Linear Filtering
and Decimation & Expansion
**Need for Filtering – Decimation**

*Recall:* M-Fold Decimation has no aliasing if original signal is an “Mth Band signal”

→ Usually need to ensure this before decimating
→ Pre-filter w/ M-Band D-T LPF (i.e. a D-T anti-alias filter)
Need for Filtering – Expansion

Recall: Expansion leaves high frequency images that shouldn’t be there

→ Need to remove these after expanding
→ Post-filter w/ L-Band D-T LPF (an “anti-image” filter)
Need for Filtering – Expansion (cont.)

So… the **ideal** post-filter converts the inserted zeros into **interpolated** samples:

> Expansion-Filtering is called “**Interpolation**”
Time-Domain Description – Filter/Decimate

To get a time-domain description of filter/decimate:

\[ \hat{x}[n] = \sum_{i} x[i]h[n - i] \]

\[ \hat{x}_{(\downarrow M)}[n] = \hat{x}[nM] = \sum_{i} x[i]h[nM - i] \]
Time-Domain Description – Expand/Filter

To get a time-domain description of expand/filter:

\[ \hat{x}_{(\uparrow L)}[n] = \sum_{i} x[i]h[n - Li] \]

Compare to filter/decimate:

\[ \hat{x}_{(\downarrow M)}[n] = \sum_{i} x[i]h[nM - i] \]

We’ll see later where this comes from.

Staggered Input Index
Staggered Output Index