Notes on
"Rate-Distortion Methods for Image and Video Compression,"

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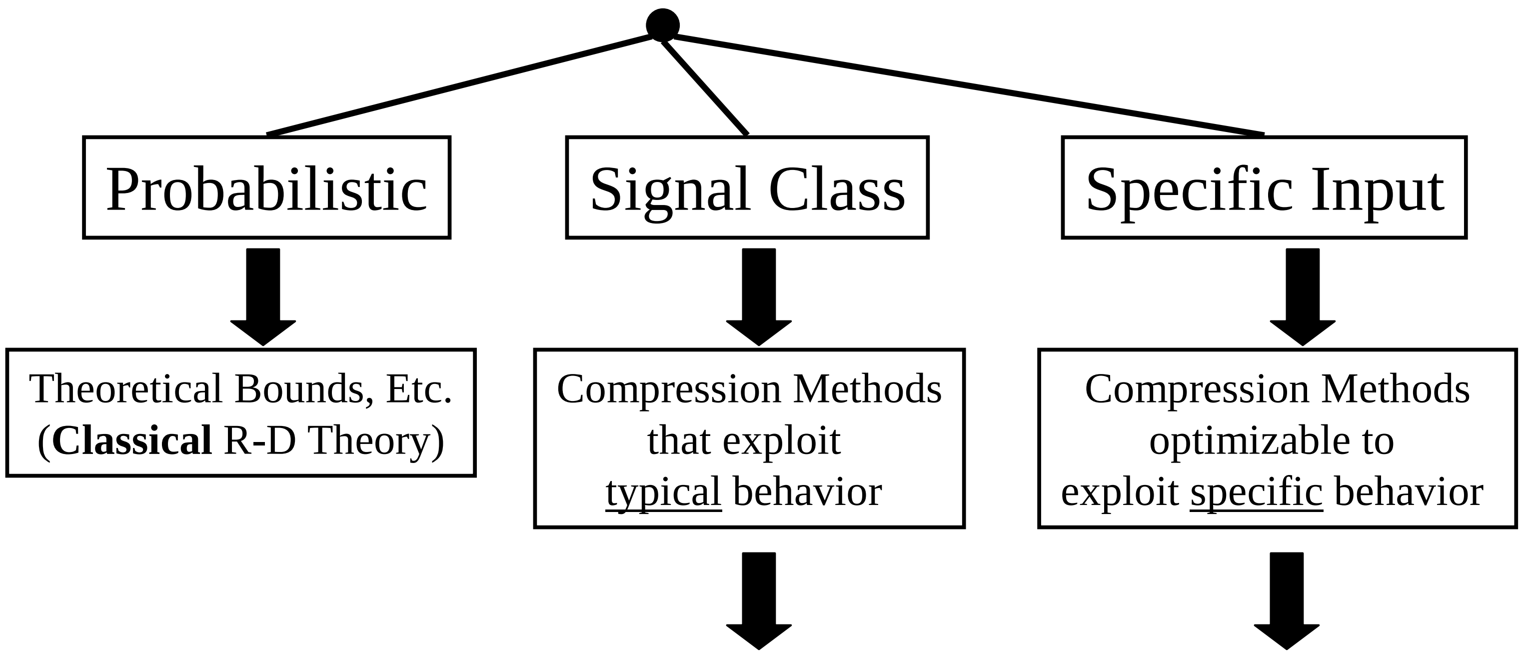
I. From Shannon Theory to MPEG Coding

I-A. Classical R-D Theory

- Concerned with representing a source with the smallest number of bits possible for a given reproduction quality

Three Views of "Source"



- Coding Scheme (framework): design based on typical features
- Coding Parameters: chosen on input-by-input basis to optimize to a particular input

I-B. Distortion Measures

- Elusive Goal: finding a general, easily computed measure of perceptual quality
- Workable approach: apply simple, perceptually-sound design rules
- Example: Not all frequencies are equally important to hearing/vision
  - Use a perceptually-weighted MSE criteria

$$MSE_{PW} = \int_{-\pi}^{\pi} \left| W(\Omega) \left[X(\Omega) - \hat{X}(\Omega) \right] \right|^2 d\Omega$$

  - After perceptual weighting, use optimized encoder to minimize
- Note: perceptual weighting works well
  - Tests of proposals made for JPEG-2000 showed that those that minimized some perceptually-weighted MSE criteria were judged best

I-C. Optimality & R-D Bounds

- **Classical**: Given a statistical model, find the lower bound on R-D
  - Limited to:
    - Simple Statistical Models
    - Asymptotic Results (large block or high rate)

- **Practice**: Optimizing performance consists of:
  1. Given a particular type of data, what is the appropriate model for that type of data (probabilistic or otherwise)?
  2. Given the chosen model (in #1), and any applicable bounds, how close can a practical algorithm get to the bound?

Both steps are equally important

<u>Box #1: Experiments on Statistical Models</u>

- **<u>Two experiments</u>** to explore the impact of choice of model on compression

- **<u>Experiment #1</u>**: <u>Actual R-D of a method</u> vs. <u>R-D bound for simple model</u>
  - Method = SPIHT applied to "Lena"
  - Simple Model = i.i.d. zero-mean Gaussian model for each subband
    - Uses empirically measured variance for each subband
    - "Shannon R-D Bound" – uses infinitely-long vectors (asymptotic result) ➔ infinite complexity!
  - **<u>Result</u>**: Choice of model is very important!!!
    - SPIHT model + SPIHT low-complexity <u>suboptimal coder</u>: **<u>better</u>**
    - IID Gaussian Model + infinite-complexity <u>optimal coder</u>: **<u>worse</u>**

Box #1: Continued

- **Experiment #2**: See how well various statistical models can synthesize image.
  - Create a random realization of wavelet coefficients using some statistical model with parameters set using measured values from “Lena”
  - Synthesize the “the image” using inverse wavelet transform
- **Model #1** (Global Subband Variances, No Sign Info)
  - Measure variance in each subband
  - Use i.i.d. Laplacian (+/–) with measured subband variances
- **Model #2** (Global Subband Variances, With Sign Info)
  - Measure variance in each subband
  - Use i.i.d. Laplacian model (+) for magnitudes w/ measured subband variances
  - Random coefficient signs are set to true values for “Lena” coefficients
- **Model #3** (Local Subband Variances, No Sign Info)
  - Measure local variances in each subband (spatially/spectrally varying variances)
  - Use i.i.d. Laplacian model (+/–) with measured local variances
- **Model #4** (Local Subband Variances, With Sign Info)
  - Measure local variances in each subband (spatially/spectrally varying variances)
  - Use i.i.d. Laplacian model (+) for magnitudes w/ measured local variances
  - Random coefficient signs are set to true values for “Lena” coefficients

II. Operational R-D in Practical Coder Design

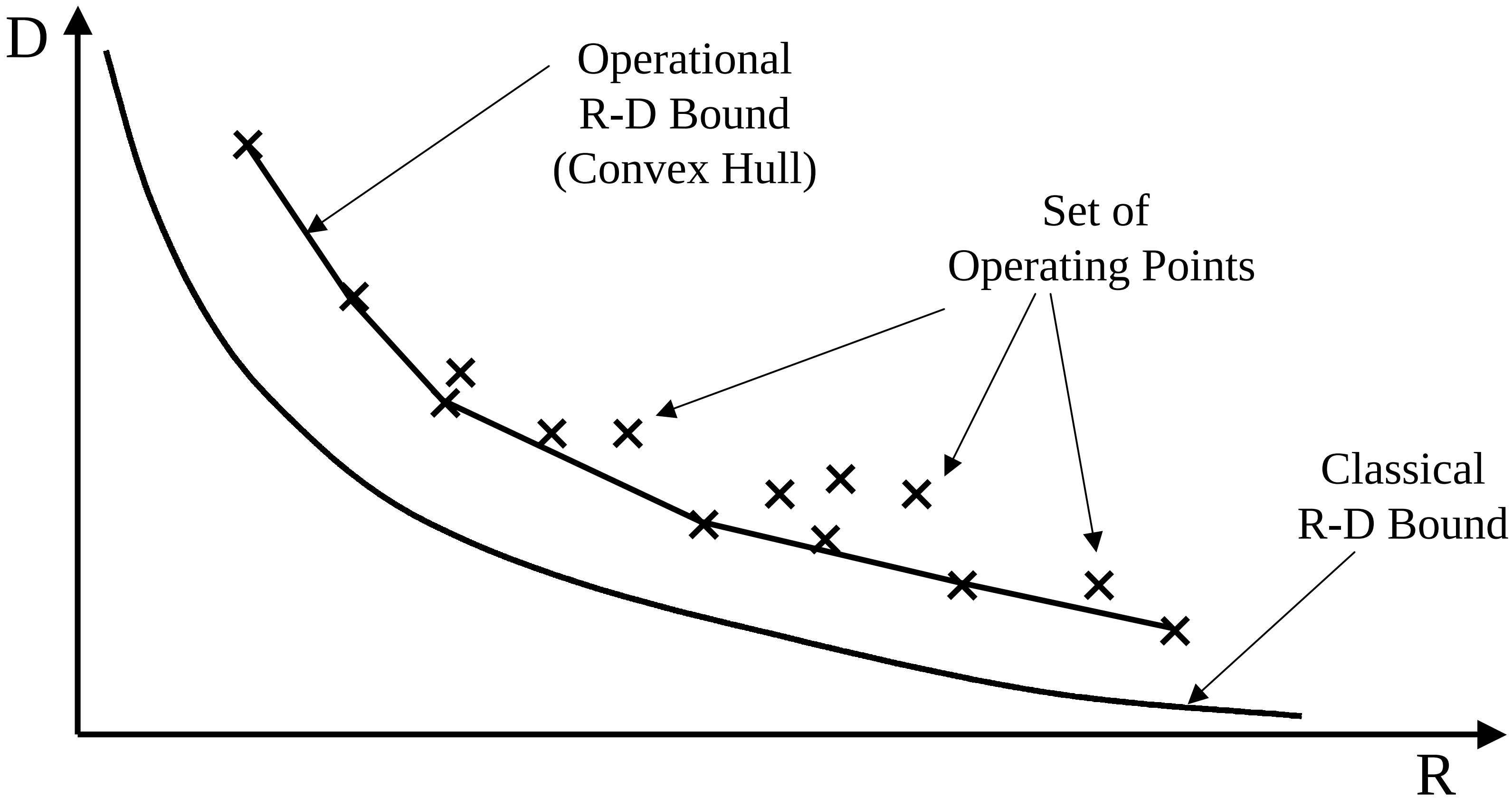
II-A. Choosing Parameters of Concrete System: Operational R-D

- Abandon (classical) search for best unconstrained R-D performance
- Adopt the following operational approach :
  - Choose a specific coding scheme
    - Efficiently capture relevant statistical dependencies of source type of interest
    - Satisfies system requirements (complexity, delay, memory, etc.)
  - Search for the best operating points for that specific system

- Consider Optimality in the Operational Sense
  - Given our choice compression framework:
  - Find best achievable performance for a given source
    - Source is described by a training set or given statistical model
      - Training set is most practical (because useable closed-form models aren’t usually known for real sources)

An Operational R-D Characteristic

- Composed of all possible operating points obtained by applying admissible coding parameters to each of the elements in a particular set of test data



II-B. Choosing a Good Model: Transform Coding

- Main challenge in achieving good R-D performance is finding a model that is
  - Simple enough that good performance can be achieved with reasonable "cost"
  - Complex enough to capture main characteristics of source
- Many choices are available:
  - Scalar
  - Vector
  - Transform (which one?)
  - Predictive
  - Signal-Model Based
    - E.g., speech compression models speech as AR and sends AR parameters rather than samples or transform coefficients
  - What PDF?
  - Local vs. Global Variance Estimates
  - Spatial Redundancy Structure (e.g., trees as in EZW & SPIHT)
  - Etc.

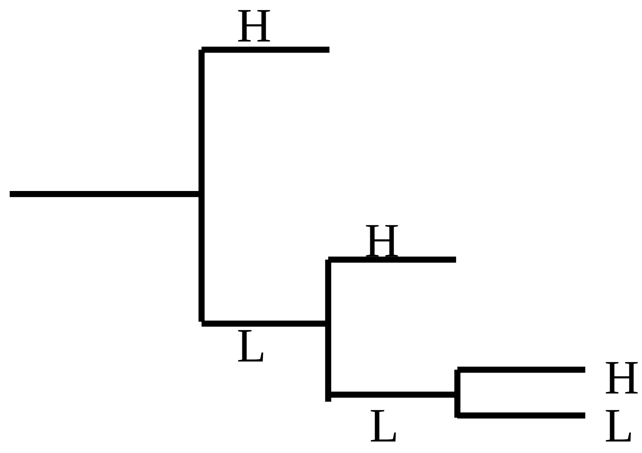
Box #2: Duping JPEG in Operational R-D Sense

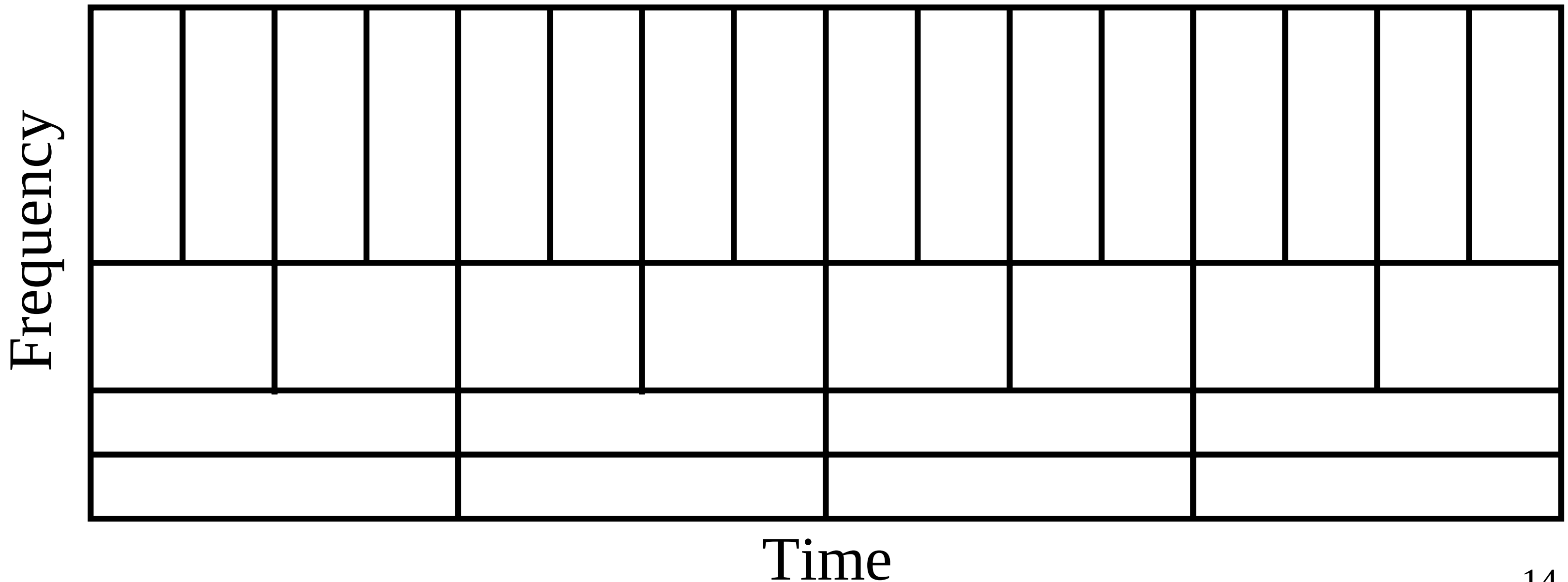
- Although we described JPEG from the encoder point of view:
  - JPEG standard is actually syntax-specified from the decoder point of view
  - ➔ Encoder has flexibility to deviate from “standard” operation
- Simplest Way:
  - use custom quantization matrix and entropy tables on per-image basis
- More devious way:
  - Encoder “dupes” the decoder in an optimal R-D way while meeting syntax
  - Example: a small non-zero value can breakup otherwise long runs of zeros
    - They are Expensive from R-D view
    - Encoder lies to the decoder: says this non-zero is zero
      - ➔ code as long run of zeros; more efficient coding
      - Encoder does this if it improves the R-D
      - Decoder “doesn’t know any better”
    - Research results: gains on order of 25% in compression efficiency
- This is an example of Operationally Optimal R-D
  - Given JPEG syntax as framework
  - Optimize R-D over the parameters

Box #3: Adaptive Transforms from Wavelets

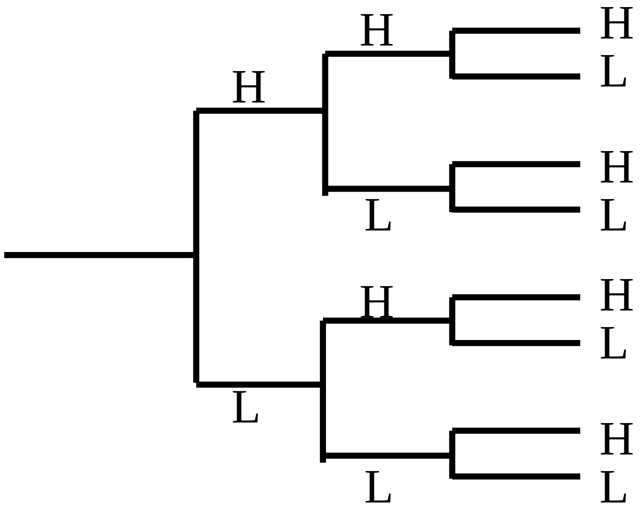
- General transform coding framework:
  - Transform, quantizer, entropy coder
  - We've talked about encoder adapting quantizer and/or entropy coder
    - i.e., via bit allocation
  - But, could also adapt the choice of transform
- Example: Wavelet is actually a family of flexible transforms
  - Adaptively choose a mother wavelet
    - e.g. choose a particular filter from set of allowable wavelet filters
  - Adaptively choose the number of subbands used
- Even more flexibility comes from generalizations of wavelets
  - Called wavelet packets
  - Come about from modifying wavelet filterbank structure
    - Don't always "leave HP channel, split LP channel"

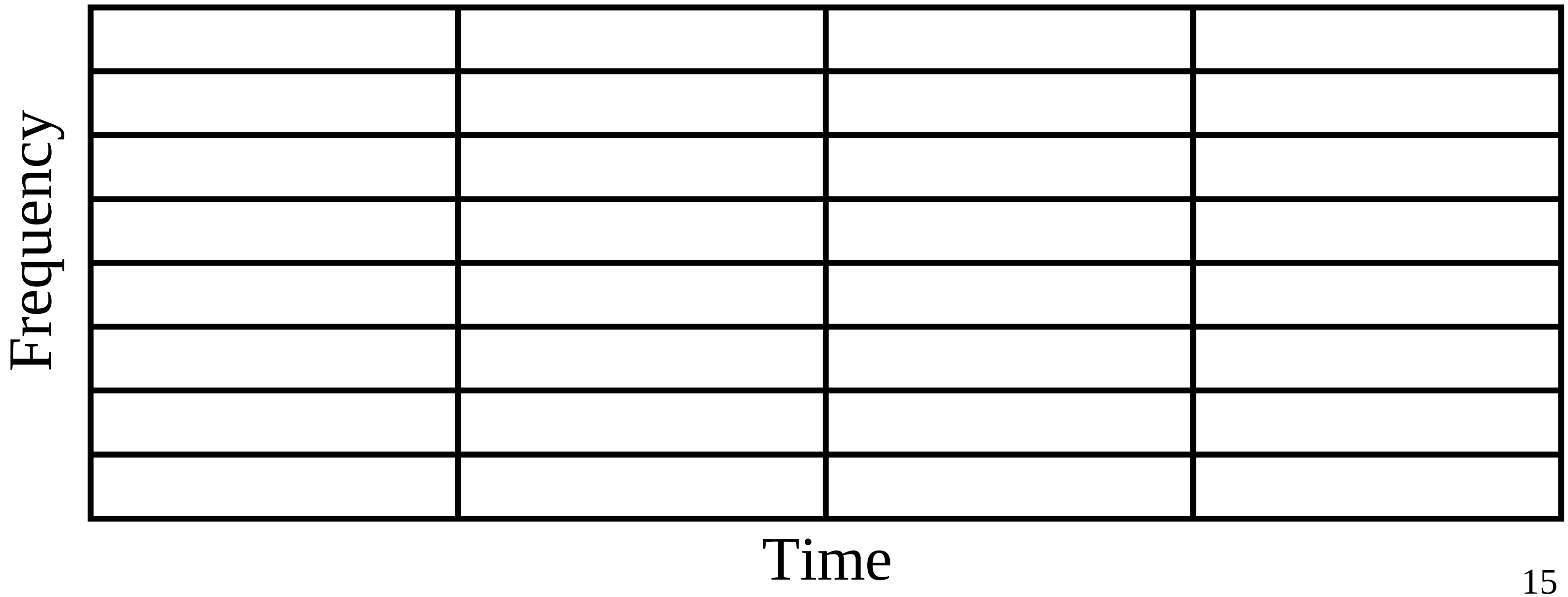
Box #3: Wavelet Tree and T-F Tiling



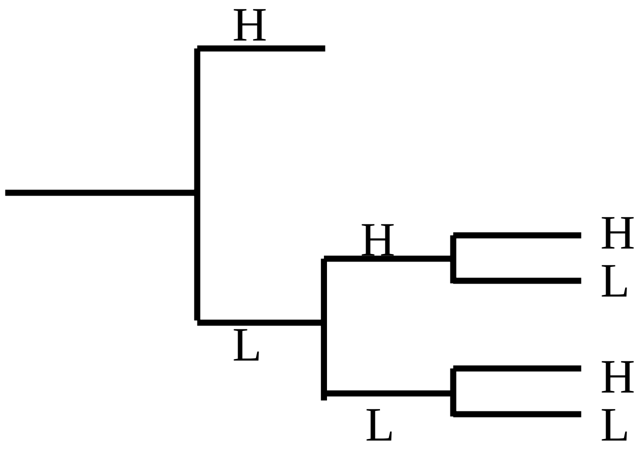


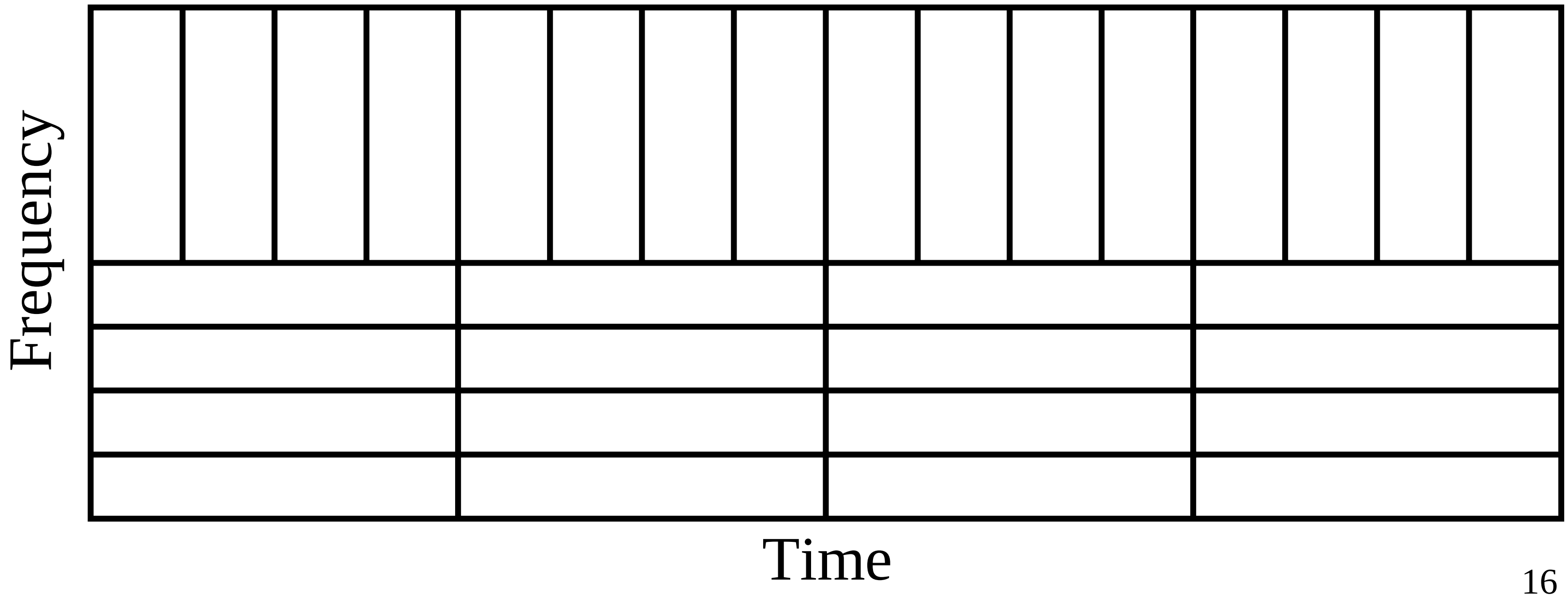
Box #3: STFT Tree and T-F Tiling



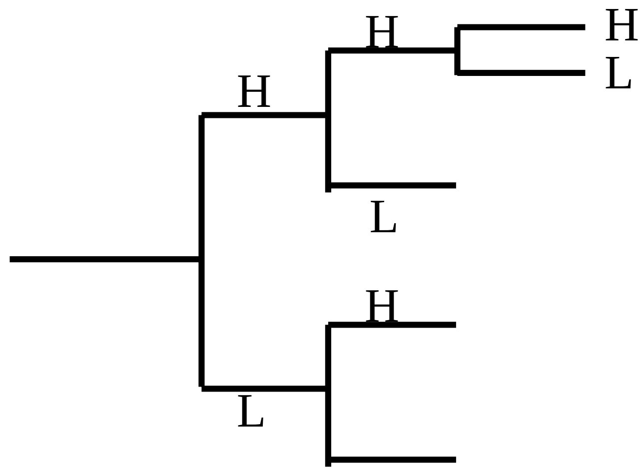


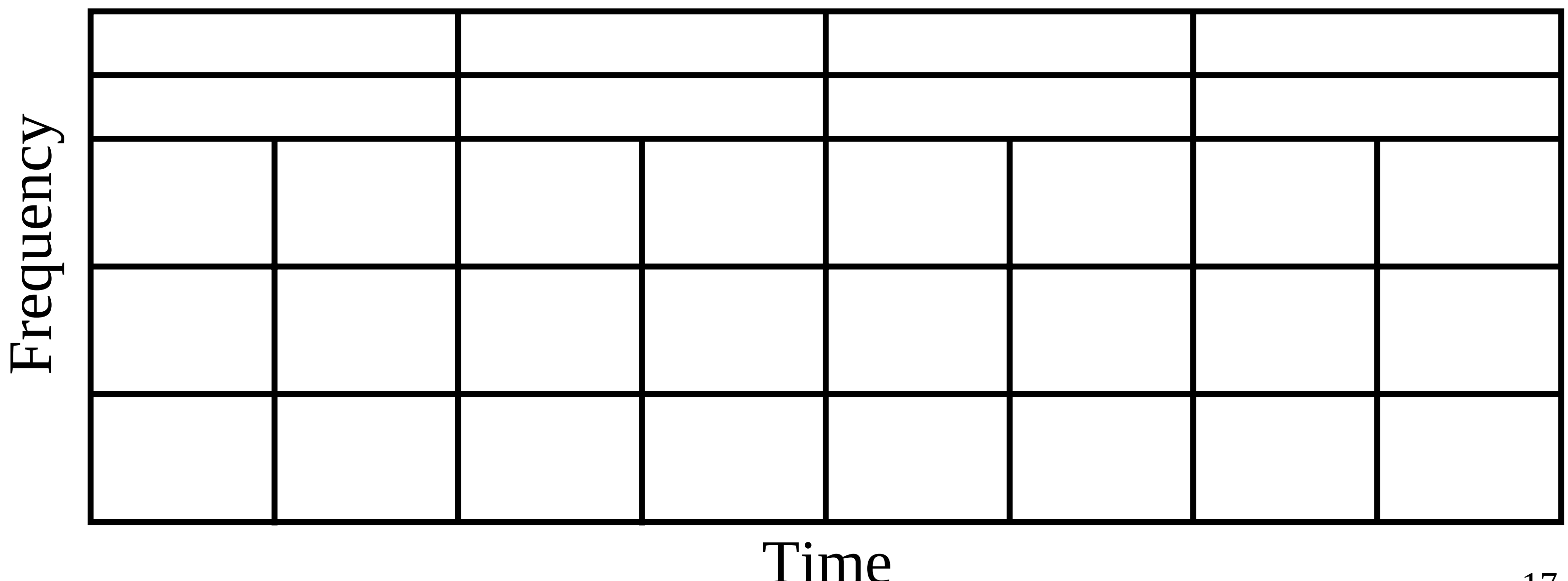
Box #3: *A* Wavelet Packet Tree and T-F Tiling





Box #3: *Another* Wavelet Packet Tree and T-F Tiling





II-C. Standards-Based Coding: Syntax-Constrained R-D Optimization

- Compression standards provide an agreed upon bit stream syntax
  - Needed to ensure interoperability
  - Any standard-compliant decoder can then decode bit stream
- Goal: Syntax-Constrained Optimization
  - Encoder's task: select the best operating point from a discrete set of options agreed upon *a priori* by a fixed decoding rule (i.e. the decoder syntax)
  - Selected Operating Point is Side Information
    - Sent to decoder (typically in the header)
    - Trade-offs:
      - Flexibility vs. Amount of Side Info
      - Flexibility vs. Computational Complexity

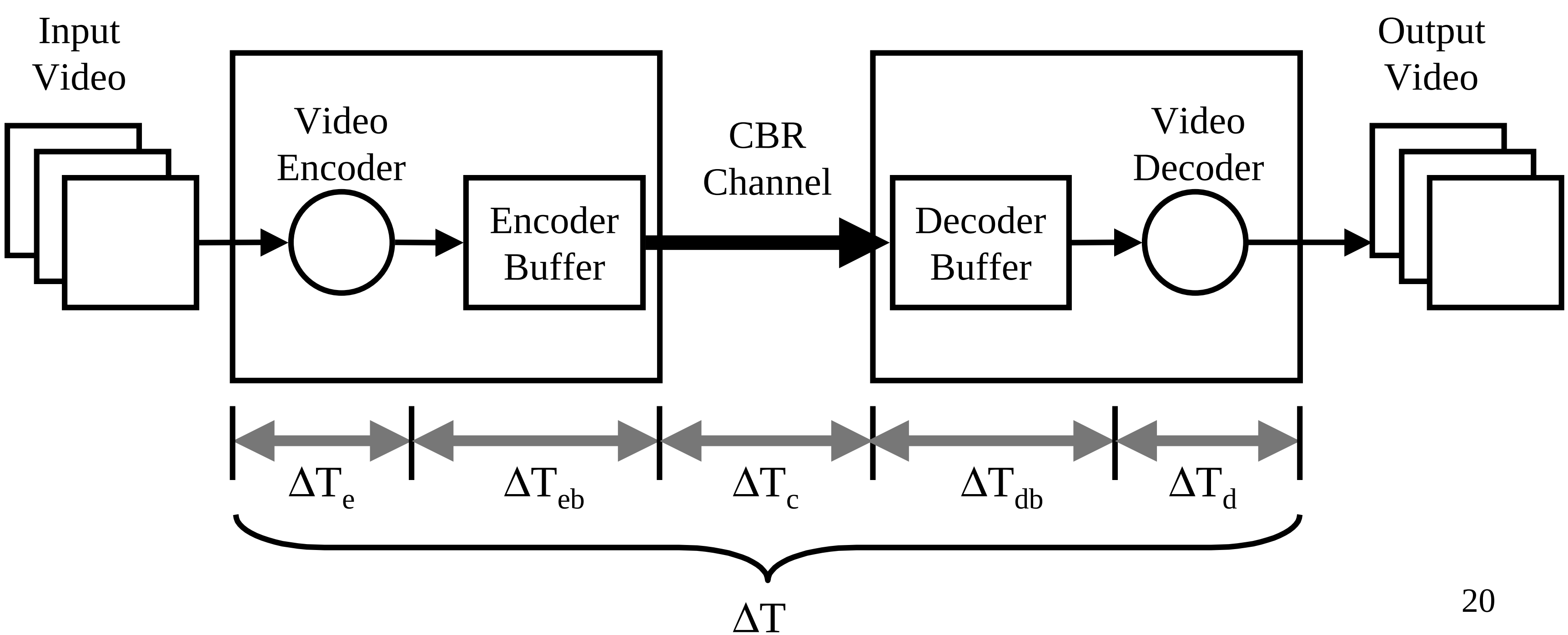
<u>II-C. Standards-Based Coding (cont.)</u>

<u>Formulation #1</u> – General Discrete R-D Optimization:
Given a specific encoding framework where the decoder is fully defined, optimize the encoding of a ***<u>particular</u>*** image or video sequence in order to meet some rate/distortion objectives

- Note: optimizing for a ***<u>particular</u>*** input
- Caution: selection of the coding framework is key to performance
  - Bad Approach: poor framework & sophisticated optimization method
  - Recall Exp. #1 in Box #1: i.i.d. Gaussian & Shannon Coding = Bad
- Optimal Solution = the operating point giving best objective function value
- Since there are finite number of operating points (coding choice)
  - Could do an exhaustive search
  - But, strive for efficient non-brute-force optimization approach

Box #4: Delay Constrained Transmission & Buffer Control

- Coding of Video Sequences results in a variable bit rate
  - Need an Encoder Buffer to connect the variable bit rate (VBR) coded stream to the constant bit rate (CBR) channel
  - Need Decoder Buffer to connect the CBR channel to the VBR decoding stream
- Video Input Rate = Video Output Rate ➔ constant ΔT (end-to-end delay)
  - Frame coded at time t must be decoded at time $t + \Delta T$



Box #4: Delay Constrained (Cont.)

- Encoder/Decoder Delays, ΔT_e and ΔT_d, assumed constant due to processing considerations
- Channel Delay, ΔT_c, is assumed constant because of CBR channel
- Thus, only buffer delays, ΔT_{eb} and ΔT_{db}, are variable
- Constraint on end-to-end delay ΔT ➔ Need for **<u>encoded rate control</u>**
  - Constraint on ΔT puts an upper bound on buffer size B_{max} (in bits)
    - Need $B_{max} < C\,\Delta T$ where C = channel rate in bits/s
    - Otherwise bits going in when buffer is nearly full would take more than ΔT to come out at the emptying rate of C
  - Range of Variation in Coded Rate puts lower limit on B_{max}
    - Otherwise we could overflow the buffer during high-rate segments
  - Thus we either have to:
    - Use large buffers to deal with rate variation (causes excessive delay)
    - Use shorter buffers to meet delay and reduce variation using rate control to ensure buffers don't overflow
- Note that MPEG (and other methods) have rate control capabilities
  - Research Issue: Operational R-D Optimized Rate Control

III. Typical Allocation Problems

- Two Basic Classes
  - Compression for Storage
    - Rate Budget Constraint
  - Compression for Transmission
    - Delay Constrained
    - Buffer Constrained

Several Practical Issues to Address

- **Selection of Basic Coding Unit**
  - Coding Unit = entity for which encoder parameters can be set
    - Sample, Block, Image, Subsequence of Frames, Etc.
  - Example: Video
    - Might use Coding Unit = Video Frame
    - Measure frame-wise rate-distortion & decide operating point per frame
  - Example: Image
    - Might use Coding Unit = 8×8 block of pixels (JPEG)
  - Optimization could be over a single coding unit or multiple units

- **Complexity** – Two main sources:
  - R-D itself may have to be measured from data (several encodes/decodes)
    - Can ease this by using models or approximations of R-D
  - Finding the Optimal operating point

Several Practical Issues (Cont.)

- **Cost Function –** May include both rate and distortion
  - Easily computed for each coding unit
  - But, when allocating among several units:
    - Overall cost requires careful definition
    - There are several options
  - Example: Long Video Sequence
    - consider cost = average distortion over all units
    - Is this really a desirable cost function?
      - Could have large peak distortion in some frames
    - Might it be better to minimize worst-case distortion?
      - So-called minimax criterion
      - Could have larger average distortion
  - Also should consider perceptually-weighted versions
- **Notation**
  - N coding units (i = 1, 2, …, N) Each having M operating points (j = 1, 2, ..

Notation

- Assume N coding units ($i = 1, 2, \ldots, N$)
- Each coding unit has M operating points ($j = 1, 2, \ldots, M$)
- For the i^{th} coding unit when using the j^{th} "quantizer" we have
  - Rate: r_{ij}
  - Distortion: d_{ij}
- "Quantizer" indices j are listed in order of increasing coarseness
  - $j = 1$ is the finest quantizer (highest rate, lowest distortion)
  - $j = M$ is the coarsest quantizer (lowest rate, highest distortion)
- Will formulate problems under two types of constraint
  - Total Bit Budget (e.g., storage applications)
  - Transmission Delay (e.g., video transmission)

III-A. Storage Constraints: Budget-Constrained Allocation

- Here, rate is constrained by a restriction on the maximum total number of bits
  - Total Number of Bits = R_T
  - Must allocate the R_T bits among the *N* coding units
  - Allocation should minimize some overall distortion metric
- Examples:
  - Allocate bits among 8×8 blocks of pixels in an image
  - Allocate bits among a set of images to be compressed into an archive
    - Here we may care about the aggregate quality of the set of images

III-A. Budget-Constrained Allocation (Cont.)

Formulation #3 – Budget Constrained Allocation:
Find the optimal quantizer (i.e., operating point) $j(i)$ for each coding unit i such that

$$\sum_{i=1}^{N} r_{ij(i)} \le R_T$$

and some metric $f\,(d_{1j(1)}, d_{2j(2)}, \ldots, d_{Nj(N)})$ is minimized.

- Example Metric: Minimum Average Distortion Metric (i.e., MMSE)

$$f\,(d_{1\,j(1)}, d_{2\,j(2)}, \ldots, d_{Nj(N)}) = \sum_{i=1}^{N} d_{ij(i)}$$

- Note: Formulation #3 with the Minimum Average Distortion Metric is nothing more than the bit allocation problem we already looked at
  - Where we assumed:
    - Each quantizer's input was i.i.d. with some known variance

Alternative Metrics for Formulation #3

- **Minimax (MMAX) Approach**
  - Minimize the maximum distortion over the coding units
  - That is, for all possible operating points, the optimal point is the one with the smallest maximum distortion
  - Example showing only three possible operating points:
    - $d_{18} = 113$, $d_{27} = 91$, $d_{33} = 34$, $d_{45} = 47$ MSE = 285 MAX = 113
    - $d_{16} = 97$, $d_{25} = 95$, $d_{34} = 50$, $d_{44} = 50$ MSE = 292 **MAX = 97**
    - $d_{17} = 103$, $d_{24} = 86$, $d_{36} = 90$, $d_{44} = 55$ MSE = 334 MAX = 103
    - First one is MMSE solution; Second one is MMAX solution
  - Is a good alternative to MMSE
    - MMSE can result in some really bad distortion in a small number of units
    - MMAX tries to put a limit on the "worst that can happen"

Alternative Metrics for Formulation #3 (Cont.)

- **Lexicographically Optimal (MLEX) Approach**
  - Sort quantizers used into decreasing order of index (i.e. decreasing MSE)
  - Use sorted indices to form the digits of a number (one per operating point)
  - The optimal point is the one with the smallest such number
  - Example showing only three possible operating points (same as above):
    - $d_{18} = 113$, $d_{27} = 91$, $d_{33} = 34$, $d_{45} = 47$ LEX = 8753 MSE = 285
    - $d_{16} = 97$, $d_{25} = 95$, $d_{34} = 50$, $d_{44} = 50$ **LEX = 6544** MSE = 293
    - $d_{17} = 103$, $d_{24} = 86$, $d_{36} = 90$, $d_{44} = 55$ LEX = 7644 MSE = 334
    - Second one is MLEX solution

  - MLEX is a generalization of MMAX
  - MLEX tends to equalize distortion across all coding units
    - Gives the coded data a more uniform appearance

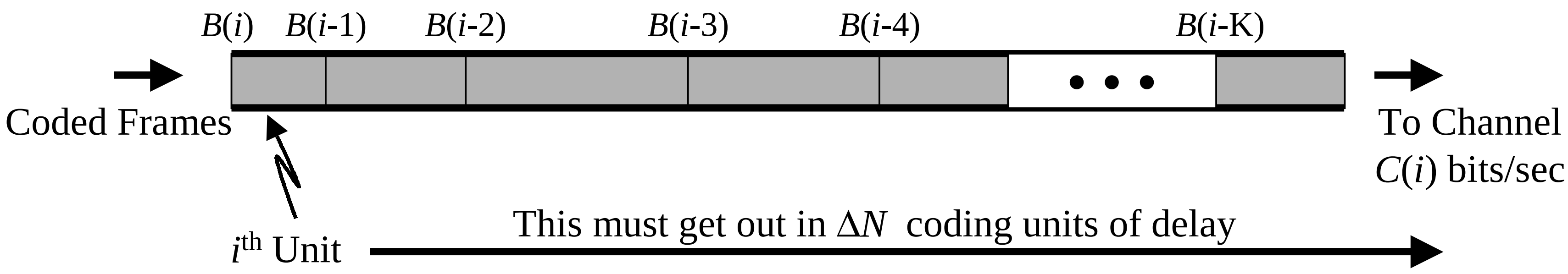
III-B. Delay-Constrained Allocation & Buffering

- Formulation #3 can't handle case where coding units (e.g., video frames) are streamed across a link
- The constraint here is: each coding unit is subject to a delay constraint
  - Let a coding unit be coded at time t
  - It must be available at the decoder at time $t+\Delta T$ (assumes fixed decode time)
    - Where ΔT is the end-to-end delay
  - Can express coding delay in terms of "coding units"
    - If each coding unit lasts t_u seconds, then
      - $\Delta N = \Delta T / t_u$ is the coding delay in "coding units"
    - So, at any time there will be ΔN coding units in the system stored in:
      - Encoder buffer, in transit, decoder buffer
    - Ex: For 30 frames/sec and $\Delta T = 2$ sec
      - Then have $\Delta N = 2 / (1/30) = 60$ stored frames

Formulation #4 – Delay Constrained Allocation:
Find the set of quantizers $j(i)$ such that each coding unit i coded at time t_i is at the decoder at time $t_i + \delta_i$ while minimizing a distortion metric $f(d_{1j(1)},\ldots, d_{Nj(N)})$.
For ease, often assume that $\delta_i = \Delta T$ is the same for all coding units.

Impact of Delay Constraint on Buffer

- What impact does this delay constraint have on buffer constraints?
  - Assume a variable channel rate: $C(i)$ bits/sec during the i^{th} coding interval
  - Then encoder buffer state at time i is:
    - $B(i) = \max\left\{ [B(i\text{-}1) + r_{ij(i)} - C(i)] ,\ 0 \right\}$ w/ initial state $B(0) = 0$
    - Also, $B(i)$ can't grow larger than buffer physical size: $B(i) \le B_{\max}$
    - <u>**BUT**</u>, there is another constraint on the buffer:



- How many bits can be emptied in ΔN coding units? $$B_{eff}(i) = \sum_{k=i+1}^{i+\Delta N} C(k)$$
- Thus, to get the i^{th} unit to the decoder in ΔT seconds using this channel there can be no more than $B_{eff}(i)$ bits in the buffer after the i^{th} unit is put in the buffer

Delay Constraint Leads to Buffer Constraint

Formulation #5 – Buffer Constrained Allocation:
Find the set of quantizers $j(i)$ such that the buffer occupancy $B(i)$ doesn't exceed the effective buffer size $B_{eff}(i)$ while minimizing a distortion metric $f(d_{1j(1)},\ldots, d_{Nj(N)})$; where

$$B_{eff}(i) = \sum_{k=i+1}^{i+\Delta N} C(k) \qquad B(i) = \max\left\{ \left\lfloor B(i-1) + r_{ij(i)} - C(i) \right\rfloor, 0 \right\}$$

- Note: constraints depend on the channel's future rates!!!
  - If user can choose the rates (e.g. transmission over a network):
    - What is the best combination of channel rates and compression rates?
  - If future rates are uncertain:
    - Can't know deterministically what the effective buffer state is
    - Thus, need a good model of expected channel rates
      - Need some probabilistic model

IV. The R-D Optimization Toolbox

- Two Types of Problems
  - Independent Problems
    - R-D operating points can be measured indep. for each coding unit
  - Dependent Problems
    - R-D operating points for a coding unit depend on choices made for the others

IV-A. Independent Problems

- Here, r_{ij} and d_{ij} can be measured independently for each coding unit
  - Example: JPEG coding of AC DCT components; coding unit = block
  - Not Independent: anytime prediction between coding units is used
    - Example: MPEG frames using motion-compensated prediction
    - Sometimes ignore dependence to speed up encoding
- Goal: Given some chosen coding framework:
  - Compute or obtain the achievable R-D operating point data
  - Use some optimization method to choose the optimal operating point according to some appropriate formulation
- Two main optimization methods discussed:
  - Lagrangian
    - Operational form of "equal slope" solution we discussed earlier
  - Dynamic Programming
    - Trellis-based solution (similar to Viterbi Algorithm in digital comm.)

Lagrangian Method

- Recall the Equal Slope result we derived in class (more detailed than text)
  - Optimal allocation must be such that $dR_i/dD_i = -\lambda$ for all i
  - But what value for λ?
    - The one that results in: total # bits = bit budget
      - **λ controls the total rate** and is set to meet the budget

- We derived a closed-form optimal bit allocation result:
  - Used a closed-form "high-rate" R-D result for scalar quantizer

    $$\sigma_k^2 = \alpha 2^{-2R_k}\,\sigma_{c_k}^2$$

    - This is not an operational R-D approach
  - Got a result for the R_k that depended on λ

    $$R_k = \frac{1}{2}\log_2\left[2\alpha \ln 2\sigma_{c_k}^2\right] - \frac{1}{2}\log_2[\lambda]$$

  - Plugged them into Total Rate constraint

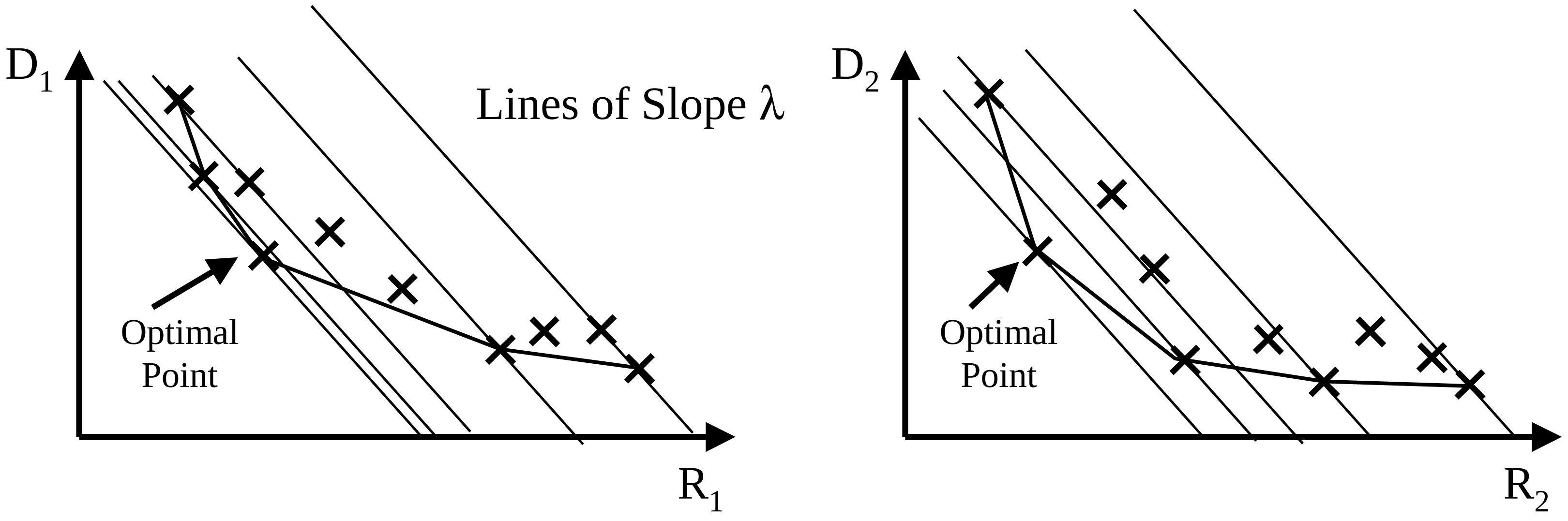
    $$R = \sum_{k=1}^{M} R_k$$

  - Solved for λ and put result into R_k result

    $$R_k = \frac{R}{M} + \frac{1}{2}\log_2\left[\frac{\sigma_{c_k}^2}{\prod_{l=1}^{M}\sigma_{c_l}^2}\right]$$

Lagrangian Method (cont.)

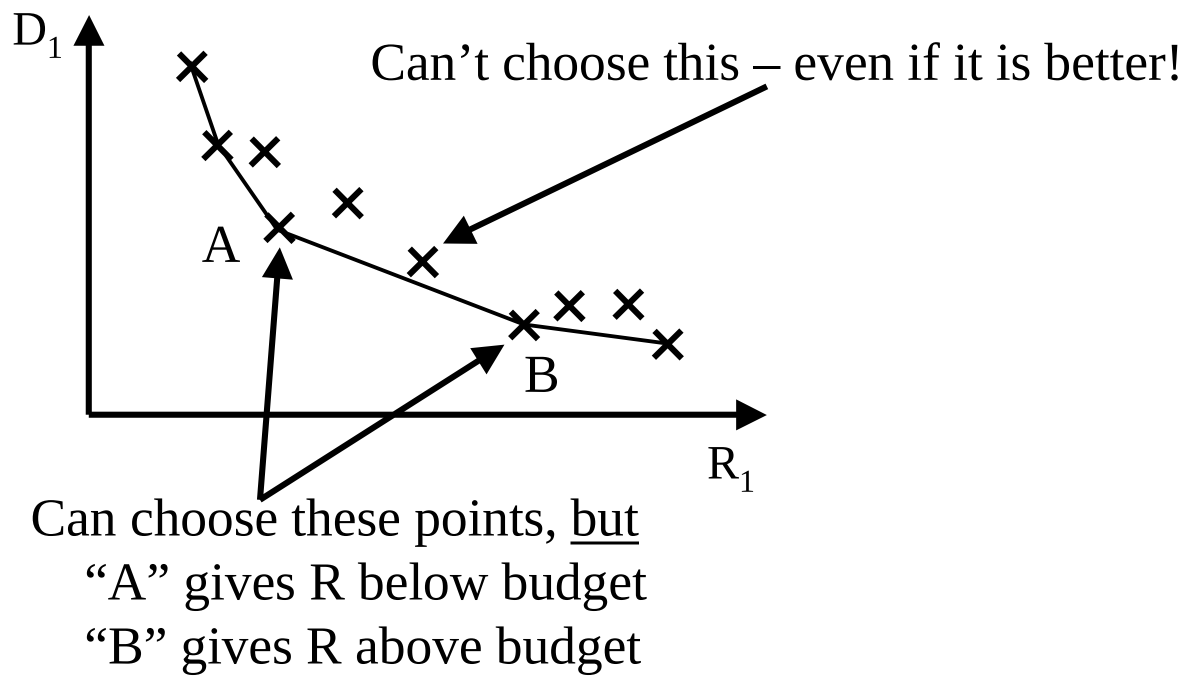
- What we want is a way to use Lagrangian method on <u>operational</u> R-D data
- Key points to draw from the non-operational allocation result:
  - <u>Equal Slopes Condition</u> gives <u>Optimal Allocation</u>
  - Slope = $-\lambda$
  - Value of λ is adjusted to obtain the desired total rate R that meets budget
- For a given λ and a given set of operational R-D data it is easy to find the point on the convex hull that is the optimal operating point:



- Challenge: finding the right λ to meet the budget

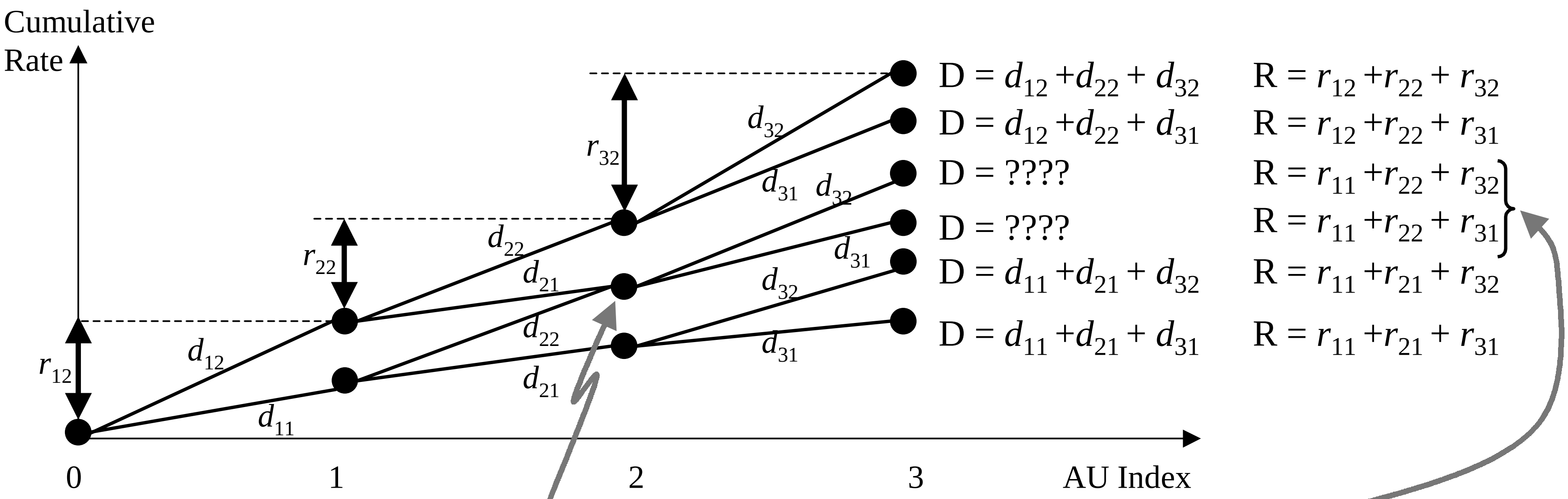
Lagrangian Method (cont.)

- Challenge: finding the right λ to meet the budget
  - Easy to do
  - Can be done independently for each coding unit
  - Uses the so-called “Bisection Search” Method (see references in paper)
- Complexity of Lagrangian method is low
- But, can have problems when only a few operating points are on convex hull:
  - Lagrange can’t choose a point off the convex hull



Dynamic Programming Method

- For the Operational R-D problem:
  - Create a trellis (i.e., a tree) with each stage being a allocation unit (AU)
    - M quantizers $\equiv$ M allocation units; each has P operating points
    - Thus, you have P^M possible allocations
  - Going from stage-to-stage: a branch for each of the P operating points
  - Each branch is labeled with the distortion achieved by that operating point
  - The node that each branch goes into is at height = cumulated rate of path
- The **non-pruned** trellis looks like (for M = 3, P = 2):

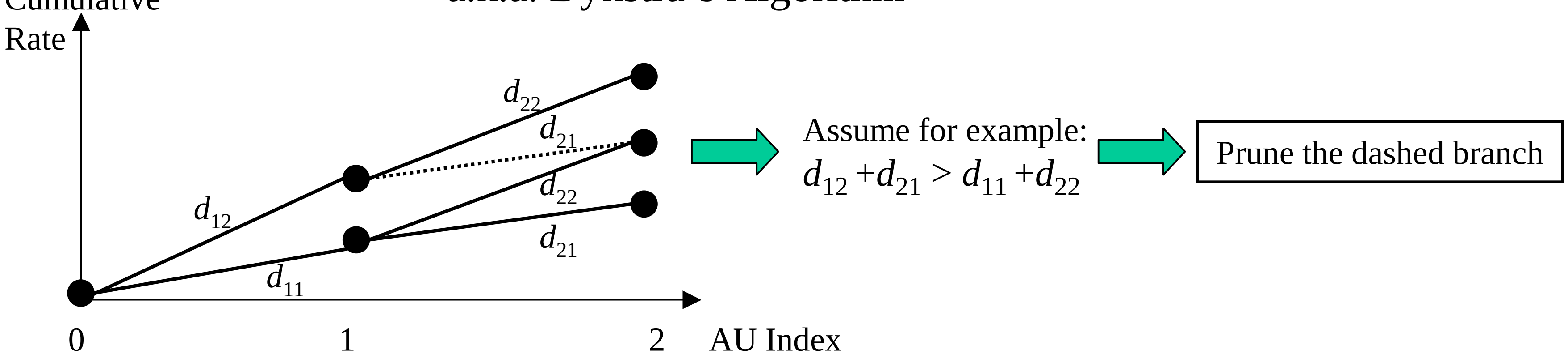


For this example: $r_{11} + r_{22} = r_{12} + r_{21}$

Dynamic Programming Method (cont.)

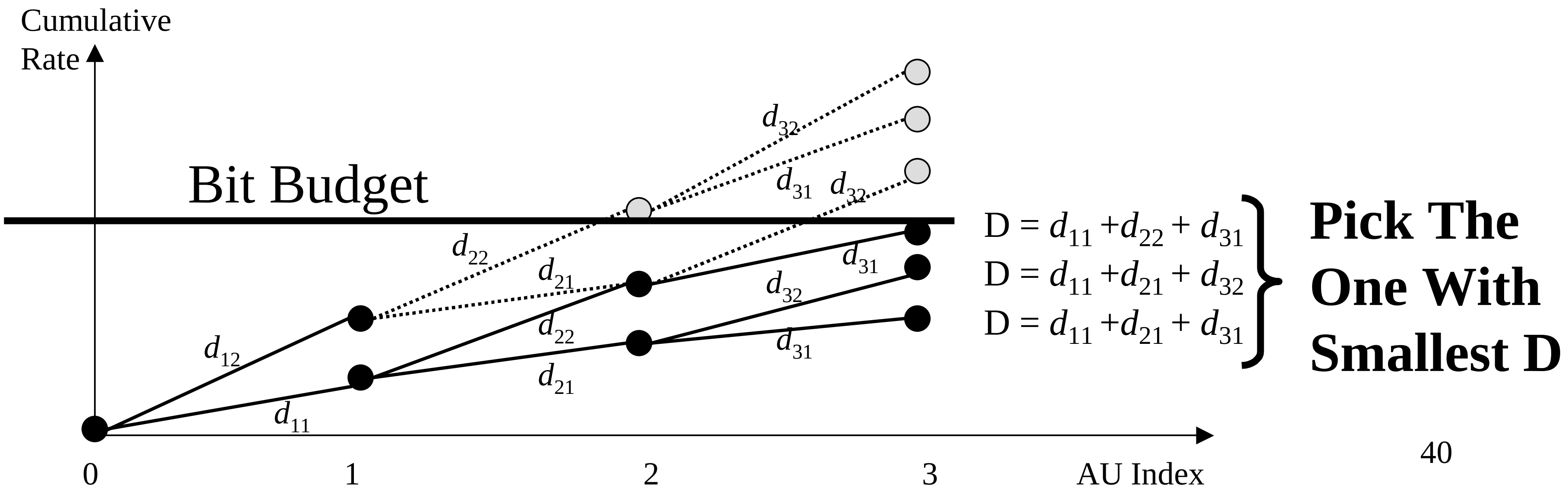
- **<u>Prune</u>** the trellis as it is built
  - Prune it to optimize
  - Prune it when it exceeds a constraint

- Pruning to optimize
  - If two branches go to the same node (i.e. have the same cumulative rate)
    - Prune the one with the larger distortion
    - Retains minimum distortion for that node
      - This is Bellman's Optimality Principle
        - » a.k.a. Viterbi Algorithm
        - » a.k.a. Dykstra's Algorithm



Dynamic Programming Method (cont.)

- Pruning to meet constraints
  - Prune a branch if it exceeds the Total Rate Constraint (Bit Budget)
    - Trellis can't grow above a "ceiling"
  - Prune a branch it it exceeds the Buffer Constraint
    - Would need to keep track of Buffer Size of Each Branch
      - On each branch, put a second "tag" along side its distortion tag
- Con: Computationally Complex
- Pro: Method can achieve operating points that are not on convex hull
- When points are dense on convex hull Lagrangian method can give nearly as good result at much less complexity

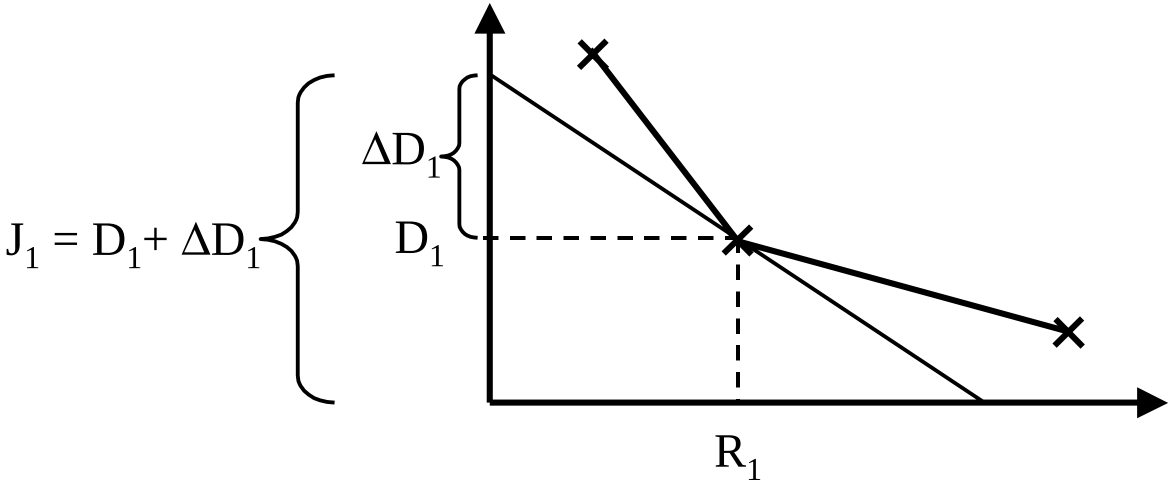


IV-B. Dependent Problems

- In some scenarios we can't make decisions independently on each coding unit
- One example of this is in predictive-based coding:
  - Assume that the i^{th} coding unit is predicted from the $(i\text{-}1)^{th}$ coding unit
  - Prediction is done using the past quantized data
    - Must use quantized data since that is what the decoder has available
      - Otherwise there is a growth in the quantization error variance
      - See equation (10.8) in textbook
    - But this use of quantized data causes a dependency between coding units when you try to find the optimal operating points
- To see how this works we first need to revisit the Lagrangian cost

Revisit Lagrangian Cost

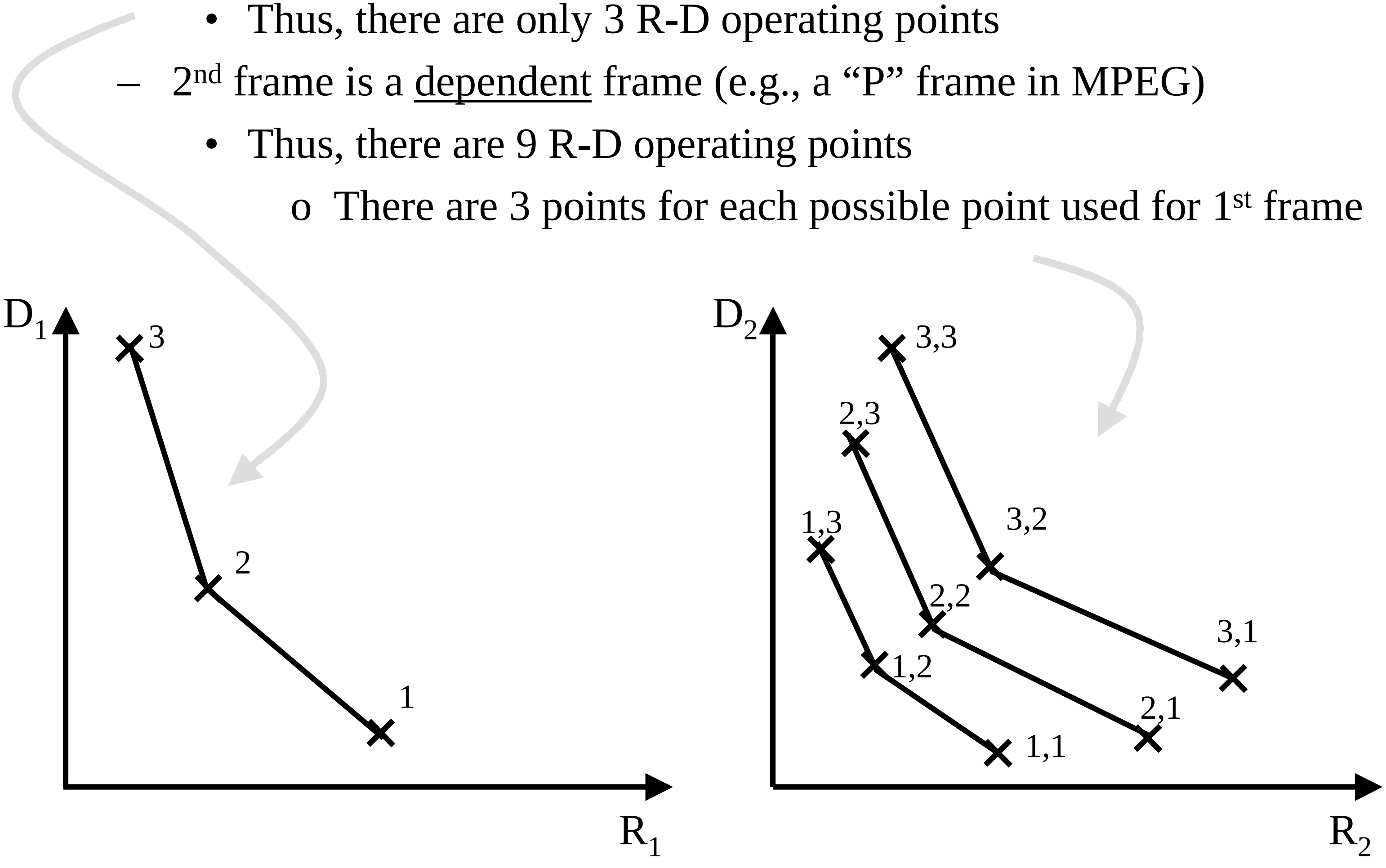
- Lagrangian Cost: $$J = D + \lambda R = \sum_i D_i + \lambda \sum_i R_i$$ ➔ $$J_i = D_i + \lambda R_i$$
- Let's re-interpret what J_i is:
  - Recall that $-\lambda$ is the slope of the "operating line"
  - Thus, $\lambda = \Delta D_i / R_i$ ➔ $J_i = D_i + (\Delta D_i / R_i) R_i = D_i + \Delta D_i$



- Thus, we are minimizing the sum of the y-intercepts of the "λ-slope lines"

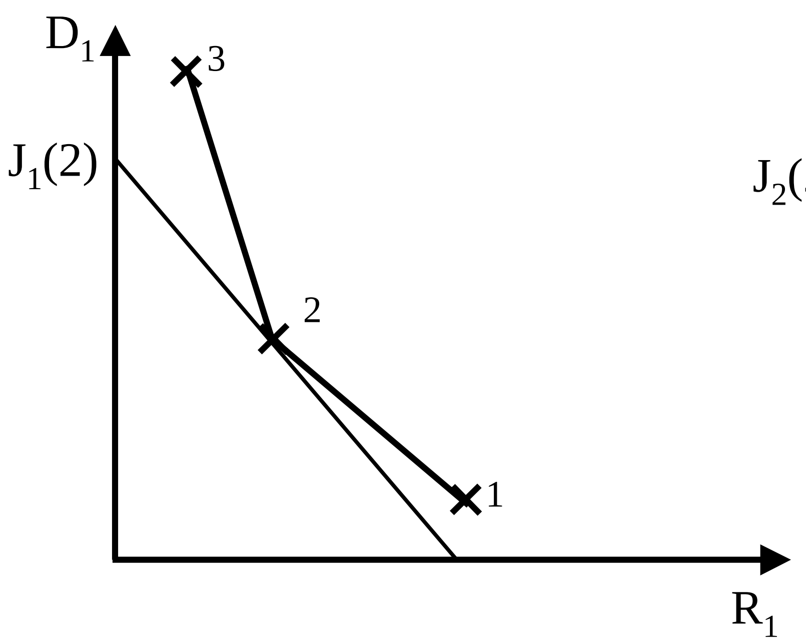
Dependency Between Coding Units

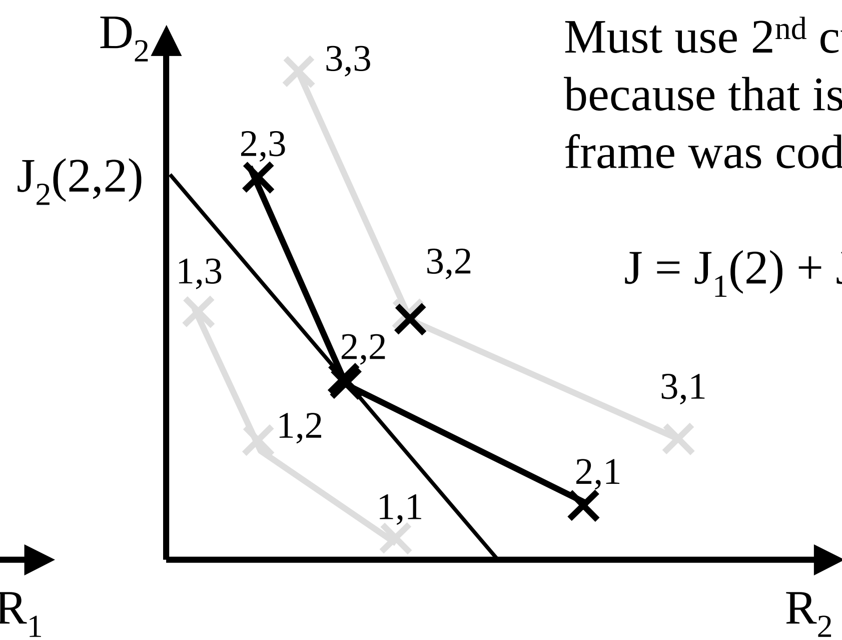
- Consider this simple video example of two frames:
  - Assume each frame can be coded at 3 different quantizer settings
  - 1st frame is an <u>independent</u> frame (e.g., an “I” frame in MPEG)
    - Thus, there are only 3 R-D operating points
  - 2nd frame is a <u>dependent</u> frame (e.g., a “P” frame in MPEG)
    - Thus, there are 9 R-D operating points
      - o There are 3 points for each possible point used for 1st frame



Ignoring Dependency

- If in this this example we ignored the dependency and coded each frame independently:
  - The 1st frame would be coded at operating point #2 (for the given λ)
  - 2nd frame operating points must be chosen from the 2nd curve because that is how the 1st frame is coded
    - This <u>ignores dependency</u>
    - You might be able to get a better cost J by <u>jointly optimizing</u> over all the operating points



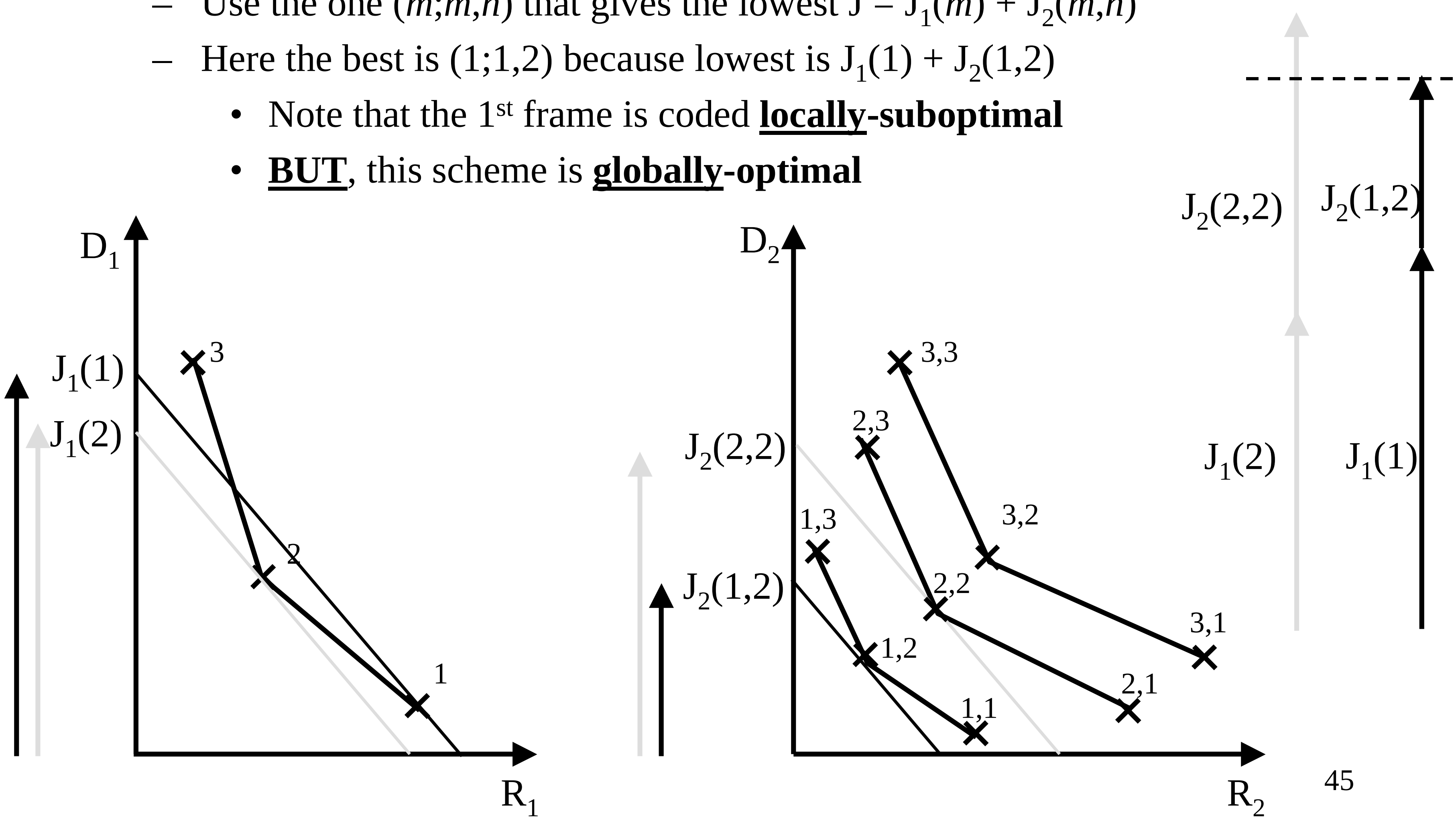


Must use 2nd curve here because that is how 1st frame was coded

$J = J_1(2) + J_2(2,2)$

Considering Dependency

- If in this this example we consider dependency:
  - Have to try all 9 possibilities (for the given λ)
    - (1;1,1) (1;1,2) (1;1,3) (2;2,1) (2;2,2) (2;2,3) (3;3,1) (3;3,2) (3;3,3)
  - Use the one (m;m,n) that gives the lowest $J = J_1(m) + J_2(m,n)$
  - Here the best is (1;1,2) because lowest is $J_1(1) + J_2(1,2)$
    - Note that the 1st frame is coded **<u>locally</u>-suboptimal**
    - **<u>BUT</u>**, this scheme is **<u>globally</u>-optimal**



Handling Dependency

- Complicates the computing of the R-D operating points
  - Can sometimes use analytic models of the dependent R-D characteristics
    - Then don't need to compute all possible operating R-D values
- Necessitates the use of dynamic programming
  - Use trellis-based approaches that capture the dependence
- Full trellis structure has exponential growth in number of combinations
  - Possible to make approximations that simplify the search
  - Possible to embed suboptimal approaches into the trellis
    - Prune at each stage use e semi-greedy approach

V. Application to Basic Components in Image/Video Coding Algorithms

- Budget Constraint Problems
  - Fixed-Transform-Based Case
  - Adaptive Transform-Based Case
- Delay Constraint Problems
- Role of R-D in Joint Source-Channel Coding

This section of the paper gives a tour of the types of problems that have been addressed in the recent literature and gives thorough pointers to relevant, high-quality references.