Note Set #3

- What are Discrete-Time Signals???
- Reading Assignment: Section 1.2 of Kamen and Heck
Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).
1.2 Discrete-Time (D-T) Signals

Recall from Note Set #1 that a common scenario in today’s electronic systems is to do most of the processing of a signal using a computer.

A computer can’t directly process a C-T signal but instead needs a stream of numbers… which is a D-T signal.
What is a discrete-time (D-T) signal?

A discrete time signal is a sequence of numbers indexed by integers

Example: $x[n] \rightarrow n = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Brackets indicate discrete-time signal. Recall… we used parentheses to indicate a C-T signal.

A stem plot emphasizes that the signal does not exist in-between integer n values

Remember: for our point of view, D-T signals are needed to allow us to process signals (i.e. information) using D-T systems rather than only Continuous-Time systems

Sometimes we violate this and plot with line segments connecting the dots.
D-T systems allow us to process information in much more amazing ways than C-T systems!

Sensor

\[ x(t) \]

C-T system (RLC, etc.)

\[ y(t) \]

Analog to Digital Converter (ADC)

\[ y[n] \]

D-T system (computer)

\[ z[n] \]

“sampling” is how we typically get D-T signals

In this case the D-T signal \( y[n] \) is related to the C-T signal \( y(t) \) by:

\[
    y[n] = y(t) \bigg|_{t=nT} = y(nT)
\]

\( T \) = time spacing between samples (seconds)

\( T \) is “sampling interval”

\( 1/T \) = sampling rate \( (F_s) \) in samples/second

\( F_s \) is “sampling rate”
Ex: CD audio is sampled at 44,100 samples per second

⇒ \( T = \frac{1}{44,100} \approx 22.68 \text{ μsec} \)

Major Question: How fast should we sample a specific signal?

(We can’t answer that until Chapter 5!!)

Hint: You may know that humans can’t hear frequencies above approximately 20kHz. Therefore, audio signals typically are limited to have no frequencies above 20kHz. Note that CD audio uses a sampling rate that is slightly more than twice this “highest” frequency!!
**Digital Signals**

-A practical ADC not only gives a D-T signal but also one that is “digital”

-An ADC represents each sample $y[n]$ using a finite number of bits (typically 8 to 16 bits/sample)

-ADC’s have a min/max input voltage

  -If the ADC uses $B$ bits per sample and $V = \text{max} = -\text{min}$ volts
  
  Then there are $2^B$ levels or values that are spaced $\frac{2V}{2^B} = V2^{(-B+1)}$ volts apart

This “quantization” of values causes degradation in the quality of the representation of $y[n]$  

For this course we will ignore the quantization! Our D-T signals can take on any value!
Some Common D-T Signals

Much of what we learned about C-T signals carries over to D-T signals

The **D-T Unit Step** is defined in an obviously similar way that the C-T Unit Step was defined. The D-T unit step is just a sampled version of the C-T unit step.

The same holds true for the **D-T Unit Ramp**.

**However... there are a Few Exceptions...**
Unit Pulse: $\delta[n]$  

“D-T Impulse” or “D-T Delta”

\[
\delta[n] = \begin{cases} 
1, & n = 0 \\
0, & n \neq 0 
\end{cases}
\]

Note: $\delta[n]$ is not a sampled version of $\delta[t]$
Sifting Property for D-T Delta Function

Note: $\delta[n]$ works inside summations the same way $\delta(t)$ works inside integrals

\[
\sum_{n=-\infty}^{\infty} \delta[n] = 1 \quad \text{Compare} \quad \int_{-\infty}^{\infty} \delta(t) \, dt = 1
\]

\[
\sum_{n=-\infty}^{\infty} x[n] \delta[n-n_0] = x[n_0] \quad \text{Compare} \quad \int_{-\infty}^{\infty} x(t) \delta(t-t_0) \, dt = x(t_0)
\]
D-T Rectangular Pulse

Often in practice we need to use pulses to model real-world signals…

One definition of DT version of this is as follows.

Let $q$ be an integer, then a centered pulse of $2q+1$ samples is

\[
p_q[n] = \begin{cases} 
1, & n = -q, \ldots, -1, 0, 1, \ldots, q \\
0, & \text{otherwise} 
\end{cases}
\]

$q$ specifies pulse +/- extent

**Warning:** My defn. is different from the book!
D-T Sinusoids

\[ x[n] = A \cos(\Omega n + \theta) \]

Use “upper case omega” for frequency of D-T sinusoids

What is the unit for \( \Omega \)?

\( \Omega n + \theta \) must be in radians \( \Rightarrow \Omega n \) in radians

\( \Rightarrow \) \( \Omega \) is “how many radians jump for each sample”

\( \Rightarrow \) \( \Omega \) is in radians/sample

Other than the above difference… D-T sinusoids are pretty much like C-T sinusoids.