EECE 301  
Signals & Systems  
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Note Set #17

• C-T Systems: Frequency-Domain Analysis of Systems
• Reading Assignment: Section 5.1 of Kamen and Heck
• We’re jumping over Ch. 4 for now… we’ll come back later
Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).

Ch. 1 Intro
- C-T Signal Model
  - Functions on Real Line
- System Properties
  - LTI
  - Causal
  - Etc
- D-T Signal Model
  - Functions on Integers

Ch. 3: CT Fourier Signal Models
- Fourier Series
- Periodic Signals
- Fourier Transform (CTFT)
- Non-Periodic Signals

Ch. 4: DT Fourier Signal Models
- DTFT
  - (for “Hand” Analysis)
- DFT & FFT
  - (for Computer Analysis)

Ch. 2 Diff Eqs
- C-T System Model
  - Differential Equations
- D-T Signal Model
  - Difference Equations

Ch. 2 Convolution
- C-T System Model
  - Convolution Integral
- D-T System Model
  - Convolution Sum

Ch. 5: CT Fourier System Models
- Frequency Response
  - Based on Fourier Transform

Ch. 5: DT Fourier System Models
- Freq. Response for DT
  - Based on DTFT

Ch. 6 & 8: Laplace Models for CT Signals & Systems
- Transfer Function

Ch. 7: Z Trans. Models for DT Signals & Systems
- Transfer Function

New Signal Models
New System Model
Powerful Analysis Tool
Ch. 5 Frequency-Domain Analysis of Systems

Our main interest in this chapter is:

- How do we use the FT to analyze LTI systems?

  We’ll focus on the zero-state response here…

  (The zero-input response can be found using the characteristic equation method or the more complete methods we’ll study later)

We’ll look first at CT systems using three steps:

  5.1: Find out how sinusoids go through a C-T LTI

  5.2: Because a periodic signal is a sum of sinusoids we use linearity to extend section 5.1 results to periodic signals.

  5.2: Non-periodic signals also can be viewed as a sum (really an integral) of sinusoids so we can extend the result again!

Later we’ll essentially do the same things for D-T systems.

In between we’ll look at “Ideal C-T Filters” and “Sampling” to convert C-T signals into D-T signals
5.1 Response to a sinusoidal input:

In the notes for Section 3.1 (when we motivated WHY we were studying FS) we saw that it is easy to state how a complex sinusoid goes through a C-T LTI system:

\[ x(t) = Ae^{j(\omega_0 t + \theta)} \]

We now know that this is the FT of the system’s impulse response, evaluated at \( \omega = \omega_0 \):

\[ y(t) = A e^{j(\omega_0 t + \theta)} \int_{-\infty}^{\infty} h(\tau) e^{-j\omega_0 \tau} d\tau = H(\omega_0) \]

\[ y(t) = A H(\omega_0) e^{j(\omega_0 t + \theta)} \]

\[ y(t) = |H(\omega_0)| A e^{j(\omega_0 t + \theta + \angle H(\omega_0))} \]

Same frequency sinusoid comes out… the system just changes the input sinusoid’s amplitude and phase.

An LTI acts to change a complex sinusoid’s amplitude and phase.
We also saw how a real sinusoid goes through a C-T LTI System

\[ x(t) = A \cos(\omega_0 t + \theta) \quad \Rightarrow \quad h(t) \quad y(t) = A|H(\omega_0)| \cos(\omega_0 t + \theta + \angle H(\omega_0)) \]

The only thing an LTI system does to a real sinusoid is change its amplitude and its phase!!!!

Of course, you already knew that from circuits!!

So… The big result is:

\[ h(t) = \text{impulse response} \quad \Rightarrow \quad H(\omega) = \text{frequency response} \]

\[ A \cos(\omega_0 t + \theta) \quad \Rightarrow \quad h(t) \quad H(\omega) \quad \Rightarrow \quad |H(\omega_0)| A \cos(\omega_0 t + \theta + \angle H(\omega_0)) \]

\[ H(\omega) \text{ is called the “frequency response” of the system} \]
**Example:** Connecting these general ideas to sinusoidal analysis of circuits.

\[ x(t) = A \cos(\omega t + \theta) \]

To go from the circuit view to the system view… we need \( H(\omega) \)

\[ A \cos(\omega_0 t + \theta) \rightarrow h(t) \rightarrow y(t) = ? \]

When you did sinusoidal analysis in Circuits you did this!!!
Sinusoidal Analysis of Circuit gives the System’s Frequency Response $H(\omega)$

$$x(t) = A \cos(\omega t + \theta)$$

1. Convert capacitor into impedance: $Z_c(\omega) = \frac{1}{j\omega C}$
   - Small impedance at high $\omega$
   - Large impedance at low $\omega$

2. Write input as phasor: $Ae^{j\theta} = \tilde{x}$
   - Phasor captures amplitude and phase of cosine… the only things the system can change!!

3. Now analyze the circuit as if it were a DC circuit with a complex voltage in (the phasor) and complex resistors (the impedances):

Now find the output phasor as a function of the input phasor… Here this is easiest using voltage divider!
Voltage Divider: \[ \bar{y} = \frac{Z_c(\omega)}{R + Z_c(\omega)} \bar{x} = \left[ \frac{1}{1 + j\omega RC} \right] \bar{x} = H(\omega) \bar{x} \]

Output Phasor: \[ \bar{y} = H(\omega)\bar{x} = |H(\omega)|e^{j\angle H(\omega)} \bar{x} = |H(\omega)|e^{j\angle H(\omega)} A e^{j\theta} = \left( |H(\omega)| A \right) e^{j(\theta + \angle H(\omega))} \]

4. Convert the “phasor solution” into the “sinusoidal solution”:

Remember that a phasor is a complex number that holds:

- sinusoid’s amplitude in its magnitude
- sinusoid’s phase in its angles

\[ \bar{y} = \left( |H(\omega)| A \right) e^{j(\theta + \angle H(\omega))} \Rightarrow y(t) = |H(\omega)| A \cos(\omega t + \theta + \angle H(\omega)) \]
To see how different frequencies are affected by the RC circuit we plot

$$\left| H(\omega) \right| & \angle H(\omega)$$

Input has equal amounts at the 2 frequencies...

$$x(t) = \cos(100t) + \cos(3000t)$$

Output has almost all of the low frequency component but much reduced high frequency component!

$$y(t) = 0.995 \cos(100t - 0.097) + 0.316 \cos(3000t - 1.249)$$

$$H(100) = 0.995e^{-j0.097}$$

$$H(3000) = 0.316e^{-j1.249}$$
So what have we seen:

• We can find the frequency response function $H(\omega)$ by doing a simple sinusoidal analysis of the circuit

• The frequency response function tells how a circuit changes the input sinusoid’s amplitude and phase

• The amount of change in each of these is different for different input frequencies… and a plot of $H(\omega)$ magnitude and phase shows this dependence

• RLC circuits can be used to allow certain frequency components to pass mostly unchanged while others are drastically reduced in amplitude
  – We can “filter out” undesired frequency components