# Z Transform Table

| Time Signal | Z Transform |
|---|---|
| $\delta[n]$ | $1$ |
| $\delta[n-q], \quad q = 1, 2, \ldots$ | $\frac{1}{z^q} = z^{-q}, \quad q = 1, 2, \ldots$ |
| $u[n]$ | $\frac{z}{z-1}$ |
| $u[n] - u[n-q], \quad q = 1, 2, \ldots$ | $\frac{z^q - 1}{z^{q-1}(z-1)}, \quad q = 1, 2, \ldots$ |
| $a^n u[n], \quad a$ real or complex | $\frac{z}{z-a}, \quad a$ real or complex |
| $nu[n]$ | $\frac{z}{(z-1)^2}$ |
| $(n+1)u[n]$ | $\frac{z^2}{(z-1)^2}$ |
| $n^2 u[n]$ | $\frac{z(z+1)}{(z-1)^3}$ |
| $na^n u[n], \quad a$ real or complex | $\frac{az}{(z-a)^2}$ |
| $n^2 a^n u[n], \quad a$ real or complex | $\frac{az(z+a)}{(z-a)^3}$ |
| $n(n+1)a^n u[n], \quad a$ real or complex | $\frac{2az^2}{(z-a)^3}$ |
| $\cos(\Omega_o n)u[n]$ | $\frac{z^2 - \cos(\Omega_o)z}{z^2 - 2\cos(\Omega_o)z + 1}$ |
| $\sin(\Omega_o n)u[n]$ | $\frac{\sin(\Omega_o)z}{z^2 - 2\cos(\Omega_o)z + 1}$ |
| $a^n \cos(\Omega_o n)u[n]$ | $\frac{z^2 - a\cos(\Omega_o)z}{z^2 - 2a\cos(\Omega_o)z + a^2}$ |
| $a^n \sin(\Omega_o n)u[n]$ | $\frac{a\sin(\Omega_o)z}{z^2 - 2a\cos(\Omega_o)z + a^2}$ |

# One-Sided Z Transform Properties

| Property Name | Property | |
|---|---|---|
| Linearity | $ax[n]+bv[n]$ | $aX(z)+bV(z)$ |
| Right Time Shift (**Causal** Signal) | $x[n-q], \quad q>0$ | $z^{-q}X(z)$ |
| Right Time Shift (Non-**Causal** Signal) | $x[n-1]$<br>$x[n-2]$<br>$x[n-q], \quad q>0$ | $z^{-1}X(z)+x[-1]$<br>$z^{-2}X(z)+x[-2]+z^{-1}x[-1]$<br>$z^{-q}X(z)+x[-q]+z^{-1}x[-q+1]+\cdots$<br>$\cdots+z^{-q+1}x[-1]$ |
| Multiply by $n$ | $nx[n]$ | $-z\dfrac{d}{dz}X(z)$ |
| Multiply by $n^2$ | $n^2x[n]$ | $z\dfrac{d}{dz}X(z)+z^2\dfrac{d^2}{dz^2}X(z)$ |
| Multiply by Exponential | $a^nx[n], \quad a$ real or complex | $X(z/a), \quad a$ real or complex |
| Multiply by Sine | $\sin(\Omega_o n)x[n]$ | $\dfrac{j}{2}\left[X(e^{j\Omega_o}z)-X(e^{-j\Omega_o}z)\right]$ |
| Multiply by Cosine | $\cos(\Omega_o n)x[n]$ | $\dfrac{1}{2}\left[X(e^{j\Omega_o}z)+X(e^{-j\Omega_o}z)\right]$ |
| Summation (**Causal** Signal) | $\sum_{i=0}^{n}x[i]$ | $\dfrac{z}{z-1}X(z)$ |
| Convolution in Time | $x[n]*h[n]$ | $X(z)H(z)$ |
| Initial-Value Theorem | $x[0]=\lim_{z\to\infty}\left[X(z)\right]$<br>$x[1]=\lim_{z\to\infty}\left[zX(z)-zx[0]\right]$<br>$x[q]=\lim_{z\to\infty}\left[z^{N}X(z)-z^{q}x[0]-z^{q-1}x[1]-\cdots-zx[q-1]\right]$ | |
| Final-Value Theorem | If $X(z)$ is rational and the poles of $(z-1)X(z)$ are inside unit circle<br>Then $\lim_{n\to\infty}x[n]=\left[(z-1)X(z)\right]_{z=1}$ | |