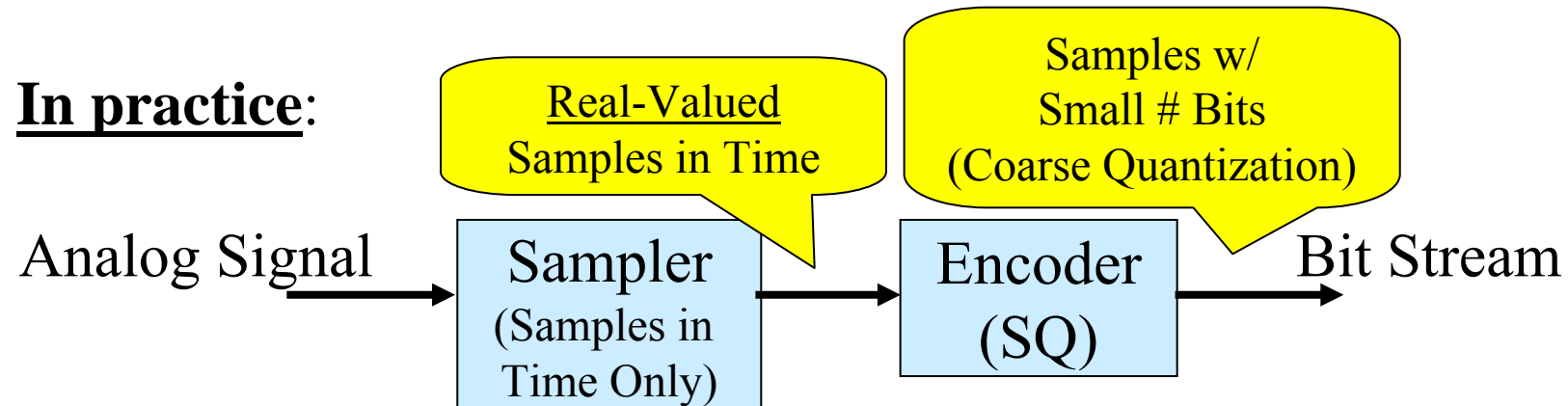
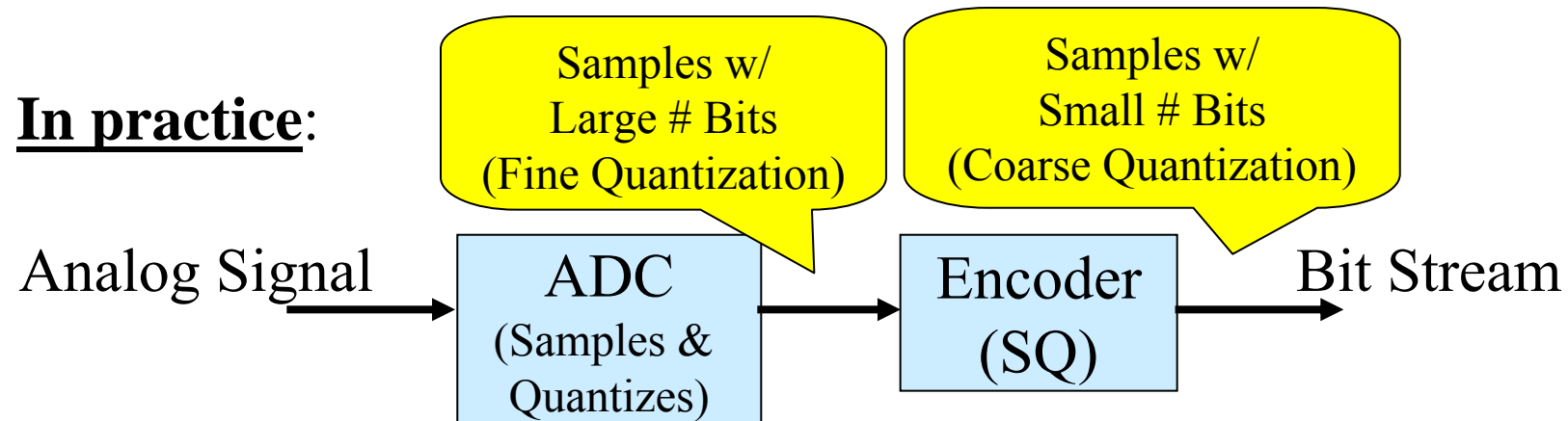
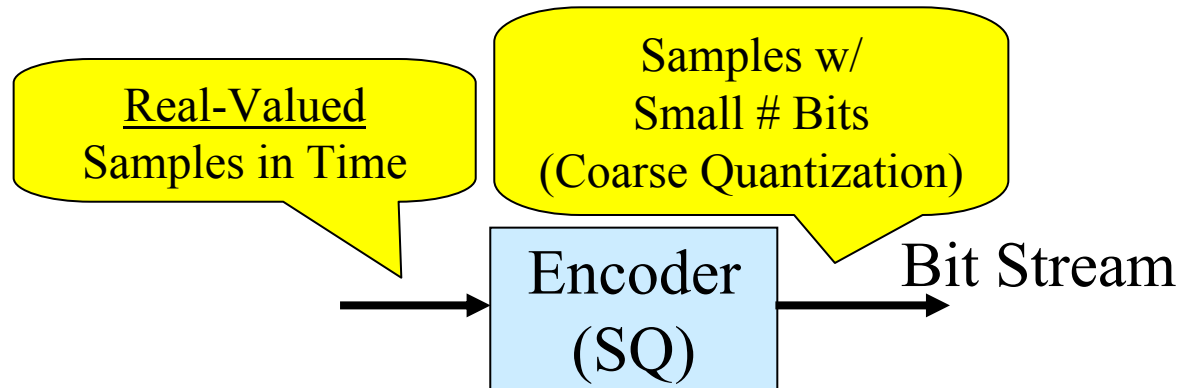


Ch. 9 Scalar Quantization

Encoder: Scalar Quantization

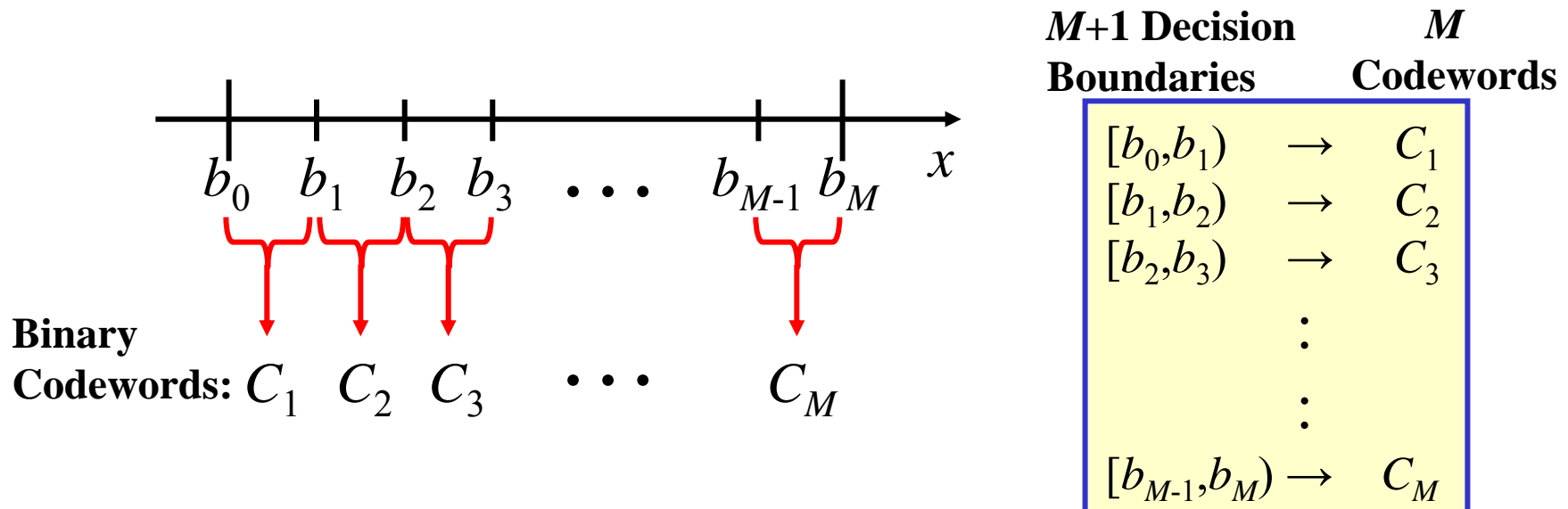




Encoder: Maps real #s into finitely many binary codewords

$$\mathcal{R} \rightarrow \text{Binary Code}$$

Assigns M codes to M input intervals



Decoder: Scalar Quantization

Bit Stream → **Decoder (SQ)** → Quantized DT Signal

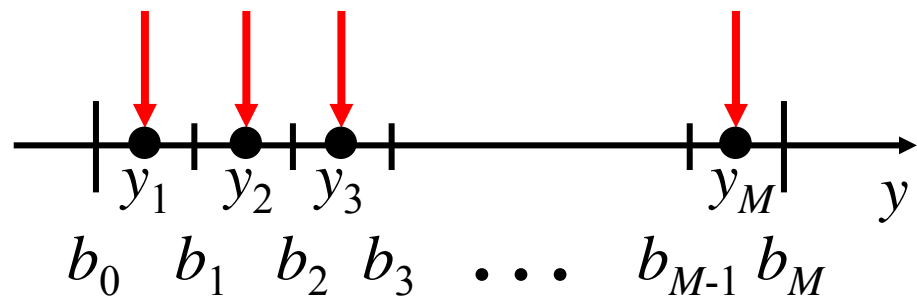
Real #s... but only finitely many possible values

M Codewords M Reconstruction Levels

| | | |
|-------|---|-------|
| C_1 | → | y_1 |
| C_2 | → | y_2 |
| C_3 | → | y_3 |
| ⋮ | | |
| ⋮ | | |
| C_M | → | y_M |

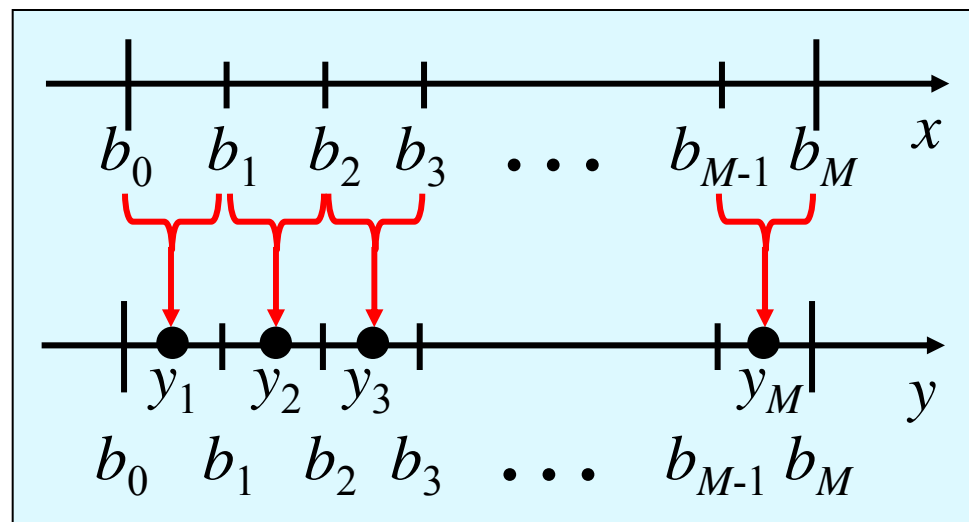
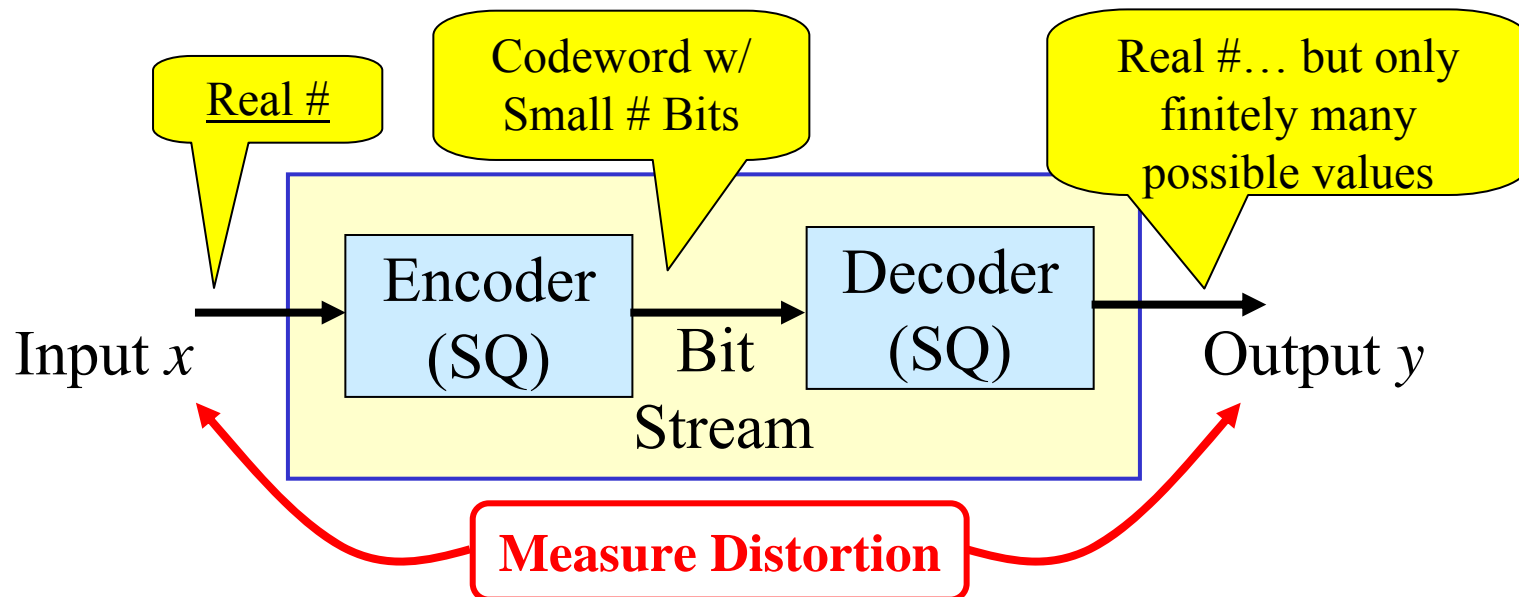
Binary

Codewords: C_1 C_2 C_3 ⋯ C_M



Encoder-Decoder = Quantizer

For analysis: View Encoder & Decoder as a Pair



Scalar Quantizer Input-Output Map

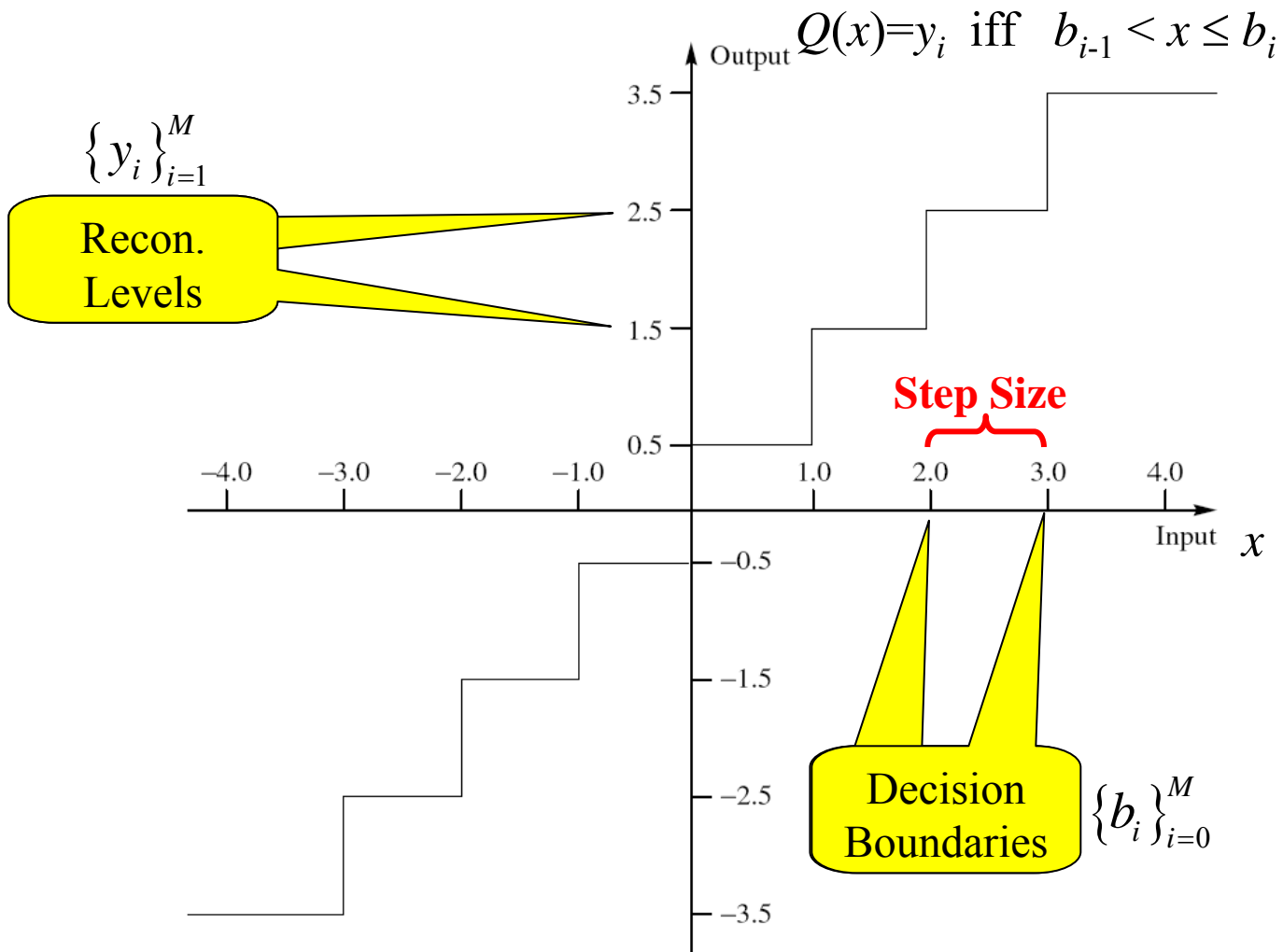


FIGURE 9.3 Quantizer input-output map.

Quantizer Design

- Specify:**
1. Decision Boundaries (DBs)
 2. Codewords
 3. Reconstruction Levels (RLs)

DBs Impact Rate if Entropy Coding is used

Set Rate

Set Distortion

- To Meet:**
1. Distortion Criteria
 2. Rate Criteria

Sometimes just MSE

Distortion We'll use Mean Square Quantization Error (MSQE)

Theory:

$$\begin{aligned}\sigma_q^2 &= \int_{-\infty}^{\infty} [x - Q(x)] f_X(x) dx \\ &= \sum_{i=1}^M \int_{b_{i-1}}^{b_i} [x - y_i] f_X(x) dx\end{aligned}$$

DBs

RLs

We use this theoretical view in the design process.

Experimental: For a sequence of signal values $s[n]$, $n= 0, 1, 2, \dots$

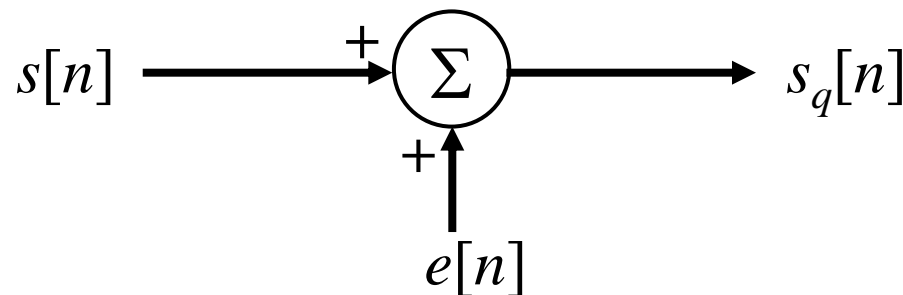
$$s_q[n]$$
$$\hat{\sigma}_q^2 = \frac{1}{N} \sum_{i=1}^N [Q(s[n]) - s[n]]^2$$

$e[n]$ is called Quantization Error
...or...

$$s_q[n] = s[n] + e[n]$$

Quantization Noise

Leads to a view of the quantization noise being added to the original signal... not really what happens but a nice viewpoint sometimes.



Rate We'll use Avg # of bits/sample

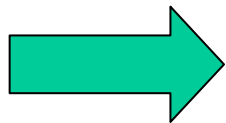
Let l_i be the length of the binary code for RL y_i .

Theory: Then the theoretical Rate is: $R = \sum_{i=1}^M l_i P(y_i)$

This depends on how likely it is to encode the signal at RL y_i

That probability depends on the choice of the DLs:

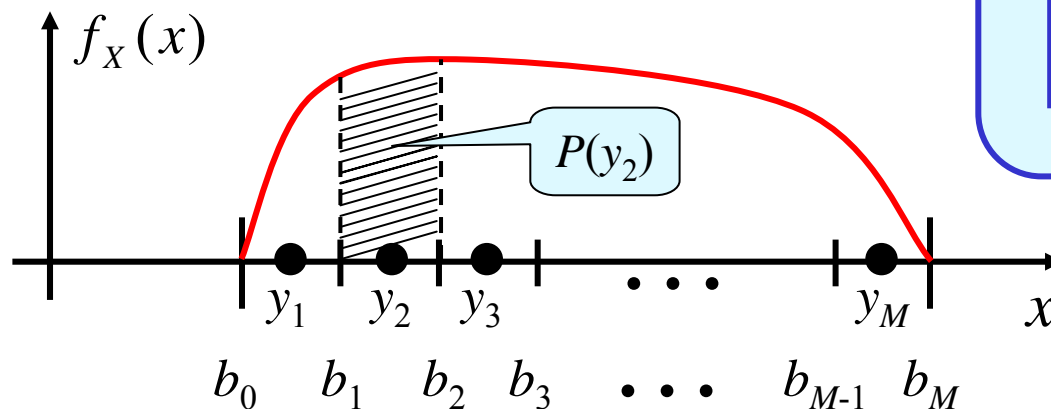
$$P(y_i) = \int_{b_{i-1}}^{b_i} f_X(x) dx$$



$$R = \sum_{i=1}^M \left[l_i \int_{b_{i-1}}^{b_i} f_X(x) dx \right]$$

Experimental:

$$\hat{R} = \frac{1}{N} \sum_{i=1}^N \text{Length} \{Q(s[n])\}$$



Quantizer Design Problem

Two “Flavors” of the problem:

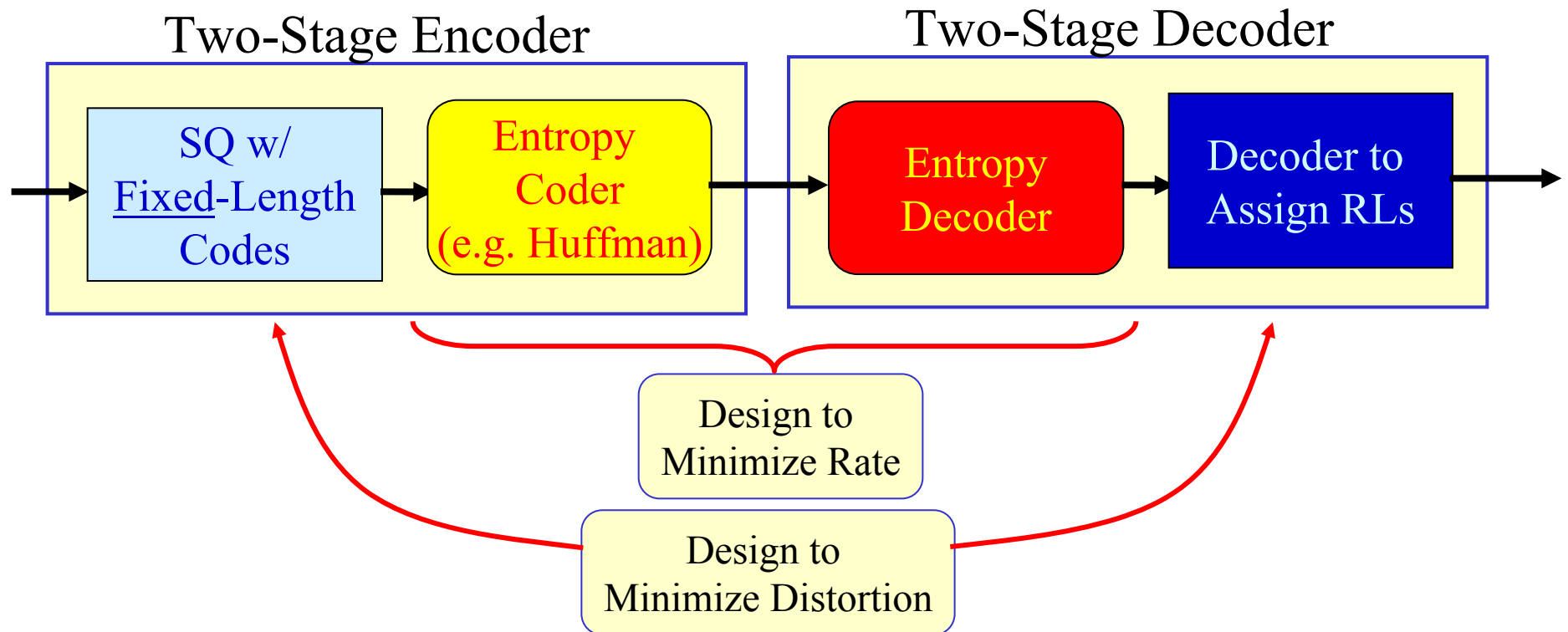
1. Given a Distortion Constraint $\sigma_q^2 \leq D^*$
Find DBs, RLs, & Codes to minimize rate while satisfying
2. Given a Rate Constraint $R \leq R^*$
Find DBs, RLs, & Codes to minimize Dist. while satisfying

In general, solving this is pretty complex... So, in practice it is quite common to use fixed-length codes rather than trying to optimally choose code lengths as part of the design:

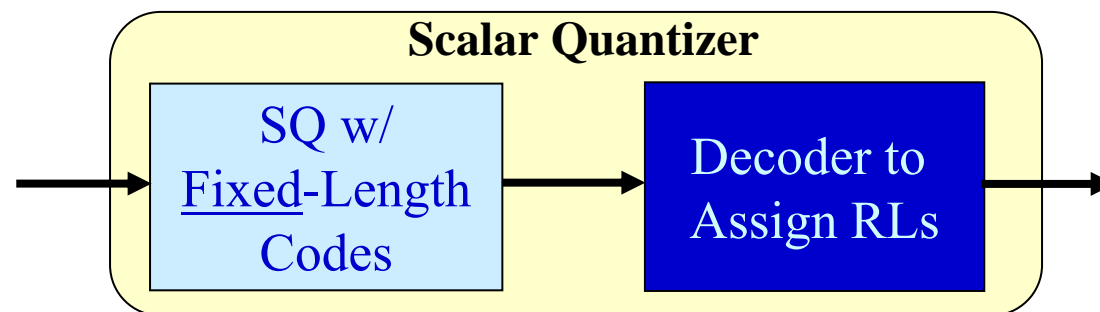
Given $R = \lceil \log_2 M \rceil$ *bits / sample*

Find DBs & RLs to minimize distortion

Tractable Quantizer Design



We'll first focus on outside blocks... and design to minimize MSQE:



Hierarchy of SQ Types

Scalar Quantizers

Uniform Quantizers
(Equal Step Sizes)

Non-Uniform Quantizers
(Unqqal Step Sizes)

PDF Optimized
Select Δ to min.
MSQE for assumed
PDF

Adaptive Uniform
 Δ adapted to signal's
local PDF

PDF Optimized
- Lloyd-Max Q
- Iteratively find DB/RL
- Suffers mismatch

Compander w/ UQ
- Use nonlinear
compander around UQ
- Robust to mismatch

Forward Adaptive
- Uses current block to
choose Δ
- Must send side info

Backward Adaptive
Uses past values to
choose Δ for current
sample

Uniform Jayant
Uses single past
sample to adapt

Non-Uniform Jayant
Uses single past
sample to adapt