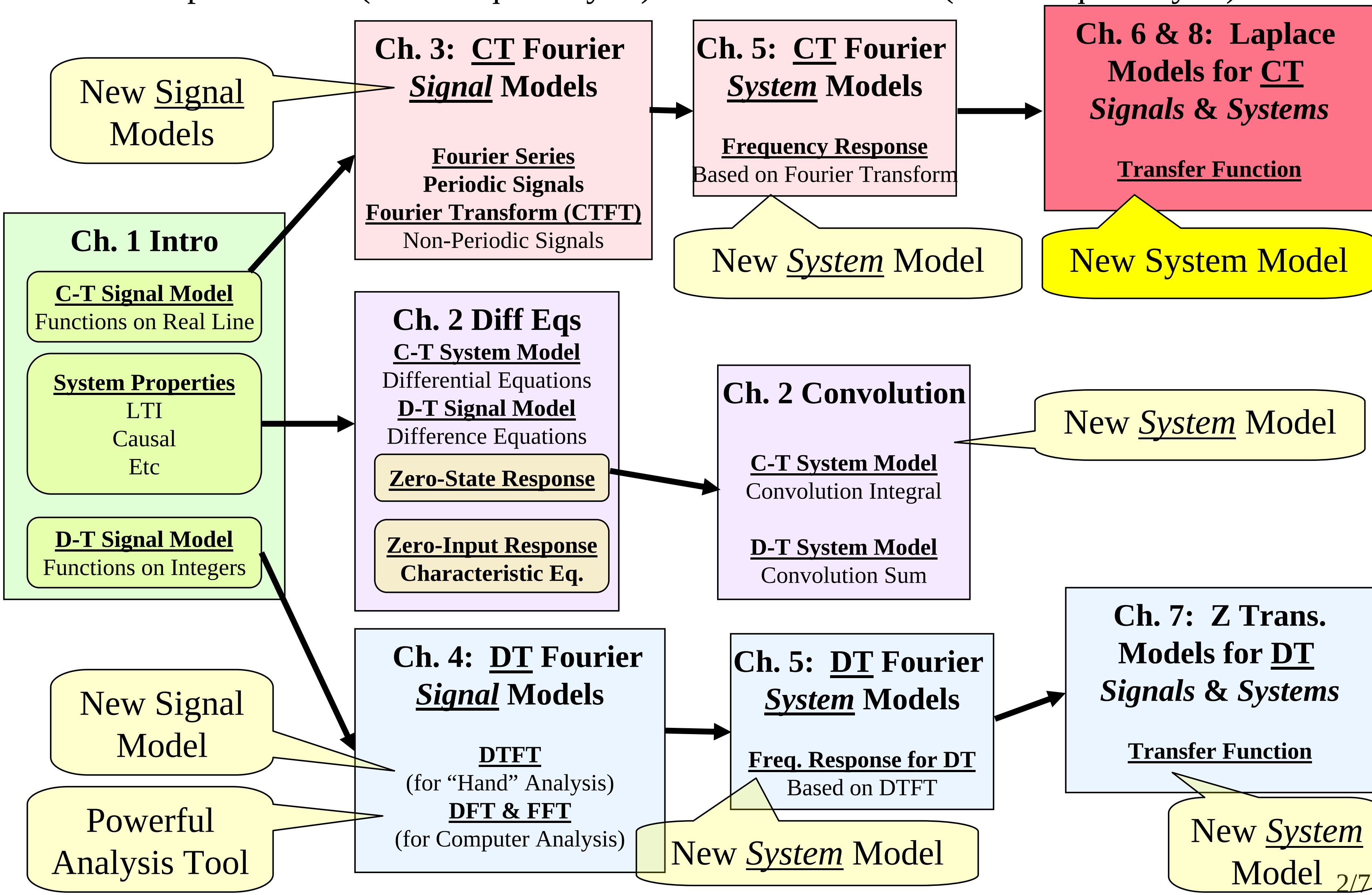
EECE 301
Signals & Systems
Prof. Mark Fowler

<u>Note Set #31</u>

- C-T Systems: Laplace Transform… and System Response to an Input
- Reading Assignment: Section 8.4 of Kamen and Heck

Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).



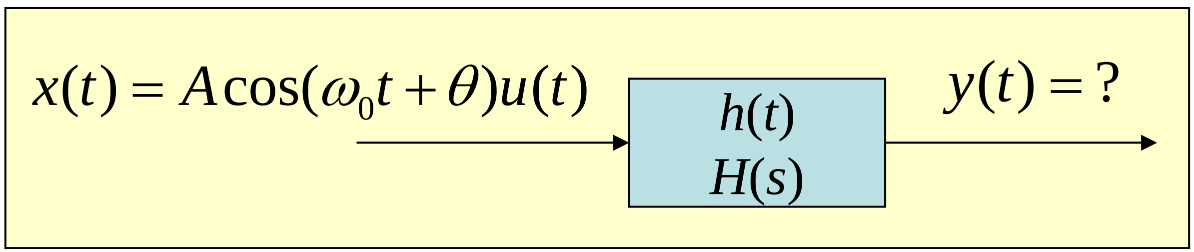
8.4: Response to Sinusoids and Arbitrary Signals

Sinusoidal input: Before, we used FT methods to answer this question…but there we assumed the sinusoid extended infinitely in both directions:

$$x(t) = A\cos(\omega_0 t + \theta) \qquad -\infty < t < \infty$$

For our studies of LT we have considered causal signals which are more practical!

$$x(t) = \begin{cases} A\cos(\omega_0 t + \theta), & t \geq 0 \\ 0, & t < 0 \end{cases}$$



For $x(t) = A\cos(\omega_0 t)u(t)$ we have (Table 8.2)

$$X(s) = \frac{As}{s^2 + \omega_0^2} = \frac{As}{(s + j\omega_0)(s - j\omega_0)}$$

For ease, we'll let $\theta = 0$, but we can handle the case of $\theta \neq 0$ using:

$$A\cos(\omega_0 t + \theta) = [A\cos(\theta)]\cos(\omega_0 t) - [A\sin(\theta)]\sin(\omega_0 t)$$ and linearity

Let $H(s) = \dfrac{B(s)}{A(s)}$ (for the finite-dimensional system case)

Assume system has no initial stored energy (i.e., no ICs) then we have:

$$Y(s) = H(s)X(s) = \frac{As\,B(s)}{A(s)(s + j\omega_0)(s - j\omega_0)}$$

Use Partial Fraction Expansion

$$Y(s) = \frac{\gamma(s)}{A(s)} + \left[\frac{(A/2)H(j\omega_0)}{s - j\omega_0} + \frac{(A/2)\overline{H(j\omega_0)}}{s + j\omega_0} \right]$$

Some Polynomial

ILT

ILT

$$y(t) = y_t(t) + A\left|H(j\omega_0)\right|\cos(\omega_0 t + \angle H(j\omega_0)) \qquad \underline{t > 0}$$

Looks like what we got before using "double-sided" sinusoid. BUT here it starts at time $t = 0$…

… and we have this term… what does it look like?

Note that $Y_t(s) = \dfrac{\gamma(s)}{A(s)}$ …and we know that the den. $A(s)$ sets the behavior!

<u>Note</u> that $A(s)$ is the system characteristic poly…. it sets the system poles.

$\Rightarrow$ System poles determine the behavior of $y_t(t)$

If system is stable $\Rightarrow$ the poles are in the LH plane

$\Rightarrow y_t(t)$ consists of <u>decaying</u> terms (might also oscillate if poles are complex)

So… $y_t(t)$ is "the <u>transient</u> response"

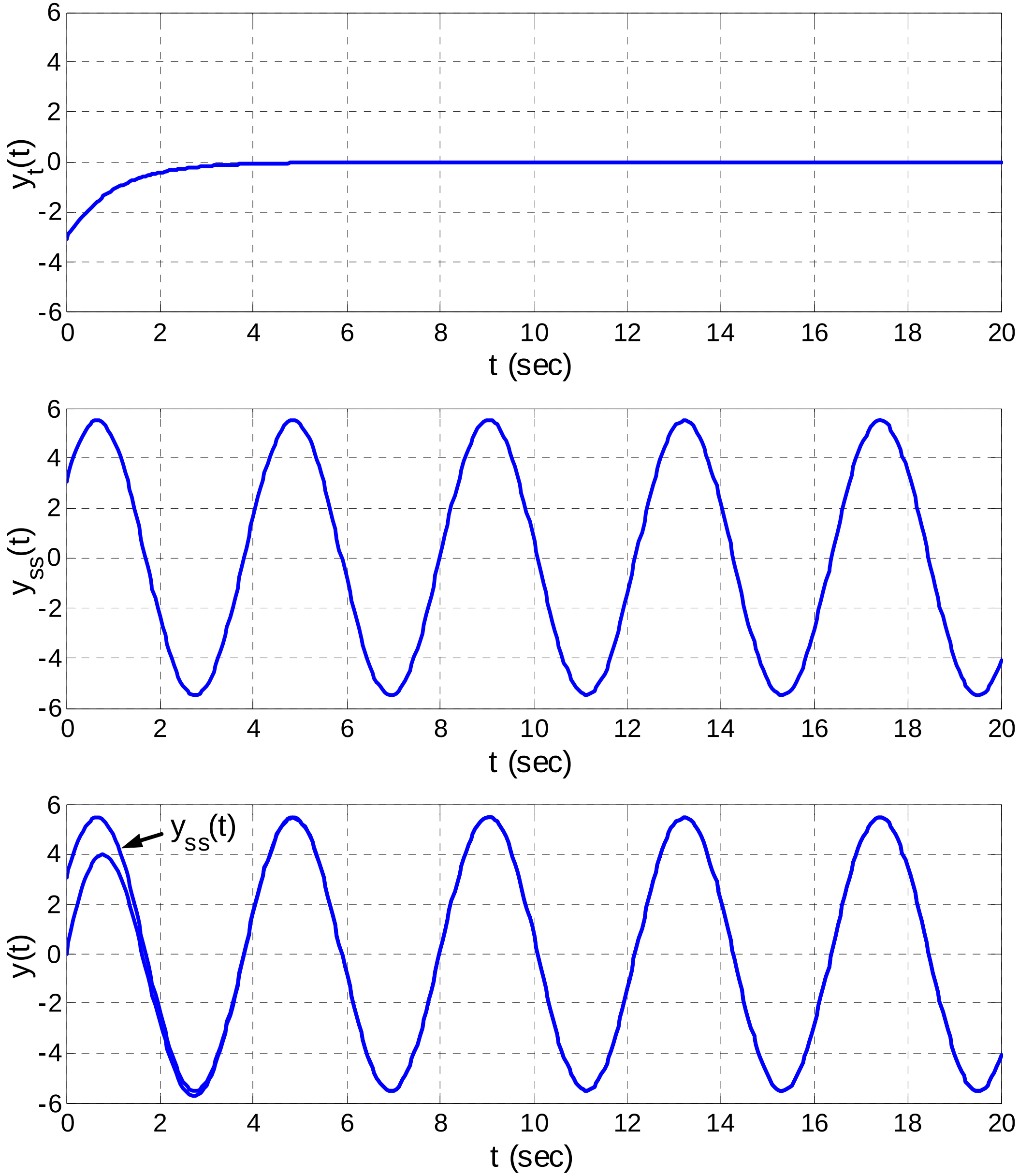
And…

- How fast it dies out depends on the real parts of the poles
- The pole closest to $j\omega$ axis will "dominate" (it takes the longest to die out)
- After enough time, all that is effectively left is:

"The steady-state response"

$$y_{ss}(t) = A|H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0)), \quad \underline{t \geq 0}$$

Plots for Example 8.16: RC circuit with causal sinusoid applied



RC = 1 second

Note that the transient has completely died out by "5 time constants" (actually, even before that)

Arbitrary Inputs

$$X(s) = \frac{N_X(s)}{D_X(s)} \quad \longrightarrow \quad H(s) = \frac{B(s)}{A(s)} \quad \longrightarrow \quad Y(s) = \frac{B(s)C(s)}{A(s)D(s)}$$

If there are no common poles between *X*(*s*) & *H*(*s*):

$$Y(s) = \frac{E(s)}{A(s)} + \frac{F(s)}{D_X(s)}$$

$$Y_t(s) = \frac{E(s)}{A(s)}, \qquad Y_{ss}(s) = \frac{F(s)}{D_X(s)}$$

System Den.

Signal Den.

If common poles, then *Y*(*s*) has repeated poles and you know how to modify for that case

E(*s*) & *F*(*s*) are "some polynomials"… they come from the math while factoring

So… if the system has poles in the "left-half plane" then the time-domain terms that arise from *A*(*s*) will decay:

$$y(t) = y_t(t) + y_{ss}(t)$$

Will die out if system is stable

Will persist if input persists