BINGHAMTON
UNIVERSITY
State University of New York

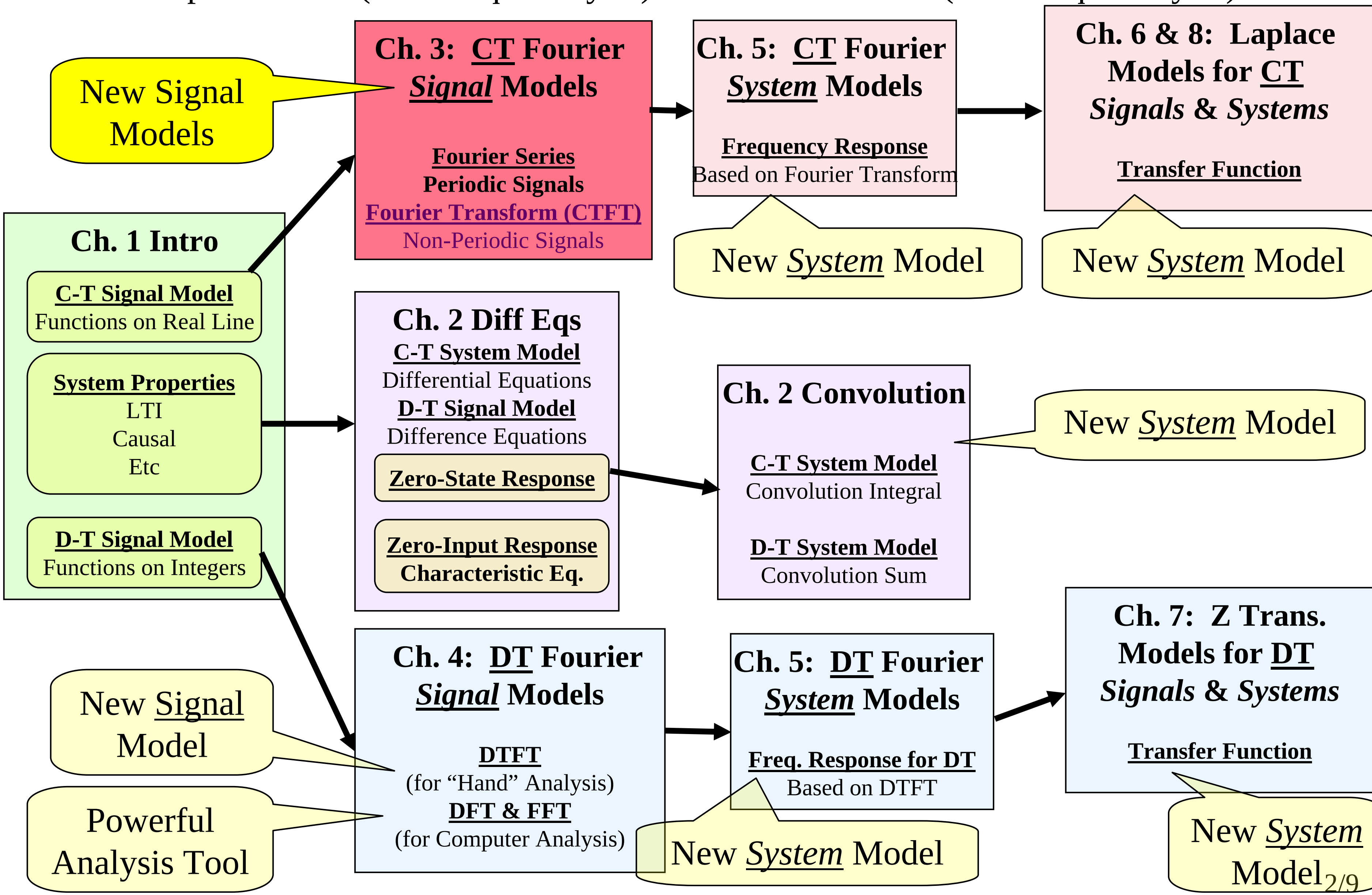
EECE 301
Signals & Systems
Prof. Mark Fowler

Note Set #16

- C-T Signals: Generalized Fourier Transform
- Reading Assignment: Section 3.7 of Kamen and Heck

Course Flow Diagram

The arrows here show conceptual flow between ideas. Note the parallel structure between the pink blocks (C-T Freq. Analysis) and the blue blocks (D-T Freq. Analysis).



Generalized FT

This section allows us to apply FT to an even broader class of signals that includes the periodic signals and some other signals.

The trick is to allow the delta function to be a part of a valid FT

But first we start "backwards"… with the delta function in the time domain.

Q: What is the FT of $\delta(t)$?

A: First… think it through! $\delta(t)$ is "the narrowest pulse"

And… a narrow pulse has a broad FT…

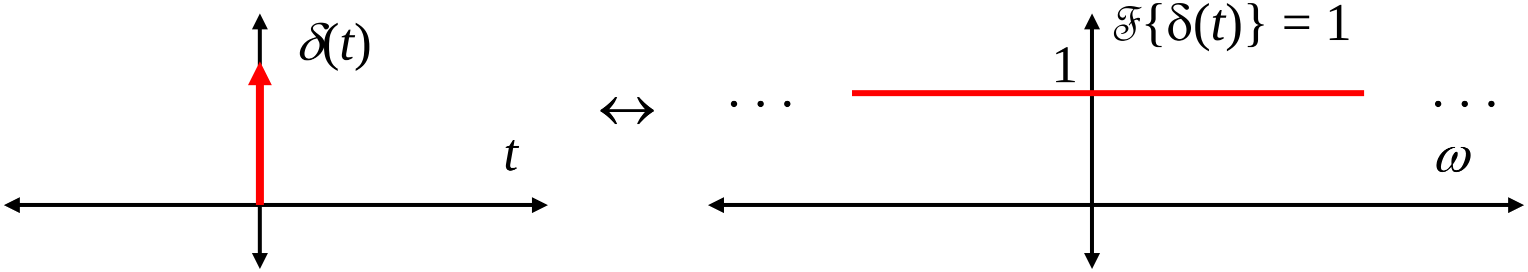
So… the narrowest pulse should have in some sense the broadest FT

Now… work the math:

$$\mathfrak{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = e^{-j\omega \cdot 0} = 1$$

Sifting property

$$\delta(t) \leftrightarrow 1$$

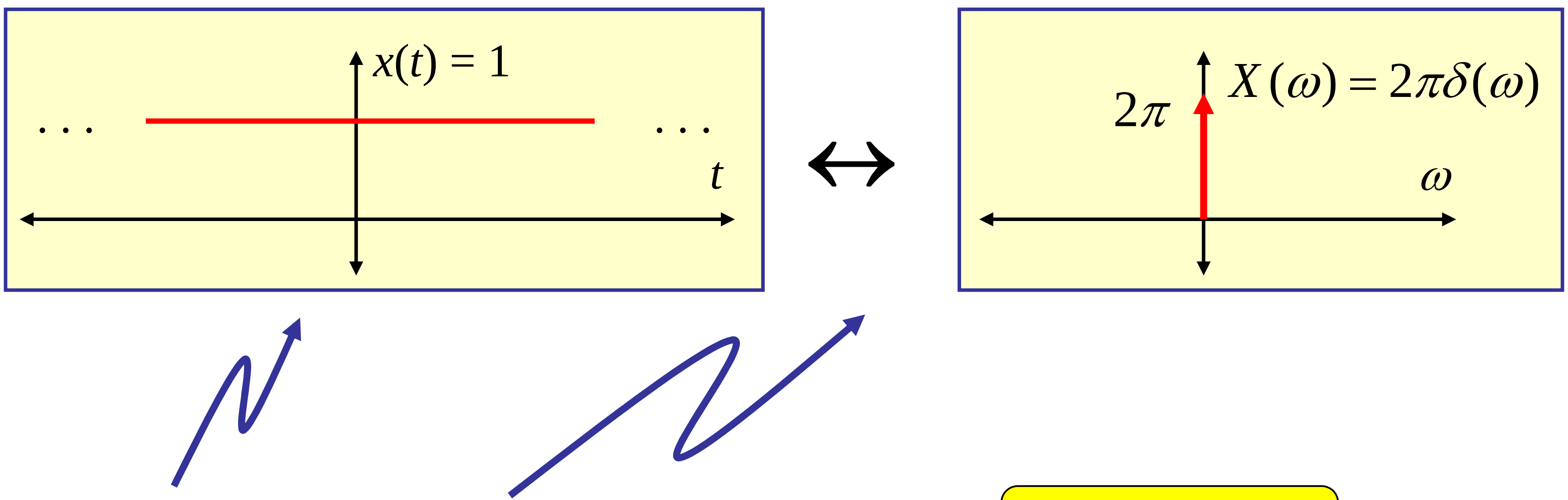


Now we can use the duality property to get another FT Pair:

$$\delta(t) \leftrightarrow 1$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

So we now know:



"A DC signal" has FT concentrated at 0 Hz

DC = 0 Hz

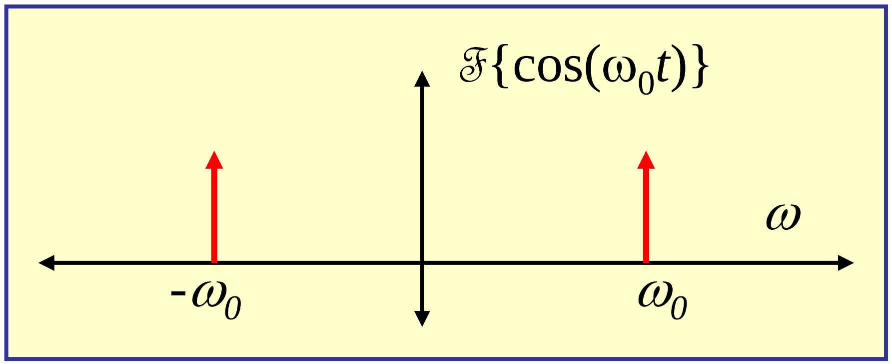
Now we can get ***another*** pair by using this last result and the real modulation property:

$$1 \leftrightarrow 2\pi\delta(\omega)$$

Multiply by cosine

Frequency Shift Up & Down

$$\cos(\omega_0 t) \times 1 \leftrightarrow \frac{1}{2}\left[2\pi\delta(\omega+\omega_0) + 2\pi\delta(\omega-\omega_0)\right]$$



Note: This says you only need the components at $+\omega_0$ and $-\omega_0$ (i.e., $\exp\{j\omega_0 t\}$ and $\exp\{-j\omega_0 t\}$) to build $\cos(\omega_0 t)$

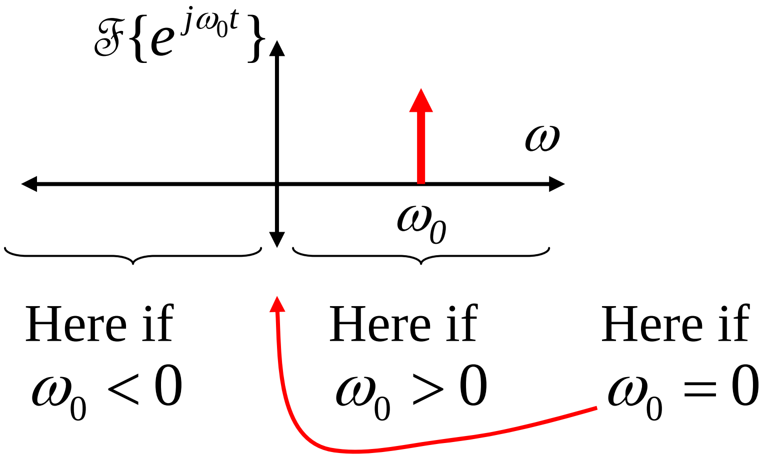
Can do similar thing for sine:

$$\cos(\omega_0 t) \leftrightarrow \pi\left[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right]$$

$$\sin(\omega_0 t) \leftrightarrow j\pi\left[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right]$$

Similarly… By the complex mod. property:

$$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$$



Note: This says you need only exp{j$\omega_0 t$} to build exp{j$\omega_0 t$}!!! **Duh!!!**

Note that we have now used the FT to analyze cosine and sine… which are ***PERIODIC*** signals!!! Before we used the Fourier ***Series*** to analyze ***periodic*** signals… Hmmm… it seems possible to use the FT instead of the FS!!

i.e. the FT subsumes the FS!

FT of periodic signal:

If $x(t)$ is periodic then we can write the FS of it as: $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$

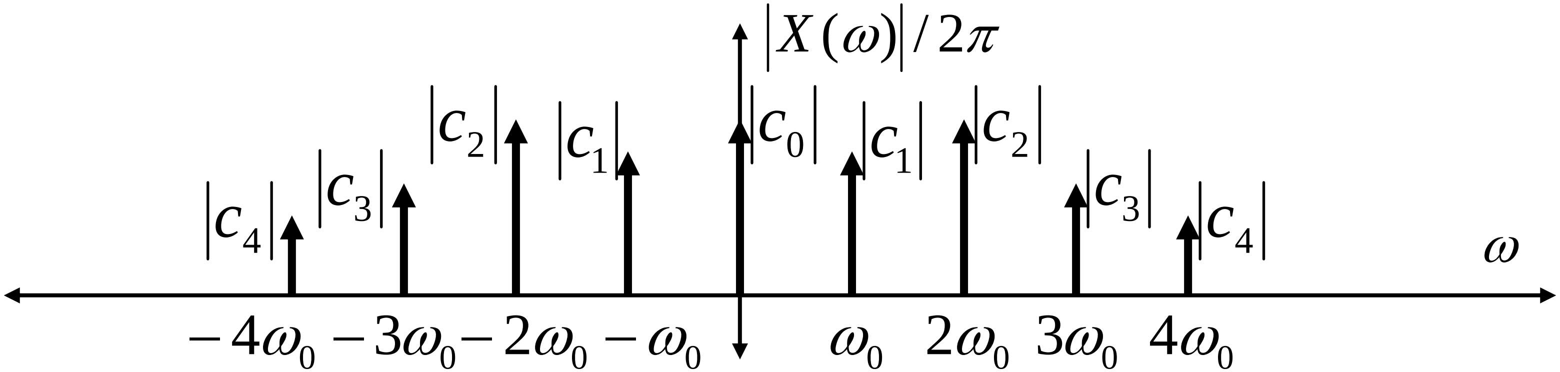
Now we can take the FT of both sides of this: $\mathscr{F}\{x(t)\} = \mathscr{F}\left\{ \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} \right\}$

$$= \sum_{k=-\infty}^{\infty} c_k \underbrace{\mathscr{F}\left\{ e^{jk\omega_0 t} \right\}}_{2\pi\delta(\omega - k\omega_0)}$$

FT of a Periodic Signal

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi c_k \delta(\omega - k\omega_0)$$

Note: the FT is a bunch of delta functions with "weights" given by the FS coefficients!



So the FT of a periodic signal is zero except at multiples of the fundamental frequency ω_0, where you get impulses.

We call these spikes "Spectral Lines"

See the book for FT of unit step, which contains a delta function