GEOMETRY-ADAPTIVE DATA COMPRESSION FOR TDOA/FDOA LOCATION

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ABSTRACT

The location of an emitting target is estimated by intercepting its emitted signal and sharing them among several sensors to measure the time-difference-of-arrival (TDOA) and the frequency-difference-of-arrival (FDOA). Doing this in a timely and energy efficient fashion, which is especially important for wireless sensor network applications, requires effective data compression. Since the commonly used MSE distortion measure is only weakly related to optimal TDOA/FDOA estimation, in this paper, we derive a new class of non-MSE distortion measures for TDOA/FDOA estimation using the concept of Fisher information. We then use these new distortion measures to compress the data using a wavelet packet transform and show that it improves TDOA/FDOA estimation accuracies relative to using the MSE-based compression. Finally, the scheme of applying our algorithms in a wireless sensor network is proposed, and energy efficiency and accuracy enhancement of the proposed scheme over that of traditional scheme using MSE is shown through the simulations.

1. INTRODUCTION

A common way to locate an electromagnetic emitter is to measure the time-difference-of-arrival (TDOA) and the frequency-difference-of-arrival (FDOA) between signals received at pairs of sensors [1],[2]. This requires that the samples of one signal are sent over a data link, where data compression can reduce latency and save energy. Some past results are available on the issue of compression for TDOA/FDOA applications [3]–[7], but only recently have researchers begun to explore other than the standard mean-square-error (MSE) distortion measure [5]–[7]. We introduced a new non-MSE distortion measure [5] that uses a CRLB-based measure for the TDOA-only problem. However, we have found [5] that optimizing this measure was difficult because the form that the CRLB-based distortion metric takes depends on the relationships between the SNRs at the two sensors.

More recently we have shown [6] that a Fisher-information-based approach avoids these difficulties and we developed a general method for a single-parameter problem. A key advantage for this approach is that if the noise is uncorrelated between sensors, then the total Fisher information is the sum of the Fisher information from each sensor. Compression only impacts one sensor so we can avoid the complicating cross-sensor couplings.

In this paper we attack the two-parameter TDOA/FDOA problem by extending our single-parameter ideas [6] and explore trade-off issues that arise. We assume that the TDOA/FDOA estimation processing and compression processing are not jointly designed – this is motivated by our belief that sensors are likely be called to provide data to other processing systems that are independently designed. With our proposed compression scheme, we develop a new cooperative scheme for the sensor nodes in the wireless sensor network to adapt the compression scheme to the sensor-target geometry.

2. FISHER-INFORMATION-BASED DISTORTION

Fisher information matrix (FIM) is a well-known concept in estimation theory [8]. It quantifies how much information a data set provides about the parameters to be estimated. Let $\mathbf{x} = \mathbf{s}(\boldsymbol{\theta}) + \mathbf{w}$ denote a real random vector consisting of a deterministic signal vector $\mathbf{s}(\boldsymbol{\theta})$ parameterized by 2×1 parameter vector $\boldsymbol{\theta}$, and corrupted by a white noise vector $\mathbf{w}$ with variance σ^2. The FIM for this $\boldsymbol{\theta}$ is the 2×2 matrix $\mathbf{J}(\boldsymbol{\theta})$ with elements given by

$$J_{ij} = \frac{1}{\sigma^2}\left(\frac{\partial \mathbf{s}(\boldsymbol{\theta})}{\partial \theta_i}\right)^T \left(\frac{\partial \mathbf{s}(\boldsymbol{\theta})}{\partial \theta_j}\right). \qquad (1)$$

The FIM specifies an information ellipse – the larger the better – with semi-axes along the FIM's eigenvectors and whose lengths are the square roots of the eigenvalues.

Lossy compression of the data vector $\mathbf{x}$ changes the FIM. Namely, it reduces the on-diagonal elements J_{ii}, which makes the post-compression information ellipse smaller; it is unclear what effect it would have on the cross-information and hence the tilt of the ellipse. In the single-parameter case [6] the way to proceed was clear: compress so as to minimize the reduction in J_{11} for a given bit budget. But how should we proceed in the two-parameter case? There are several possibilities, but our choice is to minimize the impact on the information ellipse's semi-axis lengths while neglecting the impact on the ellipse's tilt: this implies minimizing the reduction of the FIM's eigenvalues. A simple, effective measure is to minimize the reduction of the sum of the eigenvalues, which is equivalent to minimizing the reduction of the trace of the FIM. However, sometimes TDOA accuracy is more important than FDOA accuracy, or vice versa; so, we use a weighted trace. Thus, our goal is to seek an *operational* rate-distortion method that minimizes the reduction in $\alpha J_{11} + (1-\alpha)J_{22}$ with $0 \le \alpha \le 1$ while satisfying a budget on the total number of bits.

The signals at sensors S_1 and S_2 with unknown TDOA of n_d and FDOA of v_d can be modeled by:

$$\begin{aligned} x_1[n] &= s[n-(n_0+n_d/2)]e^{j(v_0+v_d/2)n} + w_1[n] \\ x_2[n] &= s[n-(n_0-n_d/2)]e^{j(v_0-v_d/2)n} + w_2[n] \\ n &= -N/2, -N/2+1, \ldots, N/2 \end{aligned} \quad (2)$$

where $s[n]$ is a complex baseband signal, v_0 and n_0 are unknown nuisance parameters that need not be estimated, and $w_1[n]$ and $w_2[n]$ are uncorrelated complex Gaussian white noise with variances σ_1^2 and σ_2^2, respectively. The signal-to-noise ratios (SNR) for these two received signals are denoted by SNR_1 and SNR_2, respectively. We assume here that $x_1[n]$ is the signal that gets compressed.

As shown in [6], the TDOA Fisher information depends on the DFT coefficients (with the DFT frequencies running over both negative and positive frequencies) while the FDOA Fisher information depends on the signal samples themselves.

After quantization of the DFT coefficients the data-computable TDOA Fisher information measure is

$$\tilde{J}_{n_d n_d} = \sum_{m=-N/2}^{N/2} \tilde{J}_{n_d n_d}(m), \quad (3)$$

with

$$\tilde{J}_{n_d n_d}(m) = \begin{cases} \dfrac{2\pi^2 m^2 |X_1[m]|^2}{\sigma_1^2 + q_m}, & \text{if } b_m > 0 \\ 0, & \text{if } b_m = 0 \end{cases} \quad (4)$$

where $X_1[m]$ is the normalized DFT of the data at sensor S_1 and q_m is the quantization noise variance of the m^{th} DFT coefficient when quantized to an allocated b_m bits. The tilde (~) in (3) and (4) indicates that the quantity is based on the noisy data rather than the unavailable underlying signal.

After quantization of the signal samples the data-computable FDOA Fisher information measure is

$$\tilde{J}_{v_d v_d} = \sum_{n=-N/2}^{N/2} \tilde{J}_{v_d v_d}(n), \quad (5)$$

with

$$\tilde{J}_{v_d v_d}(n) = \begin{cases} \dfrac{2\pi^2 n^2 |x_1[n]|^2}{\sigma_1^2 + q_n}, & \text{if } b_n > 0 \\ 0, & \text{if } b_n = 0 \end{cases} \quad (6)$$

where $x_1[n]$ is the signal data at sensor S_1 and q_n is the quantization noise variance of the n^{th} signal data sample when quantized to an allocated b_n bits.

3. ALGORITHM DEVELOPMENT

The quantities $\tilde{J}_{n_d n_d}$ and $\tilde{J}_{v_d v_d}$ capture the impact of compression on TDOA and FDOA accuracies, so an operational R-D method can be developed based on maximizing the weighted sum of these two Fisher information (i.e., the weighted trace of the FIM) under a bit constraint. Notice that the TDOA Fisher information depends on frequency-domain characteristics whereas the FDOA Fisher information depends on time-domain characteristics. It is difficult to optimize $\tilde{J}_{n_d n_d}$ and $\tilde{J}_{v_d v_d}$ jointly unless we transform the data $x_1(n)$ into a domain where frequency resolution and time resolution are jointly provided. Hence, an orthonormal wavelet packet transform has been used. Given an orthonormal wavelet packet basis set $\{\psi_n\}$ with coefficients $\{c_n\}$ for the data vector $\mathbf{x}_1$, we wish to select a subset Ω of coefficient indices and an allocation of bits $B=\{b_i | i \in \Omega\}$ to those selected coefficients such that the selected/quantized signal is

$$\tilde{\mathbf{x}}_1 = \sum_{i \in \Omega} \tilde{c}_i \psi_i \quad (7)$$

where $\{\tilde{c}_i \,|\, i \in \Omega\}$ are the quantized version of the selected coefficients, maximizing $\tilde{J}_{n_d n_d}$ and $\tilde{J}_{v_d v_d}$ while meeting a constraint on the total number of bits.

In our implementation we do not code coefficients individually, but rather operate on blocks of coefficients. The wavelet packet coefficients c_i are grouped into M blocks, where each block contains coefficients at the same frequency and over a short contiguous temporal range. The joint TDOA/FDOA distortion measure is then

$$\tilde{J}(\alpha) = \alpha \sum_{j=1}^{M} \left(\frac{f_j^{\,2} \sum_{i \in \{j\ block\}} |c_i|^2}{\sigma_1^2 + q_j^2} \right) + (1-\alpha) \sum_{j}^{M} \left(\frac{t_j^{\,2} \sum_{i \in \{j\ block\}} |c_i|^2}{\sigma_1^2 + q_j^2} \right) \quad (8)$$

where j is the block index, q_j^2 is the quantization noise variance in the j^{th} block, and f_j and t_j are the frequency and central time, respectively, of the j^{th} block. Bits are allocated to the coefficients using the method of [9] to maximize (8) for a given rate constraint. To compare the proposed scheme with a traditional scheme, we also allocated bits to the wavelet packet blocks to minimize MSE under the bit constraint.

Although the developed scheme is applicable to all varieties of signals, we use a linear FM radar signal to illustrate the method. A 3-level wavelet packet transform is performed and 8 subbands are produced; each subband is partitioned into 8 blocks. Moreover, to focus attention on the lossy compression performance, no entropy coding is applied after quantization.

Simulation results are shown in Figure 1 and Figure 2. In Figure 1 we see the inherent trade-off that is controlled by the choice of α, whose value controls whether the algorithm favors TDOA accuracy, FDOA accuracy, or balances them to achieve the closest operation to the no compression case. Figure 2 illustrates how changing the compression ratio affects the TDOA and FDOA accuracy under several scenarios: the results labeled "Goal Attained" illustrate the performance when the impact on TDOA and FDOA is balanced. In these figures the advantage of using our distortion measures is made clear.

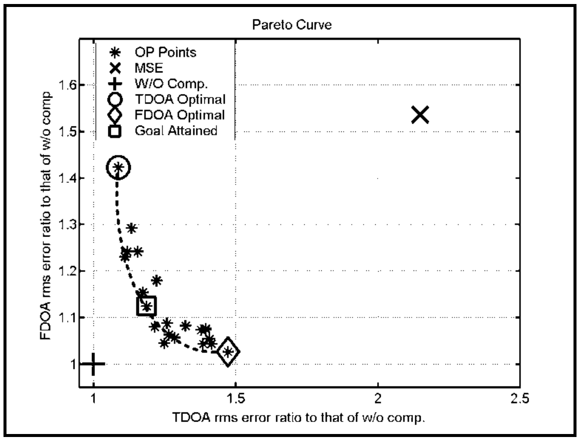

Figure 1: Trade-off between TDOA and FDOA accuracies as α is varied for compression ratio 3:1 and SNR_1 = 15 dB & SNR_2 = 15 dB; symbol □ denotes the operational point (α=0.5) closest to that without compression.

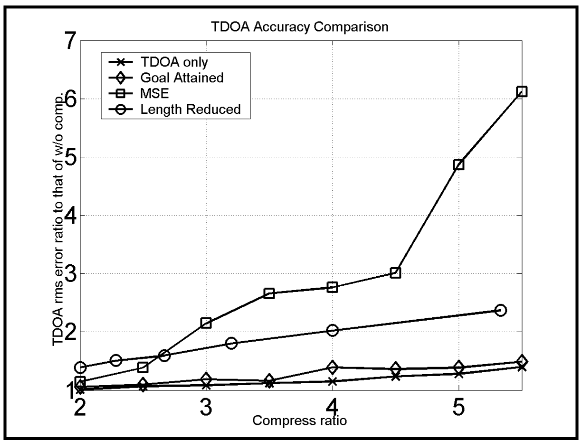

Figure 2a

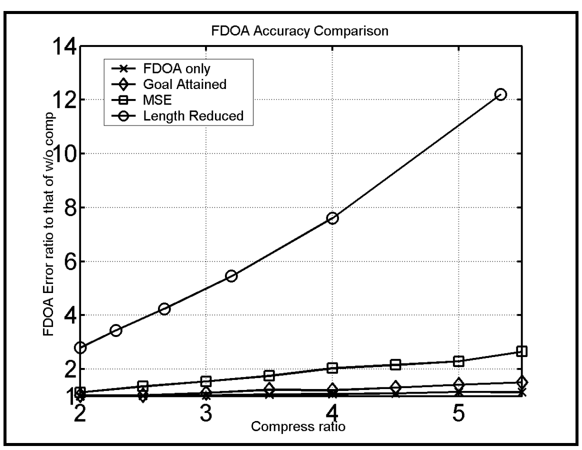

Figure 2b

Figure 2: Effect of compression ratio on (a) TDOA and (b) FDOA performance. A comparison is also made to the case of simply sending less data ("Length Reduced") rather than compressing the data.

4. SCHEME FOR GEOMETRY ADAPTATION

Besides the TDOA/FDOA accuracies, the location estimation accuracy also strongly depends on the geometry of the emitter and the sensors [1]. For the 3-D X-Y-Z location case, the 3×3 location error covariance matrix is [1]

$$\mathbf{P} = (\mathbf{G}^T \mathbf{N}^{-1} \mathbf{G})^{-1} \tag{9}$$

where matrix **G** is determined by the emitter-sensors geometry and **N** is the diagonal matrix of the TDOA/FDOA estimation error variance after compression. Diagonalizing $\mathbf{P}^{-1}$ using $\mathbf{A}^{\mathbf{T}}\mathbf{P}^{-1}\mathbf{A} = diag\{\lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1}\}$ and letting $\boldsymbol{\zeta} = \mathbf{A}^{\mathbf{T}}(\boldsymbol{\xi} - \mathbf{m})$, where **m** is the real location of target, leads to

$$R = \{\boldsymbol{\xi} : \sum_{\mathbf{i}=1}^{3} \frac{\zeta_{\mathbf{i}}^{\mathbf{2}}}{\lambda_{\mathbf{i}}} \le \boldsymbol{\kappa}\}\ , \tag{10}$$

where region R is the interior of the error ellipse with the i^{th} principal axes length of $2(\kappa\lambda_i)^{½}$.

In most sensor-target geometries, the relative importance of TDOA and FDOA is set by the target-sensors geometry. Figure 1 shows how α controls the compression trade-off between the TDOA and FDOA accuracies achieved by our method. However, to choose a proper value of α requires knowledge of the target's location – which, unfortunately, is precisely what we are trying to estimate. However, we can first send a small amount of data – enough to *roughly* determine the geometry; then that rough geometry can be used to estimate an appropriate α value that would be fed back to the compressing sensor and used to compress the remainder of the signal. In fact, this can even be done to provide repeated updates of α as more data is compressed and sent; this corresponds to updating the operating point along the curve shown in Figure 1.

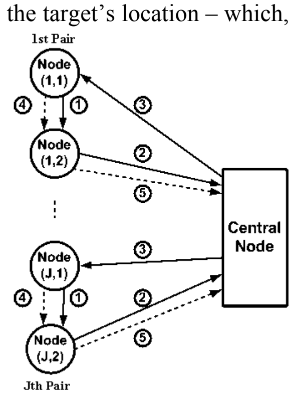

Figure 3: System Scheme

This directly leads to the following simple scheme (not the only one) where the compression can be adapted to the geometry. The scheme is shown in Figure 3, where circled numbers correspond to the actions of the steps described below.

This new geometry-adaptive scheme for applying the Fisher information-based algorithm can be described in five steps:

Step 1: The central node determines the compression ratio and its associated operational compression points based on the requirement of energy consumption and latency. Then it randomly picks J pairs of nodes to get J measurements of TDOA and FDOA. In the beginning, one of the nodes $(i,1)$, $1 \le i \le \mathrm{J}$ sends a small length of its data to the other $(i,2)$, where a rough measurements of TDOAs $\{\tilde{n}_{d,i}\}$ and FDOAs $\{\tilde{\nu}_{d,i}\}$ are measured.

Step 2: $\{\tilde{n}_{d,i}\}$ and $\{\tilde{v}_{d,i}\}$ are sent to the central nodes, where the rough prediction of the geometry matrix G is estimated.

Step 3: In term of the estimated geometry matrix G, the central node determines the optimal α and informs the nodes $(i,1)$ about this choice and the compress ratio.

Step 4: The nodes $(i,1)$ compress the data according the compression ration and the chosen α and send the compressed version of its sensed data to the nodes $(i,2)$.

Step 5: The nodes $(i,2)$ measure the $\{n_{d,i}\}$ and $\{v_{d,i}\}$ with the received data from node $(i,1)$ in Step 1 and decompressed data in Step 4. The resulting $\{n_{d,i}\}$ and $\{v_{d,i}\}$ are sent to the central node, where the final location of the target is determined.

Simulations are performed as follows: A target can appear anywhere in a 2-D 5x5 km^2 area, within which a large number of sensors is spread. Four sensors are randomly picked to form two pairs to estimate TDOA/FDOA. 256 samples are sent in Step 1 to roughly estimate the geometry. Finally, 4096 samples are compressed and shared between each pair. For comparison, the 4096 data samples are compressed using the MSE measure. In the simulation, we use the ratio of the area of circular error probable (CEP) [1] with compression to that without compression minus 1 (which corresponds to the relative increase in CEP) as the metric. 2000 experiments were performed for each compression ratio and the average values are shown in Figure 4. CEP is estimated by $0.75\sqrt{\lambda_1 + \lambda_2}$ [1].

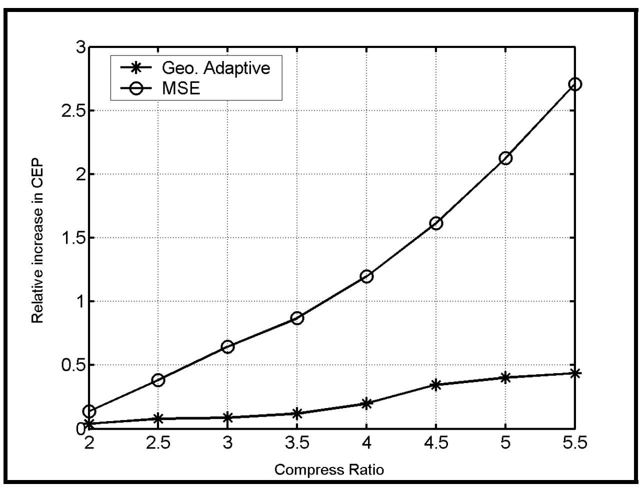

Figure 4: Effect of compression ratio on CEP.

5. CONCLUSION

Data compression can be used as a tool for TDOA/FDOA location application. But, instead of using traditional MSE distortion measure, this paper proposed a new distortion measure using the trace of the FIM, which affects the semi-axis lengths of the Fisher information ellipse. As a means of handling the case where the two parameters may have differing importance we extended this trace-based measure to a weighted trace measure.

This distortion measure was directly used to develop an efficient lossy operational R-D compression algorithm for multi-sensor emitter location application using a wavelet-packet-based block coding scheme. Our simulation results show that lossy compression using our distortion measure gives tremendous improvement over the compression with traditional MSE in estimating TDOA/FDOA. Furthermore, by adjusting the value of α we have shown that it is possible to trade between TDOA accuracy and FDOA accuracy. In addition, by realizing that geometry information of the emitter and the sensors affect the differing importance of TDOA/FDOA accuracies, we proposed a geometry-adaptive scheme that uses the cooperation of the sensors to determine and adapt the value of α during the compression processing and compress the data optimally. The approach of using Fisher information to derive distortion measures can be extended to other practical estimation applications where the lossy compression is needed either in the wireless sensor network application or the real-time micro-sensor system.

6. REFERENCES

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