EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #22
• D-T Signals: Frequency-Domain Analysis
• Reading Assignment: Section 4.1 of Kamen and Heck
Course Flow Diagram
The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).
Ch. 4: Fourier Analysis of D-T Signals

In this chapter we do for D-T signals what we did for C-T signals in Ch. 3:

– Define a D-T FT (DTFT) for D-T signals and see that it works pretty much like the FT for C-T signals (CTFT)

But… we also do something we can’t do for CTFT-based ideas:

– Develop a computer-processing version of the DTFT… called the Discrete Fourier Transform (DFT) that will allow you to use the computer to numerically compute a “view” of the DTFT

Order of Coverage:

– Sect. 4.1: DTFT & It’s Properties
– Sect. 4.2 & 4.3: The DFT & DFT-Based Signal Analysis
– Note: Section 4.4 is NOT covered
– Sect. 4.5 provides some applications of DFT analysis… we’ll cover some other applications in class
4.1 Discrete-Time FT (DTFT)
Recall: Sampling Analysis

As long as $F_s \geq 2B$ then we can clearly “see”… a view of $X(f)$ in $\tilde{X}(f)$

But we “did” this using a FT of a signal inside the DAC… Is there some other way to do this by using the *samples*?
Motivation for D-T Fourier Transform (DTFT)

$x(t)$ → ADC → “Hold” → $x[n]$ → Impulse Gen → $\tilde{x}(t)$ → DAC → CT LPF → $\hat{x}(t)$

Sample at $t = nT$

$\tilde{X}(f)$
Recall Fourier Transform of $\tilde{x}(t)$

$$\tilde{x}(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$= x(t) \delta_T(t)$$

FS of $\delta_f(t)$

$$\tilde{x}(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} x(t) e^{jk2\pi F_s t}$$

FT & Mod. Prop

$$\tilde{X}(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(f + kF_s)$$

Tells what $\tilde{X}(\omega)$ looks like!
Take An Alternate Path to the DTFT!

\[ \tilde{x}(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT) = x(t)\delta_T(t) \]

\[
\tilde{x}(t) = \sum_{n=-\infty}^{\infty} x[n] \delta(t - nT)
\]

\[
\tilde{X}(\omega) = \mathcal{F} \left\{ \sum_{n=-\infty}^{\infty} x[n] \delta(t - nT) \right\}
= \sum_{n=-\infty}^{\infty} x[n] \mathcal{F} \left\{ \delta(t - nT) \right\}
\]

\[
\tilde{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-jn\omega T}
\]

FT & Mod. Prop

FS of \( \delta_T(t) \)

Tells what \( \tilde{X}(\omega) \) looks like!

Uses Samples!!

Tells how to compute \( \tilde{X}(\omega) \)!
Re-Define to Get The DTFT!

\[ \tilde{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-jn\omega T} \]

Let \( \Omega = \omega T \) where \( T = 1/F_s \)

\[ X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-jn\Omega} \]

\( \Omega \) is called “D-T Frequency”

\[ \Omega = \omega T: \text{(rad/sec)} \times \text{(sec/sample)} = \text{rad/sample} \]

\( \tilde{X}(\omega) \) and \( X(\Omega) \) are really the same thing...

\( \omega: \text{rad/sec} \)

\( \Omega: \text{rad/sample} \)
DTFT $X(\Omega)$ shows...

$\tilde{X}(f)$ which shows...

"CTFT" $X(f)$

If sampling was done right!!!
Physical Relationship of DTFT

\[ x(t) \rightarrow \text{Sample at } t = nT \rightarrow \text{"Hold"} \rightarrow x[n] = x(nT) \rightarrow \text{Impulse Gen} \rightarrow \tilde{x}(t) \rightarrow \hat{x}(t) \]

CTFT of \( x(t) \)

\[ X(f) = \text{CTFT of } x(t) \]

DTFT of \( x[n] \)

\[ X(\Omega) = \text{DTFT of } x[n] \]

CTFT of \( \tilde{x}(t) \)

\[ \tilde{X}(f) = \text{CTFT of } \tilde{x}(t) \]

\[ \Omega = -4\pi, -2\pi, -\pi, 0, \pi, 2\pi, 4\pi \]

\[ f = -2F_s, -F_s, -F_s/2, F_s/2, F_s, 2F_s \]
Motivating D-T System Analysis using DTFT

$\begin{align*}
x(t) &\xrightarrow{\text{ADC}} x[n] \xrightarrow{\text{D-T System}} y[n] \xrightarrow{\text{DAC}} y(t)
\end{align*}$

$X(f) \xrightarrow{\text{CTFT of } x(t)} X(\Omega) \xrightarrow{\text{DTFT of } x[n]}$

$y(t) \xrightarrow{\text{CTFT of } y(t)} y[n] \xrightarrow{\text{DTFT of } y[n]}$