EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #6

• System Modeling and C-T System Models
• Reading Assignment: Sections 2.4 & 2.5 of Kamen and Heck
**Course Flow Diagram**

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).

- **Ch. 1 Intro**
  - C-T Signal Model
    - Functions on Real Line
  - System Properties
    - LTI
    - Causal
    - Etc
  - D-T Signal Model
    - Functions on Integers

- **Ch. 2 Diff Eqs**
  - C-T System Model
    - Differential Equations
  - D-T Signal Model
    - Difference Equations
    - Zero-State Response
    - Zero-Input Response
    - Characteristic Eq.

- **Ch. 3: CT Fourier Signal Models**
  - Fourier Series
    - Periodic Signals
  - Fourier Transform (CTFT)
    - Non-Periodic Signals

- **Ch. 4: DT Fourier Signal Models**
  - DTFT
    - (for “Hand” Analysis)
  - DFT & FFT
    - (for Computer Analysis)

- **Ch. 5: CT Fourier System Models**
  - Frequency Response
    - Based on Fourier Transform
  - Zero-State Response

- **Ch. 5: DT Fourier System Models**
  - Freq. Response for DT
    - Based on DTFT
  - Zero-State Response

- **Ch. 6 & 8: Laplace Models for CT Signals & Systems**
  - Transfer Function

- **Ch. 7: Z Trans. Models for DT Signals & Systems**
  - Transfer Function

- **New Signal Models**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model

- **New System Model**
  - New System Model
System Modeling

To do engineering design, we must be able to accurately predict the quantitative behavior of a circuit or other system.

This requires **math models:**

**Circuits**
- **Device Rules**
  - R: $v(t) = Ri(t)$
  - L: $v(t) = L(di(t)/dt)$
  - C: $dv(t)/dt = 1/Ci(t)$
- **Circuit Rules**
  - KVL
  - KCL
  - Voltage Divider
  - etc.
- **Differential Equation**

**Mechanical**
- **Device Rules**
  - Mass: $M(d^2p(t)/dt^2)$
  - Spring: $k_x p(t)$
  - Damping: $k_d(dp(t)/dt)$
- **System Rules**
  - Sum of forces
  - etc.
- **Differential Equation**

Similar ideas hold for hydraulic, chemical, etc. systems…

“differential equations rule the world”
Simple Circuit Example:

Sending info over a wire cable between two computers

Computer #1  --  Computer #2

Two conductors separated by an insulator
⇒ capacitance

**Two practical examples of the cable**

“Twisted Pair” of Insulated Wires

- Typical values: 100 Ω/km
- 50 nF/km

Coaxial cable

- Conductors separated by insulator

Recall: resistance increases with wire length
Simple Model:

Driver’s Thevenin Equivalent Circuit (Computer #1)

Cable Model

Receiver’s Thevenin Equivalent Circuit (Computer #2)

Infinite Input Resistance (Ideal)

Zero Output Resistance (Ideal)

Effective Operation:

$5V$

$x(t)$

$t$

$0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1 \ ...$

$y(t)$
Use Loop Equation & Device Rules:

\[ x(t) = v_R(t) + y(t) \]
\[ v_R(t) = Ri(t) \]
\[ i(t) = C \frac{dy(t)}{dt} \]

This is the Differential Equation to be “Solved”:

\[ \frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} x(t) \]

Recall: A “Solution” of the D.E. means…

The function that when put into the left side causes it to reduce to the right side

Differential Equation & System

… the solution is the output
Now... because this is a **linear** system (it only has $R$, $L$, $C$ components!) we can analyze it by **superposition**.

Decompose the input...
**Input Components**

-5v

+  

5v

-5v

+  

5v

-5v

+  

5v

-5v

+  

5v

-5v

**Output Components (Blue)**

Standard Exponential Response

Learned in “Circuits”:

-5v

+  

5v

-5v

+  

5v

-5v

+  

5v

-5v
Output is a “smoothed” version of the input… it is harder to distinguish “ones” and “zeros”… it will be even harder if there is noise added onto the signal!
Progression of Ideas an Engineer Might Use for this Problem

**Physical System:**

**Schematic System:**

**Mathematical System:**

**Mathematical Solution:**

\[
\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} x(t)
\]
Automobile Suspension System Example

\[ y(t) = \text{Output: Frame’s Position} \]

\[ x(t) = \text{Input: Tire’s Position} \]

Results in 4\textsuperscript{th} order differential equation:

\[
\frac{d^4 y(t)}{dt^4} + a_3 \frac{d^3 y(t)}{dt^3} + a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = F[x(t)]
\]

The \( a_i \) are functions of system’s physical parameters:

\[ M_1, M_2, k_s, k_d, k_t \]
Again… to find the output for a given input requires solving the differential equation

Engineers could use this differential equation model to theoretically explore:

1. How the car will respond to some typical theoretical test inputs when different possible values of system physical parameters are used

2. Determine what the best set of system physical parameters are for a desired response

3. Then… maybe build a prototype and use it to fine tune the real-world effects that are not captured by this differential equation model
So… What we are seeing is that for an engineer to analyze or design a circuit (or a general physical system) there is almost always an underlying Differential Equation whose solution for a given input tells how the system output behaves.

So… engineers need both a qualitative and quantitative understanding of Differential Equations.

The major goal of this course is to provide tools that help gain that qualitative and quantitative understanding!!!
Linear Constant-Coefficient Differential Equations

General Form: \((N^{th} - \text{order})\)

\[
y^{(N)}(t) + \sum_{i=0}^{N-1} a_i y^{(i)}(t) = \sum_{i=0}^{M} b_i x^{(i)}(t)
\]

Input: \(x(t)\)

Output: \(y(t)\)

Solution of the Differential Equation

Recall: Two parts to the solution

(i) one part due to ICs with zero-input ("zero-input response")

(ii) one part due to input with zero ICs ("zero-state response")

Characteristic Polynomial: \(\lambda^N + a_{n-1}\lambda^{N-1} + \cdots + a_1\lambda + a_0\)

\(N\) roots: \(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_N\)

\(N\) "modes": Assuming distinct roots... \(e^{\lambda_1 t}, e^{\lambda_2 t}, \ldots, e^{\lambda_N t}\)

\[
\Rightarrow y_{ZI}(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \ldots + C_N e^{\lambda_N t}
\]

Then: \(y(t) = y_{ZI}(t) + y_{ZS}(t)\) \((y_{ZS}(t) \text{ is our focus, so we will often say ICs = 0})\)
So how do we find \( y_{ZS}(t) \)?

If you examine the zero-state part for all the example solutions of differential equations we have seen you’ll see that they all look like this:

\[
y_{ZS}(t) = \int_{t_0}^{t} h(t - \lambda)x(\lambda)d\lambda
\]

So we need to find out:

1. Given a differential equation, what is \( h(t-\lambda) \)
   
   See Ch. 3, 5, 6, 8

2. How do we compute & understand the convolution integral
   
   See Ch. 2

3. Are there other (easier? more insightful?) methods to find \( y_{ZS}(t) \)
   
   See Ch. 3, 5, 6, 8