De-Noising by Wiener Filter and Wavelet Based Methods for Emitter Localization Systems

Mohammad Pourhomayoun, *Student Member, IEEE* and Mark L. Fowler, *Senior Member, IEEE* [1]

Abstract—**The Cross Ambiguity Function (CAF) used in signal location estimation is a 2-dimensional complex-valued function of TDOA and FDOA. In TDOA/FDOA systems, pairs of sensors share data to compute the CAF. In practice, the received signals are noisy and this noise perturbs the CAF from its ideal shape which is a big main lobe and some small side lobes. At low SNRs, the CAF main lobe is buried in the noise and the location estimation accuracy is poor. In this paper, by exploiting some of the CAF properties, we de-noise the CAF itself to increase the estimation performance. We use Wiener filter and wavelet based methods for de-noising. The impact of such de-noising methods on the overall location accuracy is assessed via simulations.**

Index Terms— **Ambiguity Function (CAF), FDOA (Frequency Difference of Arrival), TDOA (Time Difference of Arrival), Wiener Filter, Wavelet Transform.**

I. Introduction

The Cross Ambiguity Function (CAF) is a 2-dimensional complex-valued function of time-difference-of-arrival (TDOA) and frequency-difference-of-arrival (FDOA). In TDOA/FDOA localization systems, pairs of sensors share data to compute the CAF. The computed CAFs can be used to estimate the TDOA/FDOA for each pair of sensors by finding the peak of the CAF magnitude in the classic location estimation method [1]-[3]. Recently, some new methods based on TDOA/FDOA emitter location have been proposed that estimate the emitter location in one stage without extracting the TDOA/FDOA in a separate step. The goal of these methods is to improve the overall accuracy of the emitter location estimate. The main idea of the recent methods is that all pairs of sensors have to share their computed CAFs to each other or they have to send the CAFs to a common site to estimate the emitter location [4]-[7]. It would be also desirable to use some data compression methods to reduce the amount of data transmission in these methods [8]-[10].

The CAF in the continuous-time case is given by:

$$A_{12}(\tau,\omega)=\int_{-\infty}^{+\infty} s_1(t)s_2^{*}(t-\tau)e^{j\omega t}dt \tag{1}$$

where $s_1(t)$ is the lowpass equivalent (LPE) of the received signal at the first sensor and $s_2(t)$ is LPE of the received signal at the second sensor. CAF measures the correlation between $s_1(t)$ and a Doppler-shifted by ω and delayed by τ version of $s_2(t)$. In order to switch to discrete (or sampled) time domain, let $t=nT_s$ and $\omega=2\pi k f_s/N$ where $f_s=1/T_s$ is the sampling frequency and N is total number of samples, then:

$$A_{12}(m,k)=\sum_{n=0}^{N-1} s_1(n)s_2^{*}(n-m)e^{j2\pi\frac{pn}{N}} \tag{2}$$

II. Problem Description

In reality, the received signals are noisy. Thus, the received signal at the first sensor can be shown by $s_1(t)+n_1(t)$ and the received signal at the second sensor will be $s_2(t)+n_2(t)$, where $n_1(t)$ and $n_2(t)$ are the white additive noise. This noise perturbs the CAF from its true shape which is a big main lobe and several small side lobes. When the SNR is low it is even impossible to find the peak of the CAF magnitude.

In this paper, we propose to de-noise the CAF exploiting some of its properties and then use the de-noised version of that in the estimation process; we will see that this method will improve the estimation performance. We used linear minimum mean square error (Wiener filter) and also wavelet based de-noising methods for our purpose.

Wiener filter is a linear filter that removes additive noise of given variance from a noisy signal. Wiener filter gives the optimal *linear* estimator for the signal in the minimum Bayesian mean square sense (Linear Minimum Mean Square Error) [11].

Suppose that X is the vector of the noisy CAF points made by noisy signals and θ is the vector of noise-free CAF points with the same length and ξ is the vector of total noise elements, then we have the following model:

$$X=\theta+\xi \tag{3}$$

Assuming that X_i, θ_i and ξ_i are the i$^{\text{th}}$ elements of X, θ and ξ respectively, then we have:

The authors are with Department of Electrical and Computer Engineering, Binghamton Univ., P.O. Box 6000, Binghamton, NY 13902-6000, Phone:607-777-6973, mpourho1@binghamton.edu, mfowler@binghamton.edu

$$\theta_i = \sum_{n=0}^{N-1} s_1(n) s_2^*(n-k) e^{j\alpha n} \tag{4}$$

$$\xi_i = \sum_{n=0}^{N-1} [s_1(n) n_2^*(n-k) + n_1(n) s_2^*(n-k) + n_1(n) n_2^*(n-k)] e^{j\alpha n} \tag{5}$$

The Linear Minimum Mean Square Error (LMMSE) estimator for the model in equation (3) is [11]:

$$\hat{\theta} = E\{\theta\} + C_{\theta X} C_{XX}^{-1} (X - E\{X\}) \tag{6}$$

where C_{XX} is the covariance matrix for the observation and $C_{\theta X}$ is cross covariance between observation and our parameter. Putting (4) and (5) in (3) and using Mixed Central Moments of jointly Gaussian random variables, we have

$$C_{XX}(i,j) = E\{X_i X_j^*\} - E\{X_i\} E\{X_j^*\} \tag{7}$$

where,

$$\begin{aligned} E\{X_i X_j^*\} &= C_{\theta\theta}(i,j) + \\ &+ E\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} [s_1(n) s_1^*(m) n_2^*(n-k) n_2(m-l) e^{-j\alpha n} e^{j\beta m}] \\ &+ E\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} [n_1(n) n_1^*(m) s_2^*(n-k) s_2(m-l) e^{-j\alpha n} e^{j\beta m}] \\ &+ E\sum_{n=0}^{N-1}\sum_{m=0}^{N-1} [n_1(n) n_1^*(m) n_2^*(n-k) n_2(m-l) e^{-j\alpha n} e^{j\beta m}] \end{aligned}$$

Thus,

$$\begin{aligned} E\{X_i X_j^*\} &= C_{\theta\theta}(i,j) + \\ &+ \sigma_{N2}^2 R_s(l-k) e^{j\beta(l-k)} \sum_{n=0}^{N-1} e^{-j(\alpha-\beta)n} \\ &+ \sigma_{N1}^2 R_s(l-k) \sum_{n=0}^{N-1} e^{-j(\alpha-\beta)n} \\ &+ \sigma_{N1}^2 \sigma_{N2}^2 \delta_{kl} \sum_{n=0}^{N-1} e^{-j(\alpha-\beta)n} \\ &= C_{\theta\theta}(i,j) + \\ &+ R_s(l-k) \delta_{\alpha\beta} (\sigma_{N1}^2 + \sigma_{N2}^2 e^{j\beta(l-k)}) + \delta_{kl} \delta_{\alpha\beta} \sigma_{N1}^2 \sigma_{N2}^2 \end{aligned} \tag{8}$$

Using (7) and (8), we have

$$\begin{aligned} C_{XX}(i,j) &= C_{\theta\theta}(i,j) + R_s(l-k) \delta_{\alpha\beta} (\sigma_{N1}^2 + \sigma_{N2}^2 e^{j\beta(l-k)}) \\ &+ \delta_{kl} \delta_{\alpha\beta} \sigma_{N1}^2 \sigma_{N2}^2 - R_s(k) R_s(l) \delta_\alpha \delta_\beta \end{aligned} \tag{9}$$

where $R_s(l-k)$ is the autocorrelation of the signal, σ_{N1}^2, σ_{N2}^2 are variance of the received noise at first and second sensor and δ_{kl} is the impulse function with zero value when $(l \neq k)$. It is also straight forward to show that $C_{\theta X} = C_{\theta\theta}$.

In another approach, we applied wavelet-based methods for CAF de-noising. The basic idea behind non-linear wavelet de-noising (selective wavelet reconstruction) is to choose a relatively small number of wavelet coefficients to represent the signal [10]. Donoho and Johnstone [12][14] showed that the noise spread out among all wavelet coefficients of the noisy (empirical) signal but, only relatively few of them consist of significant signal data. Thus, it is natural to reconstruct the signal using only the largest empirical wavelet coefficients in an attempt to de-noise the signal [10].

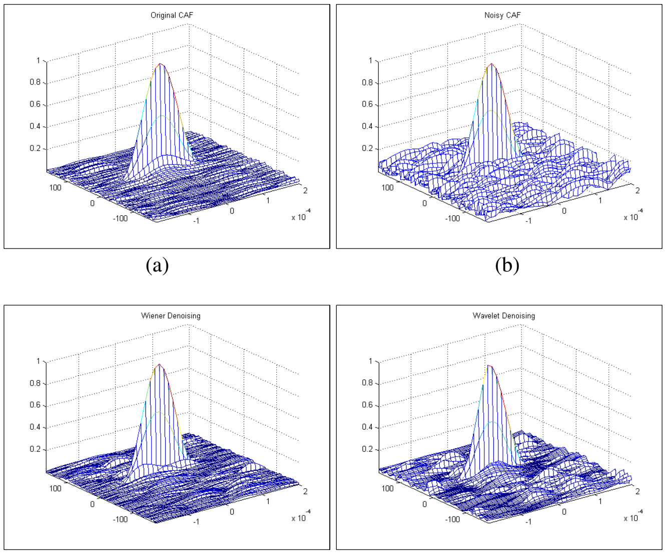
It is important to note that Cross Ambiguity Function is a relatively slowly changing function. The fast changing parts (which are equivalent to very high frequency points) come from the effect on the CAF of the additive noise of received signals. Thus, viewed as an image, it seems that the important part to be retained is a spatially low-pass type signal that should show up in the medium and low frequency parts of the wavelet transform [10]. Thus, we preferred to use *universal thresholding method* proposed by Donoho and Johnstone with threshold factor of $\lambda = \sqrt{2\log(n)}$ in each level that gives a better result for de-noising of smooth functions.

III. SIMULATION

We examined the performance of the proposed methods and compare the results using Monte Carlo computer simulations (with 500 runs each time). In this simulation, we used direct position determination method for location estimation [6].

We assumed that 6 moving sensors receive the noisy signals from one stationary emitter and for each two of them there is a cross ambiguity function which is de-noised and transmitted to a common site to do the location estimation. Fig.1 shows a typical CAF for noise-free and noisy signals with SNR = -5dB and the effect of adding noise and de-noising: (a) Original CAF for noise-free signals, (b) The same CAF for noisy signals with SNR= -5 dB, (c) De-noised CAF by Wiener filter, (d) De-noised CAF by Wavelet thresholding. Looking closely, we can see that Wiener filter cancels the noise with minimum distortion of the main lobe, but Wavelet de-noising damages the main lobe more and also moves the peak position, which is very important in location estimation.

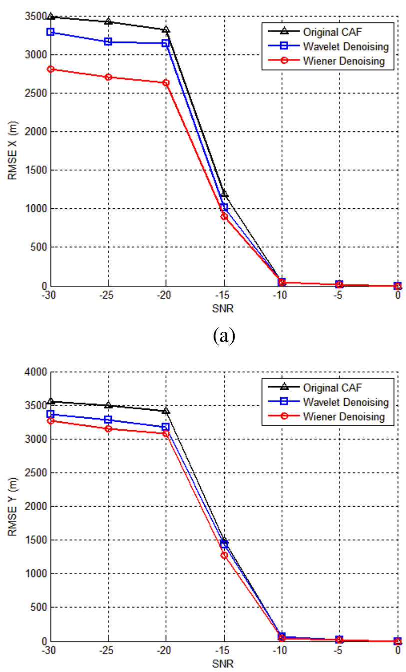
Fig.2 and Fig.3 show the effect of CAF de-noising on RMS error and standard deviation of emitter location estimation for X and Y dimensions. The plots show the results for three different cases: (i) emitter location estimation using original noisy CAF, (ii) using de-noised CAF by Wiener Filter, (iii) using de-noised CAF by Wavelet Thresholding. In this simulation, we tried both *soft* and *hard* thresholding with many different mother wavelets and different levels and finally we choose the one with the best results. Nonetheless, we can see that Wiener filter gives better results compared to Wavelet de-noising and we think that this is because of the non-linearity property of Wavelet thresholding and its effect on CAF main lobe.



(a) (b)

(c) (d)

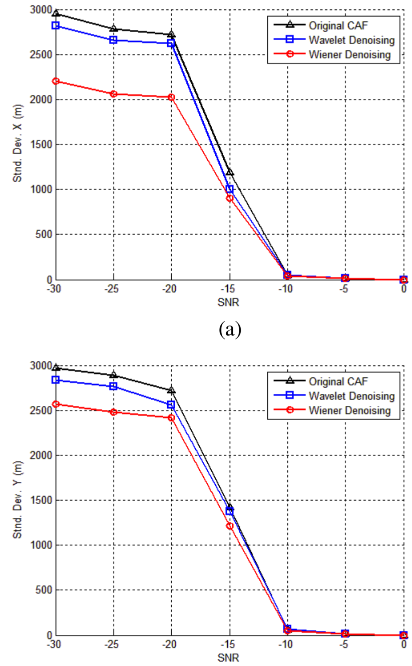
Fig.1. A typical CAF for noise free and noisy signals and the effect de-noising on that: (a) Original CAF for noise-free signals, (b) The same CAF for noisy signals with SNR= -5dB, (c) De-noised CAF by Wiener filter, (d) De-noised CAF by Wavelet thresholding.



(a)

(b)

Fig. 2. Simulation results showing RMS error for X and Y dimensions in three cases: (i) using original Noisy CAF, (ii) using de-noised CAF by Wiener Filter, (iii) using de-noised CAF by Wavelet Thresholding.



(a)

(b)

Fig. 3. Simulation results showing Standard Deviation for X and Y dimensions in three cases: (i) using original Noisy CAF, (ii) using de-noised CAF by Wiener Filter, (iii) using de-noised CAF by Wavelet Thresholding.

REFERENCES

[1] Stein, S., "Differential delay/Doppler ML estimation with unknown signals," *IEEE Transactions Signal Processing.*, 41(8), 2717–2719 (1993).

[2] Stein, S., "Algorithms for Ambiguity Function Processing," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 29(3), 588-599 (1981).

[3] D. J. Torrieri, "Statistical theory of passive location system," *IEEE Transactions on Aerospace and Electronic Systems,* vol. AES-20, no. 2, March 1984, pp. 183 – 198.

[4] Hartwell, G., D., "Improved Geo-Spatial Resolution Using a Modified Approach to the Complex Ambiguity Function (CAF)", Master's Thesis, Naval Postgraduate School (2005).

[5] Weiss, A., "Direct Position Determination of Narrowband Radio Frequency Transmitters," *IEEE Transactions Signal Process.* , 11(5), 513-516 (2004).

[6] Weiss, A., Amar, A., "Direct Geolocation of Stationary Wide Band Radio Signal Based on Delays and Doppler Shifts," *IEEE Workshop on Statistical Signal Processing,* Aug. 31 – Sept. 3, 2009, Cardiff, Wales, UK.

[7] N. Vankayalapati, S. Kay, "Asymptotically Optimal Localization of an Emitter of Low Probability of Intercept Signals Using Distributed Sensors," to appear in *IEEE Transactions on Aerospace and Electronic Systems,* vol. 48, no. 1, Jan. 2012.

[8] M. Pourhomayoun and M. L. Fowler**, "**Exploiting Cross Ambiguity Function Properties for Data Compression in Emitter Location Systems," Conference on Information Sciences and Systems, Johns Hopkins University, March 2011.

[9] M. Pourhomayoun and M. L. Fowler, "An SVD Approach for Data Compression in Emitter Location Systems", Asilomar Conference on Signals, Systems and Computers, Nov 2011.

[10] M. Pourhomayoun, M. L. Fowler, "Data compression for complex ambiguity function for emitter location", *Proceedings of SPIE - Mathematics of Data/Image Coding, Compression, and Encryption with Applications XII*, San Diego, Aug 2010.

[11] S. M. Kay, “Fundamentals of Statistical Signal Processing”, Vol.1, Estimation Theory, 1993.

[12] D. L. Donoho and I. M. Johnstone, "Ideal Spatial Adaptation by Wavelet Shrinkage," Biometrika, vol. 81, no. 3, pp. 425-455, Aug. 1994.

[13] D. L. Donoho and I. M. Johnstone, "Ideal Denoising in Orthonormal Basis from Library," Stanford University, Dept. of Statistics, Technical Report 461, September 1994.

[14] D. L. Donoho, "De-Noising by Soft-Thresholding," *IEEE Transactions on Information Theory*, vol. 41, pp. 613-627, May 1995.