Conference on Mathematics and Applications of Data/Image Coding, Compression, and Encryption IV
SPIE's International Symposium on Optical Science and Technology, San Diego, CA, July 29 – August 3, 2001

Non-MSE Wavelet-Based Data Compression for Emitter Location

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ABSTRACT

The location of an emitter is estimated by intercepting its signal and sharing the data among several platforms to measure the time-difference-of-arrival (TDOA) and the frequency-difference-of-arrival (FDOA). Doing this in a timely fashion requires effective data compression.

A common compression approach is to use a rate-distortion criterion where distortion is taken to be the mean-square error (MSE) between the original and compressed versions of the signal. However, in this paper we show that this MSE-only approach is inappropriate for TDOA/FDOA estimation and then define a more appropriate, non-MSE distortion measure. This measure is based on the fact that in addition to the dependence on MSE, the TDOA accuracy also depends inversely on the signal's RMS (or Gabor) bandwidth and the FDOA accuracy also depends inversely on the signal's RMS (or Gabor) duration. We discuss how the wavelet transform is a natural choice to exploit this non-MSE criterion. These ideas are shown to be natural generalizations of our previously presented results showing how to determine the correct balance between quantization and decimation.

We develop a MSE-based wavelet method and then incorporate the non-MSE error criterion. Simulations show the wavelet method provides significant compression ratios with negligible accuracy reduction. We also make comparisons to methods that don't exploit time-frequency structure and see that the wavelet methods far out-perform them.

Keywords: data compression, emitter location, time-difference-of-arrival (TDOA), frequency-difference-of-arrival (FDOA), wavelet transform, RMS bandwidth, Gabor bandwidth

1. INTRODUCTION

A common way to locate electromagnetic emitters is to measure the time-difference-of-arrival (TDOA) and the frequency-difference-of-arrival (FDOA) between pairs of signals received at geographically separated sites[1,2,3]. The measurement of TDOA/FDOA between these signals is done by coherently cross-correlating the signal pairs,[2,3] and requires that the signal samples of the two signals are available at a common site, which is generally accomplished by transferring the signal samples over a data link from one site to the other. An important aspect of this that is not widely addressed in the literature is that often the available data link rate is insufficient to accomplish the transfer within the time requirement unless some form of lossy data compression is employed. For the case of white Gaussian signals and noises, Matthiesen and Miller[4] established bounds on the rate-distortion performance for the TDOA problem and compared them to the performance achievable using scalar quantizers, where distortion is measured in terms of lost SNR due to the mean square error (MSE) of lossy compression. However, these results are not applicable when locating radar and communication emitters because the signals encountered are not Gaussian. A method using block adaptive scalar quantization was proposed[5] and analyzed[6] to show that it was marginally effective for various signal types. Wavelet-based methods have been proposed[7] and demonstrated[8] to give compression ratios on the order of 4 to 7 for some radar signals. A method that optimally trades between decimation and quantization has been developed for flat spectrum signals and shown to perform better than either method alone.[9] Some preliminary results[10] have shown the potential for fusing together the ideas of joint decimation-quantization and wavelet-based methods; here we improve those results and show that they give a feasible means to improve the performance of wavelet-based compression methods.

The two signals to be correlated are the complex envelopes of the received RF signals. The two noisy received signals to be processed are notated as

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$$\hat{s}(k) = s(k) + n(k)$$
$$\hat{d}(k) = d(k) + v(k) \tag{1}$$

where $s(k)$ and $d(k)$ are the complex baseband signals of interest and $n(k)$ and $v(k)$ are complex white Gaussian noises. The signal $d(k)$ is a delayed and doppler shifted version of $s(k)$. The signal-to-noise ratios (SNR) for these two signals are denoted *SNR* and *DNR*, respectively[‡]. To cross correlate these two signals one of them (assumed to be $\hat{s}(k)$ here) is compressed, transferred to the other platform, and then decompressed before cross-correlation, as shown in
Figure 1.

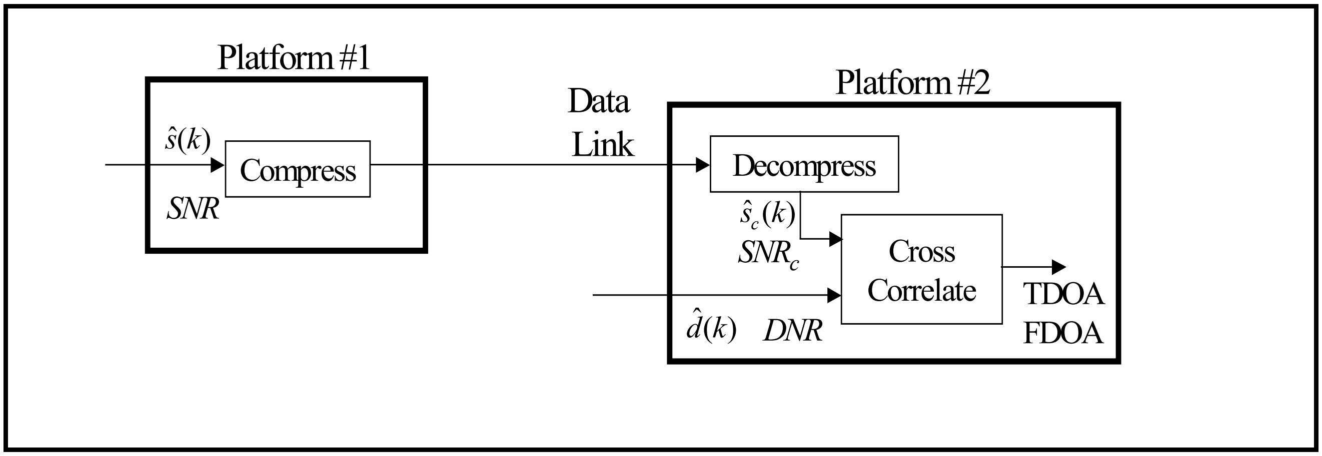

Figure 1: System Configuration for Compression

Signal $\hat{s}(k)$ has SNR of $SNR_c < SNR$ after lossy compression/decompression, and the output SNR after cross-correlation is given by

$$\begin{aligned} SNR_o &= \frac{WT}{\dfrac{1}{SNR_c} + \dfrac{1}{DNR} + \dfrac{1}{SNR_c DNR}} \\ &\triangleq WT \times SNR_{eff} \end{aligned} \tag{2}$$

where WT is the time-bandwidth product (or coherent processing gain), with W being the noise bandwidth of the receiver and T being the duration of the received signal and SNR_{eff} is a so-called effective SNR[3]. From (2) it is clear that the correlator's output SNR is set by the lower of SNR_c and DNR: when one is smaller than the other we have

$$SNR_o \approx WT \times \min\{SNR_c, DNR\}.$$

The accuracies of the TDOA/FDOA estimates are governed by the Cramer-Rao bound (CRB) for TDOA/FDOA given by[3]

$$\sigma_{TDOA} \geq \frac{1}{2\pi B_{rms}\sqrt{2SNR_o}}$$
$$\sigma_{FDOA} \geq \frac{1}{2\pi D_{rms}\sqrt{2SNR_o}} \tag{3}$$

where B_{rms} is the signal's RMS (or Gabor) bandwidth in Hz given by

‡ SNR (non-italic) represents an acronym for signal-to-noise ratio; *SNR* (italic) represents the SNR for $\hat{s}(k)$.

$$B_{rms}^2 = \frac{\int f^2 |S(f)|^2 df}{\int |S(f)|^2 df}, \tag{4}$$

with $S(f)$ being the Fourier transform of the signal $s(k)$ and D_{rms} is the signal's RMS duration in seconds given by

$$D_{rms}^2 = \frac{\int t^2 |s(t)|^2 dt}{\int |s(t)|^2 dt}. \tag{5}$$

The denominators in (4) and (5) can be considered as normalizing factors on $|S(f)|^2$ and $|s(t)|^2$, respectively, so that these equations have the form identical to the equation for variance; thus, the root-mean-squared (RMS) terminology.

2. NON-MSE DISTORTION CRITERIA

To ensure maximum performance it is necessary to employ a compression method that is designed specifically for this application. However, much of the past effort in developing general lossy compression methods has focused on minimizing the MSE due to compression; furthermore, even compression schemes specifically developed for TDOA/FDOA applications have also limited their focus to minimizing the MSE[4,5,6,7]. But when the goal is to estimate TDOA/FDOA, the minimum MSE criterion is likely to fall short because it fails to exploit how the signal's structure impacts the parameter estimates. In such applications it is crucial that the compression methods minimize the impact on the TDOA/FDOA estimation performance rather than stressing minimization of MSE as is common in many compression techniques. Achieving significant compression gains for the emitter location problem requires exploitation of how signal characteristics impact the TDOA/FDOA accuracy. For example, the CRBs in (3) show that the TDOA accuracy depends on the signal's RMS bandwidth and that the FDOA accuracy depends on the signal's RMS duration. Thus, compression techniques that can significantly reduce the amount of data while negligibly impacting the signal's RMS widths have potential. We have shown[9] that it is possible to exploit this idea for TDOA-only estimation through simple filtering and decimation together with quantization to meet requirements on data transfer time that can't be met through quantization-only approaches designed to minimize MSE. These results provide the motivation for the method proposed here.

Looking at the expressions for TDOA/FDOA accuracies it is clear that we should take as measures to maximize for a given desired rate the following weighted "RMS-distortion" SNRs:

$$\begin{aligned} SNR_{rms,TDOA} &= B_{rms}^2 SNR_o \\ &= B_{rms}^2 (WT \times SNR_{eff}) \end{aligned} \tag{6}$$

and

$$\begin{aligned} SNR_{rms,FDOA} &= D_{rms}^2 SNR_o \\ &= D_{rms}^2 (WT \times SNR_{eff}) \end{aligned}. \tag{7}$$

So the general goal is the following, expressed as transform coding[11,12] with a non-MSE distortion. Given some signal decomposition

$$\hat{s}(k) = \sum_{n=1}^{N} c_n \psi_n(k) \tag{8}$$

of the signal to be compressed, we wish to select which coefficients should be coded and transmitted to achieve a desired rate-distortion goal where distortion is measured using (6) and (7). For example, we may wish to find a subset Ω of indices such that the signal given by

$$\tilde{s}(k) = \sum_{n\in\Omega} c_n \psi_n(k) \tag{9}$$

maximizes (6) and (7) while the set $\{c_n \mid n \in \Omega\}$ can be coded using rate R. In general this selection process is quite difficult because of (i) the nonlinear, nonmonotonic relationship between the coefficients and the RMS widths, and (ii) the fact that removing a coefficient from Ω effects both the RMS widths and SNR_o. Furthermore, the simultaneous maximization of (6) and (7) can be difficult, especially given that there may be a different acceptable level of degradation on TDOA than there is on FDOA; this issue is not considered here.

Before discussing the application of the wavelet transform to this problem we first relate this general viewpoint to the simple method reported previously[9] for balancing the effect of decimation and quantization for the TDOA-only case. This method is based on a simple form of two-band subband coding[11], which can be thought of as a form of transform coding, that completely discards the upper band. The structure of the method is shown in Figure 2 where the received signal is shown to be sampled with an analog-to-digital converter (ADC) and then applied to a lowpass filter (LPF) that reduces the signal's bandwidth by a factor of M to W_f Hz, after which the signal can be decimated by a factor of M. Finally the signal is quantized to b bits, which is a coarser level than was done by the ADC. Both the decimation and the quantization act to reduce the rate and the optimal trade-off between them was determined for a flat-spectrum signal; the flat-spectrum assumption simplified the problem by making SNR_{eff} in (6) independent of the filtering/decimation operation. Thus, the correlator output SNR can be written in the separable form

$$SNR_o(W_f, b) = W_f T \; SNR_{eff}(b) \tag{10}$$

Likewise, the effect on the flat-spectrum signal's RMS bandwidth depends only on the filtering and is given by

$$B_{rms}^2 = (1.8/2\pi)^2 W_f^2 \tag{11}$$

Assuming that the signal duration T is a fixed system parameter, maximizing (6) for this case is equivalent to maximizing $W_f^3 SNR_{eff}(b)$, which can be maximized under a rate constraint.[9] Under this maximization it was shown that the joint decimation/quantization scheme outperformed quantization-only and decimation-only methods, demonstrating the usefulness of the RMS width approach. This result motivates our search for a more general way to exploit the RMS width ideas; for this we turn to the wavelet transform.

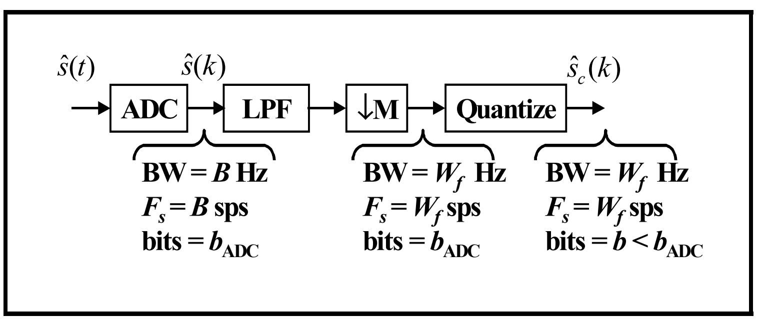

Figure 2: Compression via Combined Decimation and Quantization

3. WAVELET TRANSFORM METHODS

The wavelet transform has been found to be very useful for signal and image compression in general[11,12]. It is an extension of the Fourier transform in the sense that it provides a decomposition of a signal in terms of a set of component signals, as in (8). However, the wavelet transform decomposes a signal into a weighted sum of component signals that are localized in time as well as in frequency; this allows them to provide a more efficient representation of signals with time varying spectra. Accordingly, each wavelet coefficient conveys how much of the signal's energy is in a specific time-frequency cell. A simple example of such cells are shown in Figure 3. The rectangles in Figure 3 represent where each of the wavelet coefficients is positioned in the time-frequency plane. A particular characteristic of the wavelet transform is that it yields broad frequency resolution and narrow time resolution at high frequencies while giving narrow frequency resolution and broad time resolution at low frequencies. Thus, the highest frequency wavelet coefficients contain information about the content of the signal in the upper half of the signal's bandwidth; the lowest frequency wavelet coefficients contain information about the content of the signal over its entire duration. We first review a recently proposed method[7,8] for using the wavelet transform for the MSE-based compression for emitter location, and then show how to modify it to exploit the RMS width ideas discussed above; we will consider only the exploitation of the RMS bandwidth idea.

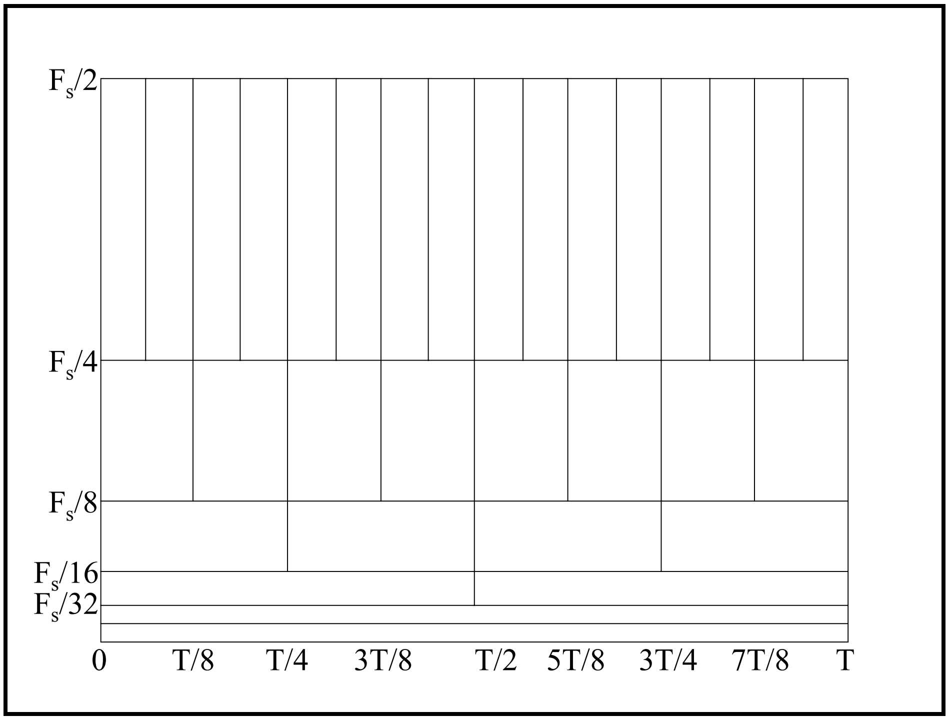

Figure 3: Wavelet Time-Frequency Cells

1. MSE-Based Wavelet Method

Wavelet-based compression based on a MSE criterion exploits the fact that a signal may be concentrated in the time-frequency plane. Signals typically have their energy concentrated in specific areas of the time-frequency plane, while large regions of the time-frequency plane may contain only very little or none of the signal's energy. A small number of bits is then spent encoding these small energy time-frequency regions, while a large number of bits is spent encoding the regions that exhibit large energy concentrations.

The MSE-based wavelet transform compression algorithm[7] consists of breaking the signal into blocks of $N = 2^p$ samples, applying an L-level wavelet transform to each block for $L<p$ (i.e., stopping the cascade of wavelet transform filter bank stages at the level where the filter outputs have $N_B = N/2^L$ elements[12]), grouping the resulting N wavelet coefficients into $K = 2^L$ subblocks of $N_B = 2^{p-L}$ samples each, and adaptively quantizing each of these subblocks. For the complex baseband signals used here, this procedure is applied independently to the real and the imaginary components.

The subblocks of the wavelet coefficients are formed within wavelet scale levels as follows: the $N/2$ wavelet transform coefficients from the first filter bank stage are grouped into 2^{L-1} subblocks of 2^{p-L} coefficients each, the $N/4$ wavelet transform coefficients from the second filter bank stage are grouped into 2^{L-2} subblocks of 2^{p-L} coefficients each, . . ., and finally the 2^{p-L} wavelet transform coefficients from the last filter bank stage form a single subblock, and the 2^{p-L} scaling coefficients from the last stage also form a single subblock.

Each one of these subblocks is quantized with a quantizer designed to achieve the desired level of quantization noise. The choice of these quantizers is made easy by the fact that the wavelet transform preserves energy; this property can be used to show that the proper choice of the quantizer cell width is given by

$$\Delta = \sqrt{\frac{12\ P_x}{SQR}}\ , \tag{12}$$

where *SQR* is the desired signal-to-quantization noise ratio and P_x is the power of the input signal $x(n)$ (in this case, either the real or imaginary part of $\hat{s}(k)$). Thus, to obtain a desired SQR, the quantizers $\{Q_1, Q_2, \ldots Q_K\}$ should each have a quantization step size given by Δ. Then the number of bits B_k used by the k^{th} quantizer is chosen to assure that the resulting quantizer covers the range of the k^{th} subblock. This leads to the rule

$$B_k = \lceil_0 \left(\log_2 \left[\max\{|W_x^k|\} \right] - \log_2 \Delta + 1 \right), \tag{13}$$

where the maximum is taken over the wavelet coefficients in the k^{th} block and the operator $\lceil_0(a)$ means "the smallest integer not less than 0 that is larger than a;" this means that when the expression in parentheses in the equation for B_k is negative we set $B_k = 0$.

In addition to sending the quantized wavelet coefficients, this scheme requires sending side information to the receiver about the number of bits used for each quantizer as well as the step size used. If the maximum number of bits used by any of the subblocks is $B_{\max}$, then the allowable quantizers are those that use between 0 and $B_{\max}$ bits, for a total of $B_{\max}+1$ different quantizers; the number of bits required to specify which of these is used for a specific subblock is $\log_2(B_{\max}+1)$ bits. Since this must be done for each of the *K* subblocks, we require $K\ \log_2(B_{\max}+1)$ bits of side information; side information on the quantizer step size also must be sent, which will be no more than the number of bits to which the original signal is quantized (we have assumed 8 bits here). So the total amount of side information is

$$R_{\text{side}} = K\ \text{x}\ \log_2(B_{\max}+1) + 8\ \text{(bits)}\ . \tag{14}$$

Simulations have shown that it is possible to limit $B_{\max}$ to 7 bits.

In this approach, the wavelet transform is used together with bit allocation to provide a means of reducing the number of bits per (real or imaginary) sample with negligible degradation of the TDOA/FDOA accuracy. This scheme accepts a specific desired signal-to-quantization ratio (SQR) and strives to minimize the number of bits needed to achieve that SQR value. In practice, the desired SQR can be set either (i) to be roughly equal to the estimated SNR of the signal to ensure that the impact of the compression on the TDOA/FDOA accuracy is negligible, or (ii) to some fixed *a priori* value.

The compression is achieved by assigning small numbers of bits to time-frequency regions where the signal has negligible energy as measured by the wavelet coefficients. Because signals that occur in practice have spectra that decay as frequency increases, the high frequencies tend to be allocated few bits – in fact, it is likely that the highest frequencies in a signal will have zero bits allocated to them. However, this contradicts the RMS bandwidth based distortion measure in (6), where it was seen that the highest frequencies contribute highly to the RMS bandwidth because of the f^2 weighting in (4). Conversely, the low frequency coefficients will be allocated a large number of bits even though they are less important from an RMS bandwidth viewpoint, again due to the f^2 weighting in (4). Thus, we seek a way to emphasize the higher frequencies and de-emphasize the lower frequencies.

2. RMS-Based Wavelet Method

Consider two cases: $SNR > DNR$ and $SNR < DNR$. For each of these cases we will examine how to select wavelet coefficients in order to maximize the RMS-width-based measures of (6) and (7).

If $SNR > DNR$, then $SNR_o \approx WT \times DNR$ is independent of *SNR* at least as long as *SNR* remains significantly greater than *DNR*. Then the process of retaining only those coefficients in some set Ω will not effect the value of *SNR* as long as the selection of Ω doesn't reduce *SNR* below *DNR*. Similarly, assume that the quantization of the retained coefficients does not further reduce *SNR* such that it is below *DNR*. Then in (6) and (7) we need only focus on B_{rms}^2 and D_{rms}^2 and therefore we would want to compress the signal in such a way that B_{rms}^2 and D_{rms}^2 are left unaffected. Doing this is an open issue at this time.

If $SNR < DNR$, then $SNR_o \approx SNR$ (as long as *SNR* remains significantly less than *DNR*). But we know that

$$SNR \propto \frac{\int |S(f)|^2 df}{N_o W} = \frac{\int |s(t)|^2 dt}{N_o W} \tag{15}$$

so that

$$\begin{aligned} SNR_{eff,TDOA} &= B_{rms}^2 SNR_o \\ &\propto \left[\frac{\int f^2 |S(f)|^2 df}{\int |S(f)|^2 df}\right]\left[\frac{\int |S(f)|^2 df}{N_o W}\right] \\ &= \frac{\int f^2 |S(f)|^2 df}{N_o W} \end{aligned} \tag{16}$$

and

$$\begin{aligned} SNR_{eff,FDOA} &= D_{rms}^2 SNR_o \\ &\propto \left[\frac{\int t^2 |s(t)|^2 dt}{\int |s(t)|^2 dt}\right]\left[\frac{\int |s(t)|^2 dt}{N_o W}\right] \\ &= \frac{\int t^2 |s(t)|^2 dt}{N_o W} \end{aligned} \tag{17}$$

Thus if $SNR < DNR$ we only have to consider the square-weighted spectrum and the square-weighted signal, each weighted by the noise power, as our RMS-distortion measures. In the rest of the paper we consider only the $SNR < DNR$ case and use only (16) as a guide for modifying the wavelet-based method given above to include the effect of RMS bandwidth. Now the goal is to retain some subset Ω of wavelet coefficients and subsequently quantize them according to some allocation of the bits budgeted so as to maximize the last line (16).

These considerations lead to the following (suboptimal) algorithm. The first step of the algorithm is to remove wavelet coefficients that correspond to components that have little impact on the numerator of (16). This is done by weighting the squared wavelet coefficients by the square of their equivalent frequency, which is taken as the center frequency of the time-frequency cell corresponding to the coefficient. From Figure 3 it can be seen that these frequencies are (in descending order) $0.75\text{Fs}\times 2^{-1}$, $0.75\text{Fs}\times 2^{-2}$, $0.75\text{Fs}\times 2^{-3}$, $0.75\text{Fs}\times 2^{-4}$, …. Thus, we can eliminate the common factor of 0.75Fs and just use the negative powers of two as our weights. The weighted coefficients are grouped into blocks of eight in the same way as in the MSE-based WT method and groups whose largest magnitude weighted coefficient fall below a user specified threshold are set to zero. This eliminates coefficients that are insignificant on the basis of RMS bandwidth. Then bits are allocated to the remaining groups on the basis of the unweighted coefficients, as was done for the MSE-based WT method; this step attempts to minimize the increase in the denominator of (16) due to the quantization.

4. SIMULATION RESULTS

Although this method is general and applicable to all varieties of signals we focus here on radar signals. Two different linear FM (LFM) radar signals were used to illustrate the operation of the method and compare its performance to the MSE-based WT method. Signal #1 has a pulse width of PW = 40 μsec with an LFM frequency deviation of Δf =2 MHz; Signal #2 has a pulse width of PW = 5 μsec with an LFM frequency deviation of Δf =1 MHz. The pulse repetition interval (PRI) is not relevant since it is assumed that the radar signals have been gated prior to the compression processing. The compressing platform detects the individual pulses of the emitter of interest, gates around them, and keeps only the signal samples that lie inside the pulse gates; the numbers of samples removed between the pulses are also kept as side information; this process is called pulse gating and is a form of compression itself. Figure 4 shows the spectra of the two simulated signals; notice that Signal #2 has a spectrum that falls off more rapidly. In all cases the SNR of the signal not compressed was set at *DNR* = 40 dB. The SNR of the compressed signal was varied over the range 10 dB to 40 dB.

Each signal was processed by the MSE-based and RMS-based WT methods and were compared to the case where no compression is performed. A previously proposed[5] post-compression correction method was used. The TDOA/FDOA standard deviations were measured using 200 Monte Carlo runs at each SNR value. The compression ratios were measured by averaging over the values obtained over the Monte Carlo runs, and include the necessary side information. For Signal #1 both compression methods achieved TDOA/FDOA accuracies that were very close to that achieved without compression and very close to each other, but – as expected – the RMS-based method achieved a higher compression ratio. For Signal #2 both compression methods achieve higher compression ratios but have more difficulty achieving the no-compression accuracy at low SNR values. The higher compression ratio arises because of this signal's decaying spectrum. None the less, the RMS-based method still achieves an improvement in compression ratio. It should be noticed that the RMS-based method performs better on TDOA accuracy than does the MSE-based method; for FDOA accuracy the case is reversed – the MSE method does better than the RMS method. However, due to the fact that the geolocation accuracy's sensitivity to TDOA and FDOA accuracies is not a straightforward relation, it is difficult to tell which of these scenarios is better without knowing the geometry of the platforms and the emitter; it is likely thought that their geolocation accuracy would be similar in many scenarios.

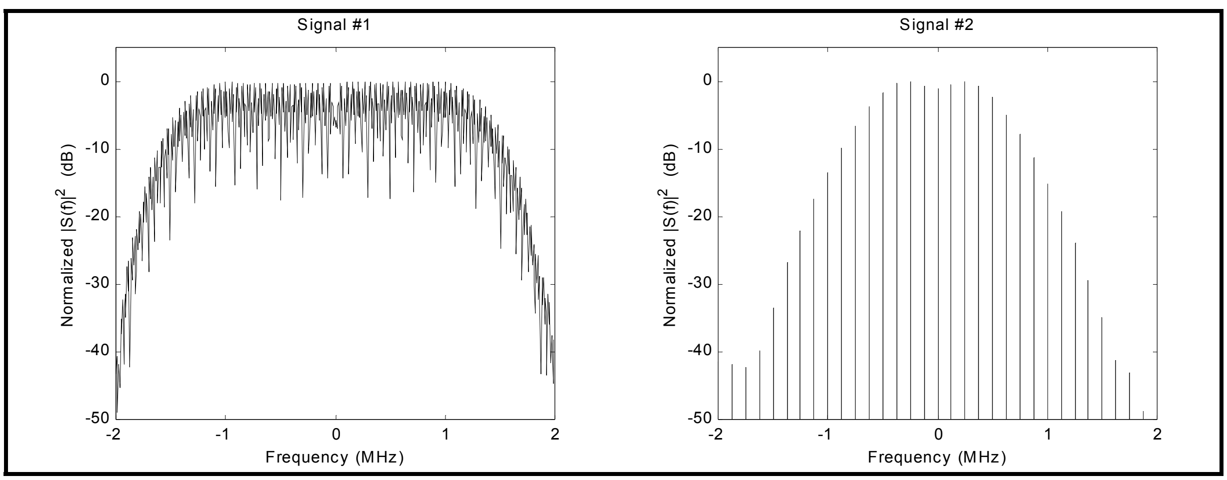

Figure 4: Fourier Transform of Signals

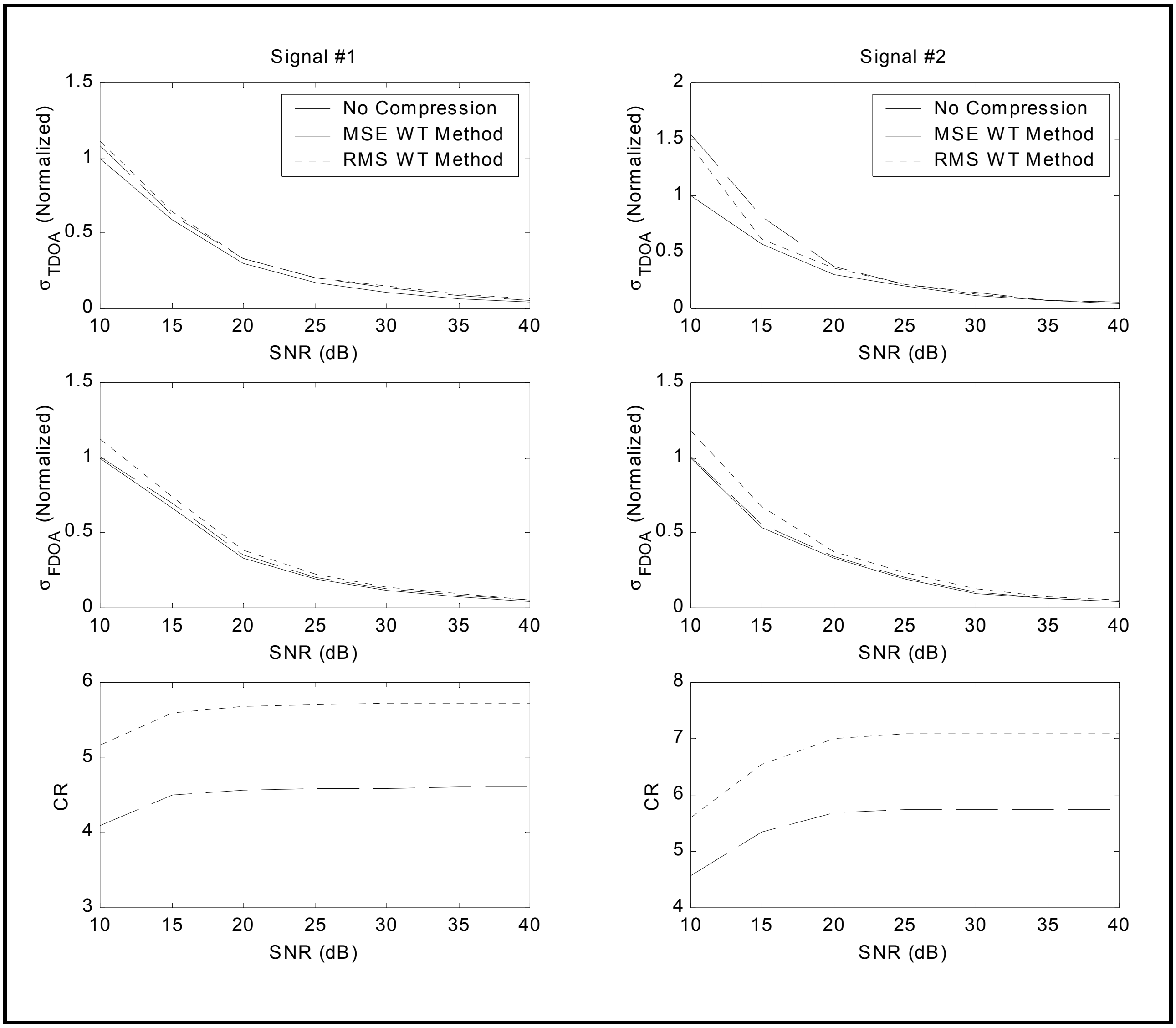

Figure 5: Simulation results for MSE vs. RMS methods for two radar signals; *DNR* = 40 dB.

5. CONCLUSIONS

The results presented here give another indication of the compression performance improvement available from RMS width methods for multiplatform emitter location applications., and give a practical way to exploit the RMS width idea – albeit a suboptimal method. The key points made here is the importance of using the appropriate distortion measure, chosen specifically for this application. By considering the form of the equations for TDOA/FDOA accuracies it is possible to derive new distortion measures that incorporate both the standard MSE measures and RMS durations/bandwidth measures. Although the resulting measures are found to be difficult to optimize we showed that it is possible to use them as a guide to develop suboptimal WT-based methods that none-the-less outperform the MSE-based WT methods. The improvements were demonstrated using two radar signals – one having a fairly flat spectrum and one that falls off more quickly. The ability to handle non-flat spectra shows that this method is more applicable than the previously reported[9] method of joint quantization and decimation. (It should be mentioned, though, that the joint quantization/decimation method is not prohibited from operating on non-flat spectra, it is simply that results describing the optimal trade-off between quantization and decimation are not currently known for other spectral shapes.)

One of the key results put forth here is that the proposed distortion measures given in (6) and (7) can be reduced to simpler forms under the conditions that either $SNR > DNR$ or $SNR < DNR$. In the first case we showed that the compression can be done solely to maintain the RMS widths and that the impact on SNR can be ignored. In the second case we showed that the quantity that must be addressed is the squared-frequency-weighted spectrum as shown in (16). In each of these cases these results make it easier to attack the optimization of the new distortion measures. Future work will address this optimization.

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