plan A

Checking using PW analysis we get:

Cash Flow				Present Worth	Present Worth
Year	A	B	B–A	At 7%	At 5%
0	−$300	−$450	−$150	−$150	−$150
1- 15	−$75	−$100	−$25	−$228	−$259
15	−$250	−$50	+$200	+$72	+$96
16- 30	−$125	−$125	$0	$0	$0
30	−$300	$0	+$300	+$39	+$69
31- 40	−$250	−$125	+$125	+$115	+$223
40	$0	+$150	+$150	+$10	+$21
Sum			+$1,375*	−$142	$0

* This is sum of −$150 − 15 ($25) + $200 ….

When the Present Worth of the B – A cash flow is computed at 7%, the NPW = –142. The increment is not desirable at i = 7%. So, as above we select Plan A.

16-21

	Existing	Plan A	Plan B	Plan C
Length (miles)	10	10	10	10.3
Number of Lanes	2	4	4	4
Average ADT	20,000	20,000	20,000	20,000
Autos	19,000	19,000	19,000	19,000
Trucks	1,000	1,000	1,000	1,000
Time Savings (minutes)				
Autos				
Trucks		2	3	5
		1	3	4
Accident Rate/MVM	4.58	2.50	2.40	2.30
Initial Cost per mil (P)	−	$450,000	$650,000	$800,000
Annual Maintenance per lane per mile	$1,500	$1,250	$1,000	$1,000
Total Annual Maintenance	$30,000	$50,000	$40,000	$41,200
EUAC of Initial Cost = (P x miles) (A/P, 5%,20)	$0	$360,900	$521,300	$660,850
Total Annual Cost of EUAC + Maintenance	$30,000	$410,900	$561,300	$702,050

Annual Incremental Operating Costs due to distance
None for Plans A and B, as they are the same length as existing road.
Plan C Autos 19,000 x 365 x 0.3 mi x $0.06 = $124,830
Trucks 1,000 x 365 x 0.3 mi x $0.18 = $19,710

Total = $144,540/yr

Annual Accident Savings compared to Existing Highway
Plan A: (4.58 − 2.50) (10^{-6}) (10 mi) (365 days) (20,000 ADT) ($1,200)
= $182,200

Plan B: (4.58 − 2.40) (10^{-6}) (10 mi) (365 days) (20,000 ADT) ($1,200)
= $190,790

Plan C: (4.58 − 2.30) (10^{-6}) (10.3 mi) (365 days) (20,000 ADT) ($1,200)
= $205,720

Time Savings Benefits to Road Users compared to Existing Highway
Plan A:
Autos 19,000 x 365 days x 2 min x $0.03 = $416,100
Trucks 1,000 x 365 days x 1 min x $0.15 = $54,750
Total = $470,850

Plan B:
Autos 19,000 x 365 days x 3 min x $0.03 = $624,150
Trucks 1,000 x 365 days x 3 min x $0.15 = $164,250
Total = $788,400

Plan C:
Autos 19,000 x 365 days x 5 min x $0.03 = $1,040,250
Trucks 1,000 x 365 days x 4 min x $0.15 = $219,000
Total = $1,259,250

Summary of Annual Costs and Benefits

	Existing	Plan A	Plan B	Plan C
Annual Highway Costs	$30,000	$410,900	$561,300	$702,050
Annual Benefits				
Accident Savings		$182,200	$190,970	$205,720
Time Savings		$470,850	$788,400	$1,259,250
Additional Operating Cost*				−$144,540
Total Annual Benefits		$653,050	$979,370	$1,320,430

* User costs are considered as disbenefits.

Benefit – Cost Ratios
A rather than Existing: B/C = $653,050/($410,900 − $30,000)
= 1.71

B rather than A: B/C= ($979,370 − $653,050)/($561,300 − $410,900)
= 2.17

C rather than B: B/C= ($1,320,430 − $979,370)/($702,050 − $561,300
= 2.42

<u>Plan C is preferred.</u>

16-22

	A	B	C
Initial Investment	$9,500	$18,500	$22,000
Annual Savings	$3,200	$5,000	$9,800
Annual Costs	$1,000	$2,750	$6,400
Salvage Value	$6,000	$4,200	$14,000

(a) Conventional B/C

	A	B	C
PW Numerator	$21,795	$34,054	$66,746
PW Denominator	$15,215	$36,463	$63,032
B/C Ratio	1.43	0.93	1.06

Here we eliminate Alternative B. Rank order is A, then C.

	Do Nothing – A	A – C
Δ Initial Investment	$9,500	$12,500
Δ Annual Savings	$3,200	$6,600
Δ Annual Costs	$1,000	$5,400
Δ Salvage Value	$6,000	$8,000
Δ PW Numerator	$21,795	$44,952
Δ PW Denominator	$15,215	$47,817
Δ B/C Ratio	1.43	0.94
Justified?	Yes	No

<u>We recommend Alternative A.</u>

(b) Modified B/C

	A	B	C
PW Numerator	$14,984	$15,324	$23,157
PW Denominator	$8,404	$17,733	$19,442
B/C Ratio	1.78	0.86	1.19

Here we eliminate Alternative B. Our rank order is A then C.

	Do Nothing – A	A – C
Δ Initial Investment	$9,500	$12,500
Δ Annual Savings	$3,200	$6,600
Δ Annual Costs	$1,000	$5,400
Δ Salvage Value	$6,000	$8,000
Δ PW Numerator	$14,984	$8,173
Δ PW Denominator	$8,404	$11,038
Δ B/C Ratio	1.78	0.74
Justified?	Yes	No

<u>We recommend Alternative A.</u>

(c) Present Worth

Year	A	B	C
0	−$9,500	−$18,500	−$22,000
1-14	$2,200	$2,250	$3,400
15	$8,200	$6,450	$17,400
Present Worth	$6,580	−$2,408	$3,715

<u>We recommend Alternative A.</u>

(d) IRR Method

	A	B	C
IRR	23%	10%	15%

Here we need the incremental analysis method. Eliminate Alternative B because IRR < MARR.

Year	Do Nothing – A	A – C
Δ 0	−$9,500	−$12,500
Δ 1 – Δ 14	$2,200	$1,200
Δ 15	$8,200	$9,200
Δ IRR	23%	8%
Justified?	Yes	No

<u>We recommend Alternative A.</u>

(e) Simple Payback

Year	A	B	C
0	−$9,500	−$18,500	−$22,000
1	−$7,300	−$16,250	−$18,600
2	−$5,100	−$14,000	−$15,200
3	−$2,900	−$11,750	−$11,800
4	−$700	−$9,500	−$8,400
5	$1,500	−$7,250	−$5,000
6	$3,700	−$5,000	−$1,600
7	$5,900	−$2,750	$1,800
8	$8,100	−$500	$5,200
9	$10,300	$1,750	$8,600
10	$12,500	$4,000	$12,000
11	$14,700	$6,250	$15,400
12	$16,900	$8,500	$18,800
13	$19,100	$10,750	$22,200
14	$21,300	$13,000	$25,600
15	$29,500	$19,450	$43,000

Alternative A (SPB) = 4 + [$700/($700 + $2,200)] = 4.32 years
Alternative B (SPB) = 8 + [$500/($500 + $1,750)] = 8.22 years
Alternative C (SPB) = 6 + [$1,600/($1,600 + $1,800)] = 6.47 years

<u>We recommend Alternative A.</u>

16-23

This problem will require some student though on how to structure the analysis. This is a situation of providing the necessary capacity when it is needed- in other words Fixed Output. Computing the cost is easy, but what is the benefit?

One cannot compute the B/C ratio for either alternative, but the incremental B/C ratio may be computed on the difference between alternatives.

Year	A: Half-Capacity Tunnel Now plus Half-Capacity Tunnel in 20 Years	B: Full-Capacity Tunnel	B – A Difference Between the Alternatives
0	−$300,000	−$500,000	−$200,000
10	−$16,000	−$20,000	−$4,000
20	−$16,000 −$400,000	−$20,000	+$396,000
30	−$32,000	−$20,000	+$12,000
40	−$32,000	−$20,000	+$12,000
50	$0	$0	$0

$\Delta B/\Delta C$ = [$396,000 (P/F, 5%, 20) + $12,000 (P/F, 5%, 30) + $12,000 (P/F, 5%, 40)]/[$200,000 + $4,000 (P/F, 5%, 10)]
= $153,733/$202,456
= 0.76

This is an undesirable increment of investment. Build the half-capacity tunnel now.

16-24

	Alt. A	Alt. B	Alt. C	Alt. D
First Cost	$9,500	$12,500	$14,000	$15,750
Annual O & M Costs	$550	$175	$325	$145
Salvage Value	$1,000	$6,000	$3,500	$7,500
PW of Denominator	$15,592	$13,874	$17,311	$16,637
Annual Benefits	$2,200	$1,500	$1,000	$2,500
Annual Disbenefits	$350	$150	$75	$700
PW of Numerator	$20,827	$15,198	$10,413	$20,265
B/C Ratio	1.34	1.10	0.60	1.22

We eliminate Alternative C from consideration. Our rank order is B, A, D.

	Do nothing: B	A - B	D - A
Δ First Cost	$12,500	−$3,000	$6,250
Δ Annual O & M Costs	$175	$375	−$405
Δ Salvage Value	$6,000	−$5,000	$6,500
PW of Δ Denominator	$13,874	$1,719	$1,045
Δ Annual Benefits	$1,500	$700	$300
Δ Annual Disbenefits	$150	$200	$350
PW of Δ Numerator	$15,198	$5,629	−$563
Δ B/C Ratio	1.10	3.28	−0.54
Justified?	Yes	Yes	No

Choose Alternative A because it is associated with the last justified increment of investment.

16-25

	1	2	3	4	5	6
AW Costs (sponsor)	15.5	13.7	16.8	10.2	17	23.3
AW Benefits (users)	20	16	15	13.7	22	25
B/C Ratio	1.29	1.17	0.89	1.34	1.29	1.07

We can eliminate project #3 from consideration. Our rank order is “Do-Nothing” (DN), 4, 2, 1, 5, and 6.

	4-DN	2-4	1-4	5-1	6-5
Δ AW Costs (sponsor)	10.2	3.5	5.3	1.5	6.3
Δ AW Benefits (users)	13.7	2.3	6.3	2	3
Δ B/C Ratio	1.34	0.66	1.19	1.33	0.48
Justified?	Yes	No	Yes	Yes	No

Choose Alternative 5 because it is associated with the last justified increment of investment.

16-26

It is important to recognize that if Net Present Worth analysis is done, then the criterion is to maximize NPW. But, of course, the NPWs must be computed at a common point in time, like Year 0.

Repair Now

$NPW_{YEAR\ 0}$ = \$5,000 (P/F, 15%, 1) + \$10,000 (P/G, 15%, 5)
+ \$50,000 (P/A, 15%, 5) (P/F, 15%, 5) − \$150,000
= \$5,000 (0.8696) + \$10,000 (5.775)
+ \$50,000 (3.352) (0.4972) −\$150,000
= −\$4,571

Repair Two Years Hence

$NPW_{YEAR\ 2}$ = \$20,000 (P/A, 15%, 3) + \$ 10,000 (P/G, 15%, 3)
+ \$50,000 (P/A, 15%, 7) (P/F, 15%, 3) − \$150,000
= \$20,000 (2.283) + \$10,000 (2.071)
+ \$50,000 (4.160) (0.6575) − \$150,000
= +\$53,130

$NPW_{YEAR\ 0}$ = \$53,130 (P/F, 15%, 2) = \$53,130 (0.756) = +\$40,172

Repair Four Years Hence

$NPW_{YEAR\ 4}$ = \$50,000 (P/A, 15%, 10) − \$10,000 (P/F, 15%, 1) − \$150,000
= \$50,000 (5.019) − \$10,000 (0.8696) − \$150,000
= +\$92,254

$NPW_{YEAR\ 0}$ = \$92,254 (P/F, 15%, 4) = \$92,254 (0.5718) = +\$52,751

Repair Five Years Hence

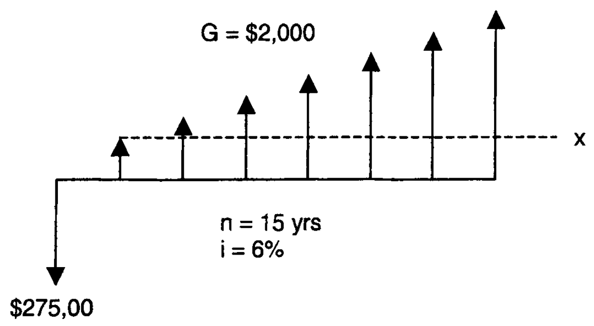
$NPW_{YEAR\ 5}$ = \$50,000 (P/A, 15%, 10) − \$150,000
= \$50,000 (5.019) − \$150,000
= +\$100,950

$NPW_{YEAR\ 0}$ = \$100,950 (P/F, 15%, 5) = \$100,950 (0.4972) = +\$50,192

To maximize NPW at year 0, repair the road four years hence.
It might be worth noting in this situation that since the annual benefits in the early years (Years 1, 2, and 3) are less than the cost times the interest rate (\$150,000 x 0.15 = \$22,500), delaying the project will increase the NPW at Year 0. In other words, we would not expect the project to be selected (if it ever would be) until the annual benefits are greater than \$22,500.

If a "repair three years hence" alternative were considered, we would find that it has an NPW at year 0 of +\$49,945. So the decision to repair the road four years hence is correct.

16-27



Compute X for NPW = 0

NPW = PW of Benefits − PW of Costs
= X (P/A, 6%, 15) + $2,000 (P/G, 6%, 15) − $275,000 = $0
= X (9.712) + $2,000 (57.554) − $275,000 = $0

X = [$275,000 − $2,000 (57.554)]/9.712 = $16,463

Therefore, NPW at year 0 turns positive for the first time when X is greater than $16,463. This indicates that construction should not be done prior to 2015 as NPW is not positive. The problem thus reduces to deciding whether to proceed in 2015 or 2016. The appropriate criterion is to maximize NPW at some point. If we choose the beginning of 2015 for convenience,

Construct in 2015

NPW_{2005} = $18,000 (P/A, 6%, 15) + $2,000 (P/G, 6%, 15) − $275,000
= $18,000 (9.712) +$2,000 (57.554) − $275,000
= +$14,924

Construct in 2016

NPW_{2006} = [$20,000 (P/A, 6%, 15) + $2,000 (P/G, 6%, 15)
− $275,000] (P/F, 6%, 1)
= [$20,000 (9.712) + $2,000 (57.554) − $275,000] (0.9434)
= +$32,404

Conclusion: <u>Construct in 2016.</u>

16-28

(a) B/C Ratio = [($550 − $35) (P/A, 8%, 20)]/[($750 + $2,750) + $185 (P/A, 8%, 20)]
= 0.95

(b) Let's find the breakeven number of years at which B/C = 1.0
1.0 = [($550 − $35) (P/A, 8%, x)]/[($750 + $2,750) + $185 (P/A, 8%, x)]

By trial and error:

X	*B/C ratio*
24 years	0.995
25 years	1.004
26 years	1.031

One can see how Big City Carl arrived at his value of "at least" 25 years for the project duration. This is the minimum number of years at which the B/C ratio is greater than 1.0 (nominally).

16-29

(a) PW of Benefits = $60,000 (P/A, 5%, 10)
+ $64,000 (P/A, 5%, 10) (P/F, 5%, 10)
+ $66,000 (P/A, 5%, 20) (P/F, 5%, 20)
+ $70,000 (P/A, 5%, 10) (P/F, 5%, 40)
= $60,000 (7.722)
+ $64,000 (7.722) (0.6139)
+ $66,000 (12.462) (0.3769)
+ $70,000 (7.722) (0.1420)
= $1,153,468
For B/C ratio = 1, PW of Cost = PW of Benefits

Justified capital expenditure
= $1,153,468 − $15,000 (P/A, 5%, 50)
= $1,153,468 − $15,000 (18.256)
= $879,628

(b) Same equation as on previous page except use 8% interest PW of Benefits = $60,000 (6.710) + $64,000 (6.710) (0.4632) + $66,000 (9.818) (0.2145) + $70,000 (6.710) (0.0460)
= $762,116

Justified Capital Expenditure
= $762,116 – $15,000 (12.233)
= $578,621

16-30

For Plan B in Problem 16-20 to be chosen, the increment B – A must be desirable. The last column in the table in Problem 16-20 shows that the B – A increment has a 5% rate of return. In other words, at all interest rates at or below 5%, the increment is desirable and hence Plan B is the preferred alternative. The value of MARR would have to be 5% or less.

16-31

(a) The conventional and modified versions of the B/C ratio will always give consistent recommendations in terms of "invest" or "do not invest". However, the magnitude of the B/C Ratio will be different for the two methods. Advocates of a project may use the method with the larger ratio to bolster their advocacy.

(b) Larger interest rates raise the "cost of capital" or "lost interest" for public projects because of the sometimes quit expensive construction costs. A person favoring a $200 M turnpike project would want to use lower i% values in the B/C ratio calculations to offset the large capital costs.

(c) A decision maker in favor of a particular public project would advocate the use of a longer project in the calculation of the B/C ratio. Longer durations spread the large initial costs over a greater number of years.

(d) Benefits, costs and disbenefits are quantities that have various amounts of "certainty" associated with them. Although this is true for all engineering economy estimates it is particularly true for public projects. It is much easier to estimate labor savings in a production environment than it is to estimate the impact on local hotels of new signage along a major route through town. Because benefits, costs, and disbenefits tend to have more uncertainty it is therefore easier to manipulate their values to make a B/C Ratio indicate a decision with your position.

16-32

(a) If the saved hour adds directly to your employment productivity then its value would be the appropriate fraction of your salary. If the saved hour enables you to spend that much more time on a personal money making adventure, then the value would be the extra money you would be able to make. If the saved hour simply adds to your "personal time" then its economic value would probably be zero.

(b) The value would be some fraction (based on usage) of the value of the total recreational activities available to the community. Check with the city's Chamber of Commerce to see if such a total number has been assigned or, if not, see if other similar cities have assigned such a number. Consult with a local bike club or, perhaps, the local YMCA to obtain an estimate of total yearly bike path usage. The value would be total number of people using the bike path divided by 300,000 times the total value of recreational activities.

(c) Determine the 100-year flood plain from the Army Corps of Engineers. Determine the value of residential and commercial property in the flood plain from the County Assessor's Office. Talk to local realtors to determine the worth of development likely to occur in the flood plain. Restrict new development at the appropriate level through zoning ordinances, and estimate the value of the property that would not be destroyed as a percentage of developed property present. Another possibility is to institute a buyout plan where each year 5% of the most vulnerable property is purchased, the buildings razed, and the land converted into a green area. This second option would be expensive. The topic of flood damage amelioration is not an easy one. Many ideas are possible.

(d) From an economic view point the value of a human life would be established by a Court of Law and a jury. Thus, consult with an attorney about precedence established in previous cases. It would be expected that the judged value would be highly dependent upon case circumstances as well as age, sex, health, occupation, earning potential, etc.

16-33

(a) Costs

1. Buy Property – Contact local realtors to get cost estimates of the properties needed.
2. Site Preparation and Construction – Contact the United States Nuclear Regulatory Commission (USNRC) about design, construction, and licensing requirements and regulations, Environmental Protection Agency (EPA) about environmental requirements and regulations, and then local contractor to get estimates of all the items associated with construction costs. There will also be state regulatory agencies that will need to be contacted as well.
3. Cooling System – Same as above except may need to include the Army Corps of Engineers if river or lake cooling water is needed.

(b) Benefits
1. Environment – If a conventional coal-fired plant is being replaced (or not being built) then the decrease (or lack of) in emissions of all types can be estimated. The actual dollar value of some, say carbon dioxide emissions, is hard to quantify, however, others, say mercury and sulfur dioxide (acid rain), could be estimated as the appropriate fraction of total economic damage to the U.S. per year. Contact the EPA for numbers and/or search the internet.
2. Jobs and Economy – Certainly wages paid during construction would be known quite accurately. The number of new employees at the plant and their wages could be estimated using employment records at existing plants. Increased tax base could be estimate by talking to the county property assessor. Income taxes and potential sales taxes could be estimated from the plant's total wages.
3. The amount of money that would be spent to operate uranium mining and enrichment facilities could be obtained by researching information published by the World Nuclear Association or, perhaps, by contacting the USNRC.

(c) Disbenefits
1. Fission Product Material – Contact USNRC about methods and costs of on-site storage.
2. Not In My Backyard – Talk to advertising agency about the cost of a media campaign extolling the benefits of nuclear power.
3. Risk of Reactor – Research the cost of the Three-Mile-Island disaster (an internet search will work) and multiply by the estimated probability (use the USNRC's estimate) of a similar event happening.
4. Loss to Economy – Contact a coal mine to find out yearly cost of the coal that will not be burned. The electricity won't be lost unless the nuclear plant generates less than the replaced (or not built) coal-fired plant. If that's the case, then estimate the value of electricity not generated using current average rates.

16-34

(a) Costs
1. Buy Property – Contact local realtors to get cost estimates of the properties needed to complete the intersection.
2. New Signage and Traffic Light – Contact the state Department of Transportation (DOT) about cost of these items.
3. Construction – Contact local contractors to get estimates of all the items associated with construction costs.

(b) Benefits

1. Increased Traffic Flow – Difficult to estimate. Check with U.S. DOT and state DOT for any modeling or studies related to time and fuel savings with increased traffic flow.
2. Increased safety – Also difficult to estimate. Search the Institute of Transportation Engineers web site to see if any publications exist related to intersection safety. Same for state DOT and U.S. DOT.
3. Increase in jobs – Certainly wages paid during construction would be known quite accurately. Difficult to estimate long-term job creation. If the improved intersection allows much easier access to a Wal-Mart, mall or other significant business, then job increase benefit could be estimated by talking to owners of these affected businesses.
4. Same as part d.

(c) Disbenefits

1. Traffic Disruption During Construction – Talk with local business owners about how much business they expect to lose.
2. Increase In Noise and Dust – Difficult to estimate. Both are annoying but their economic impact is probably minimal compared to other disbenefits.
3. Land Acquisition – Other than direct cost which is included above. If the gas station, bank and church relocate out of the area, then the lost direct and indirect sales (e.g. church goers stopping at a local restaurant for a Sunday meal) could be estimated by speaking to the owners of the gas station and bank and other local businessmen.

16-35

The time required to initiate, study, fund, and construct public projects is generally several years (or even decades). Because of this, it is not uncommon for there to be turnover in public policy makers. Politicians, who generally strive to maintain a positive public image, have been known to “stand up and gain political capital” from projects that originally began many years before they took office.

16-36

(a) Density = $1500\frac{lbs}{yd^3} = \frac{1500\ lbs}{2000\frac{lbs}{ton}} / yd^3 = 0.75\frac{ton}{yd^3}$

Design capacity = $(1{,}000{,}000\ yd^3)\ (0.75\ \frac{ton}{yd^3}) = 750{,}000\ ton$

Lifetime = $\frac{750{,}000\ ton}{120{,}000\ \frac{ton}{yr}}$ = 6.25 years

(b) Amount of MSW = $(750,000\, ton)\,(0.80) = 600,000\, ton$

Amount of C&D = $(750,000\, ton)\,(0.20) = 150,000\, ton$

MSW LFG recovery = $(3000\, \frac{ft^3}{ton})\,(600,000\, ton) = 1.8 \times 10^9\ ft^3$

C&C LFG recovery = $(1500 \frac{ft^3}{ton})\,(150,000\, ton) = 0.225 \times 10^9\ ft^3$

MSW Methane recovery = $(1.8 \times 10^9\ ft^3)\,(0.50) = 0.9 \times 10^9\ ft^3$

C&D Methane recovery = $(0.225 \times 10^9\ ft^3)\,(0.20) = 0.045 \times 10^9\ ft^3$

Total Methane recovery = $(0.9 \times 10^9) + (0.045 \times 10^9) = 0.945 \times 10^9\ ft^3$

Average annual methane production = $\frac{0.945 \times 10^9}{15} = 6.3 \times 10^7\ \frac{ft^3}{yr}$

(c) Heat per year = $(6.3 \times 10^7\ \frac{ft^3}{yr})\,(1030\, \frac{BTU}{ft^3}) = 6.489 \times 10^{10}\ \frac{BTU}{yr}$

kWh per year = $\frac{6.489 \times 10^{10}\ \frac{BTU}{yr}}{1.17 \times 10^4\ \frac{BTU}{kWh}} = 5.546 \times 10^6\ \frac{kWh}{yr}$

Dollar value per year = $(5.546 \times 10^6\ \frac{kWh}{yr})\,(0.05\, \frac{\$}{kWh}) = \$277,300$

16-37

Heating load per residential dwelling = $1.00\times10^{8}\ \frac{BTU}{yr}$.

The furnace efficiency is 0.88 so:

Heating load for 650 units = $\frac{(650)\,(1.00\times10^{8})}{0.88} = 7.386\times10^{10}\ \frac{BTU}{yr}$

Methane needed for 650 units = $\frac{7.386\times10^{10}\ \frac{BTU}{yr}}{1030\ \frac{BTU}{ft^3}} = 7.1712\times10^{7}\ \frac{ft^3 methane}{yr}$.

Since the land fill has $9.45\times10^{8}\ ft^3 methane$ (see Problem 16-36), there is plenty of methane available to heat the development for $\frac{9.45\times10^{8}}{17.712\times10^{7}} = 13.2\ years!$ To determine the economic feasibility one can calculate the dollar value per year per residential unit of the used methane if it were converted into electricity. (For details see Problem 16-36.)

Dollar value per year = $\left[\frac{(7.1712\times10^{7}\ \frac{ft^3}{yr})\,(1.030\times10^{3}\ \frac{BTU}{ft^3})}{(1.1700\times10^{4}\ \frac{BTU}{kWh})\,(650\ units)}\right](0.05\ \frac{\$}{kWh}) = \frac{\$485.62}{unit}$.

The cost for heating oil for one residential unit (furnace efficiency = 0.82) is given by

Dollar value per year = $\left[\frac{1.00\times10^{8}\ \frac{BTU}{yr}}{(0.82)\,(1.388\times10^{5}\ \frac{BTU}{gal})}\right](2.50\frac{\$}{gal}) = \frac{\$2,047}{unit}$.

The heating oil is 4.2 times more expensive than the methane, so the methane is more economically feasible. The technology exists, is available commercially, and is proven, so it would seem to be operationally feasible.

16-38

Total area = (1000 ft) (200 ft) = 200,000 ft^2. The minimum number of wells would be 10 by 2 = 20 down the length of the landfill. Coverage of 20 wells would be = (20) [π $(50)^2$] = 157,080 ft^2 or 78.5% coverage. To increase the coverage, one could expand the above 20 wells out, say, 5 feet and place 9 new wells down the middle. The new coverage would be about 90%.

Minimum cost estimate:
Construct and place well heads for 20 wells = (20) ($3,000 + $2,500) = $110,000
Pipe for the 20 wells = [900 ft + (100 ft) (10)] (35 $/ft) = $66,500
One condensate knockout (assumes one low spot) = $5,000
One blower/flare station = $500,000

Total Cost (minimum coverage) = ($110,000 + $66,500 + $5,000 + $500,000)
= $681,500

Maximum cost estimate:
Construct and place well heads for 29 wells = (29) ($3,000 + $2,500) = $159,500
Pipe for the 29 wells = [900 ft + (110 ft) (10)] (35 $/ft) = $70,000
Two condensate knockouts (assumes one low spot) = $10,000
One blower/flare station = $500,000

Total Cost (minimum coverage) = ($159,500 + $70,000 + $10,000 + $500,000)
= $739,500

Note that the increase in coverage is 90 / 78.5 = 1.146 but the increase in cost is only $739,500 / $681,500 = 1.085. One would need to calculate the value of the extra extracted methane to see if the increased construction cost is justified. The student's cost estimate will vary depending upon layout.

Chapter 17: Accounting and Engineering Economy

17-1

Engineers and managers make better decisions when they understand the "dollar" impact of their decisions. Accounting principles guide the reporting of cash flows for the firm. Engineers and managers can access this information through formal and informal education means, both within and outside the firm.

17-2

The accounting function is the economic analysis function within a company — it is concerned with the dollar impact of past decisions. It is important to understand, and account for, these past decisions from management, operational, and legal perspectives. Accounting data relates to all manner of activities in the business.

17-3

Balance Sheet – picture of the firm's financial worth at a specific point in time.
Income Statement – synopsis of the firm's profitability for a period of time.
Fundamental accounting equation - assets of a firm at the sumof their liabilities and equity.

17-4

Short-term liabilities represent expenses that are due within one year of the balance sheet, while long-term liabilities are payments due beyond one year of the balance sheet.

17-5

Assets = $1,000,000
Total liabilities = $127,000 + 210,000 = $337,000
Equity = assets – liabilities = $1,000,000 − 337,000 = $663,000

17-6

(a) Equity = Assets – Liabilities = (\$870,000 + \$430,000 − \$180,000) – (\$330,000 + \$115,000)
= \$675,000

(b) Retained Earnings = Equity – (Stock + Capital Surplus) = \$675,000 – \$305,000
= \$370,000

17-7

(a) Equity = Assets – Liabilities = (\$930,000 + \$320,000 − \$108,000) – (\$350,000 + \$185,000)
= \$607,000

(b) Retained Earnings = Equity – (Stock + Capital Surplus) = \$607,000 – \$402,000
= \$205,000

17-8

(a) Working capital = current assets − current liabilities = \$5,000,000 − 2,000,000
= \$3,000,000

(b) Current ratio = (current assets / current liabilities) = \$5,000,000/2,000,000 = 2.5

17-9

Assets = \$100,000 + 45,000 + 150,000 + 200,000 + 8,000 = \$503,000
Liabilities = \$315,000 + 90,000 = \$405,000

(a) Working capital = \$503,000 − 405,000 = \$98,000

(b) Current ratio = \$495,000/405,000 = 1.22

(c) Acid test ratio = \$295,000/405,000 = 0.73

17-10

(a) Working capital = ($90,000 + 175,000 + 210,000) − (322,000 + 87,000)
= $475K − 409K = $66,000

(b) Current ratio = ($475K/409K) = 1.161

(c) Acid test ratio = ($90,000 + 175,000)/409,000 = 0.648

17-11

(a) Working capital = current assets − current liabilities
= ($110K + 40K + 10K + 250K) − (442K) = $118,000

(b) Current ratio = current assets / current liabilities = $560K/442K = 1.27

(c) Acid test ratio = quick assets / current liabilities = $310K/442K = 0.701
A good current ratio is 2 or above, and a good acid test ratio is 1 or above. This company is in major trouble unless they move inventory quickly.

17-12

(a) Current ratio = current assets / current liabilities = (1.5million)/50,000 = 30

(b) Acid test ratio = quick assets / current liabilities = (1.0 million)/50,000 = 20
While it may be tempting to think that a higher ratio is better, this is not always the case. Such high ratios as these could mean that an excessive amount of capital is being kept on hand. Excess capital does very little for the company if it is just sitting in the bank – it could and/or should be used to make the company more profitable through investing, automation, employee training, etc.

17-13

(a) Total current assets = $1740 + 900 + 2500 − 75 = $5065
Total current liabilities = $1050 + 500 + 125 = $1675
Current ratio = $5065/1675 = 3.0238
This company's financial standing is good because the current ratio is greater than 2.0.

(b) Balance Sheet

Assets			**Liabilities**	
Current Assets			Current Liabilities	
Cash	$1,740		Accounts Pay	$1,050
Acc. Rec.	2,500		Notes Pay	500
Securities			Accrued Exp	125
Inventories	900		Tot Cur. Liab.	1,675
(minus) Bad Debt	–75		Long Term Liab	950
Tot Cur. Assets	$5,065		Total Liabilities	$2,625
Fixed Assets			Equity	
Land	475		Stock	680
Plant & Equip	3,100		Capital Surplus	45
(minus) Acc. Debt	–1,060		Retained earn	4,220
Tot. Fix. Assets	$2,515		Total Equity	4,955
Total Assets	$7,580		Total Liabilities	$7,580

(c) See table above.

17-14

(a) Current ratio = current assets / current liabilities = $2670/1430 = 1.87

This is below the recommended ratio of 2.0 and may indicate that the firm is not solvent, especially since the height of the nursery business is the spring and summer and this is a June balance sheet.

(b) Acid test ratio = (cash + accounts receivable) / current liabilities
= ($870 + 450)/1430 = 0.92

This indicates that 92% of the current liabilities could be paid out within the next thirty days, which is not a bad situation, although a little higher would be preferable.

17-15

Not necessarily. The current ratio will provide insight into the firm's solvency over the short term and although a ratio of less than 2 historically indicates there could be problems, it doesn't mean the company will go out of business. The same is true with the acid-test ratio. If the company has a low ratio, then it probably doesn't have the ability to instantly pay off debt. That doesn't necessarily indicate the firm will go bankrupt. Both tests should be used as an indicator or warning sign.

17-16

Just like the fact that today's weather is not a good basis to pack for a 3-month trip, local and recent financial data are not a complete basis for judging a firm's performance. Historical and seasonal trends and a context of industry standards are also needed.

17-17

The two primary general accounting statements are the balance sheet and the income statement. Both serve useful and needed functions.

17-18

6 days/week* 52 weeks/year = 312 days/year in operation
$1000 profit/day* 312 days/year = $312,000 profit/year
Revenues – expenses = $500,000 − 312,000 = $188,000

17-19

Profit = $50,000 − 30,000 − 5,000 = $15,000
Net income = profit – taxes = $15,000 − 2,000 = $13,000

17-20

Net profit (loss) = revenues – expenses = $100,000 − 60,000 = $40,000

17-21

Operating Revenues and Expenses	
<u>Revenue</u>	
Sales	<u>30,000</u>
Total	30,000
<u>Expenses</u>	
Administrative	2750
Cost of goods sold	18,000
Development	900
Selling	<u>4500</u>
Total	26,150
Total operating income	3,850
Nonoperating revenues & expenses	
Interest paid	<u>200</u>
Income before taxes	3650
Taxes (@27%)	985.50
Net profit (loss)	2664.50

17-22

Total revenues = \$81 + 5 = \$86 million
Total expenses = \$70 + 7 = \$77 million

(a) Net income before taxes = revenue – expenses = \$86 − 77 = \$9 million
Net profit = net income before taxes – taxes = \$9 − 1 = \$8 million

(b) Income Statement

Operating revenues and expenses	
Total operating revenues	\$81
Total operating expenses	70
Total operating income	\$11
Nonoperating revenue and expenses	
Interest payments	–7
Nonoperating income	5
Total nonoperating income	–\$2
Net income before taxes	\$9
Income taxes	1
Net profit (loss)	\$8

(c) Interest coverage = (total revenues – total expenses) / interest
= (\$86 − 70)/7
= 2.28

Net profit ratio = \$8/\$81 = 0.099 = 9.9%

This interest coverage is not acceptable because it should be at least 3.0 for industrial firms.

17-23

(a) Interest coverage = total income / interest payments
= (\$455 − 394 + 22)/22 = 3.77

This is a good ratio, indicating the company's ability to repay its debts. It should be at least 3.0.

(b) Net profit ratio = net profits / sales revenue = \$31/(395 − 15) = 0.08

This is a very small ratio, indicating that the company needs to assess their ability to operate efficiently in order to increase profits. The company should compare itself to industry standards.

17-24

(a) Plant and equipment = \$2,800,000 + \$800,000 = \$3,600,000

(b) Accumulated depreciation = \$420,000

(c) Retained earnings = \$480,000 – \$200,000 = \$280,000

17-25

(a) Plant and equipment = \$15M + \$3M = \$18M

(b) Accumulated depreciation = \$8M + \$2M = \$10M

(c) $RE_{end} = RE_{begin}$ + Net income or Loss + New Stock – Dividends
= \$60M + [(\$51M + \$35) – (\$70M + \$7M)] + 0 – 0 = \$60M + \$9M = \$69M

17-26

RLW-II will use the ABC system to understand all of the activities that drive costs in their manufacturing enterprise. Based on the presence and magnitude of the activities, RLW-II will want to assign costs to each. In doing this, RLW-II will gain a more accurate view of the true costs of producing their products. Potential categories of indirect costs that RLW-II will want to account for include costs for: ordering from and maintaining a relationship with specific vendors/suppliers, shipping, receiving, and storing raw materials, components and sub-assemblies; retrieval and all material handling activities from receiving to final shipment; all indirect manufacturing and assembly activities that support the direct costs; activities related to requirements for specific and unique machinery, tools and fixtures, and engineering and technical support; all indirect quality related activities in areas such as testing, rework and scrap; activities related to packaging, documentation and final storage; shipping, distribution and warehousing activities, and customer support/service and warranty activities.

17-27

Indirect labor cost = $15,892,000 / (64,0000 + 20,000 + 32,000) = $137/hr

Activity	**Model S**	**Model M**	**Model G**
Direct material cost	$3,800,000	$1,530,000	$2,105,000
Direct labor cost	$600,000	$380,000	$420,000
Direct labor hours	64,000	20,000	32,000
Allocated overhead	64,000 x 137 = $8,768,000	20,000 x 137 = $2,740,000	32,000 x 137 = $4,384,000
Total costs	$13,168,000	$4,650,100	$6,909,000
Units produced	100,000	50,000	82,250
Cost per unit	$132	$93	$84

17-28

(a) $60,000,000/12,000 hours = $5000/hour

(b) Total cost = $1,000,000 + $600,000 + 200hours*$5000/hour = $2,600,000

17-29

(a) Total direct labor = 50,000 + 65,000 = $115,000

Allocation of overhead

$\text{Overhead}_{\text{Standard}}$	= (50,000/115,000)(35,000)	= $15,217
$\text{Overhead}_{\text{Deluxe}}$	= (65,000/115,000)(35,000)	= $19,783
$\text{Total Cost}_{\text{Standard}}$	= 50,000 + 40,000 + 15,217	= $105,217
$\text{Total Cost}_{\text{Deluxe}}$	= 65,000 + 47,500 + 19,783	= $132,283
$\text{Net Revenue}_{\text{Standard}}$	= 1800(60) − 105,217	= $2783
$\text{Net Revenue}_{\text{Deluxe}}$	= 1400(95) − 132,283	= $717

(b) Total materials = 40,000 + 47,500 = $87,500

Allocation of overhead

$\text{Overhead}_{\text{Standard}}$	= (40,000/87,500)(35,000)	= $16,000
$\text{Overhead}_{\text{Deluxe}}$	= (47,500/87,500)(35,000)	= $19,000
$\text{Total Cost}_{\text{Standard}}$	= 50,000 + 40,000 + 16,000	= $106,000
$\text{Total Cost}_{\text{Deluxe}}$	= 65,000 + 47,500 + 19,000	= $131,500
$\text{Net Revenue}_{\text{Standard}}$	= 1800(60) − 106,000	= $2000
$\text{Net Revenue}_{\text{Deluxe}}$	= 1400(95) − 131,500	= $1500

In both cases the total net revenues equal $3500, but the deluxe bag appears far more profitable with materials-based allocation.