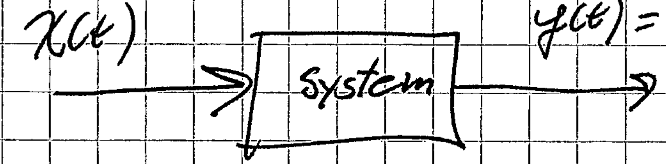
Examples for Linearity, Time-Invariance, Causality

(A)



(a) <u>Linearity</u>: Put in $x_1(t)$ get $y_1(t) = A x_1(t)$

" " $x_2(t)$ " $y_2(t) = A x_2(t)$

Now, put in $a_1 x_1(t) + a_2 x_2(t)$

and get $y(t) = A[a_1 x_1(t) + a_2 x_2(t)]$

(by defn. of the system)

Now manipulate using valid math rules:

$$y(t) = a_1 \underbrace{A x_1(t)}_{= y_1(t)} + a_2 \underbrace{A x_2(t)}_{= y_2(t)}$$

Try to link back to the "individual" outputs

$$= a_1 y_1(t) + a_2 y_2(t)$$

So it satisfies the requirement for linearity!

(b) <u>Time Invariance</u>: Put in $x(t)$ get $y(t) = A x(t)$

put in $x(t-\tau)$ get $A x(t-\tau) = y(t-\tau)$

$\Rightarrow$ satisfies TI requirement!

(c) <u>Causality:</u>

Let $x(t) = 0$ for $t < t_0$

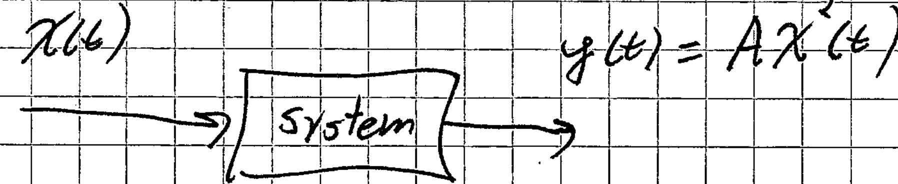
This $x(t)$ in gives output $y(t) = A\,x(t)$

$= 0 \quad t < t_0$

$\Rightarrow$ output is nonzero only after input becomes non-zero

$\Rightarrow$ Causal!

B



(a) <u>Linearity:</u> put in $x_1(t)$ get $y_1(t) = A x_1^2(t)$

" " $x_2(t)$ " $y_2(t) = A x_2^2(t)$

Now, put in $a_1 x_1(t) + a_2 x_2(t)$

and get
$$y(t) = A\left[a_1 x_1(t) + a_2 x_2(t)\right]^2$$

Now manipulate to try to get terms like these

$$y(t) = A\left[a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2a_1 a_2\, x_1(t) x_2(t)\right]$$

$$= a_1^2 \underbrace{A x_1^2(t)}_{= y_1(t)} + a_2^2 \underbrace{A x_2^2(t)}_{= y_2(t)} + 2a_1 a_2\, A x_1(t) x_2(t)$$

$$\neq a_1 y_1(t) + a_2 y_2(t) \quad \Rightarrow \text{Non linear!}$$

(b) Time - Inv.:

put in $x(t)$ get $y(t) = Ax^2(t)$

put in $x(t-\tau)$ get $Ax^2(t-\tau)$

$= y(t-\tau)$

$\Rightarrow$ TI!

(c) Causality:

Let $x(t) = 0$ for $t < t_0$

This $x(t)$ in gives $y(t) = \underbrace{Ax^2(t)}_{=0 \text{ for } t<t_0}$

$\Rightarrow$ Causal!

C

$$x(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t) = A(t)\,x(t)$$

(a) Linearity: It is linear by virtually the same arguments as in A

(b) TI: Put in $x(t)$ get $y(t) = A(t)\,x(t)$

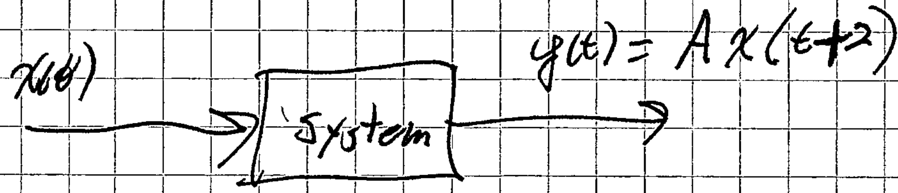
Put in $x(t-\tau)$ get $A(t)\,x(t-\tau)$

$\neq y(t-\tau)$

Not TI!

(c) Causality: It is causal by virtually the same arguments as in A

D



(a) Linear: Prove linearity as above

(b) TI: Prove TI as above

(c) Causality:

Let $x(t) = 0$ for $t < t_0$

Output is $y(t) = Ax(t+2)$

Evaluate $y(\underbrace{t_0 - 1}_{\text{before } t_0}) = A\,x(t_0 - 1 + 2)$

$$= A\,\underbrace{x(t_0 + 1)}_{\text{Non zero}}$$

$\Rightarrow$ $y(t)$ is nonzero before t_0

$\Rightarrow$ Non Causal!