Example of LT Solution

First... A word about the 3 parts of the solution:

- **Total Solution**
  - **Zero - Input Resp.**
    - This decays (transient) if the system is stable. Its behaviour is set by the Char. Poly & the IC's
  - **Transient Response**
    - This decays (transient) if the system is stable. Its behaviour is set by the Char. Poly
  - **Steady - State Resp.**
    - This might not decay. Its behaviour is set by the denominator of the Input X(s)
For this problem consider the RLC series circuit

We've seen that the Transfer Function is:

\[
H(s) = \frac{\sqrt{L/C}}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}
\]

Let \( L = 100 \text{ mH} \quad C = 8.5 \text{ mF} \quad R = 220 \Omega \)

\[
\Rightarrow H(s) = \frac{B(s)}{A(s)} = \frac{1084.7^2}{s^2 + 2200s + 1084.7^2} = \frac{1084.7^2}{(s + 916.9)(s + 1283.1)}
\]

(For this case we've seen that when \( R > \frac{2L}{VLC} \) we get distinct real roots)
The TF $H(s)$ captures the structure of the system and gives us everything we need to solve for the circuit output.

Suppose we have that:

$I_C(s): y(0^+) = 5 \quad y(0^-) = 2$

**Input:** $X(t) = 5 e^{-100t}$ u(t)

From LT table: $X(s) = \frac{5}{s + 100}$
We have seen—"Note set 33 - CT used to solve Diff Eq."—that a 2\textsuperscript{nd} - Order System has solution given by:

\[ Y(s) = \frac{y(0)s + [y(0) + a_1y(0) -]}{A(s)} + H(s)X(s) \]

\begin{align*}
\text{Zero - Input Part} & \quad \text{Zero - State Part} \\
\end{align*}

Now putting in for \( A(s) \) & \( H(s) \) gives:

\[ Y(s) = \frac{y(0)s + [y(0) + 2200y(0) -]}{s^2 + 2200s + 1084.7^2} + \left[ \frac{1084.7^2}{s^2 + 2200s + 1084.7^2} \right] X(s) \]

Note that \( A(s) \) - the system's char. Poly. Shows up here
Now plugging in the IC's gives:

\[ Y(s) = \frac{5s + 1100}{s^2 + 2200s + 1084.7^2} + \left[ \frac{1084.7^2}{s^2 + 2200s + 1084.7^2} \right] X(s) \]

Now plugging in \( X(s) \) gives:

\[ Y(s) = \frac{5s + 1100}{s^2 + 2200s + 1084.7^2} + \left[ \frac{1084.7^2}{s^2 + 2200s + 1084.7^2} \right] \left( \frac{5}{s + 100} \right) \]

\[ = \frac{5s + 1100}{s^2 + 2200s + 1084.7^2} + \frac{5,982.4 \times 10^6}{s^3 + 2300s^2 + (1.3765 \times 10^6)s + (1.1765 \times 10^8)} \]

Zero-Input Part...
has system char. Poly. \( A(s) \) in Denominator

Zero-State Part...
has combo of \( A(s) \) & den of \( X(s) \) in Denominator

\[ \text{Note: Can use 'conv' command to compute this! Just believe me... or prove it yourself!} \]

\[ \text{>> A=[1 (R/L) 1/(L*C)];} \]
\[ \text{>> format long} \]
\[ \text{>> Den=conv(A,[1 100])} \]
\[ \text{Den =} \]
\[
1.0e+008 *
0.00000001000000 0.00002300000000 0.01396470588235 1.17647058823529
\]
\[ Y(s) = \frac{11002}{s^3 + 2200s^2 + 1084.7^2} + \frac{5.8824 \times 10^6}{s^3 + 2300s^2 + (1.3715 \times 10^6)s + (1.1795 \times 10^8)} \]

\[
\begin{align*}
A &= [1 \ (R/L) \ 1/(L*C)]; \\
[R, P, K] &= \text{residue}([5\ 11002], A) \\
R &= -12.5237 \\
17.5237 \\
P &= 1.0e+003 * \\
-1.2831 \\
-0.9169
\end{align*}
\]

\[
\begin{align*}
A &= [1 \ (R/L) \ 1/(L*C)]; \\
\text{Den} &= \text{conv}([1\ 100], A) \\
[R, P, K] &= \text{residue}(5*(1/(L*C)), \text{Den}) \\
R &= 13.5763 \\
-19.6628 \\
6.0864 \\
P &= 1.0e+003 * \\
-1.2831 \\
-0.9169 \\
-0.1000
\end{align*}
\]

\[ Y(s) = \left[ \frac{-12.52}{s + 1283.1} + \frac{17.52}{s + 916.9} \right] + \left[ \frac{13.58}{s + 1283.1} + \frac{-19.64}{s + 916.9} \right] + \frac{6.09}{s + 100} \]
\[ y(s) = \left[ \frac{-12.52}{s + 1283.1} + \frac{17.52}{s + 916.9} \right] + \left[ \frac{13.58}{s + 1283.1} + \frac{-17.64}{s + 916.9} \right] + \frac{6.09}{s + 100} \]

**Zero-Input Part**

**Zero-State Part**

- **Transient Due to Input**
  - is the part of \( y(s) \) Resp. that is due to the factors of \( A(s) \)
- **Steady-State Resp to Input**
  - is the part of \( y(s) \) Resp. that is due to the factors of the Den. of \( X(s) \)

**General Rules!!**
Now use the LT table to invert:

\[ Y(s) = \left\{ \frac{-12.52}{s+1283.1} + \frac{17.62}{s+916.9} \right\} + \left\{ \frac{13.58}{s+1283.1} + \frac{-19.66}{s+916.9} \right\} + \frac{6.09}{s+100} \]

\[ y(t) = -12.52e^{-1283.1t} + 17.62e^{-916.9t} - 1283.1t + 13.58e^{-1283.1t} - 19.66e^{-916.9t} - 916.9t + 6.09e^{-100t} \]

For this example the ZI part & ZS Trans. Part die out long before the steady state part decays away.