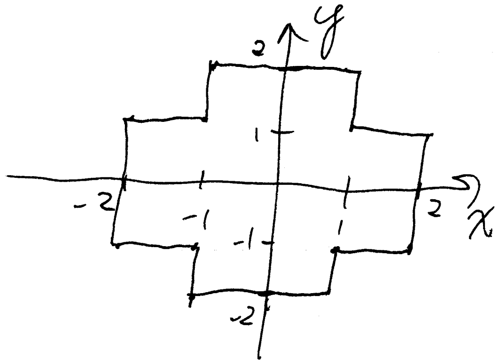
A

Example of Joint PDF of 2 Uncorrelated RV's that are not Independent

X, Y are RVs with joint PDF $P_{XY}(x,y)$ given by

$$P_{XX}(x,y) = \begin{cases} \frac{1}{12}, & x,y \in \mathcal{U} \\ 0, & \text{otherwise} \end{cases}$$

where the set $\mathcal{U}$ is shown below:

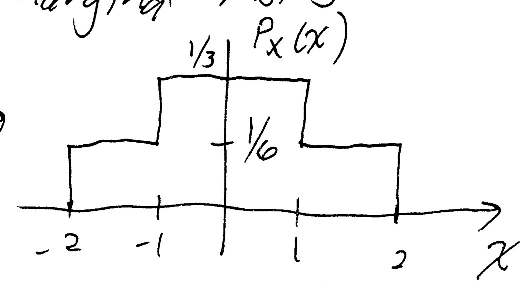


1. X & Y are Uncorrelated

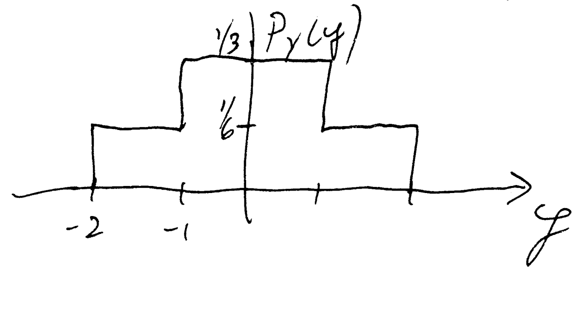
$$\sigma_{XY} = E\{(X-\bar{X})(Y-\bar{Y})\}$$

Need $\bar{X}$ & $\bar{Y}$ so need marginal PDF's

$$P_X(x) = \int P_{XY}(x,y)\,dy \longrightarrow$$



$$P_Y(y) = \int P_{XY}(x,y)\,dx \longrightarrow$$



$$\Rightarrow \bar{X} = \bar{Y} = 0$$

B

So... $\sigma_{XY} = E\{XY\}$

$$= \iint xy\, P_{XY}(x,y)\, dx\, dy$$

$$= \frac{1}{12}\iint_{\mathcal{U}} xy\, dx\, dy$$

$$= \frac{1}{12}\left[\underbrace{\iint_{QI\cap\mathcal{U}} xy\, dx\, dy + \iint_{QII\cap\mathcal{U}} xy\, dx\, dy}_{\substack{\text{negatives of each other}\\ \Rightarrow \text{cancel}}} + \underbrace{\iint_{QIII\cap\mathcal{U}} xy\, dx\, dy + \iint_{QIV\cap\mathcal{U}} xy\, dx\, dy}_{\Rightarrow \text{cancel}}\right]$$

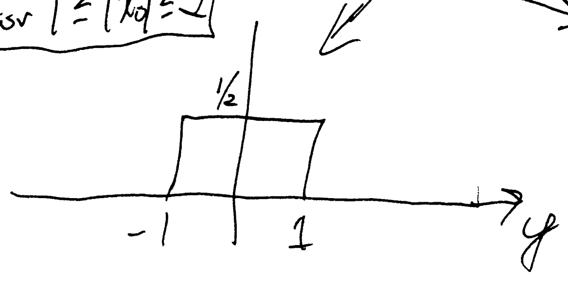
$\Rightarrow \sigma_{XY} = 0 \quad \Rightarrow$ Uncorrelated

2. X & Y are Dependent

Find $P_{Y|X=x_0}(y|x_0)$ & ~~see if~~ show that it depends on x_0 value

$$P_{Y|X=x_0}(y|x_0) = \frac{P_{XY}(x_0, y)}{P_X(x_0)}$$ ← just a # for each fixed x_0

$\boxed{\text{For } 1 \le |x_0| \le 2}$



$\boxed{\text{For } 0 \le |x_0| \le 1}$