Ch. 10 Vector Quantization

Overview

Motivation

Recall in Lossless Methods:

- Block Huffman is better than single-symbol Huffman
  - Blocks allow to exploit correlation between symbols (assuming source symbols are not independent!)
- Shannon proved that "blocking taken to the limit" achieves optimal compression… exploits correlation

Recall in Scalar Quantization:

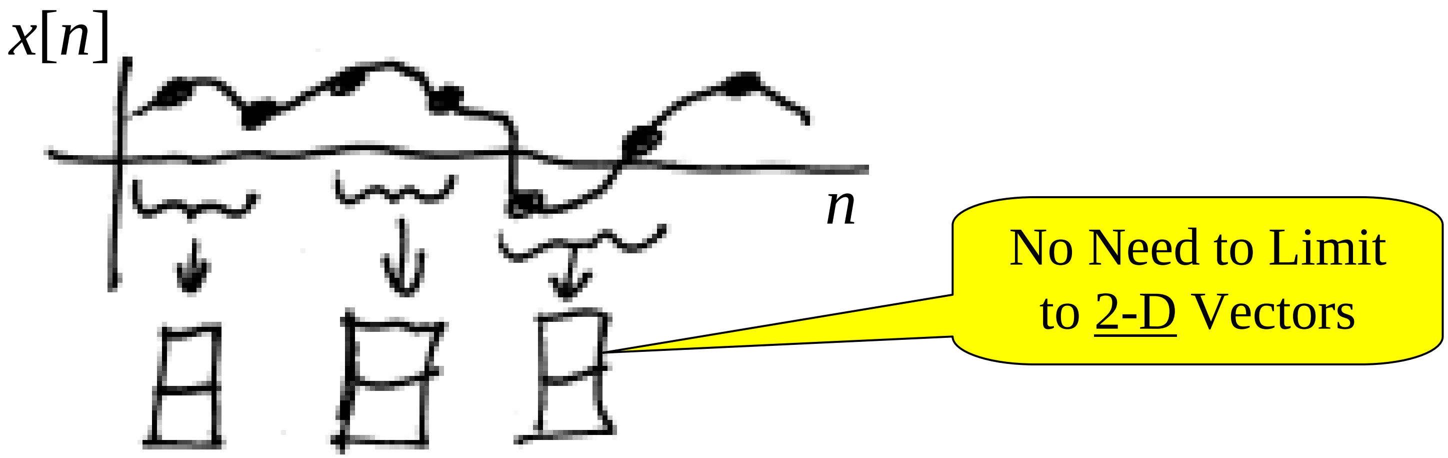
- It is the lossy version of a single-symbol method
- Shannon also proved that for lossy we can achieve the theoretical bound on compression (R-D curve) via "blocking taken to the limit"

This blocking idea motivates Vector Quantization

Main Idea of VQ

That info theory says to consider "blocking" to exploit correlation

➔ Group into vectors (non-overlapping) and "quantize" each vector



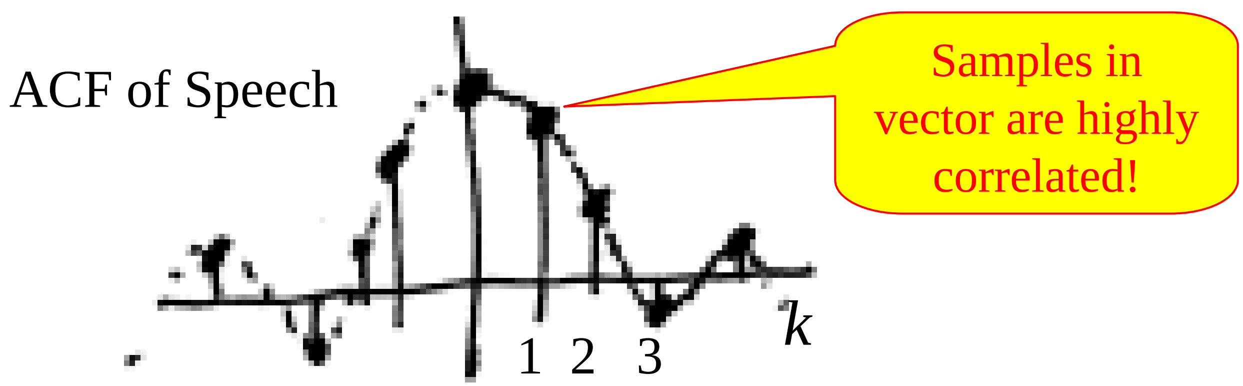
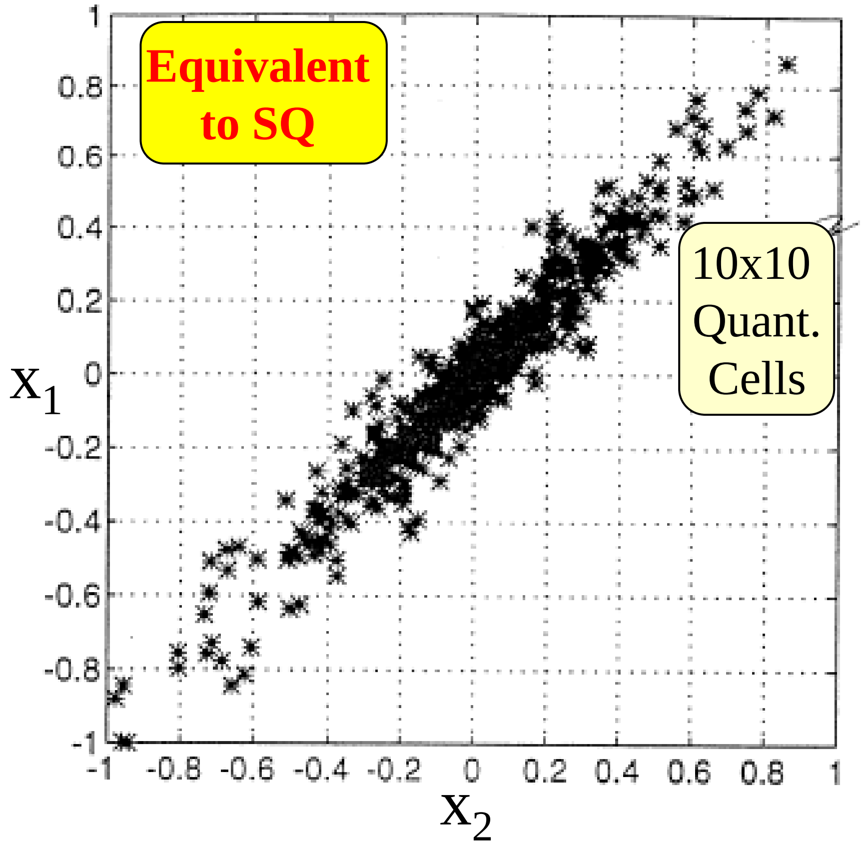
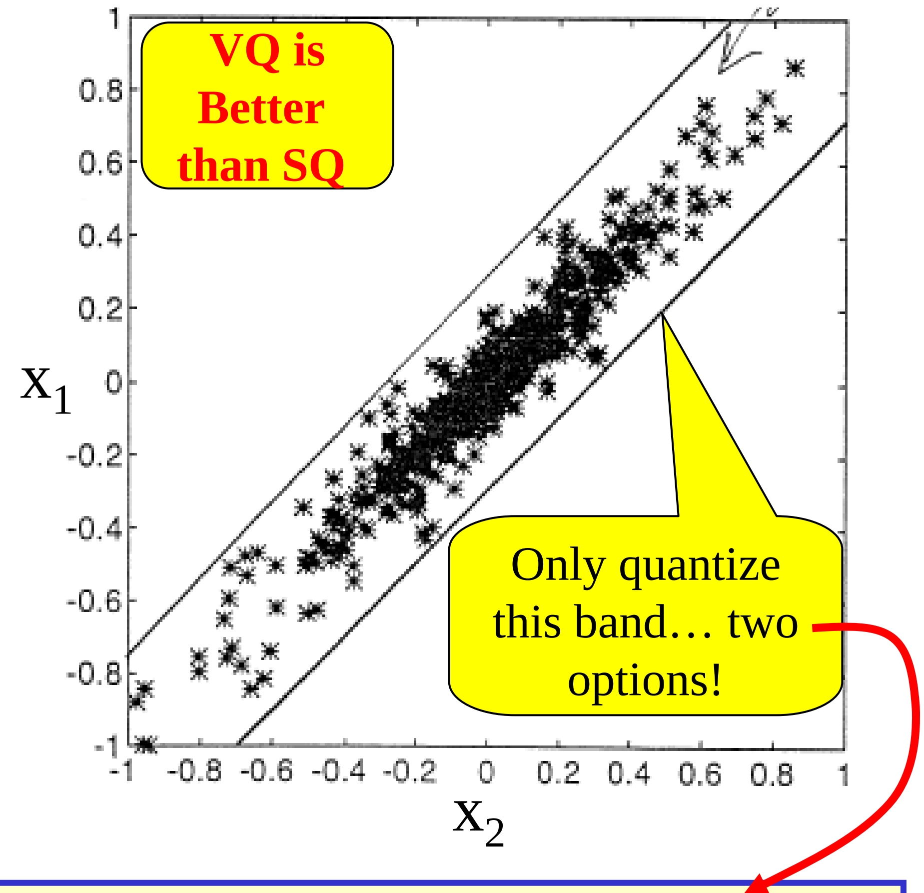


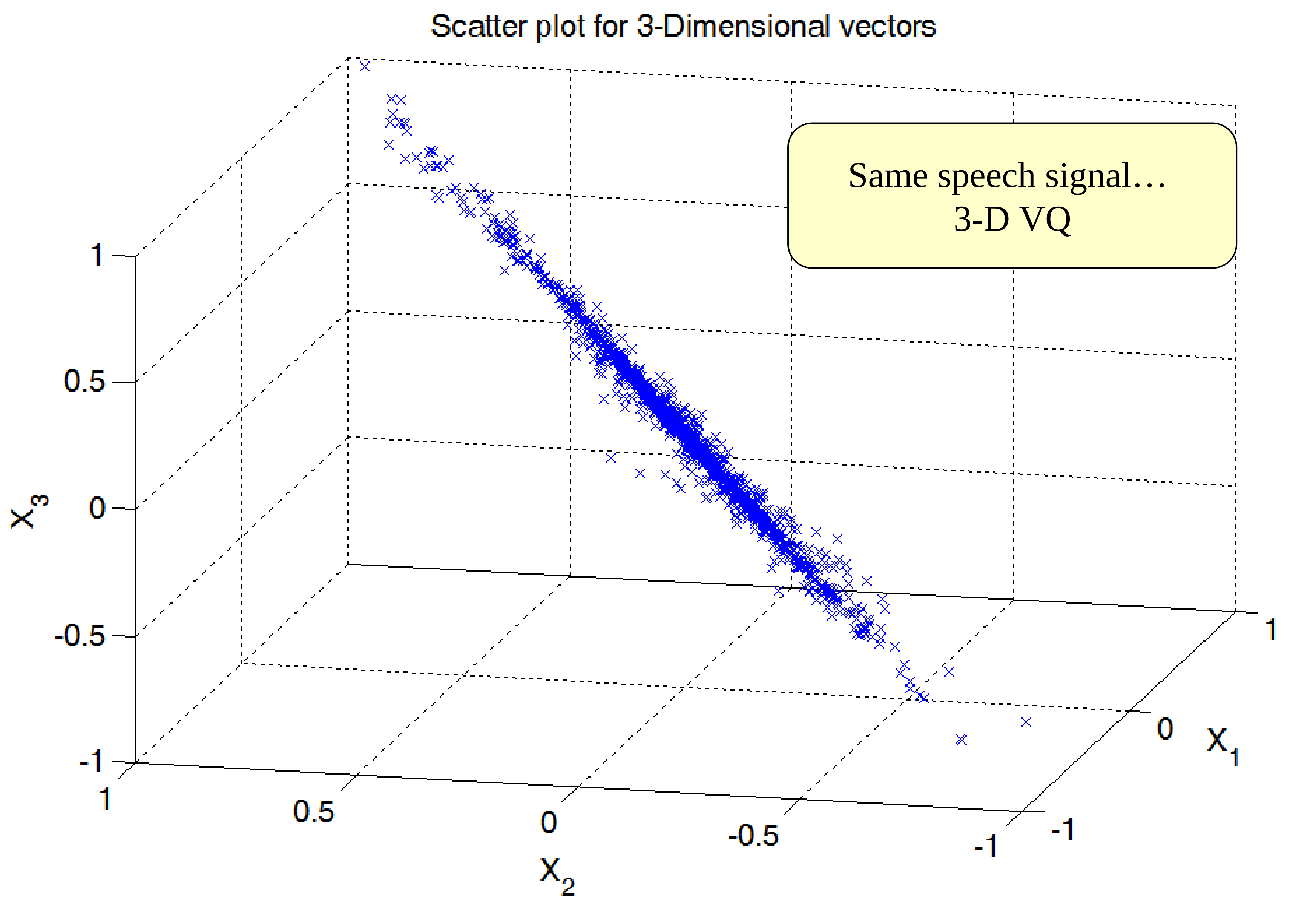
Illustration of Gain from VQ: Real Speech Data





- 100 cells smaller than SQ
  - Same Rate, Lower Distortion
- Fewer than 100 cells of same size as SQ
  - Lower rate, Same Distortion

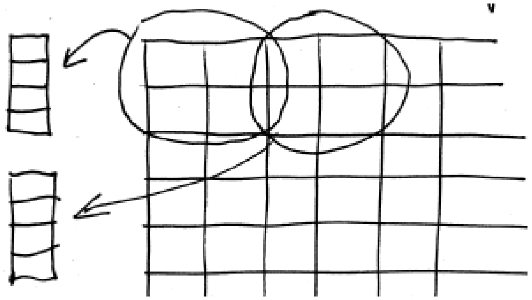
Going to higher dimension: vectors concentrated in smaller % of whole space ➔ Improved Performance



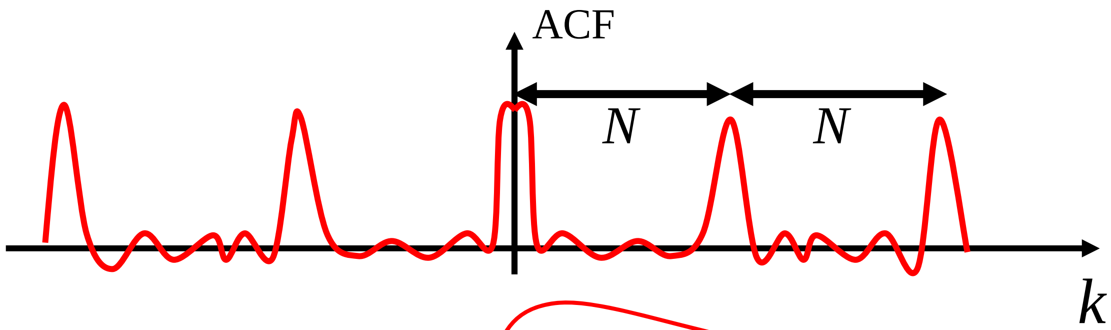
Forming Vectors

For time signals… we usually form vectors from temporally-sequential samples.

For images… we usually form vectors from spatially-sequential samples.



But… you should let the ACF structure guide your choice…



Note: Book uses upper-case italic to indicate a vector… I use the more standard lower-case bold non-italic…

$$\mathbf{x}_1 = \left[x[0] \quad x[N] \quad x[2N] \quad x[3N] \right]$$

$$\mathbf{x}_2 = \left[x[1] \quad x[N+1] \quad x[2N+1] \quad x[3N+1] \right]$$

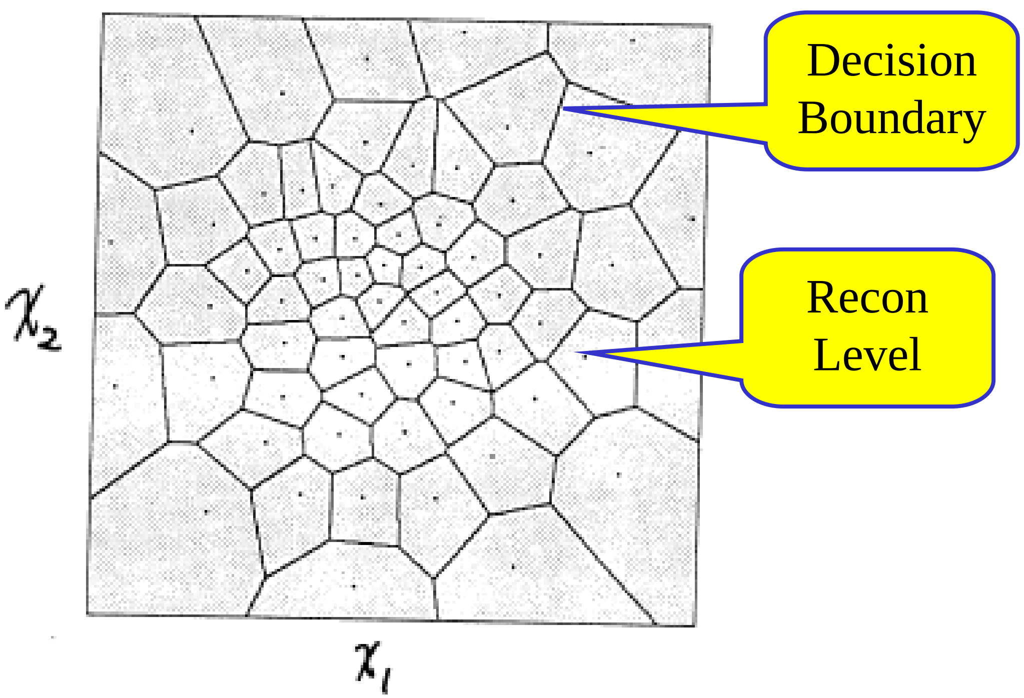
$$\mathbf{x}_3 = \left[x[2] \quad x[N+2] \quad x[2N+2] \quad x[3N+2] \right]$$

Designing VQ

Q: What does it mean to design a VQ?

A: Similar to SQ… Specify Decision Boundaries and Reconstruction Values… except now those are in *N*-D space… Goal is to minimize MSQE for a given rate (or vice versa)

Example of a VQ Design: For 2-D vectors taken from a sequence of independent Gaussian samples…



Somehow we need to design the DBs and RLs…

However, we really only need to specify the RLs…

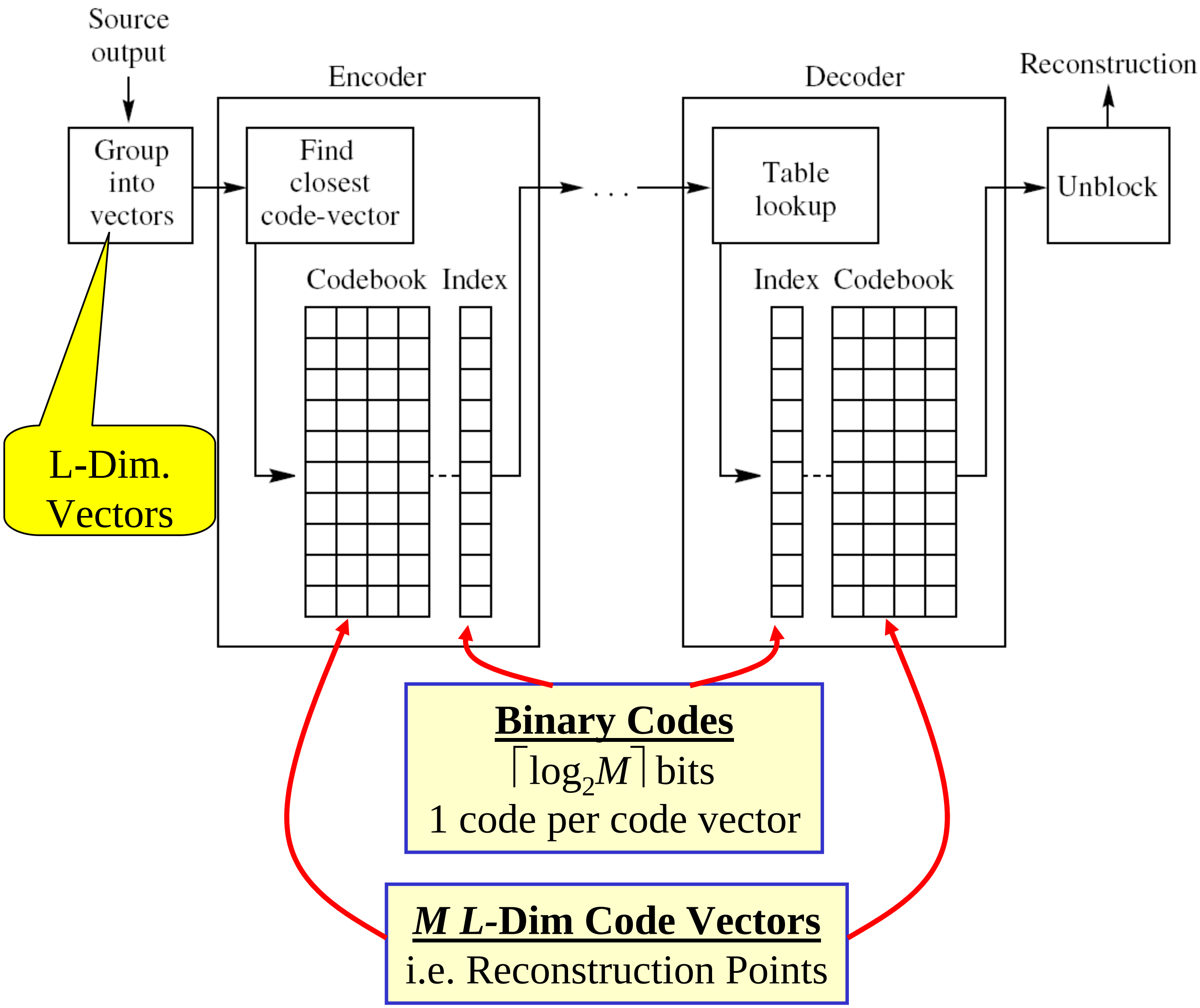
Then the i^{th} Decision Region consists of all points closer to the i^{th} RL than they are to any other RL

We'll see how to do this later…

Figure from book *Vector Quantization and Signal Compression* by Gersho & Gray

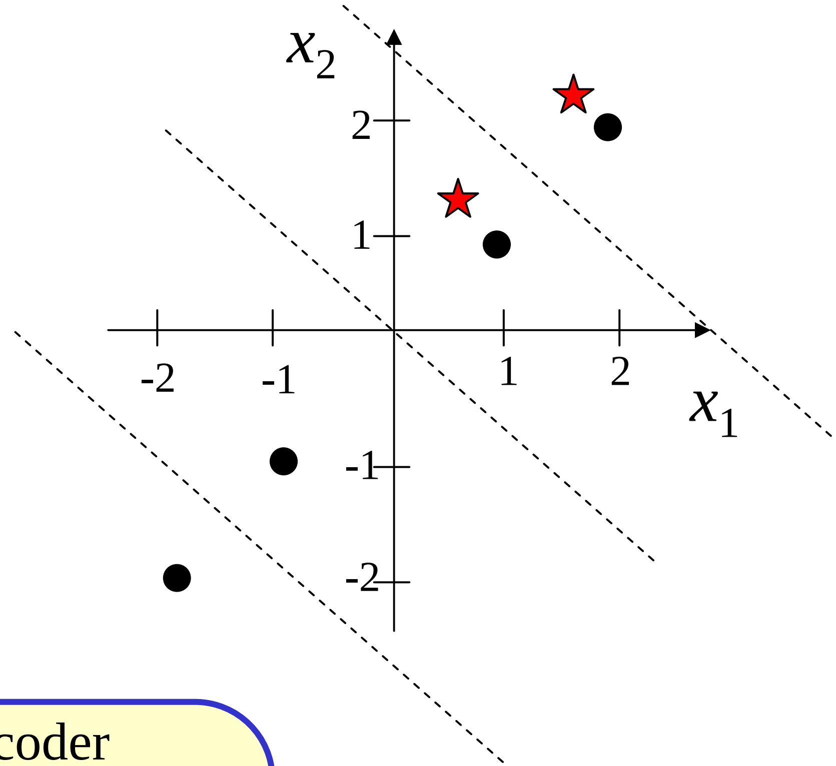
Structure of a VQ

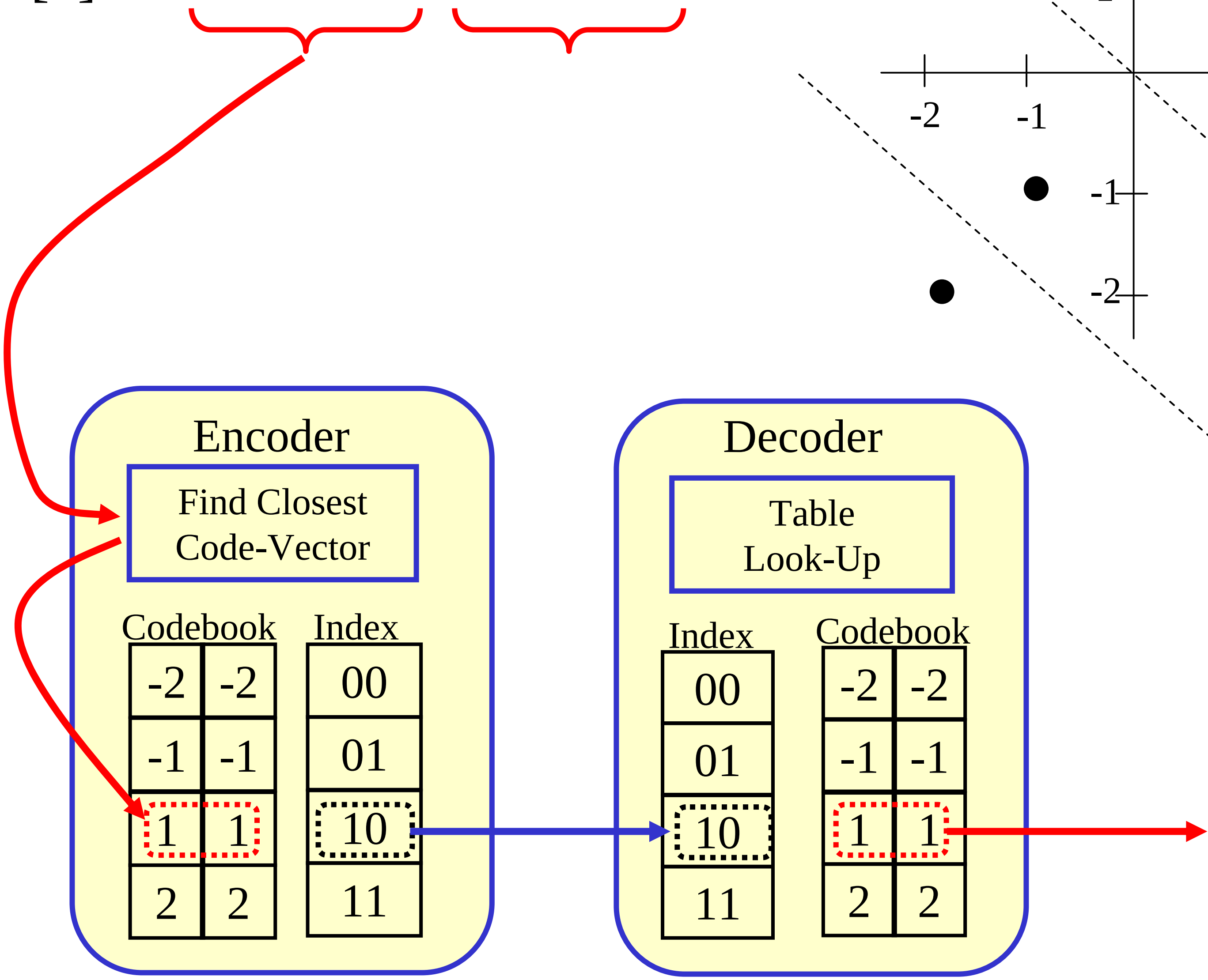
Fig. 10.1 in Textbook



Example

$x[n]$: … 0.75 1.27 1.78 2.11 …





Encoder & Decoder Operation

Encoder: ***Search*** codebook for code vector $\mathbf{y}_j$ that is closest to input vector **x**.

$$\text{Codebook } \{\mathbf{y}_i\}_{i=1}^{M} = C \quad \text{where each } \mathbf{y}_i \in \Re^L$$

x is closest to $\mathbf{y}_j$ if:

$$\|\mathbf{x}-\mathbf{y}_j\| \le \|\mathbf{x}-\mathbf{y}_i\| \quad \forall \mathbf{y}_i \in C \;\Rightarrow\; d(\mathbf{x},\mathbf{y}_j) \le d(\mathbf{x},\mathbf{y}_i) \quad \forall \mathbf{y}_i \in C$$

where $\|\mathbf{z}\| = \sqrt{\sum_{i=1}^{L} z_i^2}$ (Euclidian Norm)

for $\mathbf{z} = [z_1 \quad z_2 \quad \cdots \quad z_L]^T$

Notation: If **x** is closest to $\mathbf{y}_j$ we write $Q(\mathbf{x}) = \mathbf{y}_j$

Note: VQ cells are defined by the RLs & these equations rather than explicit boundaries!!!

Decoder: Use received binary codeword (the "index") as an address into the codebook (i.e., Table Look-Up)

Complexity of VQ

Encoder is Computationally Complex

Must check all M $\mathbf{y}_i$ for closeness… must compute M norms

M can be quite large: 256, 512, 1024, etc.

Decoder is Easy & Fast

VQ Complexity is Asymmetrical

- May not work well for Real-Time Encoding
- But Real-Time Decoding is very easy

VQ Rate & Performance vs. Rate

If L-D vectors are quantized using a VQ having M reconstruction points we need binary codewords having

$$\text{codeword length} = \lceil \log_2 M \rceil \quad \text{bits/vector}$$

$$\#\ \text{bits/sample} = \frac{\lceil \log_2 M \rceil}{L} \quad \text{bits/sample}$$

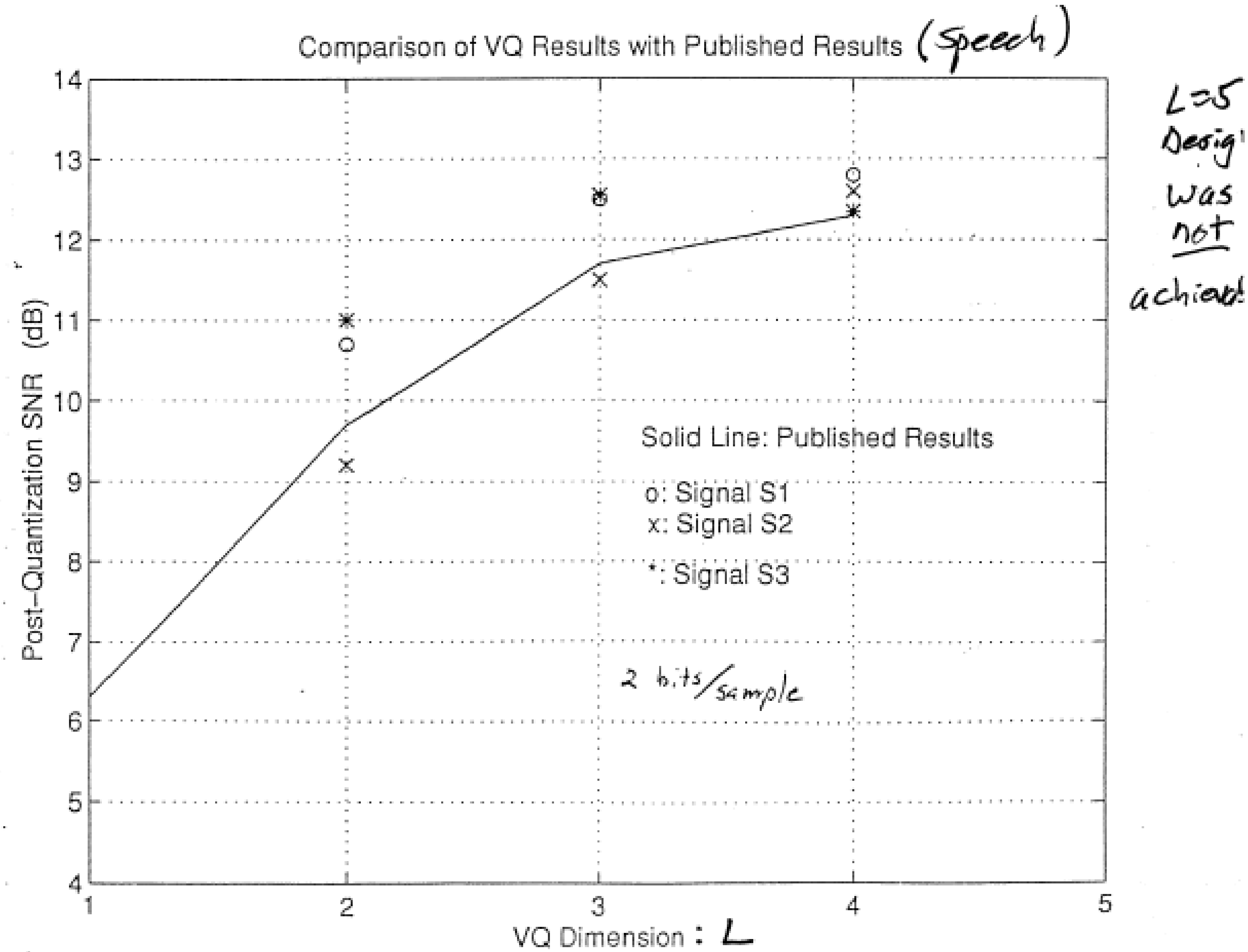
"Typical" L values:

- Images: L = 16 (4x4)
- Speech: L = 3,4,5,6

Info Theory Says: Increasing L improves the VQ performance

Practice Says: … only up to a point!

- Improvement decreases w/ increasing L
- Design gets harder w/ increasing L
- Encoder complexity grows w/ increasing L



Comparison of VQ Results with Published Results (Speech)
Post-Quantization SNR (dB)
VQ Dimension : L
Solid Line: Published Results
o: Signal S1
x: Signal S2
*: Signal S3
2 bits/sample