Exploiting Data Compression Methods for Network-Level Management of Multi-Sensor Systems

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ABSTRACT

Data compression ideas can be extended to assess the data quality across multiple sensors to manage the network of sensors to optimize the location accuracy subject to communication constraints. From an unconstrained-resources viewpoint it is desirable to use the complete set of deployed sensors; however, that generally results in an excessive data volume. Selecting a subset of sensors to participate in a sensing task is crucial to satisfying trade-offs between accuracy and time-line requirements. For emitter location it is well-known that the geometry between sensors and the target plays a key role in determining the location accuracy. Furthermore, the deployed sensors have different data quality. Given these two factors, it is no trivial matter to select the optimal subset of sensors. We attack this problem through use of a data quality measure based on Fisher Information for set of sensors and optimize it via sensor selection and data compression.

Keywords: Sensor Selection, Sensor Management, Data Compression, Sensor Networks, TDOA/FDOA Emitter Location, Fisher Information

1. INTRODUCTION

Our general interest is in achieving network-wide optimization over a large number of simultaneously deployed airborne sensors to enable more efficient and effective cooperation within the network of sensors. To provide a concrete perspective we consider the specific scenario of using the sensors to locate a non-cooperative RF emitter using TDOA/FDOA-based methods [1,2]; here TDOA is "Time-Difference-Of-Arrival" and FDOA is "Frequency-Difference-Of-Arrival", which can be jointly estimated by cross-correlating signals from a pair of the sensors [3]. The accuracy of the TDOA/FDOA estimates depend on the signal SNR and the time-frequency structure of the intercepted signal [4]; however, the accuracy of the location estimate depends also on the emitter/sensor geometry through the so-called "geometric dilution of precision" or GDOP [1]. The goal of our work is to optimize over the set of all sensor assets, under the constraint of limited network communication resources.

The sensors simultaneously intercept an RF emitter's signal data and then cooperatively share the signal data between paired sensors to estimate the TDOA/FDOA for each sensor pair, which are then used to locate the RF emitter. After data collection at the sensors, they send a small amount of data to a central node where it is possible to determine a rough estimate of the emitter location – accurate enough to assess the impact of the relative emitter-sensor geometry on the location processing task, thus allowing subsequent processing to be optimized with respect to the geometry and error sources. (An alternative to this is when the network is cued with a rough location from some cueing sensor system.) Thus, the central node then uses knowledge of the current positions and trajectories of the remaining sensors to further reduce the participating subset based on the quality and the error sensitivity of their data sets. For example, one sensor may have high-quality data but its position and trajectory give it a poor GDOP, whereas another sensor could have low-quality data but have good GDOP. By eliminating sensors that have negligible usefulness to the final outcome of the task it is possible to significantly reduce the amount of network communication needed to accomplish the task with little degradation of the location accuracy. Further reduction in the needed communication resources is then achieved through location-optimized compression when this final subset of participating sensors shares its data to support the location-estimation tasks.

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2. MATHEMATICAL CHARACTERISTICS OF THE PROBLEM

For simplicity we consider only a 2-D location scenario. For this paper we assume that the sensors have been configured into *a priori* pairs of sensors; future work will consider the task of optimally forming the pairs. This scenario is shown in Figure 1. The (*x*,*y*) positions of the emitter is $\mathbf{x} = [x_e, y_e]^T$. The 2*N* sensors positions and velocities are notated as $\mathbf{x}_i = [x_i \; y_i]^T$ and $\mathbf{v}_i = [v_{x_i} \; v_{y_i}]^T$, respectively, for $i = 1, 2, \ldots, 2N$. The i^{th} pair consists of sensor $2i - 1$ and $2i$.

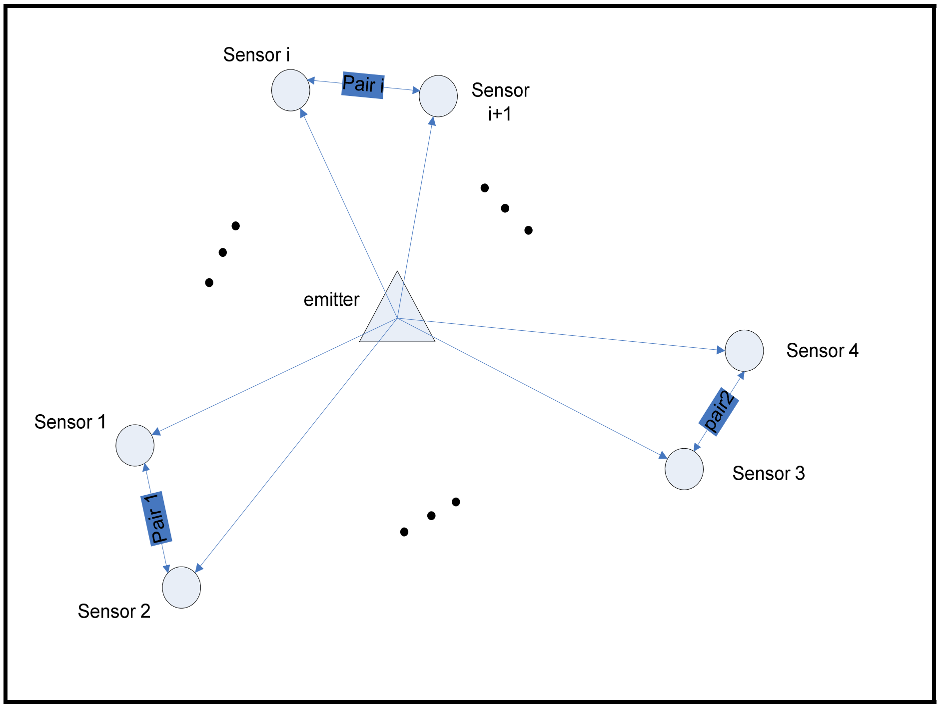

Figure 1: Illustration of the sensor scenario, where the sensors are grouped into *a priori* pairs.

For the i^{th} pair of sensors shown in Figure 2 the TDOA τ_i and FDOA ω_i between the signals received at the two sensors in the pair are given by

$$\begin{aligned} \tau_i &= \frac{1}{c}\left(\left\|\mathbf{x} - \mathbf{x}_{2i-1}\right\| - \left\|\mathbf{x} - \mathbf{x}_{2i}\right\|\right), \quad i = 1, 2, \ldots, N \\ \omega_i &= \frac{f_e}{c}\left(\mathbf{u}_{2i-1}^T \mathbf{v}_{2i-1} - \mathbf{u}_{2i}^T \mathbf{v}_{2i}\right), \quad i = 1, 2, \ldots, N \end{aligned} \tag{1}$$

where $\mathbf{u}_k$ is the unit vector pointing from the k^{th} sensor to the emitter (as shown in Figure 2)and f_e is the transmitted frequency of the transmitter (assumed estimated accurately enough for our purposes). Because the TDOA/FDOA are obtained by maximum likelihood (ML) estimator (i.e., cross correlation [3]), the asymptotic properties of ML estimators gives that the PDF of the TDOA/FDOA estimates is Gaussian with covariance matrix that is the inverse of the Fisher information matrix (FIM) [5]. Thus, the TDOA/FDOA measurements within the *N* pairs are given by

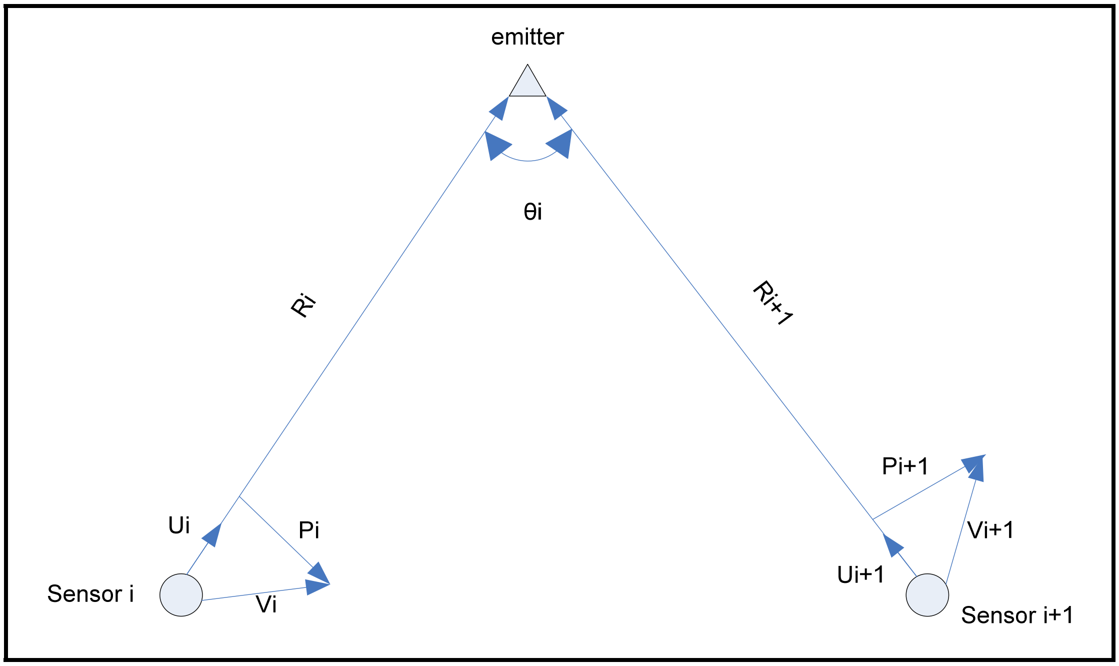

Figure 2: Definition of geometric parameters between the emitter and one sensor pair in the network.

$$\begin{bmatrix} \hat{\tau}_1 \\ \hat{\omega}_1 \\ \vdots \\ \hat{\tau}_N \\ \hat{\omega}_N \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \omega_1 \\ \vdots \\ \tau_N \\ \omega_N \end{bmatrix} + \begin{bmatrix} n_{\tau_1} \\ n_{\omega_1} \\ \vdots \\ n_{\tau_N} \\ n_{\omega_N} \end{bmatrix} \tag{2}$$

where the n_{τ_i} and n_{ω_i} are additive Gaussian noises (Gaussian due to ML estimation properties). If $\mathbf{J}_i$ is the FIM for the i^{th} TDOA/FDOA pair, then – making the reasonable assumption of independent receiver noise between disjoint sensor pairs – the Fisher information matrix for the set of TDOA/FDOA estimates within the network is given by

$$\mathbf{J} = \mathrm{diag}\{\mathbf{J}_1, \mathbf{J}_1, \cdots, \mathbf{J}_N\}. \tag{3}$$

where $\mathbf{J}_i$ is the 2×2 FIM for estimating the TDOA/FDOA at the i^{th} sensor pair given by

$$\mathbf{J}_i = \frac{2}{\sigma^2} \begin{bmatrix} \dfrac{\partial \mathbf{s}_i^H}{\partial \tau_i} \dfrac{\partial \mathbf{s}_i}{\partial \omega_i} & \mathrm{Re}\left\{ \dfrac{\partial \mathbf{s}_i^H}{\partial \tau_i} \dfrac{\partial \mathbf{s}_i}{\partial \omega_i} \right\} \\ \mathrm{Re}\left\{ \dfrac{\partial \mathbf{s}_i^H}{\partial \omega_i} \dfrac{\partial \mathbf{s}_i}{\partial \tau_i} \right\} & \dfrac{\partial \mathbf{s}_i^H}{\partial \omega_i} \dfrac{\partial \mathbf{s}_i}{\partial \omega_i} \end{bmatrix} = \begin{bmatrix} J_{i,11} & J_{i,12} \\ J_{i,21} & J_{i,22} \end{bmatrix}, \tag{4}$$

where σ^2 is the known variance of the sensor noise and $\mathbf{s}_i$ is the vector of noise-free signal samples from the i^{th} sensor pair. We have explored the effect of compression on the diagonal elements of this FIM[6].

To characterize the impact of compression and/or sensor selection we really need to look at the FIM for the geo-location estimate. The FIM of the 2-D *x-y* geo-location estimate is easily computed using the form for the general Gaussian problem[5] given by

$$\mathbf{J}_{geo} = \sum_{i=1}^{N} \mathbf{G}_i^T \mathbf{J}_i \mathbf{G}_i \; ,$$

where – again – the Gaussian structure arises due to use of the ML estimator for TDOA/FDOA); the $\mathbf{G}_i$ matrices are given by

$$\mathbf{G}_i = \begin{bmatrix} \dfrac{\partial \tau_i}{\partial x_e} & \dfrac{\partial \tau_i}{\partial y_e} \\ \dfrac{\partial \omega_i}{\partial x_e} & \dfrac{\partial \omega_i}{\partial y_e} \end{bmatrix}, \tag{5}$$

evaluated at the true emitter location; in the algorithm developed here we will be forced to evaluate this at a rough location estimate made from a small amount of initially shared data (or from a rough location from a cueing sensor system).

Now the problem of sensor pair selection can be specified. The problem of selecting $K < N$ sensor pairs (without compression) is specified by

$$\begin{aligned} &\max_{p_1,\ldots,p_N} H\left(p_1\left(\mathbf{G}_1^T \mathbf{J}_1 \mathbf{G}_1\right) + \cdots + p_N\left(\mathbf{G}_N^T \mathbf{J}_N \mathbf{G}_N\right)\right) \\ &s.t, \quad p_1 + \cdots + p_N = K < N, \quad p_i \in \{0,1\} \end{aligned} \tag{6}$$

where $p_i = 1$ when the sensor is selected and $p_i = 0$ when the sensor is not selected, and the function H is some appropriate matrix-to-scalar mapping (e.g., trace, determinant, etc.)[8, 9]. We have previously discussed the advantages of using the trace[8, 9] so we will use that here; using the trace we can re-write (6) as

$$\begin{aligned} &\max_{p_1,\ldots,p_N} \left(\sum_{i=1}^{N} p_i \left[\left(G_{i,11}^2 + G_{i,12}^2\right) J_{i,11} + \left(G_{i,21}^2 + G_{i,22}^2\right) J_{i,22} + 2\left(G_{i,11} G_{i,21} + G_{i,12} G_{i,22}\right) J_{i,12} \right] \right) \\ &s.t, \quad p_1 + \cdots + p_N = K < N, \quad p_i \in \{0,1\} \end{aligned} \tag{7}$$

which can then be optimized. Note that there are three distinct types of terms within the square brackets in (7): the term that includes $J_{i,11}$ contains the impact of the TDOA FI, the term that includes $J_{i,22}$ contains the impact of the FDOA FI, and the term that includes $J_{i,12}$ contains the impact of the TDOA/FDOA "cross-FI".

Before addressing the optimization issues we discuss some insight into the sensor selection problem that comes from (7). Direct calculation shows that the geometry factor that multiplies the TDOA FIM term is

$$\boldsymbol{G}_{i,11}^2 + \boldsymbol{G}_{i,12}^2 = \frac{2}{c^2}(1 - \cos\theta_i) \; , \tag{8}$$

where θ_i is the angle subtended at the emitter by the two sensors of the i^{th} pair. Thus, a larger angle leads to a larger contribution from the TDOA FI of that pair. Thus to increase the usefulness of TDOA measurements, we should choose pairs which have large subtended angles. A similar analysis shows that for maximum FDOA impact, we should choose pairs with small angles and with the sensors moving perpendicularly to the line-of-sight to the emitter. From this we can see there are complicated trade-offs among the geometry properties.

After selection of the sensors it is then possible to formulate a simple data compression optimization over the selected sensors for the case where the transmission within each pair is allocated the same number of bits *R*; we have addressed this data compression elsewhere[8]. The challenge is that different allocations of the *R* bits to a given sensor pair will give different trade-offs between TDOA accuracy and FDOA accuracy[8]. An example of this is shown in Figure 3 where *R* bits is allocated to optimize a weighted combination of the TDOA FI and the FDOA FI; each asterisk marks an achieved TDOA/FDOA accuracy for *R* bits allocated to optimize for a different weighting.

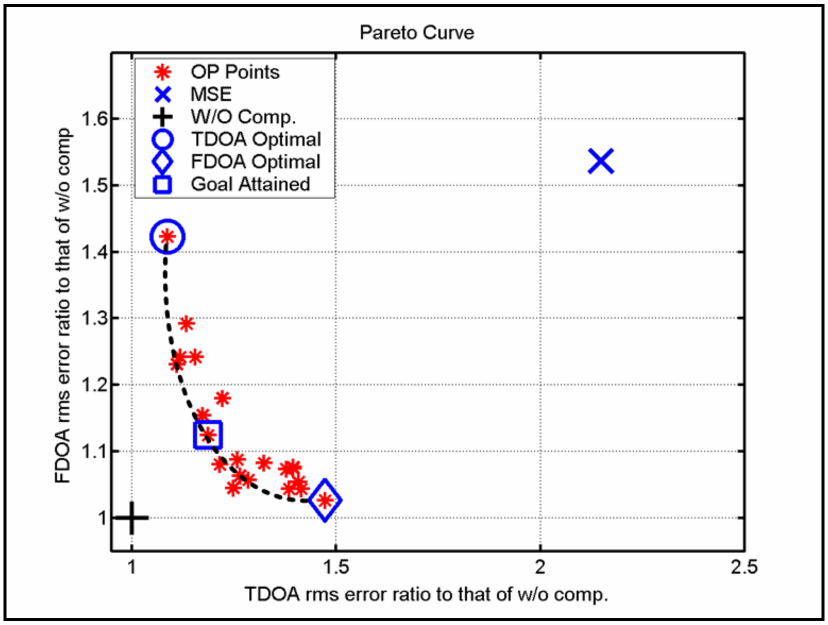

Figure 3: Trade-off between TDOA and FDOA accuracies for different optimal allocations of bits for compression ratio 3:1 and SNR_1 = 15 dB & SNR_2 = 15 dB; symbol □ denotes the operational point (α=0.5) closest to that without compression.

The specific trade-off that should be used on each pair depends on the geometry of the selected sensors[8] and can be chosen as follows. Let $\Omega \subset \{1, 2, \ldots, N\}$ be the indices of the selected sensors found to satisfy (6). Then we seek to solve allocate *R* bits to each of the *K* selected sensor pairs

$$\max_{allocate\ R}\left(\sum_{i\in\Omega}\left[\left(G_{i,11}^2+G_{i,12}^2\right)J_{i,11}+\left(G_{i,21}^2+G_{i,22}^2\right)J_{i,22}+2\left(G_{i,11}G_{i,21}+G_{i,12}G_{i,22}\right)J_{i,12}\right]\right) \qquad (7)$$

3. NUMERICAL ISSUES

The FIM measures how much information is available from the data relevant to the parameter estimation. In theory, evaluation of the FIM requires knowledge of the signal and analytical results for the derivatives that characterize the data's sensitivity to the parameters. However, for the TDOA/FDOA case it can be shown that the TDOA FI can be approximately evaluated from a measured sensor signal's DFT coefficients $X[k]$ using

$$J_{11} = \sum_{k=-N/2}^{N/2-1} \frac{2\pi^2 k^2 \left|X[k]\right|^2}{\sigma^2}. \tag{9}$$

Similarly the FDOA FI can be approximately computed from a measured sensor signal's samples $x[n]$ using

$$J_{22} = \sum_{n=-N/2}^{N/2-1} \frac{2\pi^2 n^2 \left|x[n]\right|^2}{\sigma^2}. \tag{10}$$

Also, the off-diagonal terms have structure described by

$$J_{12} \sim \mathrm{Re}\left\{ \sum_{n=-N/2}^{N/2-1} \sum_{k=-N/2}^{N/2-1} kn x[n] X^*[k], \right\} \tag{11}$$

This motivates using a time-frequency decomposition such as a filter bank to gain joint access to the time and frequency characteristics of the signal. However, in this case it becomes more challenging to numerically compute the FIM elements because such time-frequency representations lack the mathematical characteristics with respect to derivatives that are needed for a solid mathematical derivation. This leads to the following conjecture, whose viability when using the Short-Time FT (STFT) for evaluation has been explored[7].

<u>Brazen Conjecture</u>: Let $S_1[n,k]$ be the short-time Fourier transform or a one-sided filter bank representation. It is important that the frequency range of the T-F representation is $[-\pi,\pi]$ rad/sample, meaning that it explicitly shows separate channels for positive and negative frequencies. Then the FIM can be approximately computed (up to a multiplicative factor) using:

$$J_{11} = \frac{1}{\sigma_1^2} \sum_{n=-N/2}^{N/2} \sum_{k=-K/2}^{K/2} f_k^2 \left|S_1[n,k]\right|^2, \tag{12}$$

$$J_{22} = \frac{1}{\sigma_1^2} \sum_{n=-N/2}^{N/2} \sum_{k=-K/2}^{K/2} t_n^2 \left|S_1[n,k]\right|^2, \tag{13}$$

$$J_{12} = J_{21} = \frac{1}{\sigma_1^2} \sum_{n=-N/2}^{N/2} \sum_{k=-K/2}^{K/2} t_n f_k \left|S_1[n,k]\right|^2, \tag{14}$$

where f_k and t_n are the frequency and time centers of the cells of the time-frequency representation.

For the sensor selection problem we can compute the FIM elements using these equations. For the data compression problem we can allocate bits to each time-frequency sample coming out of a filter bank based on the amount of Fisher information it has, as evaluated using an auxiliary STFT assessment via (12) – (14)[8, 9].

4. ALGORITHM AND SIMULATION RESULTS

4.1 Sensor selection

To demonstrate the capability of the sensor selection method we present some simulation results for the case of locating an emitter starting with 10 pairs and selecting optimal subsets of various sizes. A typical random lay-down of 10 sensor pairs is shown in Figure 4. The 10 pairs of sensors are randomly selected in a ring around the emitter's position.

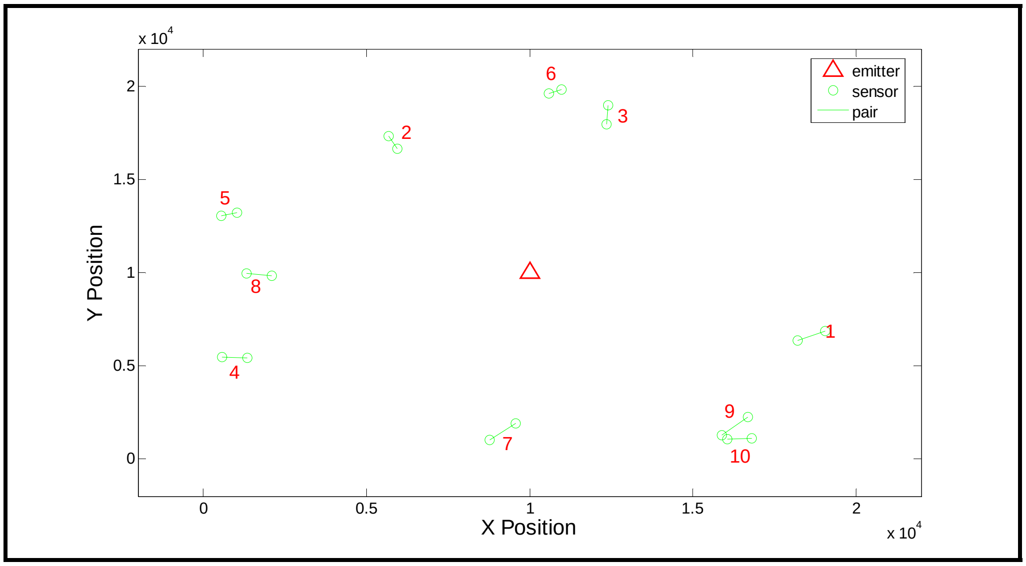

Figure 4: A typical random lay-down of 10 pairs of sensors in a 2D x-y scenario covering a 20 km by 20 km area. The axis units are in meters.

The sensor selection proceeds as follows. Each sensor intercepts the emitter signal data at SNRs in the range of 10 – 15 dB (where the SNR variation depends quadratically on the range to the emitter). The full set of sensors share a very small amount of data to obtain a rough estimate of the emitter's location; alternatively, we could consider the case where the system is cued by some other sensor system. On the basis of that rough estimate all the $\mathbf{G}_i$ are evaluated using (5). Then the FIM of each sensor pair is computed using (12) – (14). Then the N square-bracketed terms in (7) are computed and rank ordered, after which it is easy to find the K-element subset that satisfies (7) for any K. The selection results are shown in Figure 5, which shows the impact of the number of pairs selected on the normalized standard deviation of the geo-location standard deviation (i.e., root-sum-square of the x-y errors) where the normalization is with respect to the value achieved when all 10 sensor pairs are used.

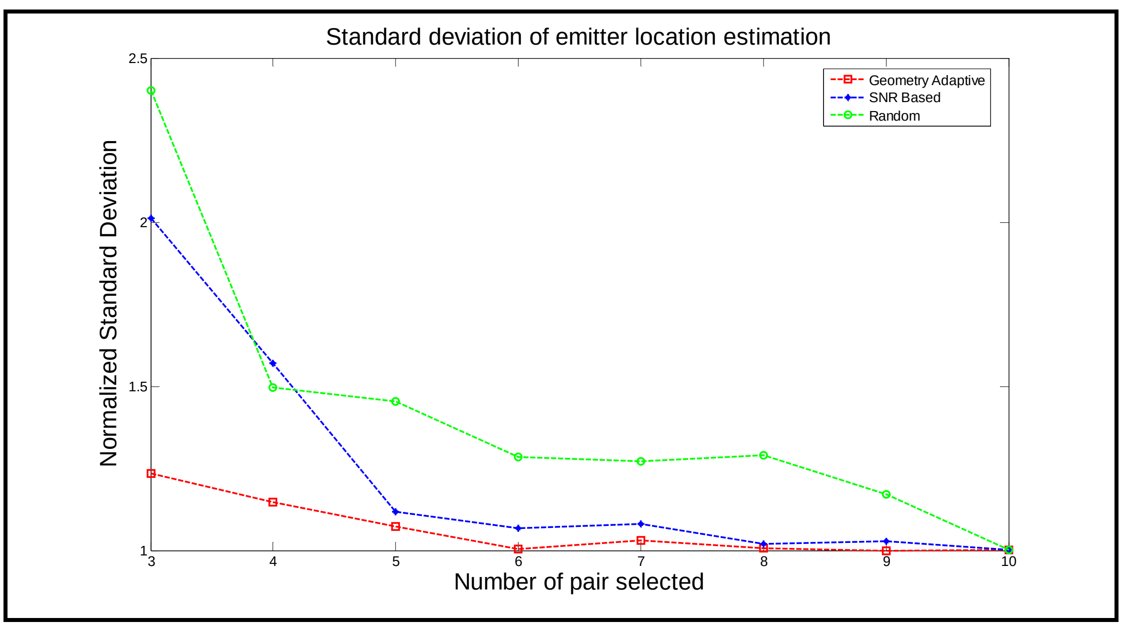

Figure 5: Performance of the sensor selection scheme of maximizing (7) – called "Geometry Adaptive" selection – showing the normalized standard deviation of the geo-location error versus the number of pairs selected. Two sub-optimal methods of sensor selection are shown for comparison: selecting pairs with largest SNR and randomly selecting pairs.

In addition to the optimal selection results based on (7), Figure 5 shows two other ad hoc methods: (i) selection by choosing the K sensor pairs with the largest SNRs and (ii) randomly selecting K pairs. These results were done for a single randomly-chosen sensor pair lay-down with 100 Monte Carlo runs over the ensemble of the noise process corrupting the sensor signals. Note that with the optimal selection method we can reduce from 10 pairs to 6 pairs with negligible impact on the geo-location accuracy. It should be noted that the kinds of sensor lay-downs considered (see Figure 4) are pre-disposed to enabling reasonable selection using SNR-based selection – thus, even in this SNR-favorable setting we can outperform the SNR-based method, especially when selecting just a few sensor pairs.

4.2 Sensor selection followed by data compression

The results in the previous sub-section did not use data compression; in a real scenario we would use compression in addition to the sensor selection method. Once the sensors are selected we can then perform "geometry-adaptive" data compression as follows. Each sensor intercepts the emitter signal data at SNRs in the range of 10 – 15 dB (where the SNR variation depends quadratically on the range to the emitter). The full set of sensors share a very small amount of data to obtain a rough estimate of the emitter's location; alternatively, we could consider the case where the system is cued by some other sensor system. On the basis of that rough estimate all the $\mathbf{G}_i$ are evaluated using (5). Then the FIM of each sensor pair is computed using (12) – (14). Then the N square-bracketed terms in (7) are computed and rank ordered, after which it is easy to find the K-element subset that satisfies (7) for any K. Then using the rough using the available rough location estimate the TDOA/FDOA trade-off (recall Figure 3) for each selected pair is determined by solving (7) using numerical Lagrange multiplier methods[10]. The results are shown in Figure 6 for a single set of selected sensors, where the vertical axis uses a common measure of emitter location accuracy called CEP (circular error probable)[1]; small CEP is better. Note that the total system compression ratio would be the compression ratio at each sensor (the horizontal axis in Figure 6) times the ratio of total pairs to the number of pairs selected.

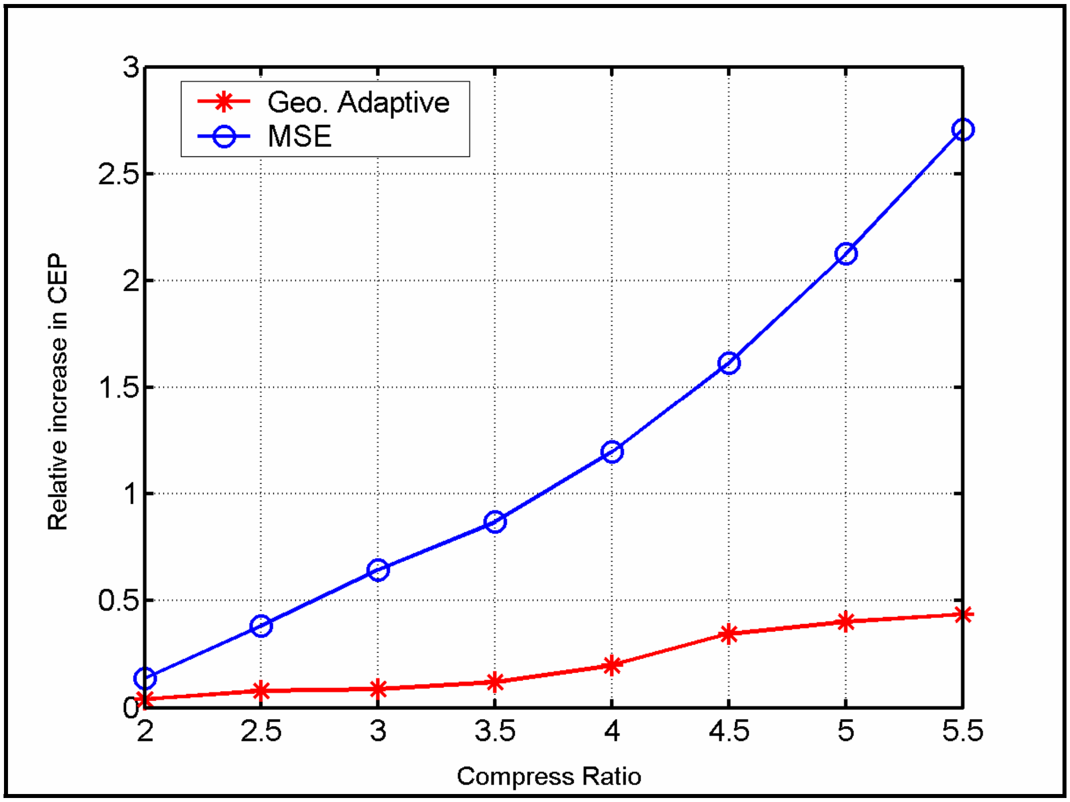

Figure 6: Simulation results using Geometry-Adaptive data compression that allocates bits to sensor pairs to maximize the geo-location Fisher information. For comparison MSE-based compression results using the same transform coding structure but optimized to minimize MSE rather than to maximize the Fisher information.

5. DISCUSSION

The results above show that it is possible to extend our previous ideas for optimizing data compression (e.g., allocating bits to maximize the Fisher information) to optimally select a subset of sensor pairs. This sensor selection optimization problem was made simple by the facts that the pairs were already specified and they were disjoint pairs (no sensor is in multiple pairs). In that case we simply need to rank-order the square-bracketed terms in (6) and then choose the K largest of them. Even for a large number of sensors this does not present much difficulty. Reducing the number of sensors that need to participate in the location processing is a form of data compression because it reduces the

However, we are currently exploring ways to attack the problem when the sensors are not paired *a priori*, which is a much more difficult problem. If there are S sensors there are C_S^2 ("S-choose-2") different pairs that can be formed from these S sensors; for the case of S = 20 sensors (like we considered above) then there are 190 ways to pair the sensors – of course one would not want to ever use all 190 of those pairs at the same time. We then need to select K of those pairs for our processing and there are $C_{C_S^2}^K$ ways to select K pairs from the C_S^2 total possible pairs; for the case of S = 20 sensors this leads to 8,145,060 different selections. To solve this problem will require more than the brute-force optimization that we have used in this paper. We have some preliminary results showing that it is possible to drastically reduce the number of possible selections.

Furthermore, in this paper we have done the sensor selection and the bit allocation separately and we have assumed that each sensor pair is allocated the same number of bits. Our current work is exploring ways to remove these limitations. In general we seek to identify only viable pairs from the C_S^2 total possible pairs and then non-uniformly allocate a total bit budget across the viable sensor pairs. Of course we would prefer this to result in the smallest number of sensors receiving allocated bit, so there should be a penalty that grows as the total number of used sensors grows.

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