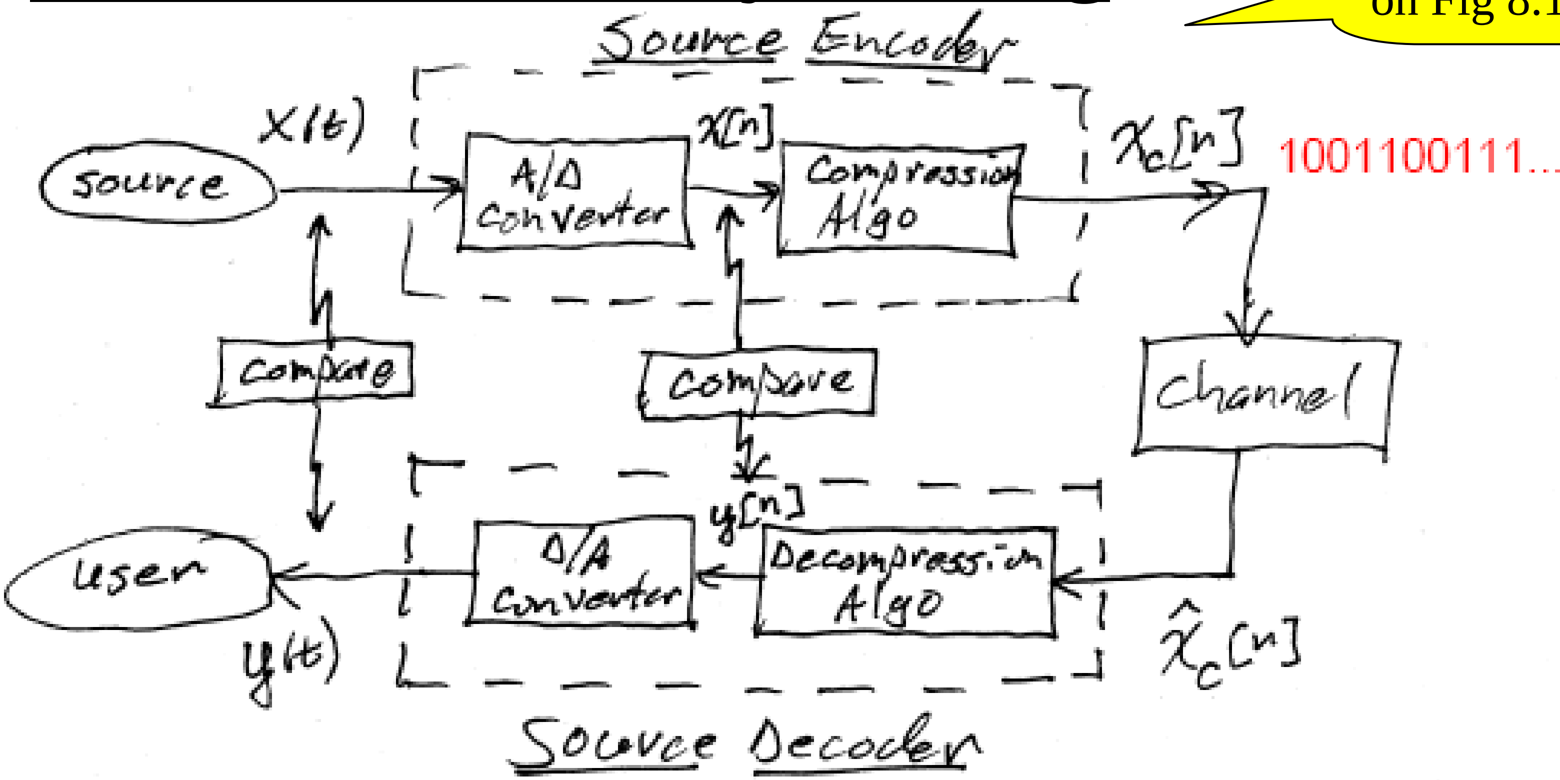
Ch. 8 Math Preliminaries for Lossy Coding

8.3 Distortion Criteria or Measure

Structure of Lossy Coding

Recall Slight variation on Fig 8.1 in textbook:



<u>In practice</u> this is how things get implemented. The A/D not only samples $x(t)$ in time but it also discretizes the values – however, the discretization is very <u>fine</u>. The Comp. Algo. often includes further, coarser discretization

<u>In theory</u>, we think of the A/D as only sampling the signal in time – the Comp. Algo. handles the discretization. Thus, we often think of:

$x[n]$ as taking values on continuum

$y[n]$ as taking discrete values

<u>Comparing Original & Compressed Signals</u>

- How do we check how close $y[n]$ is to the original $x[n]$?
- We must define a distortion measure $d(x,y)$

Square Error (SE) Measure

Most Common: $d(x,y) = (x-y)^2$

- Now, usually we have N samples to compare, so we use:

Vectors of Samples

$$d(\mathbf{x},\mathbf{y}) = \frac{1}{N}\sum_{n=1}^{N} d(x[n], y[n])$$

$$= \frac{1}{N}\sum_{n=1}^{N} \left(x[n] - y[n]\right)^2$$

If SE measure is used

An "operational" Mean Square Error (MSE) Distortion

- <u>In practice</u> we'll want to adapt comp. algo. to give the smallest value of d(**x**,**y**) for the <u>particular</u> x you are processing… <u>"Operational Distortion" Viewpoint</u>

- In Theory we don't have a particular **x** (in a theoretical setting we haven't collected a signal yet) so we strive to minimize $d(\mathbf{x},\mathbf{y})$ on average… a probabilistic average over the ensemble of **x**'s according to some probability model.

$$D = E\left\{d(\mathbf{x},\mathbf{y})\right\}$$
$$= \frac{1}{N}\sum_{n=1}^{N} E\left\{d(x[n],y[n])\right\}$$
$$= E\left\{d(x,y)\right\} \quad \text{if stationary process}$$

If SE is used (& stationary): $D = E\left\{\left(y-x\right)^2\right\} = \sigma_{err}^2$

Mean Square Error (MSE)

<u>Non-MSE Distortion Measures</u>

- MSE is the "most widely" used due to its simplicity of application (math results are fairly easy to derive)
- But SE doesn't always correspond well with visual/audio quality as perceived by humans
  - Compression algorithms intended for video/audio often use distortion measures that include ways to capture the psychology of human vision/hearing
- MSE is usually not the best choice when the decompressed signal is going to be used in statistical estimation/decision processing
  - See many of my papers posted on my web page
- **So why study MSE?**
  - Math is easy (relative to that needed for non-MSE)
  - It gives <u>decent</u> results
  - It is usually part of the non-MSE measures

Relating Distortion To SNR

SNR = Signal-to-Noise (Power) Ratio

Since… $d(x[n], y[n]) = \big(\underbrace{y[n] - x[n]}_{e[n]}\big)^2$

$$\Rightarrow \quad y[n] = x[n] + e[n]$$

Reconstructed Signal — Original Signal — "Noise" due to Compression

The reconstructed signal has an SNR of

$$SNR = \frac{power\{x[n]\}}{power\{e[n]\}} = \frac{\sigma_x^2}{\sigma_e^2} = \frac{\sigma_x^2}{D}$$

$$\Rightarrow \quad SNR = \frac{\sigma_x^2}{D}$$

SNR has the advantage that it measures distortion <u>relative</u> to the signal power

SNR is typically stated in dB form: $$SNR_{dB} = 10\log_{10}\frac{\sigma_x^2}{\sigma_e^2}$$

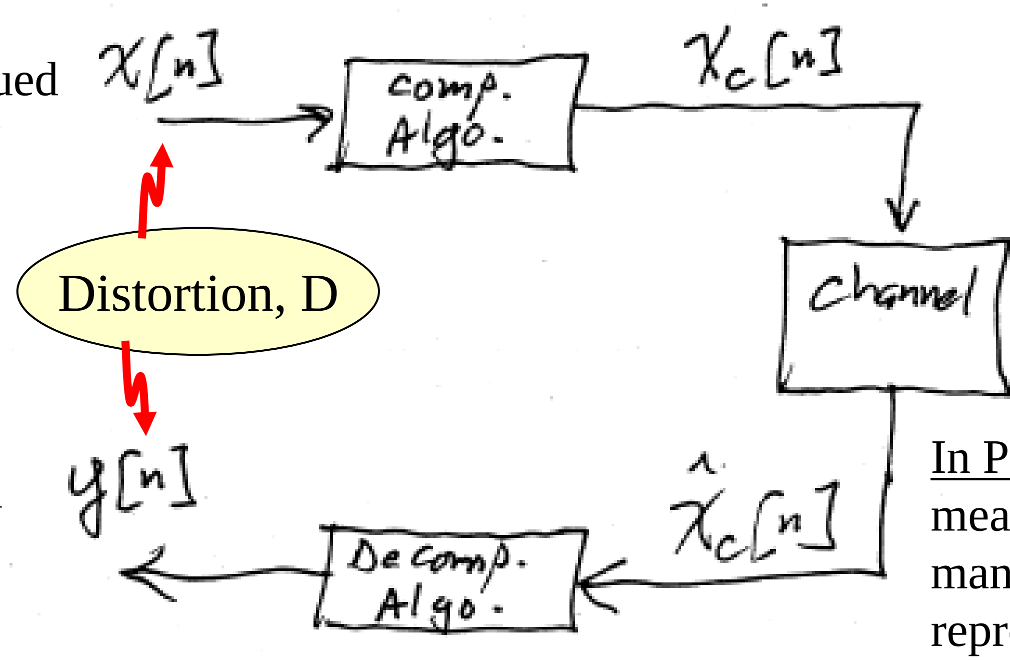
In image compression it is common to use *peak* SNR:

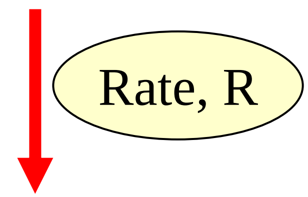
$$PSNR_{dB} = 10\log_{10}\frac{x_{peak}^2}{\sigma_e^2} \qquad x_{peak} = \max_n |x[n]|$$

Goals of Lossy Compression

Think of *x*[*n*] as DT and Continuum-Valued (in practice, *x*[*n*] is "finely" discrete-valued)

Think of *y*[*n*] as DT and Discrete-Valued





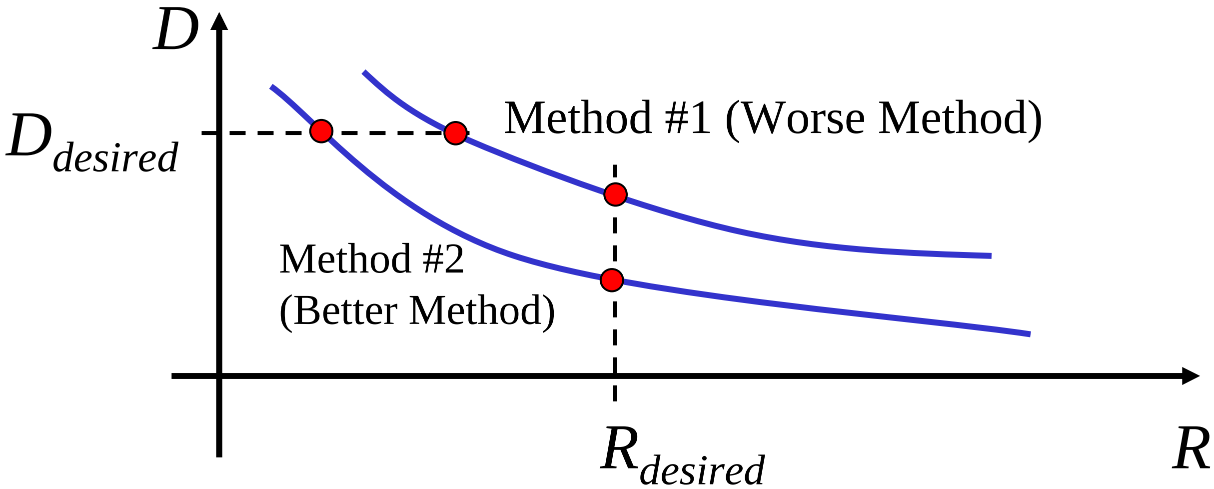
In Practice, rate is a measure of how many bits are used to represent the compressed signal:

- bits/sample
- bits/second
- bits/fixed duration
- Etc.

Goals of Compression

1. Reduce distortion for a fixed rate
2. Reduce rate for a fixed distortion

Goals of Lossy Compression



Goals of Compression

1. For a given $R_{desired}$, find a method that minimizes D
2. For a given $D_{desired}$, find a method that minimizes R

Designing Lossy Comp. Algorithms involves solving *constrained* optimization problems