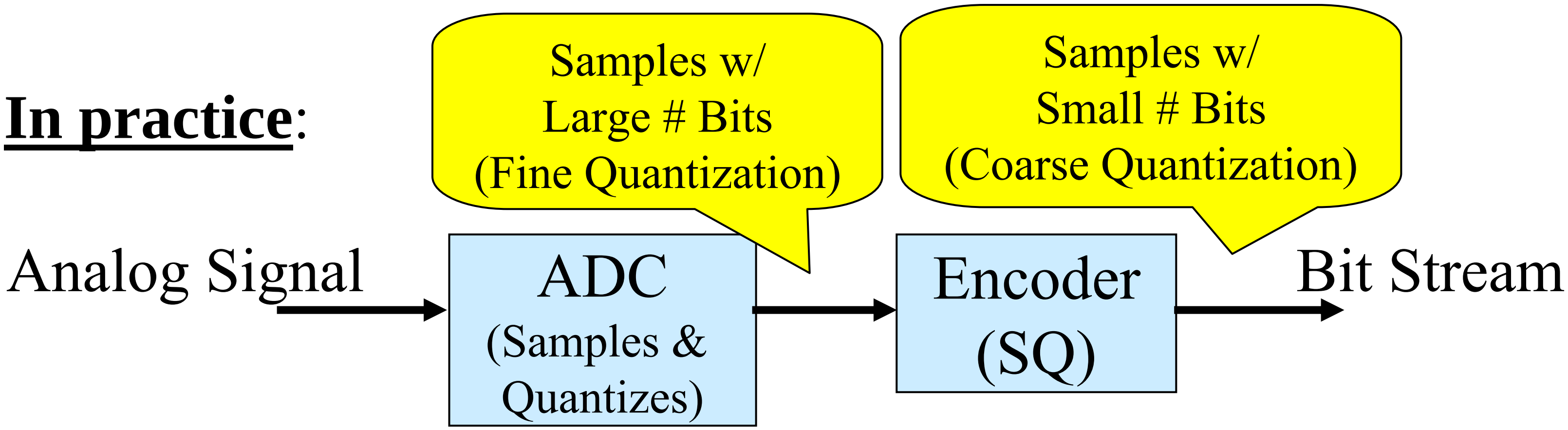
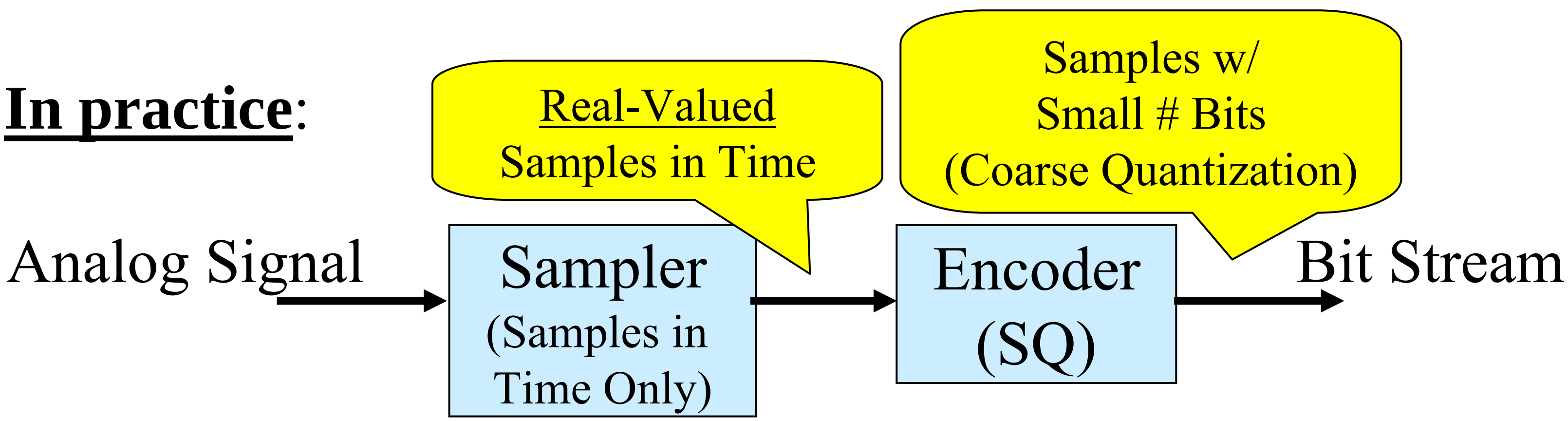
Ch. 9 Scalar Quantization

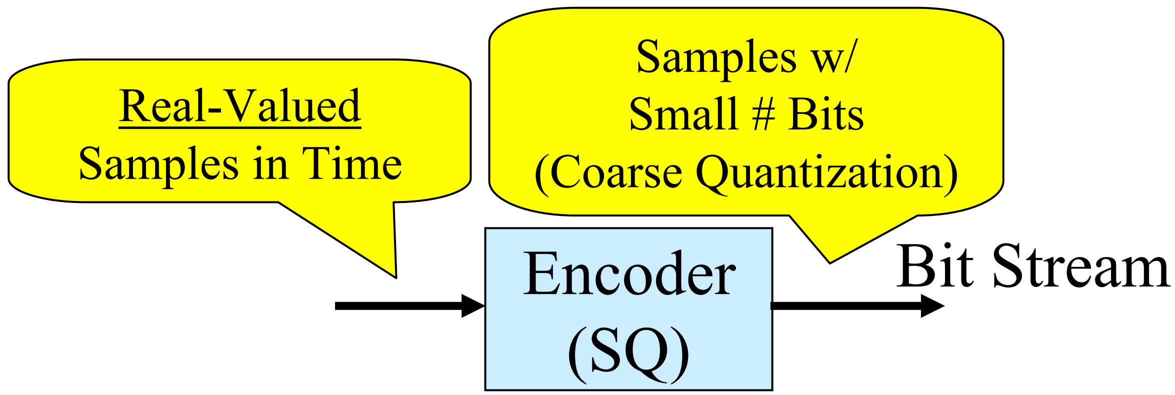
Encoder: Scalar Quantization

In practice:



In practice:

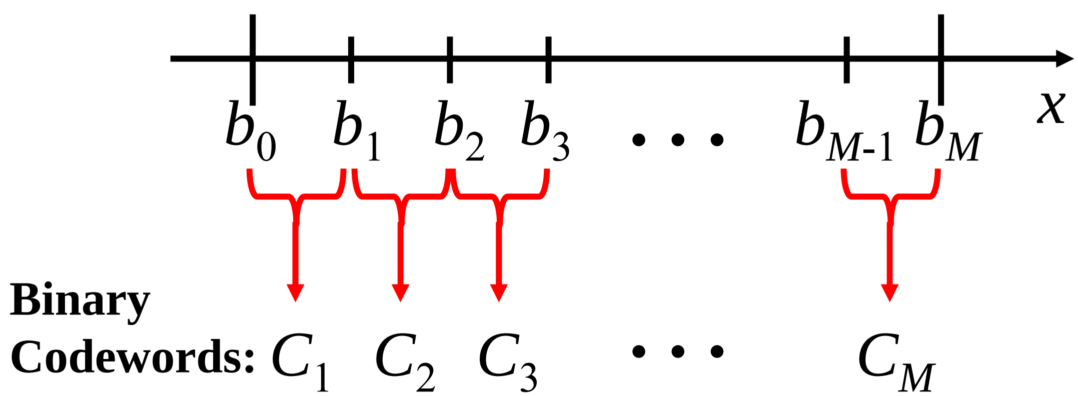




Encoder: Maps real #s into finitely many binary codewords

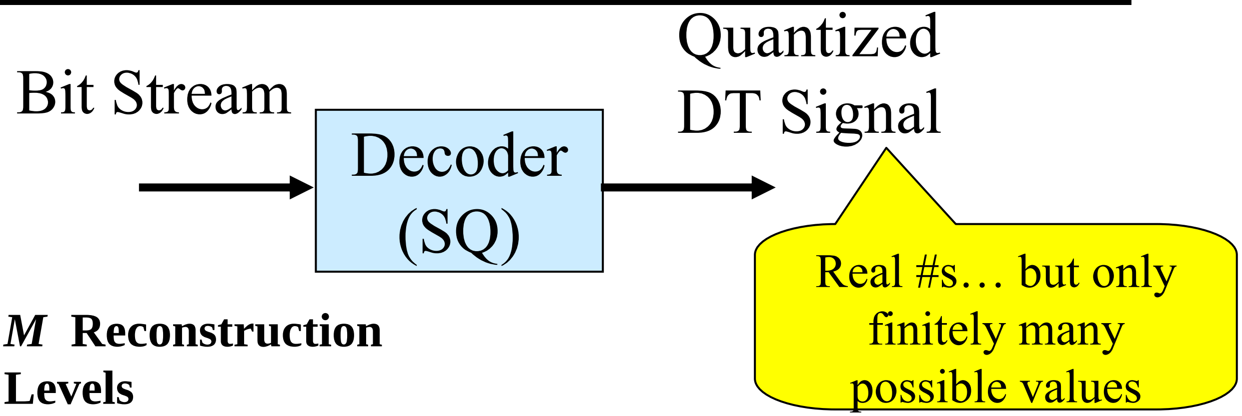
$$\Re \rightarrow Binary\ Code$$

Assigns M codes to M input intervals

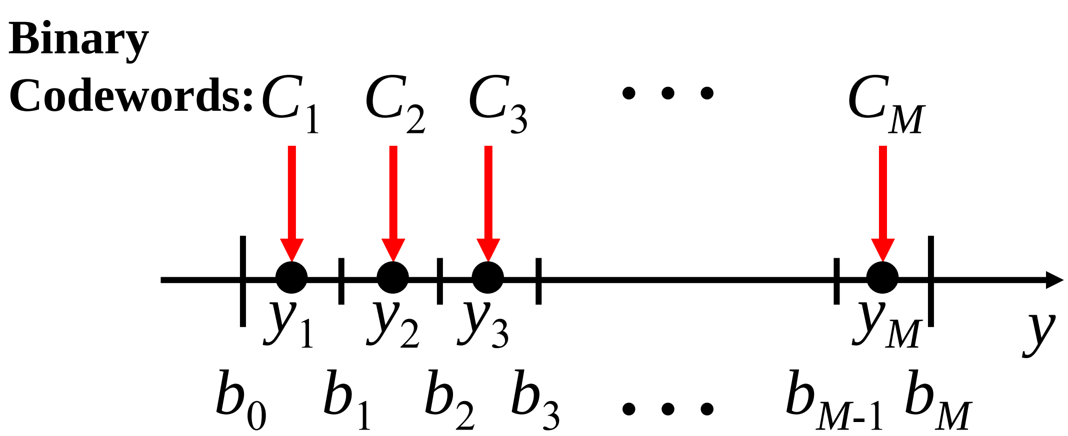


***M*+1 Decision Boundaries**		***M* Codewords**
$[b_0,b_1)$	$\rightarrow$	C_1
$[b_1,b_2)$	$\rightarrow$	C_2
$[b_2,b_3)$	$\rightarrow$	C_3
	$\vdots$	
	$\vdots$	
$[b_{M\text{-}1},b_M)$	$\rightarrow$	C_M

Decoder: Scalar Quantization

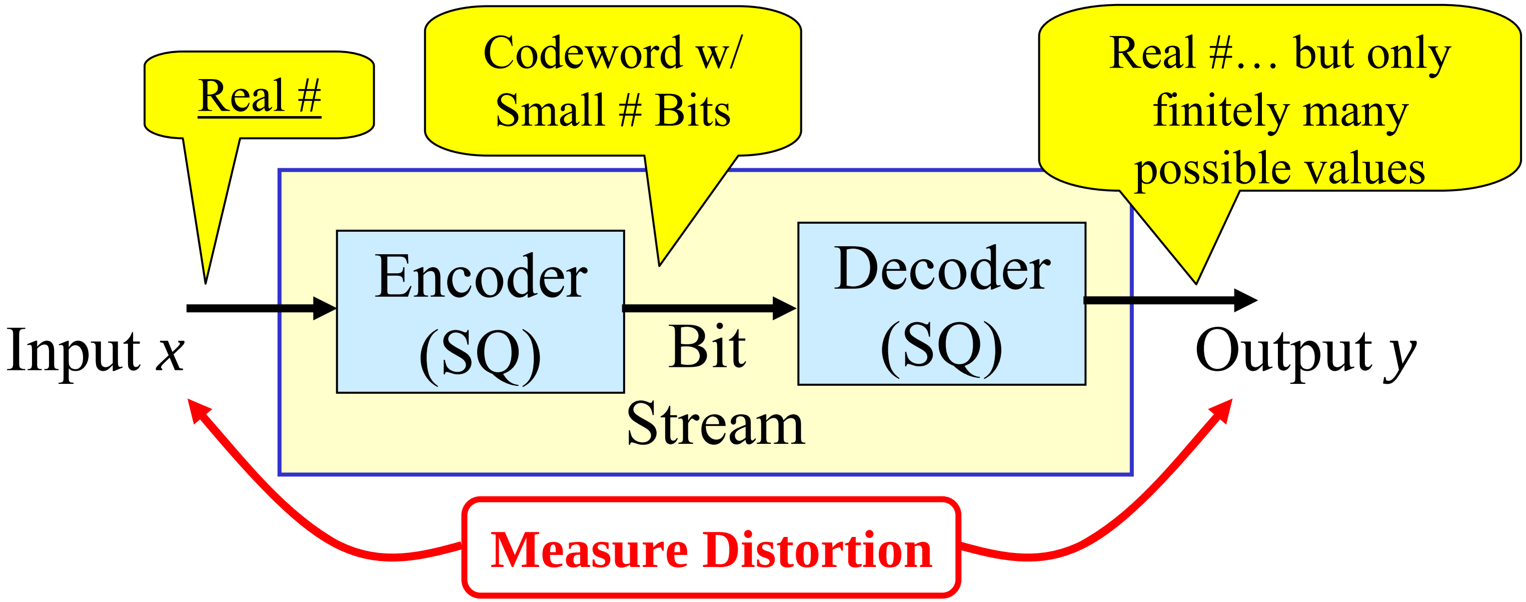


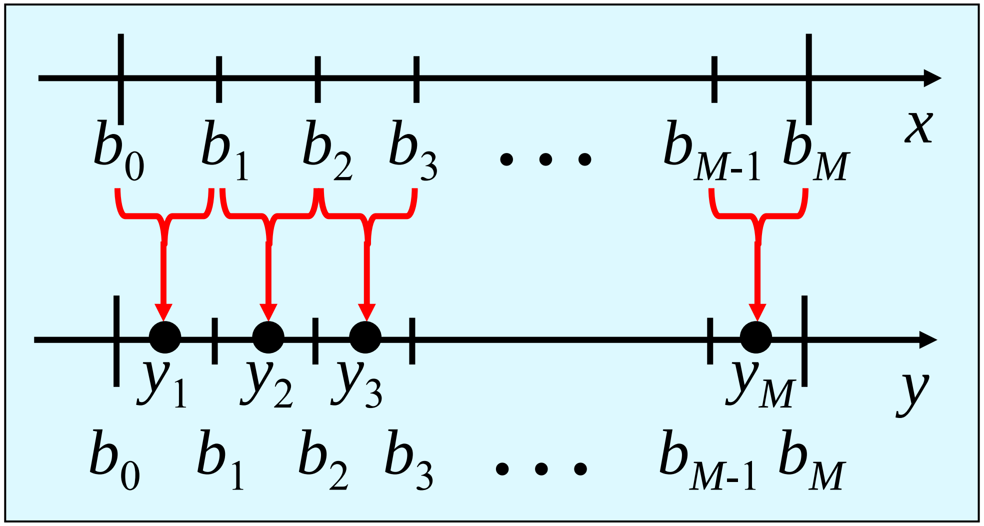
***M* Codewords**		***M* Reconstruction Levels**
C_1	$\rightarrow$	y_1
C_2	$\rightarrow$	y_2
C_3	$\rightarrow$	y_3
	$\vdots$	
	$\vdots$	
C_M	$\rightarrow$	y_M



Encoder-Decoder = Quantizer

For analysis: View Encoder & Decoder as a Pair





Scalar Quantizer Input-Output Map

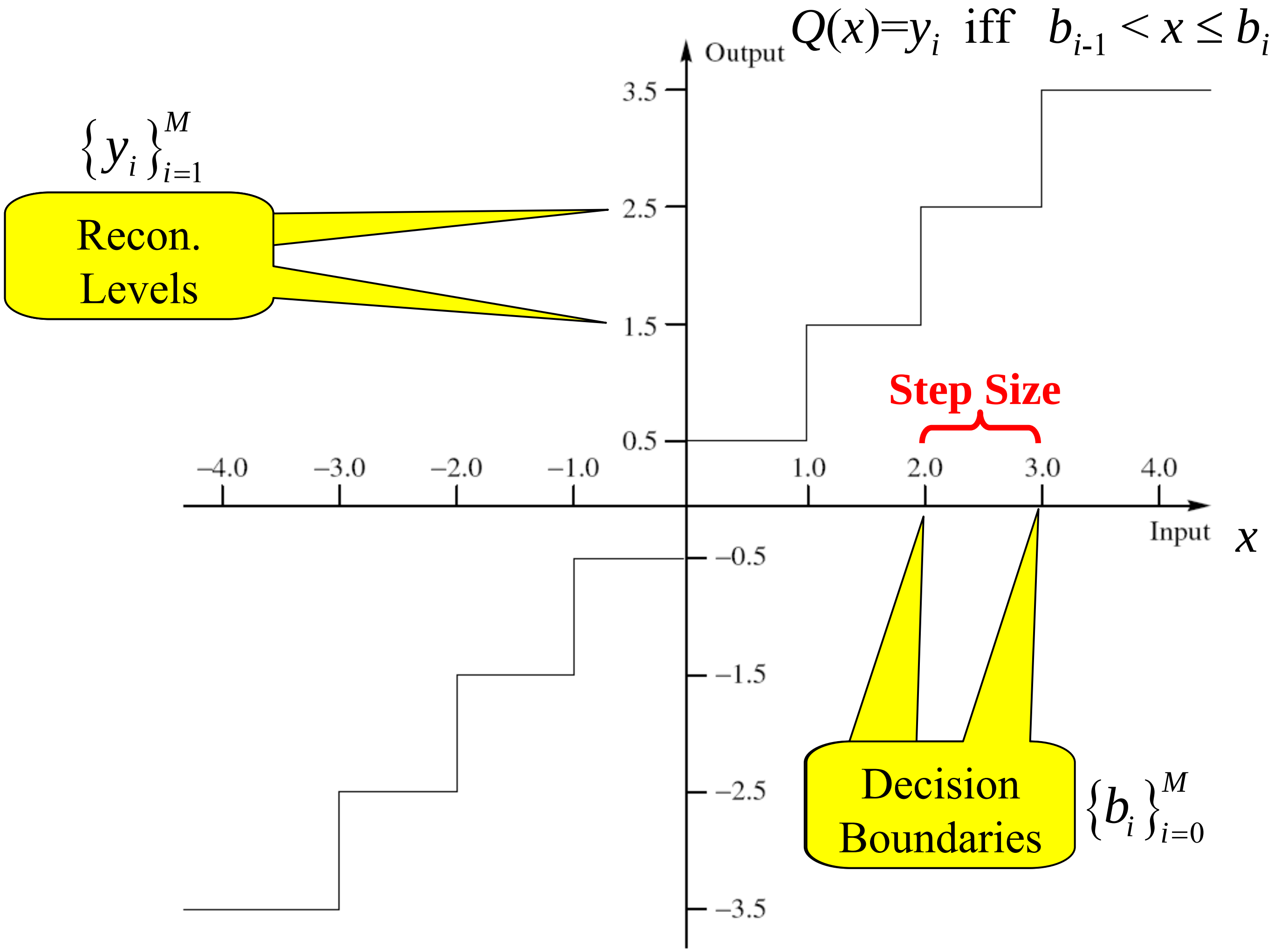


FIGURE 9. 3 **Quantizer input-output map.**

Quantizer Design

DBs Impact Rate if Entropy Coding is used

Specify:
1. Decision Boundaries (DBs)
2. Codewords
3. Reconstruction Levels (RLs)

Set Rate

Set Distortion

To Meet:
1. Distortion Criteria
2. Rate Criteria

Sometimes just MSE

Distortion We'll use Mean Square Quantization Error (MSQE)

Theory:

$$\sigma_q^2 = \int_{-\infty}^{\infty} [x - Q(x)]\, f_X(x)dx$$

$$= \sum_{i=1}^{M} \int_{b_{i-1}}^{b_i} [x - y_i]\, f_X(x)dx$$

DBs

RLs

We use this theoretical view in the design process.

Experimental: For a sequence of signal values $s[n]$, n= 0, 1, 2, …

$$\hat{\sigma}_q^2 = \frac{1}{N}\sum_{i=1}^{N}\Big[\overbrace{Q(s[n])}^{s_q[n]} - s[n]\Big]^2$$

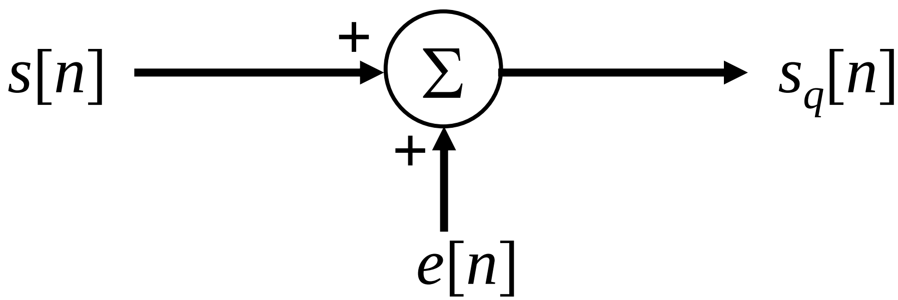
$e[n]$ is called Quantization Error

…or…

Quantization Noise

$$s_q[n] = s[n] + e[n]$$

Leads to a view of the quantization noise being added to the original signal… not really what happens but a nice viewpoint sometimes.



Rate We'll use Avg # of bits/sample

Let l_i be the length of the binary code for RL y_i.

Theory: Then the theoretical Rate is: $R = \sum_{i=1}^{M} l_i P(y_i)$

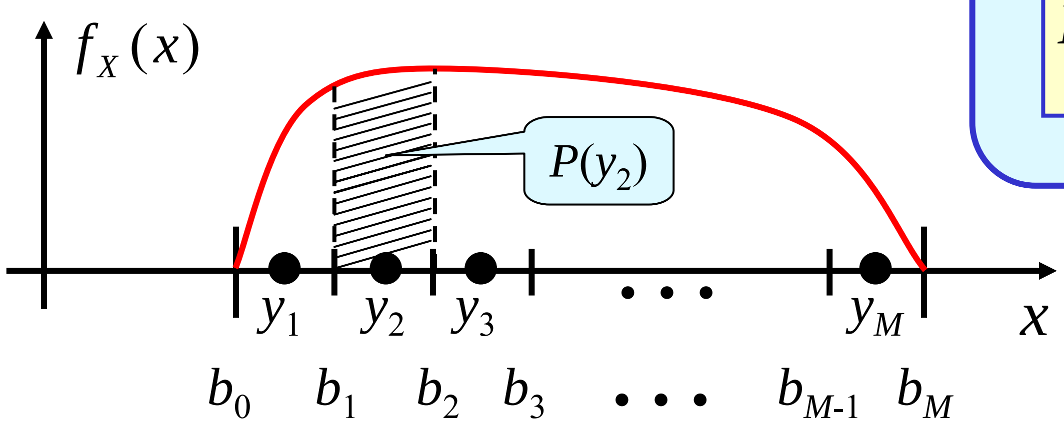
This depends on how likely it is to encode the signal at RL y_i

That probability depends on the choice of the DLs: $P(y_i) = \int_{b_{i-1}}^{b_i} f_X(x)dx$

$$R = \sum_{i=1}^{M} \left[l_i \int_{b_{i-1}}^{b_i} f_X(x)dx \right]$$

Experimental:

$$\hat{R} = \frac{1}{N} \sum_{i=1}^{N} Length\{Q(s[n])\}$$



Quantizer Design Problem

Two "Flavors" of the problem:

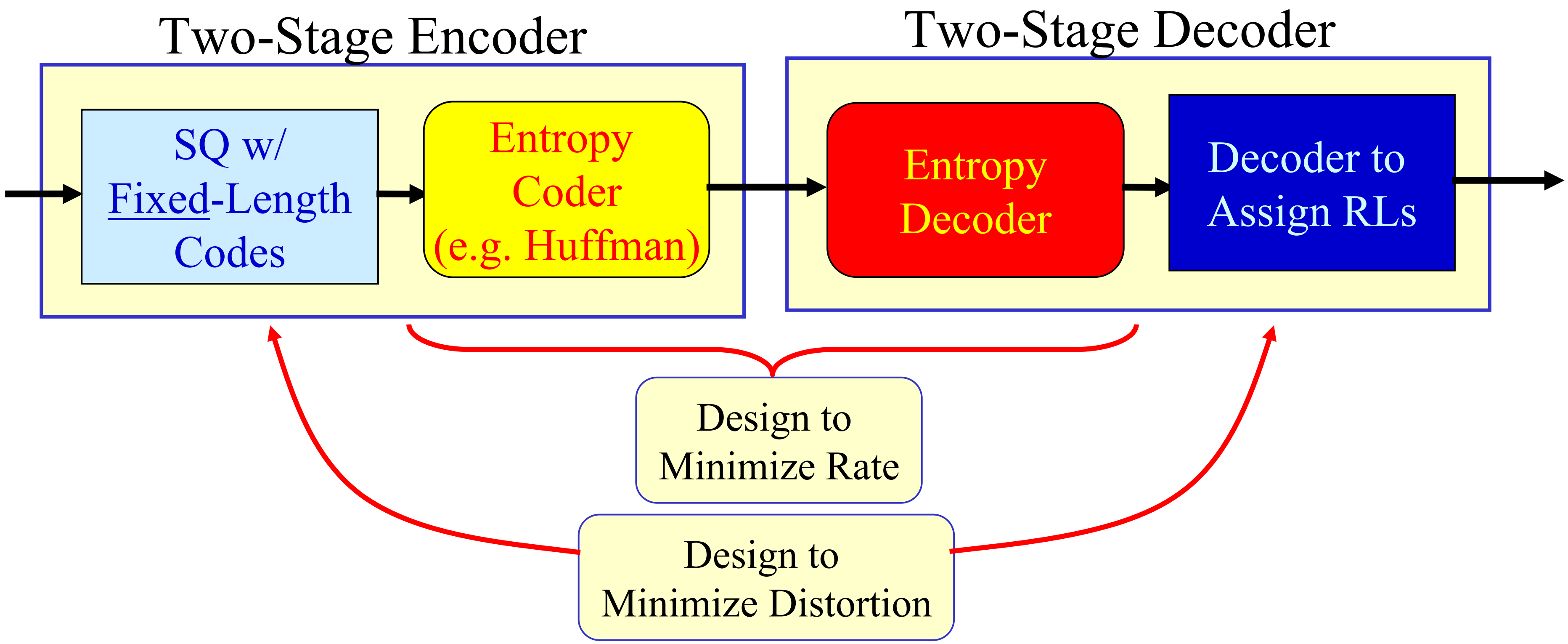
1. Given a Distortion Constraint $\sigma_q^2 \le D^*$
   Find DBs, RLs, & Codes to minimize rate while satisfying

2. Given a Rate Constraint $R \le R^*$
   Find DBs, RLs, & Codes to minimize Dist. while satisfying

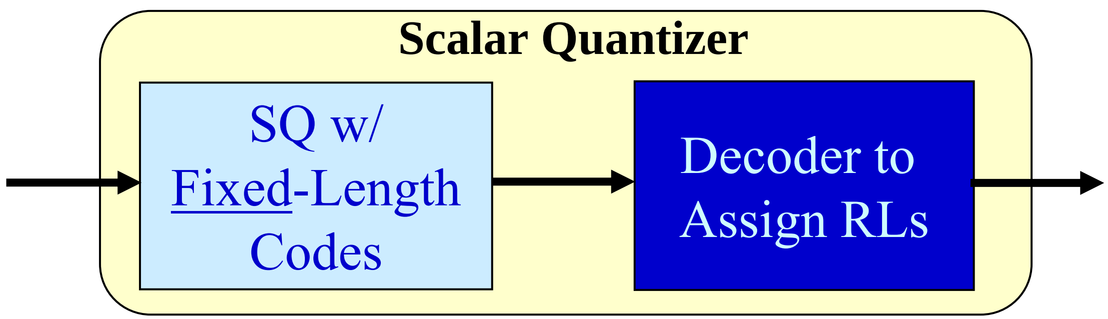
In general, solving this is pretty complex… So, in practice it is quite common to use fixed-length codes rather than trying to optimally choose code lengths as part of the design:

Given $R = \lceil \log_2 M \rceil \quad bits / sample$
Find DBs & RLs to minimize distortion

Tractable Quantizer Design



We'll first focus on outside blocks… and design to minimize MSQE:



Hierarchy of SQ Types