

Expected values (moments) of a zero-mean Gaussian Random Variable:

$$E\{x^n\} = \int_{-\infty}^{\infty} x^n p(x) dx = \int_{-\infty}^{\infty} \alpha x^n e^{-x^2/k} dx = \begin{cases} 0, & n \text{ odd} \\ \alpha \frac{n!}{2^n (\frac{n}{2})!} k^{\frac{n+1}{2}} \pi^{\frac{1}{2}}, & n \text{ even} \end{cases}$$

where $\alpha = \frac{1}{\sqrt{2\pi\sigma^2}}$, $k = 2\sigma^2$

Examples:

$$n = 0, \Rightarrow E\{1\} = 1$$

$$n = 1, \Rightarrow E\{x\} = 0$$

$$n = 2, \Rightarrow E\{x^2\} = \sigma^2$$

$$n = 4, \Rightarrow E\{x^4\} = 3\sigma^4$$

Evaluating probability of occurrences within a certain sigma:

$$\int_{A-r\sigma}^{A+r\sigma} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-A)^2/2\sigma^2} dx = Q(r) = \text{erf}\left(\frac{r}{\sqrt{2}}\right) = 1 - \text{erfc}\left(\frac{r}{\sqrt{2}}\right)$$

Use the table below to find the needed values of the Q-function:

r	$Q(r)$	$1-Q(r)$
0.0	0.00000000	1.00000000
0.1	0.07965567	$9.20344325 \times 10^{-1}$
0.2	0.15851942	$8.41480581 \times 10^{-1}$
0.3	0.23582284	$7.64177156 \times 10^{-1}$
0.4	0.31084348	$6.89156517 \times 10^{-1}$
0.5	0.38292492	$6.17075077 \times 10^{-1}$
0.6	0.45149376	$5.48506236 \times 10^{-1}$
0.7	0.51607270	$4.83927304 \times 10^{-1}$
0.8	0.57628920	$4.23710797 \times 10^{-1}$
0.9	0.63187975	$3.68120251 \times 10^{-1}$
1.0	0.68268949	$3.17310508 \times 10^{-1}$
1.1	0.72866788	$2.71332122 \times 10^{-1}$
1.2	0.76986066	$2.30139340 \times 10^{-1}$
1.3	0.80639903	$1.93600969 \times 10^{-1}$
1.4	0.83848668	$1.61513318 \times 10^{-1}$
1.5	0.86638560	$1.33614403 \times 10^{-1}$
1.6	0.89040142	$1.09598583 \times 10^{-1}$
1.7	0.91086907	$8.91309255 \times 10^{-2}$
1.8	0.92813936	$7.18606382 \times 10^{-2}$
1.9	0.94256688	$5.74331196 \times 10^{-2}$
2.0	0.95449974	$4.55002639 \times 10^{-2}$
2.1	0.96427116	$3.57288411 \times 10^{-2}$
2.2	0.97219310	$2.78068950 \times 10^{-2}$
2.3	0.97855178	$2.14482200 \times 10^{-2}$
2.4	0.98360493	$1.63950718 \times 10^{-2}$
2.5	0.98758067	$1.24193307 \times 10^{-2}$
2.6	0.99067762	$9.32237605 \times 10^{-3}$
2.7	0.99306605	$6.93394761 \times 10^{-3}$
2.8	0.99488974	$5.11026066 \times 10^{-3}$
2.9	0.99626837	$3.73162660 \times 10^{-3}$
3.0	0.99730020	$2.69979606 \times 10^{-3}$
3.1	0.99806479	$1.93520643 \times 10^{-3}$
3.2	0.99862572	$1.37427588 \times 10^{-3}$
3.3	0.99903315	$9.66848285 \times 10^{-4}$
3.4	0.99932614	$6.73858531 \times 10^{-4}$
3.5	0.99953474	$4.65258158 \times 10^{-4}$
3.6	0.99968178	$3.18217180 \times 10^{-4}$
3.7	0.99978440	$2.15599467 \times 10^{-4}$
3.8	0.99985530	$1.44696088 \times 10^{-4}$
3.9	0.99990381	$9.61926880 \times 10^{-5}$
4.0	0.99993666	$6.33424837 \times 10^{-5}$
4.1	0.99995868	$4.13150138 \times 10^{-5}$

r	$Q(r)$	$1-Q(r)$
4.2	0.99997331	$2.66914980 \times 10^{-5}$
4.3	0.99998292	$1.70798109 \times 10^{-5}$
4.4	0.99998917	$1.08250878 \times 10^{-5}$
4.5	0.99999320	$6.79534625 \times 10^{-6}$
4.6	0.99999578	$4.22490941 \times 10^{-6}$
4.7	0.99999740	$2.60161491 \times 10^{-6}$
4.8	0.99999841	$1.58665630 \times 10^{-6}$
4.9	0.99999904	$9.58366553 \times 10^{-7}$
5.0	0.99999943	$5.73303144 \times 10^{-7}$
5.1	0.99999966	$3.39653481 \times 10^{-7}$
5.2	0.99999980	$1.99288526 \times 10^{-7}$
5.3	0.99999988	$1.15802681 \times 10^{-7}$
5.4	0.99999993	$6.66408970 \times 10^{-8}$
5.5	0.99999996	$3.79791249 \times 10^{-8}$
5.6	0.99999998	$2.14351805 \times 10^{-8}$
5.7	0.99999999	$1.19807428 \times 10^{-8}$
5.8	0.99999999	$6.63149196 \times 10^{-9}$
5.9	1.00000000	$3.63501573 \times 10^{-9}$
6.0	1.00000000	$1.97317529 \times 10^{-9}$